TIME-DEPENDENT STOCHASTIC SHORTEST PATH(S) ALGORITHMS FOR A SCHEDULED TRANSPORTATION NETWORK

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Abstract: Following on from our work concerning travellers' preferences in public transportation networks (Wu and Hartley, 2004), we introduce the concept of stochasticity to our algorithms. Stochasticity greatly increases the complexity of the route finding problem, so greater algorithmic efficiency becomes imperative.
Public transportation networks (buses, trains) have two important features: edges can only be traversed at certain points in time and the weights of these edges change in a day and have an uncertainty associated with them. These features determine that a public transportation network is a stochastic and time-dependent network. Finding multiple shortest paths in a both stochastic and time-dependent network is currently regarded as the most difficult task in the route finding problems (Loui, 1983). This paper discusses the use of k-shortest-paths (KSP) algorithms to find optimal route(s) through a network in which the edge weights are defined by probability distributions. A comprehensive review of shortest path(s) algorithms with probabilistic graphs was conducted.

***Keywords*:** Stochastic, Time-dependent, K-shortest paths algorithms, Probabilistic distribution

1. INTRODUCTION

In shortest path(s) problems, different networks require different algorithms to find one or several paths optimising special cost functions in the most efficient manner.

The simplest networks are those whose edge weights are static and deterministic. If the edge weights in a network are not static but change at different times, the network is then time-dependent. The time-dependent networks are useful in transportation applications. For instance, scheduled buses and trains networks' edges can only be traversed at certain times. There are networks in which the edge weights are not a single deterministic value. Instead, each edge is assigned a random variable with a probability. This type of network is regarded as a stochastic network and is often used in transportation where there is some uncertainty regarding the travel times of traffic involved.

Standard shortest path(s) algorithms are capable of finding the shortest path(s) in a network when the cost of a path is additive and deterministic. However, travel times in real world transportation networks, especially when congestion is concerned, are unpredictable. In fact real-world transportation networks are both stochastic and time-dependent.

The purpose of this paper is to investigate solutions for finding optimal path(s) in a stochastic time-dependent network, in particular for a scheduled public transportation network.

2. NETWORK REPRESENTATION

Compared with a road network, a public transportation network is very time sensitive. Because of the stochastic and time-dependent properties of public transportation network, it is necessary to investigate how it can be best represented.

2.1 Notation and Definitions

The shortest path problem can be modelled as finding the shortest path between two nodes in a weighted and directed network. In some cases, it is of interest to compute not only the shortest path, but an ordered set of alternatives with the aim of finding the shortest one that satisfies user preferences for instance – K-shortest paths (KSP) problems.

Let (N,A) denote a given network, where N = $\{v_1, \ldots, v_n\}$ is a finite set whose elements are called nodes and A $=\{a_1,\ldots, a_m\}$ is a finite set whose elements are called arcs. Each arc a_k is a pair (v_i,v_j)

of nodes. In the context of the bus network, nodes in the graph are bus stops and arcs are links between two bus stops. The input data to the algorithm consists of a description of the bus transportation network (timetables, description of links between bus stops), the bus stop where the journey begins (the source node) and the bus stop at which the journey ends (the destination node). The objective is to find the shortest path(s) between the two specified nodes.

Bus stops are point events in a transit network. A bus stop can be identified by a street name, a street intersection with a corner name or even a street address with a house number. A bus stop is linked to a bus service through the stop sequence. A stop sequence is a many-to-many relation between the bus service and bus stops. To represent the network which contains many bus stops, a very important approach is to use bus-stop codes instead of bus-stop names. The unique bus stop codes are indicated on all bus stops and are widely advertised in the public information literature issued by the Nottingham City Transport (NCT). For example, in the timetable indicated in Figure 1, AR05 and subsequent stops AR08 AR09 AR10 are assigned to indicate the bus stops on the Front Street of Arnold through the bus route 25.

A simple approach to represent the links is to ascribe a cost to every link. The cost can be defined arbitrarily such as time taken, money spend or fuel used. In this paper, cost is defined as time taken. However, the time taken to travel from one bus stop to another is not always that shown on the timetable. This may be due to congestion, roadworks or non-availability of bus drivers. So the time taken, although related to the discrete values given in the timetable, has some uncertainty attached to it.

2.2 Modelling the Bi-modal Travel Network

By adding walking links into the transportation network, a new problem arises. Route finding for travelling on buses is timetable-based which means the length between any pair of nodes is not given directly, the information must be retrieved from the bus timetables. Route finding for travelling on foot requires the distance between any pair of nodes to be calculated according to their location (Table 1). It is essential to model the two types of arc information consistently. Again, the time taken to walk from one location to another will involve an amount of uncertainty due to the different walking speeds of travellers.

IDSTOP	LOCATION	ADDRESS	X_CD	Y_CD
Aa	CITY	ANGELRD	457050.58	339922.27
Ab	CITY	ANGELRD	457036.02	339928.82
Ac	CITY	ANGELRD	457005.98	339943.61
Ad	CITY	ANGELRD	456986.82	339951.85

Table 1 Format of Bus Locations Information

2.2.1 Modelling of Bus Timetables

To represent a bus transportation network, bus timetables need to be modelled. Figure 1 is an example of a bus timetable.

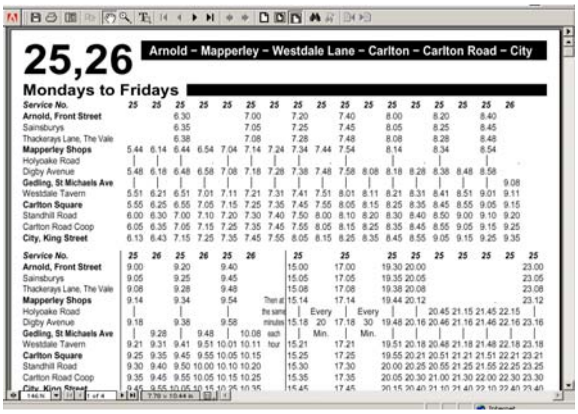

Figure 1 Timetable of Bus 25 and Bus26 (http://www.nctx.co.uk/)

To measure the expected time it takes to travel between two bus stops, a link is put into the directed graph to represent a connection between two bus stops. A bus departure time and the arrival time to the next bus stop are also needed. This departure time represents one entry in a timetable and the arrival time to the next bus stop can be taken from the same bus timetable at the next node. Each entry in a timetable is associated with the links.

2.2.2 Modelling of Walking Links

The walking links must be modelled consistently to the bus links, so that the information for both travel on buses or on foot can be used by the algorithms. This can be done by translating bus location information (Table 1) into usable time data. The physical distances between two bus stops can be translated into expected walking time information. For example, the physical distance between node i and j can be calculated by the equation:

distance(i,j)=sqrt((x(i)-x(j))**2+(y(i)-y(j))**2) (1)

Then by assuming average walking speed as 5km/hour, the expected walking time between i and j is:

$$walktime(i,j)=distance(i,j)/(5000/60) \quad (2)$$

Now the two types of information – travelling on buses and travelling on foot – are consistent. Dependant on the stochastic algorithm used and the available information, a probabilistic distribution will be used to represent the stochasticity of this information.

2.3 Bi-modal Network Representation

With the above definitions and modelling, the public transportation network can be represented as follows. The network for the route finding problem in a bus system is represented as a graph G = (N, A) where N is a finite set of *n* nodes and A is a finite set of *m* arcs. Each arc (*i,j*) Є A also has a length (or weight) $l_{ij} \geq 0$. In the route finding context, the network is the transportation network. Nodes are bus-stops and arcs represent the time taken to travel between each pair of nodes either by bus or on foot. The task is to find one or a series of ranked shortest paths between two nodes in this transportation network. A graph representation of such a network is shown in Figure 2, where the nodes are shown as numbered circles and the arcs are represented by lines and arrows linking the nodes.

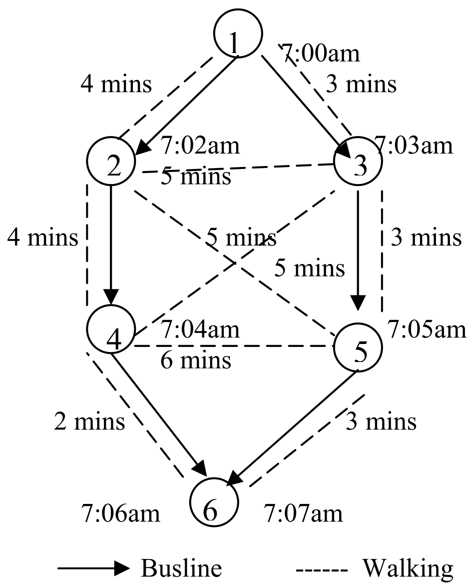

Figure 2 Representation of a Transportation Network

However, as discussed above, the network representation and subsequent route finding problem becomes much more complex when taking into consideration the stochasticity of the links.

3. STOCHASTIC SHORTEST PATH(S)

The following scenario illustrates the problems encountered when finding the shortest path under stochastic time uncertainty.

Suppose a passenger wants to travel from Nottingham to London by train. The transportation network for this case is shown in Figure 3. At 10am, there are two options for the trip from City to Train Station: Taking the new Tram service or taking Bus 79 to the Train Station. The Tram is reliable and is guaranteed to arrive at the Train Station at 10:30am. However, there is uncertainty associated with Bus 79 which means that the second choice has the potential of arriving at the Train Station earlier but also has the risk of being late. The travel time of Bus 79 is uniformly distributed between 20 and 40 minutes, which means the bus will arrive at the Train Station between 10:20am and 10:40am.

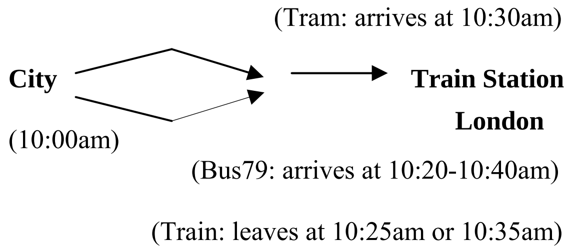

Figure 3 A transportation network with stochastic time uncertainty

By taking the Tram, the passenger is guaranteed to arrive at the Train Station at 10:30am, and by taking Bus 79, the passenger arrives between 10:20am and 10:40am. If the Train leaves the station at 10:35am, the Tram is obviously the better choice because there is 50% chance of missing the train when taking the bus. However, if the train leaves the station at 10:25am, the tram is guaranteed to miss the train and the bus has an opportunity to catch the train with 50% chance. In this example, we can see that the best route from Nottingham to London depends on the schedule from the Train Station to London.

The scenario above reveals the difficulties of route(s) finding under time-dependent uncertainty. First of all, the concept of shortest paths in a stochastic time-dependent network is not well defined. It is possible that there is a single path from an origin to destination that is shorter than all other possible paths. However, it is also likely that there are several paths having the possibility of being the shortest one. In this case, the choice of the shortest path depends on the departure and arrival time of buses (trains). The technique for determining a set of pareto optimal paths (Miller-Hooks et al., 1994) will be discussed in Section 4.

Secondly, the Optimality Principle that is required by traditional shortest path(s) algorithms is violated in this example. In the deterministic case, the best path from a to c through b must consist of the best paths from a to b and b to c. The example in Figure 1 shows that if the train departs at 10:35am, Tram is indeed the better choice. However, if the train is scheduled to depart at 10:25am, taking Tram is guaranteed to miss the train while taking Bus 79 has the potential to catch the train with a 50% possibility. So, the shortest path from City to the Train Station depends on the schedule of the Train to London. Hence the Optimality Principle is violated. For this problem, Wellman et al. (1990) proposed the concept of 'stochastic consistency' as an alternative to the deterministic network's Optimality Principle.

Furthermore, computing the shortest travel time in a stochastic but time-independent network is already extremely complex. Generating the set of pareto optimal paths (Miller et al., 1994) associated with probability distributions will further significantly increase the computational complexity. Techniques of eliminating some paths or pruning some dominant paths need to be investigated.

With the purpose of investigating potential solutions for finding the shortest path(s) in the scenario network, the author conducted the literature review on route finding algorithms for different networks.

4. LITERATURE REVIEW OF SHORTEST PATH(S) ALGORITHMS IN STOCHASTIC TIME-DEPENDENT NETWORKS

Different algorithms have been developed for finding various shortest path(s) in various networks. Bellman (1958) and Dijkstra (1959) first made efforts on efficient shortest path algorithm for networks with static and deterministic links. Significant work has been done on shortest path(s) problems for time-dependent networks. Efforts have also been made to create shortest path(s) algorithms for time-dependent stochastic non-scheduled transportation networks.

4.1 Algorithms for Time-dependent Networks

A class of algorithms has been developed for networks whose edge weights are not static but change with time. Time-dependent shortest path(s) algorithms are often used in transportation networks when, for example, travel time is different on a motorway during peak-time and off-peak time; or travel links are only available at certain time points (e.g. buses, trains).

Cooke and Halsey (1966) modified Bellman's (1958) 'single-source with possibly negative weights' algorithm to find the shortest path between any two vertices in a time-dependent network. Dreyfus (1969) made a modification to the standard Dijkstra algorithm to cope with the time-dependent shortest path problem. Orda and Rom (1990) discussed how to convert the cost of discrete and continuous time networks into a simpler model and still used traditional shortest path algorithms for the time-dependent networks. Chabini (1998) presented an algorithm for the problem that time is discrete and edge weights are time-dependent. Other algorithms deal with finding the minimum cost path in the continuous time model. Sung et al. (2000) gave a similar result using a different version of cost and improved the algorithm's efficiency. Ahuja (2001) proved that finding the general minimum cost path in a time-dependent network is NP-complete.

4.2 Algorithms for Stochastic Networks

Another class of algorithms finds the shortest path in networks where edge weights are associated with random variables with a probability function representing the possible travel costs. This class of algorithms is also useful for transportation networks where there is some uncertainty involved in the traffic so that the travel time can only be estimated.

Some research has been carried out in finding the path with the least expected travel time in a stochastic network. The very first approach in this area was done by Frank (1969) who tried to replace all the edge weights with their expected values and then solve the problem using a standard shortest path algorithm. Loui (1983) pointed out this strategy could not always be useful and would generate a sub-optimal path. He suggested using a 'utility function' to represent how relatively advantageous it would be to arrive at a certain time. After that 'maximize the expected value of a utility function' became the most common criterion to determine the optimal path. Eiger et al. (1985) showed that whenever the given utility function is linear or exponential, a Dijkstra-like algorithm could be used. Bard and Bennett (1991) approached a more complicated version of this problem and presented a heuristic for reducing the uncertainty in a stochastic network.

More recent research involves algorithms for the linear and quadratic utility function, which have first been presented by Mirchandini and Soroush (1985). Murthy and Sarkar (1996, 1997, 1998) made continuous efforts in this research area by presenting algorithms for the stochastic shortest path problem with a decreasing deadline function.

4.3 Algorithms for Stochastic Time-dependent Networks

Finally, there are algorithms for networks with edge weights that are both stochastic and time-dependent. These algorithms can be best implemented into the real-world public transportation networks.

Shortest path(s) algorithms for stochastic time-dependent networks cannot necessarily be reduced to a Dijkstra-like algorithm due to the weights changing. The algorithms require the networks to follow an Optimality Principle. Wellman et al., (1990) suggested the 'Stochastic Consistency' as an alternative to the Optimality Principle in the deterministic networks. 'Stochastic Consistency' means that the probability of arriving by a given time cannot be improved by leaving later. There is more detailed explanation of 'Stochastic Consistency' in section 5.2.

Hall (1986) first presented a solution to the problem of finding optimal route(s) in a stochastic time-dependent network by giving a high level description of an algorithm for the path with the earliest expected arrival time. Wellman et al. (1990) followed up this algorithm with an optimisation of reducing the number of paths considered on a network that obeys the principle of stochastic consistency. The implementation of Wellman et al.'s (1990) optimisation significantly reduces the running time of Hall's algorithm. Kaufman and Smith (1993) also proposed an optimisation on Hall's algorithm by using a heuristic to find upper and lower bounds on the travel time of the final path, so that many paths need not be considered. Furthermore, Wellman et al. (1995) proposed an approximation algorithm in which stochastic consistency and stochastic dominance were used to find approximate shortest paths within continuously tightening upper and lower bounds.

Not all shortest path algorithms for time-dependent stochastic graphs aim to minimize the expected arrival time. There are different desired objectives. Miller-Hooks and Mahmassani (1994) designed an algorithm focused on finding the "least possible time path", or the path with the possibility of taking the least time. This algorithm gave both the least possible time path and the probability of achieving this travel time, thus providing an alternative to the least expected time path.

5. TECHNIQUES OF FINDING SHORTEST PATH(S) FOR A STOCHASTIC TIME-DEPENDENT NETWORK

This section summaries the techniques of finding shortest path(s) for a stochastic time-dependent network from the literature review. The techniques are regarded as potential solutions and will be implemented into a scheduled transportation network.

5.1 Technique for Determining the Set of Pareto Optimal Paths Associating Probability Distributions

When there are multiple paths in a network which have the possibility of being the shortest path, Miller-Hooks et al. (1994) suggested generating a set of pareto optimal paths and then select a single shortest path by applying context dependent rules.

Miller-Hooks et al. (1994) used a specialized label correcting algorithm, in which a label or a set of labels at any node is regarded as an upper bound on the final label or set of labels until the algorithm terminates. During the node label updating, all the departure times at that node need to be stored and compared in order to maintain path information (Miller-Hooks et al., 1994). Once the pareto optimal solution set has been determined, several rules are then applied to select a single path from the set. Since there is no path among the set that is always better than the other paths, Miller-Hooks et al.'s (1994) strategy of decision-making is to choose the optimal path according to the context-dependent considerations. She listed the following possible considerations:

1) Least expected value
2) Least varience
3) Combined expected value and variance
4) A time threshold for which to find the path that maximum the probability of arriving the destination before the threshold
5) Label with largest probability of being shortest
6) Label with smallest probability of being longest

5.2 Stochastic Consistency

Kaufman and Smith (1993) proved that standard shortest path(s) algorithms work well for deterministic (both time-dependent and time-independent) networks as long as the Non-Passing

Principle (NPP) is held. The network holds the NPP when it satisfies the following consistency condition (Wellman et al., 1990): assume d_1 and d_2 are two departure times that $d_1 \leq d_2$, c_{ij} (x) denotes the travel time from node i to node j at time x. The network is consistent if for all i and j:

$$d_1 + c_{ij}(d_1) \leq d_2 + c_{ij}(d_2) \qquad (3)$$

It was proved that time-dependent transportation networks hold the consistency condition because "…although leaving later can perhaps reduce the duration of traversing an edge, it cannot decrease the ultimate arrival time…" (Kaufman and Smith , 1993).

However, stochastic networks do not hold the NPP. As shown by the example in Figure 3, the Tram has an earlier expected arrival time at the Train Station, but depending on the train schedule it may not be a part of the final shortest path from City to London.

As an alternative to the deterministic network NPP, Wellman et al. (1990) proposed the following condition. Let c_{ij} (x) denotes the travel time from node i to node j at time x, the network is stochastically consistent if for all i, j, $d_1 \leq d_2$, and z,

$$\Pr(d_1 + c_{ij}(d_1) \leq z) \geq \Pr(d_2 + c_{ij}(d_2) \leq z) \qquad (4)$$

Formula (4) means the probability of arriving by any given time z cannot be increased by leaving later. The example in Figure 3 satisfies this condition: if the train leaves at 10:25am, the shortest path from City to London is by taking Tram to the Train Station and then taking the train to London because Tram has the probability of 100% to catch the train while Bus 79 only has a 50% probability; if the train leaves at 10:35am, the shortest path is by taking bus 79 to the Train Station and then taking the train to London, because Bus 79 has a 50% probability to catch the train while Tram has no opportunity.

Wellman et al.'s (1990) proposal is based on the Stochastic Dominance concept (Whitmore and Findlay, 1978) which means "…one arrival distribution dominantes another if its accumulative probability function is uniformly greater than or equal to that of the other…". Wellman et al.'s (1990) condition gives a modified version of the shortest path algorithm. As long as the Stochastic Consistency is held, the principles of traditional shortest path(s) algorithms can be used with the stochastic networks.

5.3 Heuristics On Routes Elimination and Finding Bounds on Probability Distrobutions

More research concerning different algorithms for solving the KSP problem have been found but not many real world applications (http://liinwww.ira.uka.de/bibliography/Theory/k-path.html). The reason is the inefficiency of KSP algorithms. When addressing the 'pickup and delivery problem', Desrosiers and Soumis (1991) said, "Compared with the KSP solutions, dynamic programming takes advantages of the additional constraints and its efficiency increases with the number of constraints."

Miller-Hooks et al.'s (1994) approach for determing the set of pareto optimal paths is a specialized label correcting algorithm which requires storage of all the departure times information to a node at a certain time. A label correcting algorithm does not return a shortest path until the whole network is searched. Miller-Hooks et al. (1994) realized this inefficiency problem and in her paper she suggested some heuristic procedures for eliminating some paths from consideration. For example, paths that are stochastically dominated by an alternative path can be eliminated.

Some efforts have been made to simplify distributions over edge cost, so that traditional shortest path(s) algorithms can still be used. However, this only works for limited classes of utility functions (Kamburowski, 1985).

Loui (1983) used dominance to solve multi-attributes deterministic shortest path problems. Mirchandani and Soroush (1985) used mean/variance dominance to solve the problem when the cost distribution of arcs can be uniquely described by the mean and variance. Bard and Bennet (1991) used a stochastic dominance to reduce the search in stochastic networks but did not exploit the stochastic consistency condition.

Finally, Wellman et al.'s (1990) idea of using a consistency condition was inspired by Hall (1986) who noted the difficulty of the time-dependent stochastic shortest path(s) problem and proposed an algorithm that generated all paths in order of earliest possible arrival, terminating when this earliest-possible time exceeded the expected time of a path already found.

There is also research dealing with overlap ratio problem of KSP algorithms. Chen and Feng (1999) applied a Lagrangian Relaxation (Fisher, 1985) formula to calculate the overlap ratio for a stochastic road network.

3.4 Discussion about the Techniques

In a real-world public transportation network, links can only be traversed at certain points in time and the weights of the links change in a day and have an uncertainty associated with them. So that, finding optimal paths in such a network requires shortest path(s) algorithms that work for networks both stochastic and time-dependent. Due to the difficulty of defining the shortest path, Miller-Hooks et al.'s (1994) KSP techniques for determining a set ot pareto optimal paths is introduced as a good solution. For the Optimality Principle problem to the shortest path(s) algorithm, Wellman et al.'s (1990) 'Stochastic Consistency' is used as an alternative to the deterministic case. Finally the algorithmic efficiency problem is discussed and several heuristics are mentioned for eliminating some paths from networks.

6. CONCLUSION AND FUTURE WORK

This paper investigated some of the approaches to solving the shortest path(s) problem in a stochastic time-dependent scheduled transportation network. A comprehensive review of shortest path(s) algorithms with probabilistic graphs has been conducted.

Several potential solutions have been summarized in this paper at the start of solving the problem. At the moment, the author is focusing on the techniques for approximating the bus arrival times distributions and associating them with the KSP algorithm.

The next step of the research will be investigating the probability distributions of bus arrival times and associating them with the KSP algorithm. The algorithm will finally be implemented for a scheduled transportation network with uncertain bus arrival times.

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