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On There Being Infinitely Many Thinkable Thoughts:

A Defence of Tegmark

Introduction

Porpora (2013) offers an *a priori* argument for the conclusion that there are infinitely many thoughts that it is physically possible for us to think. That there should be such an *a priori* argument is astonishing enough. That the argument should be simple enough to teach to a first-year undergraduate class in about 20 minutes, as Porpora's is, is more astonishing still. Porpora's main target is Max Tegmark's recent argument for the claim that if current physics is right, then there are mental duplicates of us in far flung regions of the Universe.[1] His argument is directed against Tegmark's assumption that mental facts supervene upon physical facts. So, if Porpora's argument is sound then not only is Tegmark's argument unsound, but physicalism is also false. So, Porpora's argument is powerful indeed. Who would have thought that a simple *a priori* argument, together with the physical facts, could solve the issue of whether physicalism is true?

Not I. In this paper I take a closer look at Porpora's argument and show that it is fallacious. I also consider the other reasons Porpora gives for thinking there are infinitely many thinkable thoughts and find them similarly lacking.

1.

Tegmark's Argument can be stated as follows:

1. The Universe is an infinite patchwork of Hubble volumes (*viz.* physical spheres of radius $4X10^{26}$m) across which matter is uniformly spread.

2. There are only finitely many (around 10^{115}) possible combinations of physical states within a Hubble volume.

3. Mental facts about Hubble volumes supervene on the combinations of physical states within them.

C1. There are Hubble volumes that are physical duplicates of our own.

C2. There are Hubble volumes that contain mental duplicates of us.[2]

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[1] According to Tegmark each of us would likely find a duplicate of ourselves about $10^{10^{115}}$m away. (Tegmark, 2009)

[2] See Tegmark (2003, 2009). Porpora notes that Tegmark's calculation that each of us would likely find a duplicate of ourselves about $10^{10^{115}}$m away rests on the assumption that each combination of quantum states in a Hubble volume has an equal chance of occurring. This is true. But his general argument given here for the

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C1 follows from 1 and 2, and C2 follows from 3 and C1. Premises 1 and 2 state those aspects of physics that Tegmark takes to be uncontroversially true, and Porpora is happy to take Tegmark as an authority on this matter (as I am too). He is thus happy to concede C1. What he disputes is C2, and so it is premise 3 that he attacks. He argues that mental facts can differ between physical duplicate Hubble volumes and so mental facts do not supervene on physical facts. His argument is simple: Given that there are only finitely many possible combinations of physical states within a Hubble volume, if the mental facts supervene upon them, then there are only finitely many possible mental facts about Hubble volumes. But there are infinitely many possible mental facts about Hubble volumes because there are infinitely many possible thoughts that can be had by individuals within them. So, the mental facts about Hubble volumes do not supervene on the physical facts about them. (Porpora, 2013: 134) It is because minimal physicalism is captured by this very supervenience claim that, if Porpora's argument is sound, it provides a refutation not only of Tegmark's argument, but also of physicalism itself.[3]

2.

The soundness of Porpora's argument depends upon the truth of his claim that there are infinitely many possible thoughts that inhabitants of Hubble volumes like us can think. He rests his case for this claim on an argument for a more specific claim that entails it, *viz.* that there are infinitely many possible thoughts about numbers that we can think. Here is his argument in full:

> 1. Let us stipulate that what any of us thinks (explicitly or implicitly) at any point in time is a subset of a larger set of all thoughts it is possible to think (or of mental states it is possible to be in). Suppose this larger set consists of only finitely many possible thoughts (or mental states).
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> 2. If only finitely many thoughts are possible, then there are only finitely many integers of which we could possibly think.
>
> 3. If there are only finitely many integers of which we can think, then there must be some integer that is the largest we can think of. Call this largest thinkable integer N.
>
> 4. But if we can think of N, we can certainly think of N + 1 or 2N or N^2 and so on, all larger than N, which, therefore, cannot be the largest thinkable integer, contradicting the conclusion of the previous step.
>
> 5. So there is no largest thinkable integer.

conclusion that there are Hubble volumes that contain mental duplicates of us does not. All it requires is that each combination of quantum states has a non-zero probability of occurring.

[3] Physicalism can be cashed out in a variety of ways, but that the mental supervenes on the physical is a common commitment of all physicalists. See Lewis (1983: 361). So although Porpora says that his argument is directed specifically at Humean supervenience (Porpora, 2013: 135), if sound it refutes any physicalist view.

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> 6. Then there are not only finitely many integers of which we can think.
>
> 7. Then the larger set of all thoughts it is possible to think must be infinite. (Porpora, 2013: 144-145)

This argument is doubly problematic.

Firstly, I think a good case can be made for it being illegitimate to use the description 'the largest thinkable integer' as it is used in in premise 3 of the argument, i.e. to introduce a name into English. As Porpora himself admits, we can think about a number iff we can refer to it in some way.[4] So the description 'the largest thinkable integer' is materially equivalent to the description 'the largest integer that can be referred to' (i.e. they express the same satisfaction conditions). But, as is well known, descriptions containing semantic terms (and those that are materially equivalent to them) are often unfit for the job of introducing names into the language – especially, in fact, names for integers. Consider any of the descriptions which one can use in constructing Berry's paradox, e.g. 'the least integer that cannot be referred to in fewer than one-hundred syllables'. If we suppose that this description can refer, we run into paradox: Suppose that some number *n* satisfies the description. Then the description refers to *n*. But since the description has fewer than one-hundred syllables, *n* does not satisfy the description after all. To resolve this paradox, it is generally agreed, we must maintain that the description, despite being syntactically well-formed, is semantically defective in some way and so not an expression that one can use in a genuine attempt at referring (which obviously makes using it to introduce a name illegitimate).[5] And there are other well-known paradoxes involving the use of a semantically-involved description in an attempt to refer that are to be resolved by maintaining the description is defective in a similar manner (e.g. Richard's paradox and the interesting number paradox).

So, we already have reason to be suspicious of the description Porpora uses in premise 3. But now consider 'the largest number that we will ever think of on Earth'. This is equivalent to the description 'the largest number that we will ever refer to on Earth' and so is also semantically-involved. Using this description we can construct a modified version of Porpora's argument that has as its conclusion that we will think infinitely many thoughts during our time on Earth.[6] But we can hardly agree with this conclusion, for we know very well that we haven't

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[4] Indeed, throughout his paper Porpora often formulates the question of whether we can think about numbers directly in terms of whether we can refer to them. See, e.g. (Porpora, 2013: 144).

[5] There are a variety of diagnoses of precisely what the semantic defect consists in. See, for example Chihara (1979, 1984), Grover, D. (1983), French (1988). But everyone agrees that Berry's paradox requires resolution and that the semantically-involved description within it is defective.

[6] The modified argument runs as follows:

1. Let us stipulate that what any of us on Earth thinks (explicitly or implicitly) at any point in time is a subset of a larger set of all thoughts that will be thought at some time on Earth (or of mental states that someone on Earth will be in at some time). Suppose this larger set consists of only finitely many thoughts (or mental states).
2. If we will think only finitely many thoughts (on Earth), then there are only finitely many integers of which we will think (on Earth).

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the time to think that many thoughts: the Earth will come to an end at some finite point in the future (according to recent estimates, in about two billion years). So we *know* there is something wrong with this argument, and given what has been said about semantically-involved descriptions, we are justified in thinking the problem lies with the description 'the largest number that we will ever think of on Earth'. But as the modified argument is entirely analogous to Porpora's, it is entirely plausible that if there is something wrong with it, then there must be something wrong with Porpora's argument too, and so with the description 'the largest thinkable integer'.

I think we are already justified in rejecting Porpora's argument on the basis of the line of thought given above. But even if we allow that the description Porpora uses is not defective, there is a second problem with his argument that is even more substantial than this first one. The second problem is with premise 1. Premise 1 assumes for *reductio* that there are only finitely many possible thoughts we can have. Clearly the *reductio* is only effective if physicalists (and so Tegmark) are committed to this assumption. But they are only committed to it on one way of individuating thoughts. But on this way of individuating thoughts, premise 4 of Porpora's argument is false. To see this first consider:

> Oscar and Toscar are physical duplicate persons located in distinct Hubble volumes that are physical duplicates of each other. Each is entertaining a thought that he would express with the words "the oldest man in my Hubble volume is toothless". Neither have any further identifying information about the person who satisfies the description, and so neither have any particular man in mind. Now ask: Are Oscar and Toscar entertaining the same thought or different thoughts?

If we allow that descriptions can have wide content (and thus that we can entertain singular thoughts about whoever satisfies the description), and individuate thoughts in terms of such content, then Oscar and Toscar have different thoughts.[7] Suppose Ortcutt is the oldest man in Oscar's Hubble volume, and Tortcutt the oldest man in Toscar's. Then Oscar's (wide content) thought is that *Ortcutt is toothless* whilst Toscar's is that *Tortcutt is toothless*. But if

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3. If there are only finitely many integers of which we will think (on Earth), then there must be some integer that is the largest that we will think of (on Earth). Call this largest integer 'N'.
4. But we are now thinking of N, and so can now think of N + 1 or 2N or N^2 and so on, all larger than N, which, therefore, cannot be the largest integer that we will think of (on Earth), contradicting the conclusion of the previous step.
5. So there is no integer that is the largest we will think of (on Earth).
6. Then there are not only finitely many integers of which we will think (on Earth).
7. Then the larger set of all thoughts that we will think (on Earth) must be infinite.

[7] For a classic argument for the existence of wide contents see Putnam (1975). For what it's worth, I think it is implausible that descriptions such as these do have wide content. Even neo-Russellians are apt to deny this and maintain that we can only entertain singular thoughts if we are in *some sense* acquainted with the object we are thinking of (even if this is an indirect causal acquaintance). But as we will see, even if descriptions like the ones mentioned here do not have wide content, this will not save Porpora. Whether they do or not, thoughts must be individuated by narrow content in the argument he gives. Supposing that descriptions like the ones mentioned do have wide contents helps to see why.

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we individuate thoughts in terms of narrow content, they are having the same thought. Narrow contents are descriptive, so they are both having the (narrow content) descriptive thought that *there is some man who is uniquely the oldest man in my Hubble volume, and he is toothless*.

Now consider a possible world *w1* in which all that exists are an infinite series of rooms *r1*, *r2*, *r3*… *rn* stretching from left to right (the walls in the rooms are marked with an 'L' and an 'R' to signify which is left and which is right), each containing a person *p1*, *p2*, *p3*… *pn*. The walls are thick and each *p* is locked in her room so that no communication can take place between the *p*s. *p1* in *r1* (the leftmost room) is entertaining the thought that she would express using the words "1 is odd". The rest of the *p*s are all physical duplicates of each other and each is entertaining the thought she would express using the words "the integer one greater than the integer my left-hand neighbour is thinking of is odd". If we individuate the thoughts that the *p*s are having by wide content, then each *p* is thinking a distinct thought about the integer that satisfies the description she is using. So, *p2* is thinking that 2 is odd, *p3* is thinking that 3 is odd, *p4* is thinking that 4 is odd, and so on. In fact, for every integer *n*, some *p* is thinking that *n* is odd. On this way of individuating thoughts, then, there are infinitely many thoughts being thought in *w1*. This is so despite it being the case that there are only two physical states that persons in *w1* are in (*p1* is in the first physical state, and each other *p* is in the second). But it should be clear that this is perfectly consistent with physicalism. Although physicalists cannot allow that physical duplicates can differ with regard to their narrow thoughts, they can (and do) allow that they can differ with regard to their wide thoughts. So, physicalists can maintain it is possible for there to be infinitely many (widely individuated) thoughts even if there are only finitely many physical states we (and our Hubble volumes) can be in. So, physicalists are not committed to the assumption made in premise 1 of Porpora's argument if we individuate thoughts by wide content. So Porpora must individuate by narrow content instead.

Because narrow content thoughts have descriptive content, they are individuated in terms of their descriptions, and they are thus *de dicto* thoughts. And therein lies the problem. Suppose Smith is having a thought he would express using the words "the integer most would identify as their favourite is prime". Suppose also that he has no further identifying information about which number satisfies this description, and so has no particular integer in mind. Then the situation is represented here by (1) below (and *not* by (2)):

(1) Smith is thinking $\exists$x(x = the number most would identify as their favourite, and x is prime)

(2) $\exists$x(x = the number most would identify as their favourite, and Smith is thinking that x is prime)

So the narrow thought that Smith is having is a purely existential thought and so involves no genuine thought *of* or *about* any integer at all. To see this clearly note that the thought he is having does not depend in any way on which number satisfies the description. Any person in

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any possible world having a thought that they would express using the same words would be having the same narrow thought, no matter whether it was 7, or 9, or 23 that satisfied the description in their world. But now consider Porpora's premises 3 and 4 once again. In premise 3 he introduces the name 'N' using the description 'the largest thinkable integer'. Then, in premise 4, he claims that we can think about integers larger than N using the names 'N+1', '2N', 'N^2', and so on. But this is simply false. If we are individuating thoughts by narrow content, any thought involving 'N' will be purely descriptive, and so too will be any thought involving 'N+1', '2N', and 'N^2' (and any other name formed using 'N'). The thought expressed by, for example, "N is prime" is simply the thought that *there is some integer that is the largest we can think of, and it is prime*. Similarly the thought expressed by "N+1 is prime" is simply the thought that *there is some integer that is one greater than the largest we can think of, and it is prime*. But these thoughts too are purely existential thoughts, and are not thoughts *of* or *about* any particular number at all.

So, if premise 1 is to be accepted by physicalists, Porpora must individuate thoughts by narrow content. But on this way of individuating thoughts, 'thinking of N' means thinking merely that *there is* some number that satisfies the description embodied by 'N', and so does not involve a genuine thought *of* or *about* a number at all. So premise 4 of his argument is false.

3

So much, then, for Porpora's main argument. What of the other reasons he gives for thinking that there are infinitely many thinkable thoughts? There are two, and each is uncompelling.

The first reason Porpora gives is based upon the fact that thoughts are 'inexhaustible' and that there is no clear criterion of identity associated with them, and thus that it is not possible to count them. He says:

> It appears that mental states comprise a set fuzzier even than fuzzy sets for we often cannot even say clearly whether one element of this set is different from another, and new elements seem ever to arise. If such inexhaustibility is a kind of infinity, it appears that mental states are not just infinite but like the real numbers (but for a different reason) uncountably infinite. (Porpora, 2013: 142)

With regards to the first point, it is utterly unclear what it means to say that new thoughts 'seem ever to arise' (from which it is meant to follow that they are 'inexhaustible'). The best I can make of Porpora's argument here is to take him as pointing out that our phenomenological experiences are such that it seems to us that we can come up with an indefinite number of new thoughts. This he then takes to be evidence that there actually are infinitely many thoughts that we could come up with. But this is hardly convincing. It's certainly not clear to me that my experiences are like this. But even if they are, how things seem in this regard is not likely to be a good guides to how things are. There are certainly a

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huge number of thinkable thoughts – no-one would deny that. But where huge numbers of items are involved, our experiences are apt to misrepresent. Looking up at the clear night sky, new stars 'seem ever to arise', and from a fixed spot in the Sahara desert, as the wind blows and the dunes shift, new grains of sand 'seem ever to arise'. But we know there are not infinitely many visible stars, and not infinitely many grains of sand in the Sahara. At any rate, Porpora needs to say much more here than he does if his claim is to be plausible.

As for the second point, *viz*. that thoughts lack a clear criterion of identity and are therefore uncountable: this is sheer sophistry. The sense in which things that lack a clear criterion of identity are uncountable is completely different from the sense in which the real numbers are. Roughly put, the real numbers are uncountable in the sense that there are too many to count, even in principle, whilst things with no clear criterion of identity are uncountable because we cannot count them at all (so it would be better to describe the set of thoughts as being *non*-countable). We can put this less roughly by calling a *proper list* a list such that it is a determinate matter whether any given object on the list has already appeared on it. Then we can say that the sense in which the real numbers are uncountable is that any proper list of its members will be such that it is not the case that every member of it appears at some point on the list (this is just the definition of what it is for a set of things to be uncountable in this sense). The sense in which things, like thoughts, that lack a clear criterion of identity are uncountable, on the other hand, is that it is impossible to create a proper list of them at all –one would reach a point where one could not know whether a given thought on the list had already appeared on it or not.

The second reason Porpora gives fares no better. He says:

> [A]ny one belief seems to entail an infinite number of other implicit beliefs or at least belief dispositions… If I believe there are only nine planets—or maybe now eight, then I necessarily believe there are fewer than ten or eleven and so on ad infinitum…There seems in short no end to the further beliefs or belief dispositions generated by any single belief. (Porpora, 2013: 142)

The problem here is clear. An implicit belief (or a belief disposition), as Porpora is thinking of them, is a proposition that is entailed by our occurrent beliefs and is such that we would, *ceterus paribus*, assent to it were we to entertain it. But if Tegmark is right the set of implicit beliefs is a proper subset of the propositions entailed by our occurrent beliefs. That is, if Tegmark is right, there are some propositions that are entailed by our beliefs but that it is not physically possible for us to entertain. So to maintain that there are infinitely many implicit beliefs simply begs the question against Tegmark.[8]

[8] One might alternatively say that an implicit belief is a proposition that is entailed by our occurrent beliefs and is such that we would assent to it were it logically possible for us to entertain it. But this is no help to Porpora. Then the only conclusion we could then draw from the argument he gives is that there are infinitely many thoughts that it is logically possible for us to think, and this has no bearing on how many thoughts it is physically possible for us to think.

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Tegmark's argument shows that if physicalism is true, then given the physical facts, there are only finitely many thinkable thoughts, and so mental duplicates of us exist. Porpora does not mean to simply point out that if we reject physicalism then Tegmark's argument fails. That much is obvious. Rather, he means to show that physicalism itself is false and, thereby, that Tegmark's argument contains a false premise. To do this he attempts to show that there are infinitely many thinkable thoughts. But, I have argued, his main argument for this claim fails and nothing else he says gives us any reason to believe the claim either. So I conclude that Porpora has given us no reason to think that Tegmark's argument is unsound.

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