Damage Identification in a Concrete Beam Using Curvature Difference Ratio

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Abstract. Previous studies utilising changes in mode shape or curvature to locate damage rely on the fact that the greatest change occurs around the defect. However, in concrete beams this fact is undermined due to the nature of the defect as distributed multi-site cracks. In addition, differences in mode shape and curvature as ways to locate the damage is unstable because of occurrence of modal nodes and inflection points. In this paper, one interesting solution to this problem is being tested by establishing a new non-dimensional expression designated the 'Curvature Difference Ratio (CDR)'. This parameter exploits the ratio of differences in curvature of a specific mode shape for a damaged stage and another reference stage. The expression CDR is reasonably used to locate the damage and estimate the dynamic bending stiffness in a successively loaded 6m concrete beam. Results obtained by the proposed technique are tested and validated with a case study results done by Ren and De Roeck [1] also by Maeck and De Roeck [2]. Another contribution of this work is that relating changes in vibration properties to the design bending moment at beam sections as defined in Eurocode 2 specifications [3]. Linking between a beam section condition and the change in vibration data will help to give a better comprehension on the beam condition than the applied load.

1. Introduction

Although the methods of utilising vibration data as a source for structural condition identification appear simple, in principle, their effective application has proved remarkably challenging. The vibration tests are generally complemented by many issues related to creating accurate and repeatable measurements [4] [5]. In addition, the extracted vibration data are difficult to interpret with respect to the location and size of the damage. Also, except for severe cases of damage, the changes in modal parameters are usually modest [6]. In consequence, the usage of modal parameters to identify the amount of damage for concrete structures is still a subjective and not necessarily sound process or regular activity and basic problems yet to be solved in modal calibration [7][8].

Many of the active vibration-based damage identification algorithms implement sensitivity approach and utilise only natural frequencies. Major disadvantages of sensitivity based algorithms include the considerable amount of computation required and the fact that their validity is limited by

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the number of damage cases considered [6]. Even though natural frequencies are sensitive indicators of structural integrity, methods using only natural frequencies have a number of shortcomings. They cannot distinguish the damage at symmetrical locations in a symmetric structure, and the number of measured natural frequencies is generally lower than the number of unknown modal parameters ensuing in a non-unique solution [2]. In addition, damages of different scenarios may cause a same shift in natural frequencies [6] [9] [10]. Additionally, at modal nodes (points of zero modal displacements), the stress is of minimum magnitude at the specific mode of vibration. Therefore, the associated smallest change in a particular modal frequency might mean that the defect is close to the modal node [11]. Moreover, the drop in natural frequency becomes more important when the cracks emerge at regions of high curvature for the modes under consideration [11]. As a result, mode shape information should also be included to uniquely localise the damage [7] [12] [13].

From the vibration theory, reduction in the stiffness is associated with modification of the modes of vibration of the structure. Vibration mode shapes can be heavily influenced by local damage and offer a better means of locating the damage [6]. Therefore, mode shapes along with their formulation (difference between scaled mode shapes and relative difference methods) can be included in damage detection. However, modal displacements are somewhat insensitive to slight local changes in the stiffness of a section [2] [10]. Moreover, displacement of multiple damage sites usually is difficult, if not impossible, to be efficiently used for most damage location techniques. Only curvature mode shape can give an indication of multiple damage locations [6]. Furthermore, the sensitivity of mode shapes, as measured by Modal Assurance Criterion (MAC) together with a number of its formulation as other expressions for mode shapes, depends very much on the nature of the damage. If the damage is distributed, such as widespread cracking in concrete, mode shapes may change a little, although the frequency will decrease. Localised damage, on the other hand, may result in large reductions in the MAC values [14]. Also, the MAC and Coordinate Modal Assurance Criterion (COMAC) values do not detect significantly the presence of damage as they are not sensitive enough to detect the damage in earlier stages. Additionally, any differences in MAC and COMAC calculations are averaged and spread over all measurement points in case of MAC or all mode shapes in case of COMAC [10].

Neither modal nor frequency response parameters are known to be consistent in providing reliable assessment for a structure [15]. For this reason, many researchers have switched to alternative developed parameters including a combination of modal parameters and/or frequency response functions and their derived versions for damage identification. Wide range of derived damage identification parameters such as modal flexibility by Pandey and Biswas [16], uniform load surface by Zhang and Aktan [17], modal curvature by Pandey et al. [10], and modal strain energy by Stubbs and Kim [18], to mention but a few. However, besides all the abovementioned shortcomings, ill-conditioning situations, truncated modal data, and structural non-linearity are additional reasons accounting for the limitations, and difficulties allied with using these methods [15].

Despite these combined research efforts, several problems remain to need to be solved before damage detection in real structures becomes an uncomplicated practice. A need remains to improve the accuracy of damage localization and severity estimation in concrete structures by developing an easy and robust expression. To that end, modal curvatures, as they are known to be more sensitive to the local changes than modal displacements [2] [10], are adapted and successfully verified in this study. Another advantage of using curvature mode shape for damage location and severity estimation that the curvature is proportional to the bending strain, and it can be estimated by measuring strains instead of displacement or acceleration [10].

In this paper, the differences between curvature mode shapes for different damage levels are used to establish a new parameter designated 'Curvature Difference Ratio' (CDR). Then, the CDR from two damage cases is employed to obtain numerically the changes in bending stiffness. The accuracy of the results obtained by this ratio is validated with results from previous damage identification algorithms proposed by Ren and De Roeck [1] and Maeck and De Roeck [2]. A possible advantage of this parameter is that it shows rationally the decrease in dynamic bending stiffness for different damage levels and directly manages to estimate this decrease. Another potential benefit is that it is situated within the non-model based method category where there is no need to predict an analytical model. Most significantly, only one measured displacement mode shape is sufficient to create a curvature mode shape; hence, to display CDR. To the best of our knowledge, no previous work addresses directly and easily this task in concrete beams. Furthermore, in order to give a better understanding to the relationship between a load step and changes in modal parameters, the applied static load is directly correlated to the design bending moment at a section using Eurocode 2 specifications [3]. At this case, it is easier to comprehend the condition of the cross-sections of a beam under each load step and directly connect it to its corresponding vibration parameters.

2. Description of the Case Study

2.1 Reinforced Concrete Beam Model

This study utilises the final model for a concrete beam of 6.0m length which is given by Ren and De Roeck [1] to regenerate synthesis vibration data for different damage steps. The dimensions and the cross-section of the beam with all reinforcement details are shown in figure 1. A total beam mass of 750kg results in a density ρ=2500kg/m^3 of the reinforced concrete beam is adopted. Static modulus of elasticity in the test article published by Ren and De Roeck [1] is obtained from the first longitudinal natural frequency for a cylinder (300x150mm) of normal concrete and taken as E_c=33GPa.

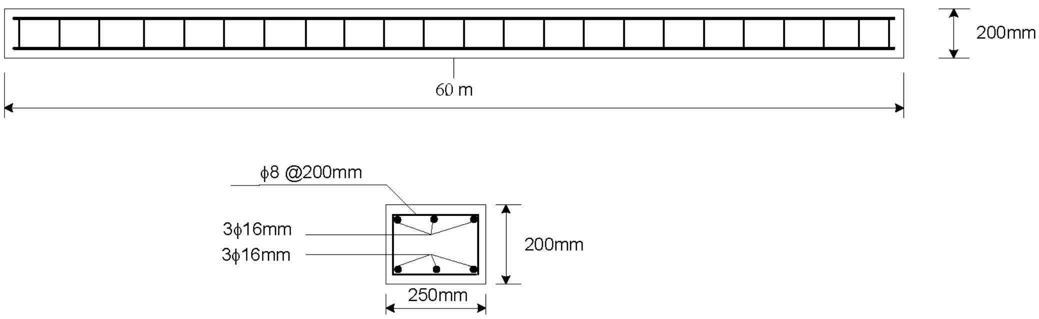


Figure 1. Beam dimensions and cross section

2.2 Static and Dynamic Testing Scheme

In the study published by Ren and De Roeck [1], the beam is subjected to two symmetrical point loads at a distance of 2.0m as a four point bending loading, as shown in figure 2. This procedure produces a middle zone of evenly distributed defects. Then, the simply supported beam is subjected to six successive increasing load steps in order to create different levels of damage. Besides the initial intact state, six load steps are conducted, as summarised in figure 2. The last step (step 6) corresponds to the ultimate damage state or plastic yielding of the reinforcing bars. At the end of each static load step, the beam surface is visually inspected to map out the crack distribution associated with that step. The approximate pattern of the observed cracks and defects for each load step is shown in figure 2. At the end of each static load step, the beam is unloaded, and the simple supports are replaced by flexible springs (distant 1.34m from each end) to create experimental modal testing on a complete freely supported beam. A dynamic force is generated by an impact hammer containing frequency components up to 1000Hz. Accelerations are vertically measured at every 0.2m with accelerometers.

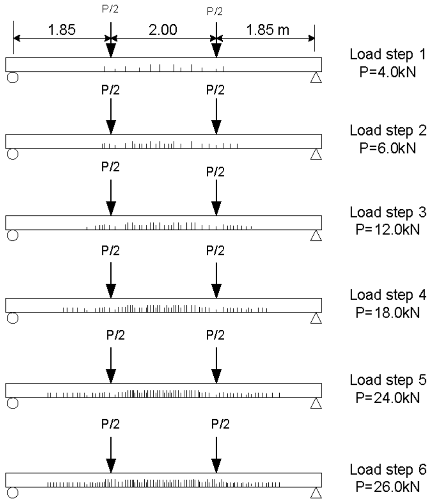


Figure 2. Successive incremental static load steps

2.3 Damage Pattern Model

The damage of the concrete beam corresponding to each loading step is identified using the mode-based damage identification method. This technique utilises the changes of natural frequencies and mode shapes to estimate the reduction in stiffness of damaged elements. Complete details about this damage identification scheme can be found in Ren and De Roeck [1]. From that study, a separated linear damage function is proposed for the tested beam as

$$\Delta a_e = \begin{Bmatrix} \frac{\alpha}{2}x & 0 \le x \le 2 \\ \alpha & 2 \le x \le 4 \\ \frac{\alpha}{2}(6-x) & 0 \le x \le 2 \end{Bmatrix} \tag{1}$$

Where the parameter α represents the degree of the damage. The distribution of decreases in the stiffness along the span of the tested beam is shown in figure 3. In addition, the amount of stiffness drop associated to each load step is given in table 1.

2.4 Regeneration of the Vibration Data and Data Verification

Making use from all published data of the simulated damage model achieved and verified by Ren and De Roeck [1], this study used a finite element method (FEM) to regenerate the synthesis dynamic characteristics and predict damage changes. For this purpose, a one-dimensional FEM dynamic analysis routine is established in the MATLAB® programming environment.

The reinforced concrete beam is equally discretised into sixty one-dimensional Timoshenko beam elements. In the data generation process, the density ρ=2500kg/m^3 is used and Young's modulus of E_c=33GPa is exploited. For the virgin reference step, the section dimensions along with the reinforcement details shown in figure 1 are employed in order to determine uncracked moment of inertia ($I_{uncracked}$=2x10^{8}mm^4). Very light suspension spring stiffness is inserted at 1.3m from each end of the beam to avoid an ill-conditioning state while performing dynamic analysis. During each load step, a new stiffness matrix is generated based on linear stiffness decrement given in Equation 1 and table 1. The first four predicted bending frequencies of the beam for each damage stage are summarised in table 2. These frequencies are compared and agreed well with the first four bending frequencies given by Ren and De Roeck [1], as shown in table 2. Moreover, the relative drop of frequencies with respect to the reference intact state is plotted in figure 4. It is found that the bending frequencies are remarkably influenced by damage.

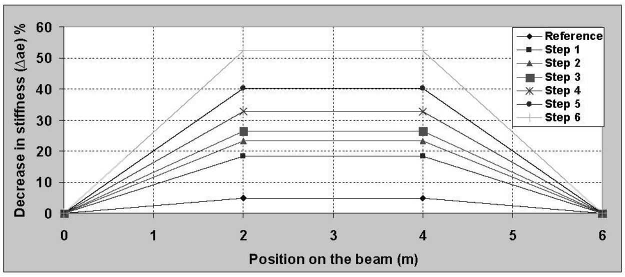


Figure 3. Decrease in stiffness of beam through loading steps

Table 1. Successive stiffness decrease

Load step	Successive %	Accumulated %
Reference	4.9	4.9
Step 1	13.52	18.42
Step 2	4.93	23.35
Step 3	3.03	26.38
Step 4	6.40	32.78
Step 5	7.35	40.13
Step 6	12.04	52.17

As shown in figure 4, there is a drop in frequency with increasing the level of damage. A maximum drop of 26% is observed for the first bending frequency. However, the drop rate becomes lower in higher modes. This observation probably attributed to that the deterioration is close to the maximum stress zone for lower modes but near the modal nodes for higher modes. The only exception, in this study we start from a completely intact reference step (i.e. 0% stiffness decrement), while in the original study by Ren and De Roeck [1], they started from a reference step of 4.9% stiffness decrement.

In order to provide a better understanding for the condition of the beam at different damage levels, theoretical bending moment is investigated. The characteristic cylinder strength of the concrete at 28 days (f_{ck}) is taken as 50N/mm^2, while the characteristic yield strength of the steel (f_{yk}) taken as 420N/mm^2. In addition, concrete grade C50/60 is utilised to determine the uncracked flexural moment using mean tensile strength of concrete (f_{ctm}) given by EC2 part1-1 specifications [3] as

$$f_{ctm,flex} = \max\{(1.6 - h/1000)\, f_{ctm}; f_{ctm}\} \qquad (2)$$

Table 2. Comparison between generated and identified bending frequencies given by Ren and De Roeck [1], Hz

Load step	Mode number 1		2		3		4	
	Present study	Ren & De Roeck	Present study	Ren & De Roeck	Present study	Ren & De Roeck	Present study	Ren & De Roeck
Intact	22.65	21.93	61.28	60.33	117.59	117.02	193.42	192.03
Step 1	20.67	20.01	56.71	56.25	110.01	110.85	181.44	181.44
Step 2	20.07	19.47	55.29	54.93	107.67	108.62	177.67	178.0
Step 3	19.70	19.19	54.44	53.19	106.21	104.36	175.36	171.67
Step 4	18.87	18.73	52.50	51.70	102.97	101.19	170.20	166.73
Step 5	17.90	18.01	50.22	50.20	99.16	98.22	164.10	161.88
Step 6	16.16	16.08	46.10	47.49	91.73	93.72	152.31	150.85

Where h is a beam depth in mm and for concrete grade C50/60, f_{ctm} equal to 4.1MPa. Therefore, flexural concrete tensile strength $f_{ctm,flex}$ for the tested beam using Equation 2 equals 5.7MPa.
Taking the beam self weight into consideration, the static load which causes the first crack is P=7KN. On the other hand, failure bending moment according to EC2 specifications [3] is 31.8kN.m, which is

almost close to the experimental failure bending moment caused by the beam self weight along with a static two point load of P=26kN.

3. Damage Identification Using Vibration Data

3.1 Damage Identification Using Changes in Natural Frequencies

From figure 4, a notable observation is drawn as an approximately 10% drop in bending frequencies is obtained, although beam sections are in uncracked condition. This is attributed to the initial unavoidable cracks developed during casting, processing and transportation of the beam. However, a maximum drop of 26% for the first bending natural frequency is obtained at the ultimate damage state. What is more, between Step 3 and Step 5 (cracked stage) the beam experiences heavy cracking development, while the drop in frequencies is rather small (almost 10% for the first four modes). Conversely, at the ultimate damage state (Step 6), the first bending frequency falls extra 6% more because of the formation of the plastic hinge. As can be seen from figure 4, the relative drop in natural frequency looks steadier in the zone of cracked beam state than in uncracked state.

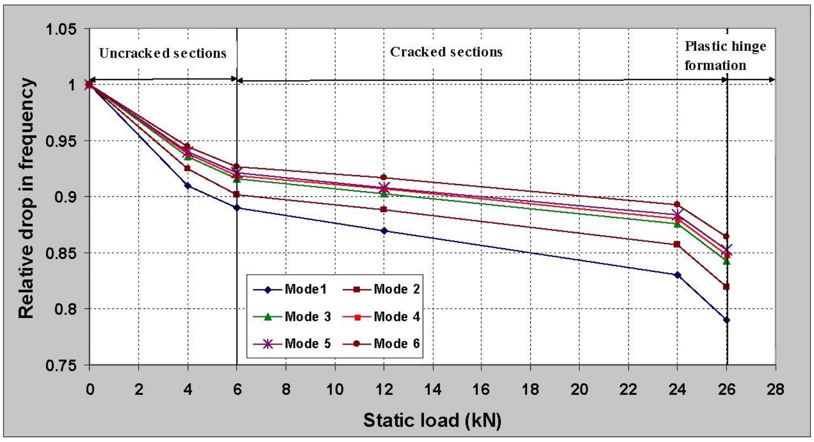


Figure 4. Relative decrease in frequencies with progressing of applied load

3.2. Damage Identification Using Mode Shapes and Their Curvatures

As normalised mode shapes and their differences failed to identify the damage of the concrete beam, as shown in figure 5, the curvature mode shape is investigated to find a better understanding.

Existence of damage in reinforced concrete beams produces almost even reduction in stiffness. The reduction in the stiffness leads to a subsequent increase in the absolute magnitude of the curvature as defined below

$$\phi'' = \frac{d^2\phi}{dx^2} = \frac{M}{EI} \tag{3}$$

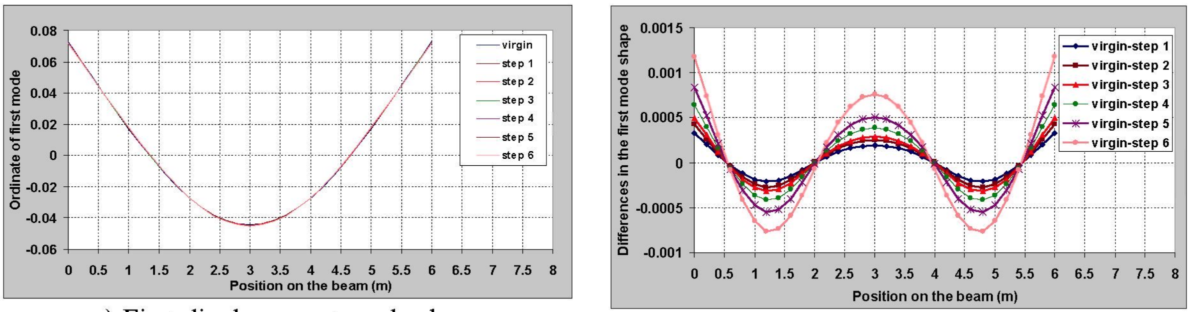
Where M is the bending moment, E is Young's modulus and I is the moment of inertia of the cross-sectional area. The curvature of continuous deflection (displacement) mode shape is obtained using a central difference approximation, which can be defined as

$$\phi_i'' = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{(\Delta x)^2} \tag{4}$$

Where Δx stands for the length of the element and Φ is the deflection mode shape at a specific position.

In general, the changes in the element system stiffness emerge as local changes of curvatures. As a result, the absolute changes in curvature mode shapes are used to detect and locate the damage. However, points of inflection, where the curvature changes from being concave upwards to concave downwards (or vice versa) results in a confusion interpretation. Basically, support points and zero curvatures which appear as inflection points often show larger changes of curvatures, of course, in addition to changes that occur due to the drop in stiffness.

For this case study, the curvature mode shapes for increasing damage level are shown in figure 6a. Although the difference in the curvature mode shape appears clearer than in the mode shape, figure 5a, the curvature mode shapes are not heavily influenced by the damage, particularly in points of inflection. Alternatively, the differences between the curvature mode shapes of the reference and different damage steps are shown in figure 6b. The maximum difference in each curvature mode shape happens in the most inflicted area. The differences in the curvature mode shapes are much bigger in the damage region, but once again inflection points interfere with this conclusion in widespread cracking sections. Additionally, the curvature mode shape ratios between any damaged level and reference intact state $(\{\Phi''\}_{damaged}/\{\Phi''\}_{virgin})$ are shown in figure 6c. Once more, these ratios are not localised to the damage zone.



a) First displacement mode shape

b)Differences in first mode shape

Figure 5. Different damage identification techniques using displacement mode shapes

4. Damage Identification Using Curvature Difference Ratio (CDR)

In order to overcome problems associated with the curvatures at support joints, which manifested as misguiding large changes in curvatures as shown in figure 6b, this study suggests to eliminate this effect by normalising the difference in curvatures between the damaged and undamaged stages. For a specific mode of vibration, the change in stiffness between to damage stages can be determined at any position on the beam from Equation 4. Therefore, for two different damage steps, a relative ratio of the difference in curvatures 'Curvature Difference Ratio' (CDR) can be established to determine the discrepancies of stiffness at any section over the beam. This non-dimensional expression is defined as

$$\{CDR\}_r = \frac{\Delta\left\{ d^2\phi / dx^2 \right\}_{case\,2}}{\Delta\left\{ d^2\phi / dx^2 \right\}_{\substack{reference\\case}}} \tag{5}$$

For any two damage steps, CDR for a specific mode (r) may be written as

$$\{CDR\}_r = \left| \frac{\left[\{\phi''_o\}_r - \{\phi''_{d(i+1)}\}_r \right]_{case2}}{\left[\{\phi''_o\}_r - \{\phi''_{d(i)}\}_r \right]_{\substack{reference\\case}}} \right| \tag{6}$$

Where Φ_o'' represents the curvature of the intact step and $\Phi_{d(i)}''$ represents the curvature of the first damage step, while $\Phi_{d(i+1)}''$ represents the curvature of any subsequent damage step.
Alternatively, in terms of the bending moment at a section *(M)* and flexural stiffness of a beam cross-section *(EI)* for a specific mode shape *(r)*, CDR can be written as

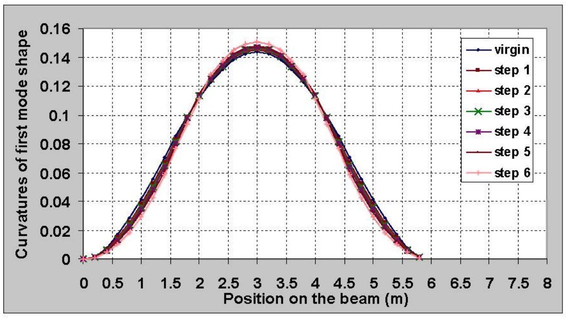
$$\{CDR\}_r = \left| \frac{\left[\left\{ \frac{M}{EI_o} \right\}_r - \left\{ \frac{M}{EI_{i+1}} \right\}_r \right]_{case\ 2}}{\left[\left\{ \frac{M}{EI_o} \right\}_r - \left\{ \frac{M}{EI_i} \right\}_r \right]_{\substack{reference\\ case}}} \right| \tag{7}$$

Remarkably, the inertia bending moments for any two different damage steps at the same section are almost identical. Therefore, by eliminating the parameter *M* from the numerator and denominator, CDR can be rewritten as

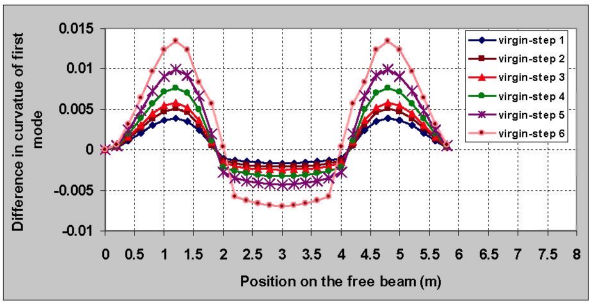
$$\{CDR\}_r = \left| \frac{\left\{ \frac{EI_o - EI_{i+1}}{EI_{i+1}} \right\}^r_{case\ 2}}{\left\{ \frac{EI_o - EI_i}{EI_i} \right\}^r_{\substack{reference\\ case}}} \right| \tag{8}$$

Equation 8 can be used to obtain the numerical stiffness at a specific beam section based on the known reference stiffness value. This method overcomes difficulties associated with calculating modal inertia forces, which are the source of bending moments over beam sections. The proposed technique relies on the fact that difference between modal bending moments at a section for two different load steps is negligible because mode shapes for different damage steps are very close.

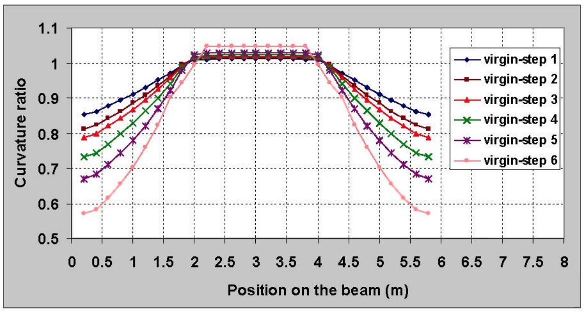
The increase of the size of the damage will result into changes in curvature mode shape. This information can be used to locate the damage, follow its trend and estimate its size. Apart from the inflection points, the last formula gives reliable and validated values for drop in stiffness, particularly for continuous curvature zones. At points of inflection, this estimate suffers numerical inconsistency due to the occurrence of an ill-condition situation (zero by zero division). To solve this problem, bending stiffness can be determined at the neighbourhood of zero curvature points.



a) Curvatures of first mode shape



b) Differences in curvatures of first mode

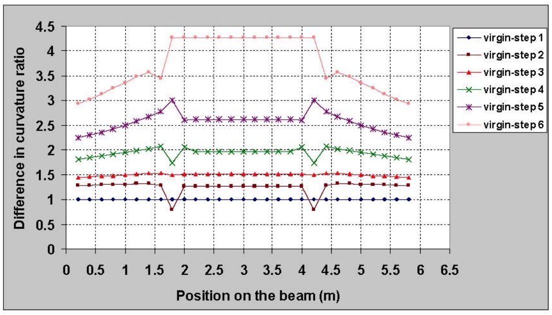


c) Different curvatures ratio of first mode

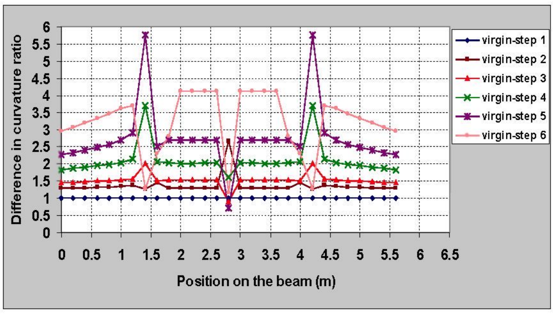
Figure 6. Different damage identification techniques using curvature mode shapes

5. Results Validation and Discussion

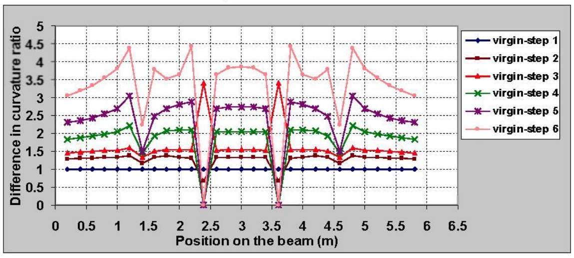
The beam zone between 2m and 4m is a zone of constant bending moment, which is expected to be of approximately same bending stiffness. As can be seen in figure 7, results from the first four mode curvatures trace the trend of stiffness drop correctly. Also, using bending Curvature Difference Ratio CDR, the numerical drop of stiffness in the mid span zone at each load step is estimated and given in table 3. The results found from each curvature mode are very similar to those used in the analytical model. Additionally, calculated decrease values in stiffness based on CDR for each step are very close to those given by Ren and De Roeck [1] and Maeck and De Roeck [2]. Estimation of bending stiffness by using this technique is no longer satisfied for all positions. From figure 7, it can be noted that positions of zero modal curvature create an unstable condition (division of zero by zero). However, as higher modes exhibit more sections with zero curvature, this technique can be efficiently used to estimate bending stiffness at the vicinity of zero curvature points. Most importantly, the first curvature mode is relevant to be applied in this method with a least degree of inconsistency. For the beam zones between 0-2m and 4-6m, which are zones where the bending stiffness descends linearly, the observed trend of the CDR is comparable to the damage distribution. Tested results of dynamic bending stiffness at sections from these zones are very close to the simulated stiffness.



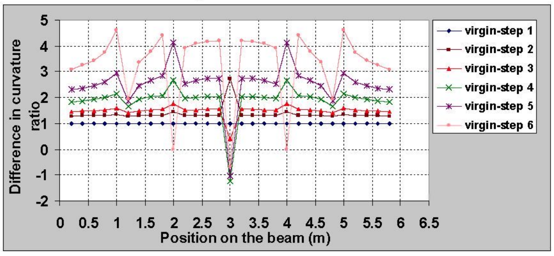
a) First mode



b) Second mode



c) Third mode



d) Forth mode

Figure 7. Curvature Difference Ratios for different curvature mode shapes

Table 3. Successive stiffness decrease based on the proposed Curvature Difference Ratio (CDR)

Load Step	Accumulated (CDR) % 1st mode	Accumulated (CDR) % 2nd mode	Accumulated (CDR) % 3rd mode	Accumulated (CDR) % 4th mode	Accumulated (Ren & De Roeck)[1] %	Accumulated (Maeck & De Roeck)[2] %
Reference	---	---	---	---	4.9	---
Step 1	18.40	18.40	18.40	18.40	18.42	17
Step 2	22.30	22.65	23.07	22.98	23.35	21
Step 3	25.37	25.60	25.76	25.72	26.38	22
Step 4	30.74	31.29	31.70	31.53	32.78	26
Step 5	37.19	37.88	38.16	38.03	40.13	33
Step 6	51.04	48.24	46.58	48.58	52.17	48

6. Conclusions

This study presents a technique to locate the damage and then obtain the drop in dynamic bending stiffness of a concrete beam subjected to an incremental static load.

The estimation of stiffness is based on Curvature Difference Ratio (CDR). The suggested expression utilises the curvature mode shape along with the relationship of the bending moment-curvature formula. The advantage of this technique is that the numerical estimation of the modal internal bending moment can be avoided, and the bending stiffness can be determined through the ratio of the difference in curvatures for various damage levels. Another potential benefit is that this method works within the non-model based method category where no need to predict an analytical model to the tested structure. Additionally, only one measured displacement mode shape will be sufficient to create curvature mode shape; hence, to display CDR. The results of bending stiffness obtained by this expression are tested and verified with results from previous damage identification algorithms proposed by Ren and De Roeck [1] and Maeck and De Roeck [2]. The results of the current study show significant accuracy with results of the validation cases. In addition, as another contribution, the study relates the changes in vibration data with the design bending moment associated with each static load increment. This attempt provides a better understanding for the condition of beam sections at each vibration parameter change.

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