

Article

Large deformation finite element analyses for 3D X-ray CT scanned microscopic structures of polyurethane foams

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Abstract: Polyurethane foams have unique properties that make them suitable for a wide range of applications, including cushioning and seat pads. The foam mechanical properties largely depend on both the parent material and the foam cell microstructure. Uniaxial loading experiments, X-ray tomography and finite element analysis can be used to investigate the relationship between the macroscopic mechanical properties and microscopic foam structure. Polyurethane foam specimens were scanned using X-ray computed tomography. The scanned geometries were converted to 3D CAD models using open source, and commercially available CAD software tools. The models were meshed and used to simulate compression tests using the implicit finite element method. The calculated uniaxial compression tests were in good agreement with experimental results for strains up to 30%. The presented method would be effective in investigating the effect of polymer foam geometrical features in macroscopic mechanical properties, and guide manufacturing methods for specific applications.

Keywords: Polyurethane foam; Structure–property relationships; Finite element analysis; Microscale analysis; X-ray computed tomography

1. Introduction

Polyurethane foams have many unique properties such as elasticity, softness and ease of forming. These properties make polyurethane foams attractive to automotive seat designers since they can effectively support the human body and distribute the body pressure. The improvement of the mechanical properties of the foams is an important challenge. Controlling the mechanical properties of foams would be useful for designing seats that are more comfortable and potentially at lower cost. The mechanical properties of polyurethane foams largely depend on their microstructures (Figure 1). The foam structure consists of a cluster of bubbles and struts at the edges of the cells. Figure 1 shows an example of an open-cell foam in which the bubbles are linked together. The macroscopic stress-strain relationship depends on the mechanical properties of the parent material, of which the struts are made, and the geometrical structure of cells and struts [1]. Understanding the relationships between the microscopic geometrical structures and the macroscopic mechanical properties is essential for developing foam products with superior mechanical properties.

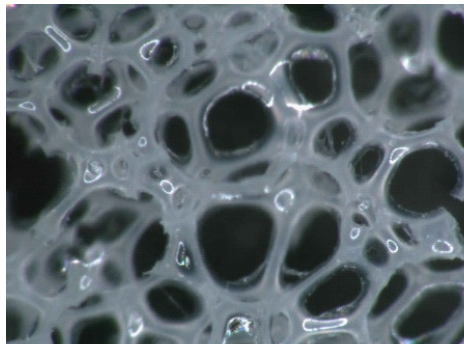


Figure 1. An example of optical microscope images of polyurethane foams

28 Three main regions can be identified in the stress-strain curve for the compressive deformation
 29 of elastomeric foams [1]. The typical stress-strain curve under the uniaxial compression of foams is
 30 shown in Figure 2. Linear elasticity is shown in the small strain region followed by a collapse plateau,
 31 and then densification appears accompanied by a rapid increase in the stress. Firstly, the struts bend
 32 and the macroscopically linear elastic behaviour is shown. Next, due to the increase of the macroscopic
 33 stress, some struts start buckling and the slope of the curve decreases. Finally, the slope of the curve
 34 increases again up to the same value as the matrix material, because of the contact between struts. The
 35 contribution of microstructures to macroscopic properties depends on these deformation mechanisms.

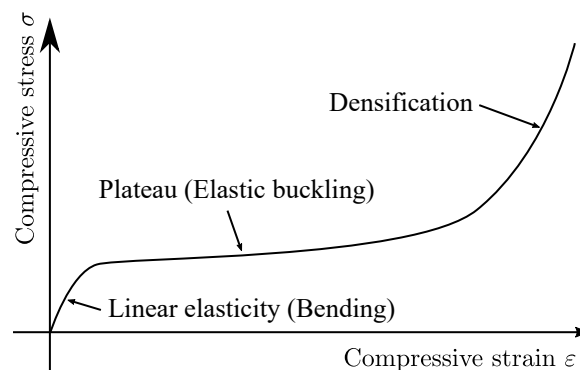


Figure 2. The typical stress-strain relationship of elastomeric foams under the uniaxial compressive stress

36 To investigate the effect of microstructures on macroscopic properties, cell structure geometries are
 37 virtually generated, and their deformations are analysed [2]. The cells were postulated to have same
 38 size and the shape of the Kelvin tetrakaidecahedron. The edges of the polyhedron were assumed to be
 39 struts represented by Euler-Bernoulli beams and the macroscopic elastic properties were analytically
 40 calculated. This approach was also expanded to the large compressive strain range up to 70% [3,4]
 41 and creep deformations [5]. Other researchers repeated the calculations of Zhu et al. [2], employing
 42 a finite element approach, while still making use of Kelvin's cell shape and Euler-Bernoulli beams
 43 [6–10]. As the Kelvin's cell has anisotropic mechanical properties, Okumura et al. [11] and Takahashi
 44 et al. [12] analysed the mechanical responses in the [001], [011] and [111] directions. Furthermore,
 45 closed cell foams have been analysed with shell elements [13]. Modelling the microscopic structures
 46 of polyurethane foam materials using the Kelvin's cell is thought to be a simple and effective way to
 47 investigate the deformation behaviour.

48 The Kelvin cell approach assumes that the microstructure is homogeneous; however, in contrast
 49 cell structures are generally heterogeneous. This is a significant disadvantage of the repeated unit
 50 cell modelling approach [14]. To model the inhomogeneous structures of foams, the 2D and 3D
 51 Voronoi tessellations were employed and the Voronoi edges were regarded as struts [14–17]. Moreover,
 52 faces in Voronoi polyhedrons were assumed as cell membranes in closed cell foams [18,19]. The

53 elastic properties in the small strain region and the compressive stress-strain curves on the plateau
54 region were calculated by the finite element method using beam elements. Furthermore, although the
55 cross-sectional area of a strut is often assumed constant, the central parts of struts are thinner than
56 other parts. The effect of this necking can be taken into account by using solid elements [11,12,20–27]
57 or beam elements with variable cross-sectional properties [23,28–33]. In addition, the curvature of
58 struts were modelled [34]. Models that consider the heterogeneity of foams are thought to show better
59 results than Kelvin cell models with straight struts. Dynamic crushing behaviour [35,36] and multiaxial
60 crushing [37] were also analysed.

61 One effective method to obtain a more adequate model that represents actual foam microstructures,
62 is to use X-ray computed tomography (CT). The X-ray CT has been performed in order to observe
63 the microstructures of various kinds of porous materials, for example, biomaterial scaffolds [38,39],
64 soil materials [40] and polyurethane foams [41]. Therefore, the X-ray CT has also been used to
65 generate the geometries for finite element analyses. For example, finite element models for the
66 microstructure of a trabecular bone was generated based on micro-CT[42]. For artificial foam materials,
67 Jeon et al. [43] analysed closed-cell aluminium foams with finite element models meshed with solid
68 tetrahedron elements. The compressive stress-strain curves of the foam were calculated and compared
69 to experimental results and the 20.86% volume error was shown up to 5.31% strain. Similarly, linear
70 elastic properties under the small strain regions were obtained from X-ray CT scanned finite element
71 models for ceramic foams [44] and a rigid organic foam [45]. Models obtained from the X-ray CT have
72 been effectively used to investigate the mechanical properties of foams under small deformations.

73 For cushioning products such as automotive seat pads or bed mattresses, the mechanical
74 properties in the plateau regions are more important than the linear elastic regions. As the slope
75 of the stress-strain curve decreases in the plateau region, elastic foams soften and help to distribute
76 body pressure. Most studies employ tetrahedron meshing due to the complexity of the geometry,
77 however, this makes analysing large deformations difficult. To analyse the deformation within the
78 plateau region, hexahedron meshing is required as it is more suitable for large deformation problems.

79 This study aims to use X-ray CT scans of foam specimens in order to construct validated finite
80 element (FE) models that can be used to study and manipulate the foam microstructure for achieving
81 desirable stress-strain behaviour in the plateau region. The microstructures of elastic polyurethane
82 foams for automotive seat pads are scanned using X-ray computed tomography and converted to STL
83 files. The STL files are smoothed and converted to solid CAD files with commercial CAD software
84 so that they can be meshed with a hexahedron dominant solid mesh. The uniaxial compressive
85 deformation of the models are analysed with a finite element method and compared with the
86 experimental results.

87 2. Materials and Methods

88 The methodology to analyse the deformation of X-ray CT scanned foam materials and the
89 materials supplied to validate its accuracy are explained here. The specimens were scanned using
90 X-ray CT, converted to CAD models and analysed with the implicit finite element method. The tools
91 used for this study is either commercially available CAD or open-source software. Moulded elastic
92 polyurethane foams were investigated using the presented method and physically tested to compare
93 with the result of the analyses.

94 2.1. Materials

95 The tested materials were supplied by Bridgestone Corporation. Polyols, isocyanates, water and
96 low amounts of other materials were mixed and poured into a $400 \times 400 \times 100(\text{mm}^3)$ sized mould and
97 then expanded and polymerized. After demoulding, the foams were crushed between rollers so that
98 cell membranes were broken and resulted in open-cell foams. The foams were left at least 24 hours
99 before proceeding to any other process of the investigation to let the chemical reactions be completed.

100 The foam materials investigated in this study are mainly used for automotive seat pads by moulding
101 in product shaped moulds.

102 2.2. Scanning by the X-ray computed tomography

103 Specimens from the centre of larger samples were cut into $5 \times 5 \times 5(\text{mm}^3)$ sized cubes. The X-ray
104 tomography equipment employed for this study was the ScanXmate RA150S145/2Be, a product of
105 Comscantecno Co.,Ltd. Figure 3, shows an example X-ray CT scan image of the foam. The white parts
106 indicate the foam struts and the black parts are the pores. The size of the pixel was $7.5(\mu\text{m})$. The cross
107 section images were taken by rotating the specimens every 0.18deg so that the cell structures could be
108 observed in three dimensions.

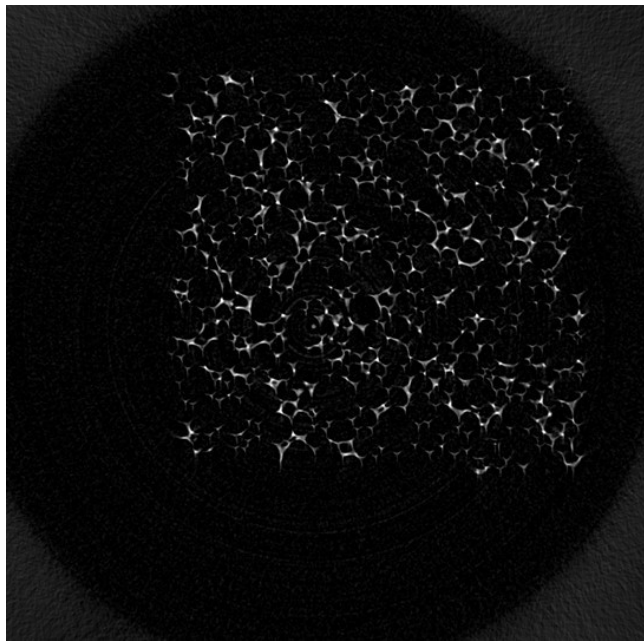


Figure 3. An example of the X-ray CT scanned images for the polyurethane foams

109 2.3. Converting the scanned images to 3D STL files

110 The cross sectional 2D images were converted to 3D STL files by Fiji [46], a distribution of Image
111 J2 [47]. Firstly, the scanned images were binarized to black and white images using a threshold of the
112 brightness. The threshold was determined using Otsu's method [48] and verified by comparing the
113 relative densities measured with the actual specimen and calculated from the computational models.
114 The borders between the black and white pixels were regarded as the surfaces of the struts. Triangles
115 were then applied to the strut surfaces and the resulting surfaces exported as STL files. An example
116 STL file is shown in Figure 4(a).

117 2.4. Converting to smoothed solid CAD models

118 The STL formatted files consist of only triangle surfaces and the triangle edges are sharp. When
119 dividing STL files to finite elements directly, the triangle surfaces are divided into further small
120 elements resulting in a considerable number of nodes and elements. Therefore, the vertices of the
121 triangle surfaces should be interpolated by mathematically smooth surfaces. This smoothing can be
122 performed using Recap® and Fusion 360® software, both products of Autodesk, Inc. Firstly, the STL
123 files with the triangle meshing were converted to surface models with quad meshing (Figure 4(b)). The
124 quad meshed surfaces were then interpolated and smoothed by T-spline surfaces (Figure 4(c)). Finally,
125 boundary representation solid models were generated based on the T-spline surface models (Figure
126 4(d)). The resulting solid models were then capable of being analysed in commercial FEA software.

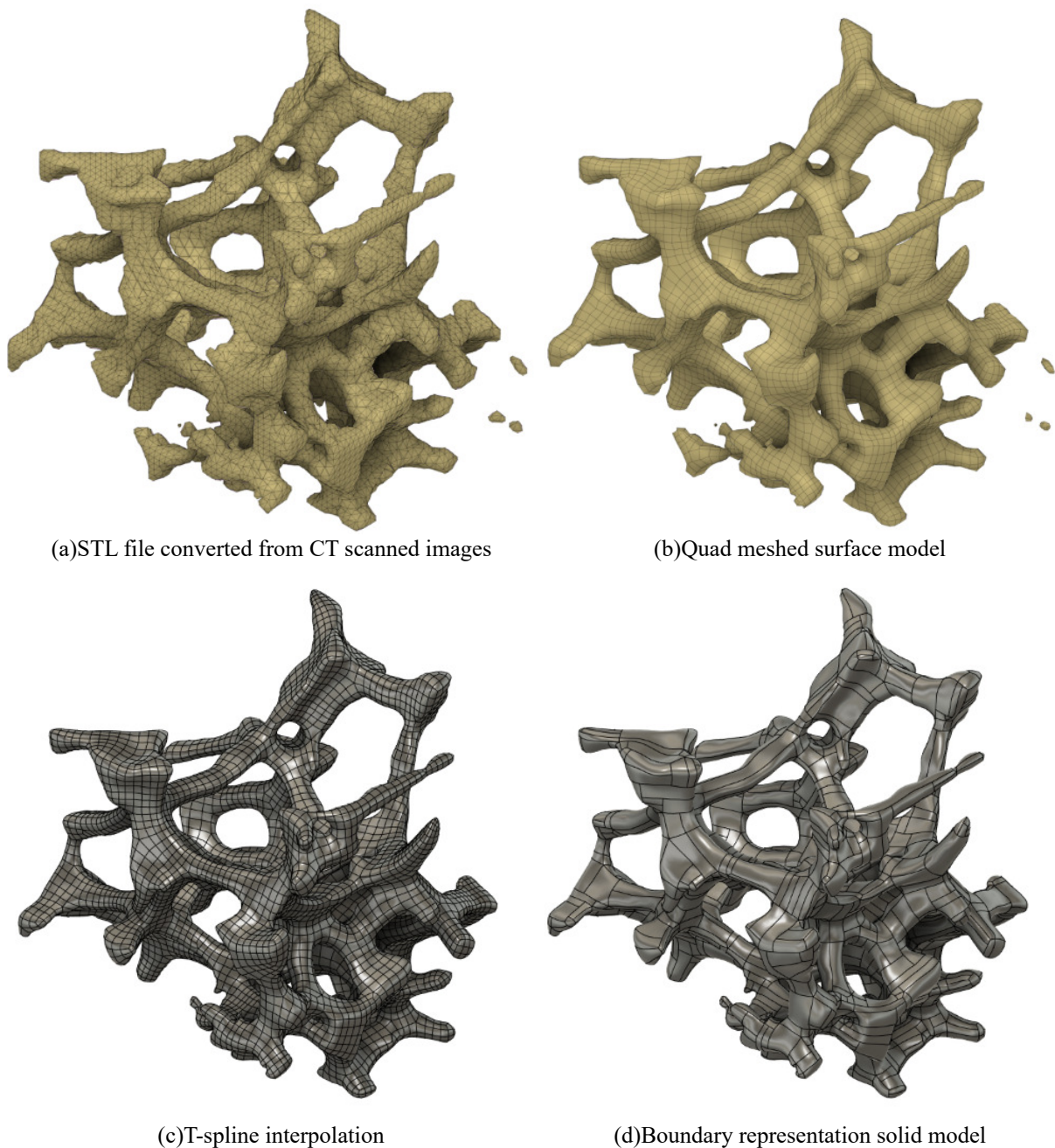


Figure 4. Conversion from the STL files to the boundary representation solid models

127 2.5. Hexahedron dominant meshing

128 Although the geometries were smoothed by the T-spline interpolation, they were still too complex
129 for hexahedron meshing to be applied. Therefore, mixed hexahedron and tetrahedron meshing was
130 employed. These two kinds of elements were joined by the pyramid mesh elements. The mesh
131 divisions were performed using Ansys® Academic Research Meshing, Release 19.2 [49]. In this study,
132 three representative geometric models were analysed. The mesh divisions of these models are shown
133 in Figure 5 and Table 1 summarises the numbers of the nodes and the elements in each. Where possible
134 the models were meshed with hexahedron or pyramid elements.

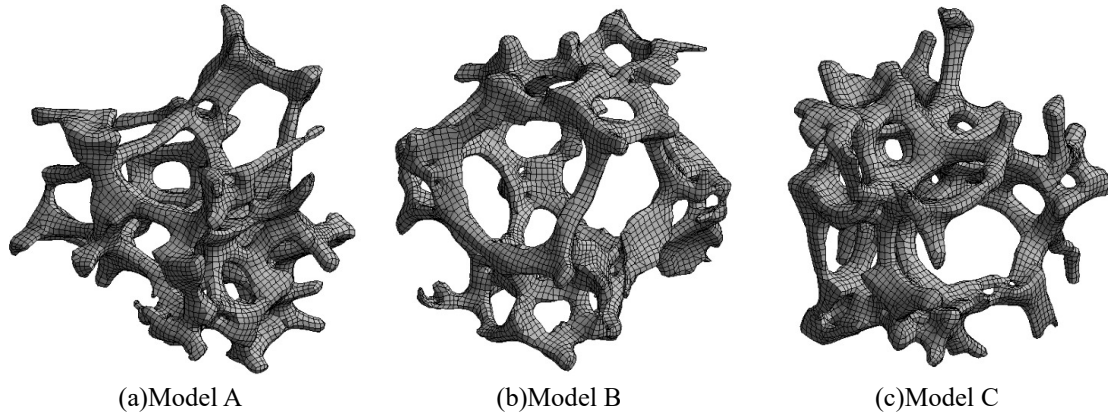


Figure 5. Mesh divisions for the models

Table 1. Numbers of nodes and elements for the models

	Model A	Model B	Model C
Nodes	27730	24937	30081
Tetrahedron elements	10873	11586	12041
Pyramid elements	16202	16795	17083
Hexahedron elements	13231	13746	15089

135 2.6. Finite element analyses

136 Deformation behaviour of 3 different specimen models was calculated with the commercial FEA
 137 software Ansys® Academic Research Mechanical, Release 19.2 [50]. To analyse the deformations
 138 up to the plateau region, the large deflection was taken into account. As this study focused on the
 139 static mechanical properties of polyurethane foams, the static implicit method was employed and the
 140 damping or the dynamic characteristics were neglected.

141 2.7. Strut material model

142 In order to measure the stress-strain relationship of the matrix material, a specimen without pores
 143 is needed. The diameters of the struts are less than 0.1mm and form a complex microstructure. Foam
 144 was compressed between plates heated to 150°C in order to obtain a parent material specimen without
 145 pores. The original thickness of the foam was 50mm and the compressed specimen had a thickness of
 146 0.7mm. The measured density of the specimen was 1200kg/m³.

147 Tensile testing was performed to obtain the tensile stress-strain relationship. The test equipment
 148 was a universal testing machine AGS-X 10kN with a 500N load cell, products of SHIMADZU
 149 CORPORATION. The specimen was cut into 50 × 5mm² rectangular shape specimens and then
 150 a tensile test was performed under the strain rate 0.01s⁻¹. The difference between the grippers was
 151 regarded as the elongation of the specimen.

The measured nominal stress-strain curve is shown in Figure 6. The experimental result is approximated by the Neo-Hookean (Equation (1)) and Mooney Rivlin (Equation (2)) hyper elastic models respectively.

$$W = C_{10} (\bar{I}_1 - 3) + \frac{1}{d} (J - 1)^2 \quad (1)$$

$$W = C_{10} (\bar{I}_1 - 3) + C_{01} (\bar{I}_2 - 3) + \frac{1}{d} (J - 1)^2 \quad (2)$$

152 W is the strain energy density, \bar{I}_1 and \bar{I}_2 are the first and second deviatoric strain invariants, J is
 153 the determinant of the deformation gradient. The material constants C_{10} , C_{01} and d are shown in
 154 Table 2. Because the matrix material is thought to be incompressible, d was calculated to let the initial

155 Poisson's ratio ν equal to 0.48. The Mooney-Rivlin model was employed in this study as it shows
 156 better agreement with the experimental result than the Neo-Hookean model.

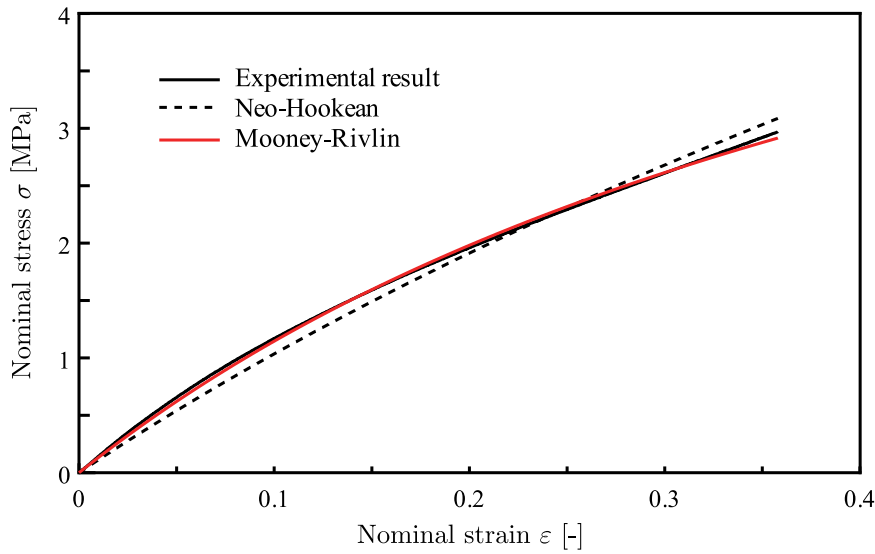


Figure 6. The result of the tensile test for the matrix material and its approximations by hyperelastic models

Table 2. Material constants for the matrix material

Hyperelasticity models	C_{10} [MPa]	C_{01} [MPa]	d [MPa ⁻¹]
Neo-Hookean	1.89	-	0.0661
Mooney-Rivlin	0.476	1.78	0.0554

157 2.8. Boundary conditions

158 The foam model specimen was uniaxially compressed between two rigid shell plates Figure 7.
 159 The lower plate was fixed preventing any translational or rotational displacements. Translational
 160 displacement was applied to the upper plate whilst all other degrees of freedom were constraint.
 161 Frictionless contacts between the foam model and the rigid walls were defined using the penalty
 162 method with a stiffness factor 0.01. Self-contacts between the struts were not considered as this
 163 study focuses on the buckling behaviour in the transitions to the plateau regions. Finally, remote
 164 displacements were used to constraint the specimen lateral boundaries from rigid translational and
 165 rotational movement. The average values of the displacements of the nodes on the boundaries
 166 corresponding to these directions were fixed. This would allow deformation but not rigid body
 167 movement.

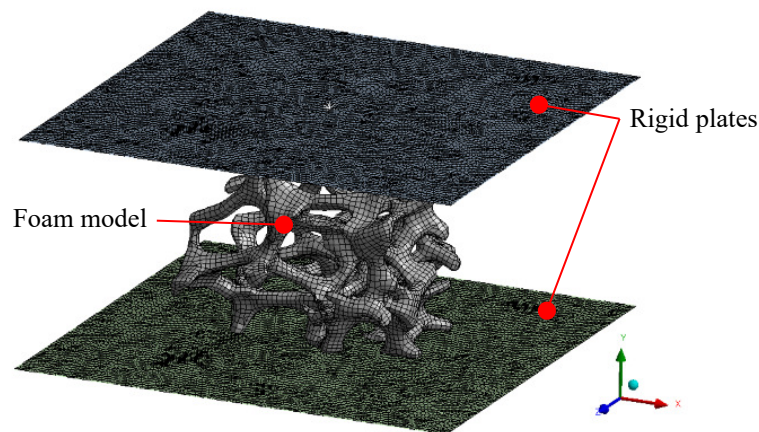


Figure 7. Boundary condition for the uniaxial compression analyses of the foam models

168 2.9. Experimental measurement for the macroscopic stress strain relationships

169 The uniaxial compression tests for the actual foam specimens were performed to compare with
 170 the FEA results. The testing method was similar to ISO3386-1 [51]. The $25 \times 25 \times 10(\text{mm}^3)$ sized
 171 specimens were cut from the centre parts of the moulded foams. The specimens were set into the
 172 same equipment as the section 2.7 with the compression plates. The lower plate was perforated by
 173 6mm holes arranged in a latticed pattern with 20mm distances so that the air in the foam could be
 174 ventilated. Firstly, the foams were compressed to achieve 75% nominal strain with the speed 50mm/s
 175 as the pre-compression. Then, the load was taken off with the same speed and the foams were left
 176 for 60s. After that, the foams were compressed again with the same speed and compressive strain to
 177 measure the load and the displacement.

178 3. Results

179 3.1. The deformed shapes of the models

180 Figure 8 shows the deformed shapes of the different specimen models at the macroscopic nominal
 181 compressive strains $\epsilon^c = 0.05, 0.25$ and 0.50 respectively. The coloured contour represents the
 182 Von-Mises equivalent strains ϵ^{eq} . As mentioned in the section 1, the struts bend in the linear elastic
 183 region ($\epsilon^c = 0.05$). After that, some struts start to buckle, which indicates a transition to the plateau
 184 region ($\epsilon^c = 0.25$). Finally, the models gradually become denser and transfer into the densification
 185 region ($\epsilon^c = 0.50$). As self-contacts were not applied in the foam models, the struts did not touch, but
 186 instead overlapped. The results of the analyses enable the microscopic behaviour of the struts to be
 187 carefully observed.

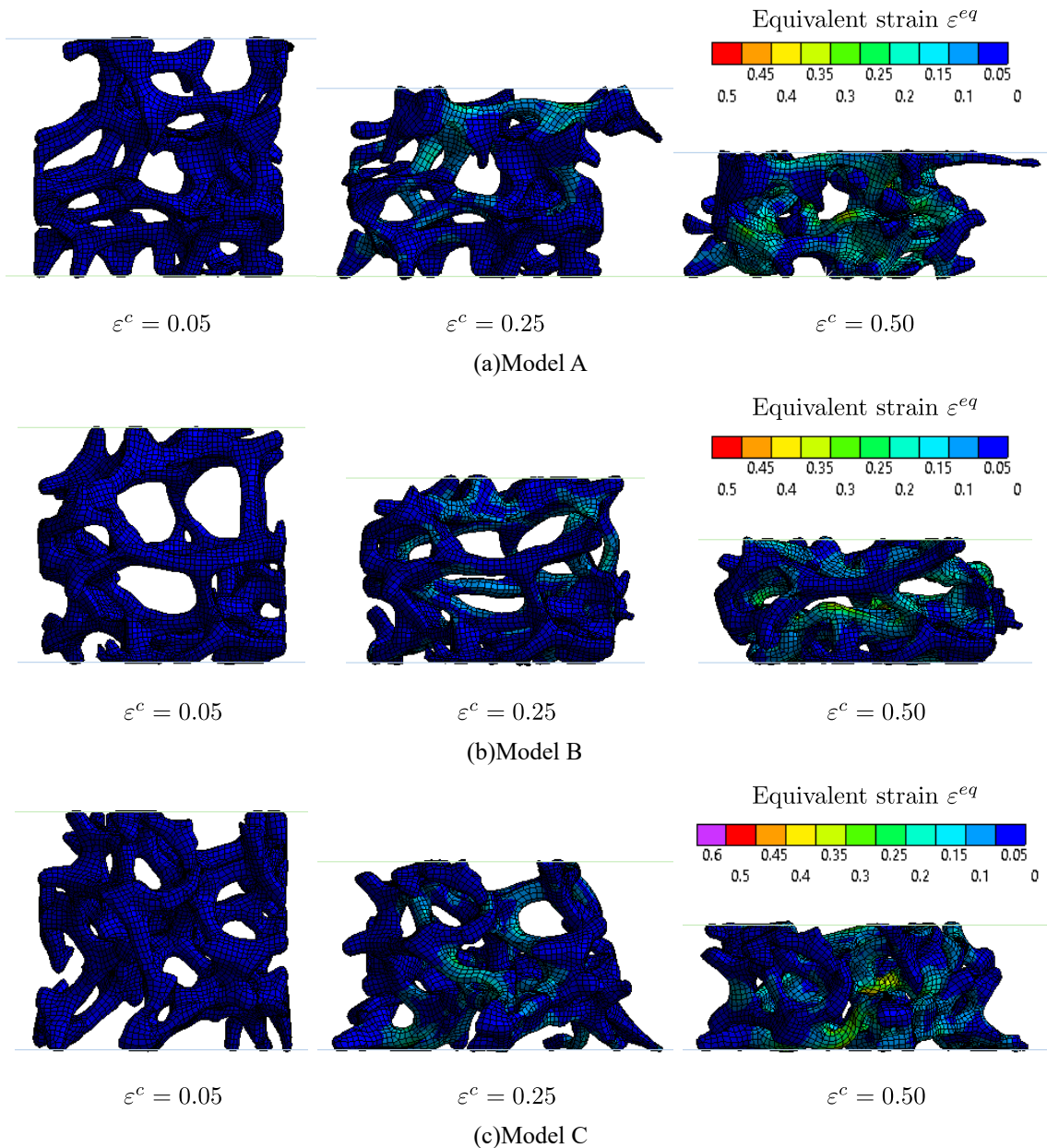


Figure 8. The deformed shapes of the models with the distributions of the equivalent strain ε^{eq}

188 3.2. Macroscopic stress-strain relationships

189 The FEA results were compared with the experiment results to validate the accuracy of the
 190 presented analysis method. Figure 9 shows the both experimental and FEA results of the relations
 191 between the nominal compressive stress and strain. The slopes of the stress-strain curves for the FEA
 192 results start decreasing in the strain region around 0.05 compared to the smaller strain region. It is
 193 thought to mean the transition from the linear elastic regions to the plateau regions.

194 The models appear to be in good agreement with experiments in the linear elastic and the plateau
 195 regions, and up to the strain of 0.30. Differences of the stresses between the experimental and FEA
 196 results at the nominal compressive strain of 0.25 were 0.1%, 16.5% and 6.6% for the models A, B and
 197 C respectively. In contrast, the FEA results are stiffer than the experimental results in larger strain
 198 regions than 0.30. After reaching the strain of 0.30, the slopes of the stress strain curves start increasing
 199 again. This behaviour looks similar to the transition to the densification regions, however, self-contacts
 200 were not enabled within the model and the stress increase occurs far too early in the strain regions.
 201 The presented method should be modified when applied for the densification region.

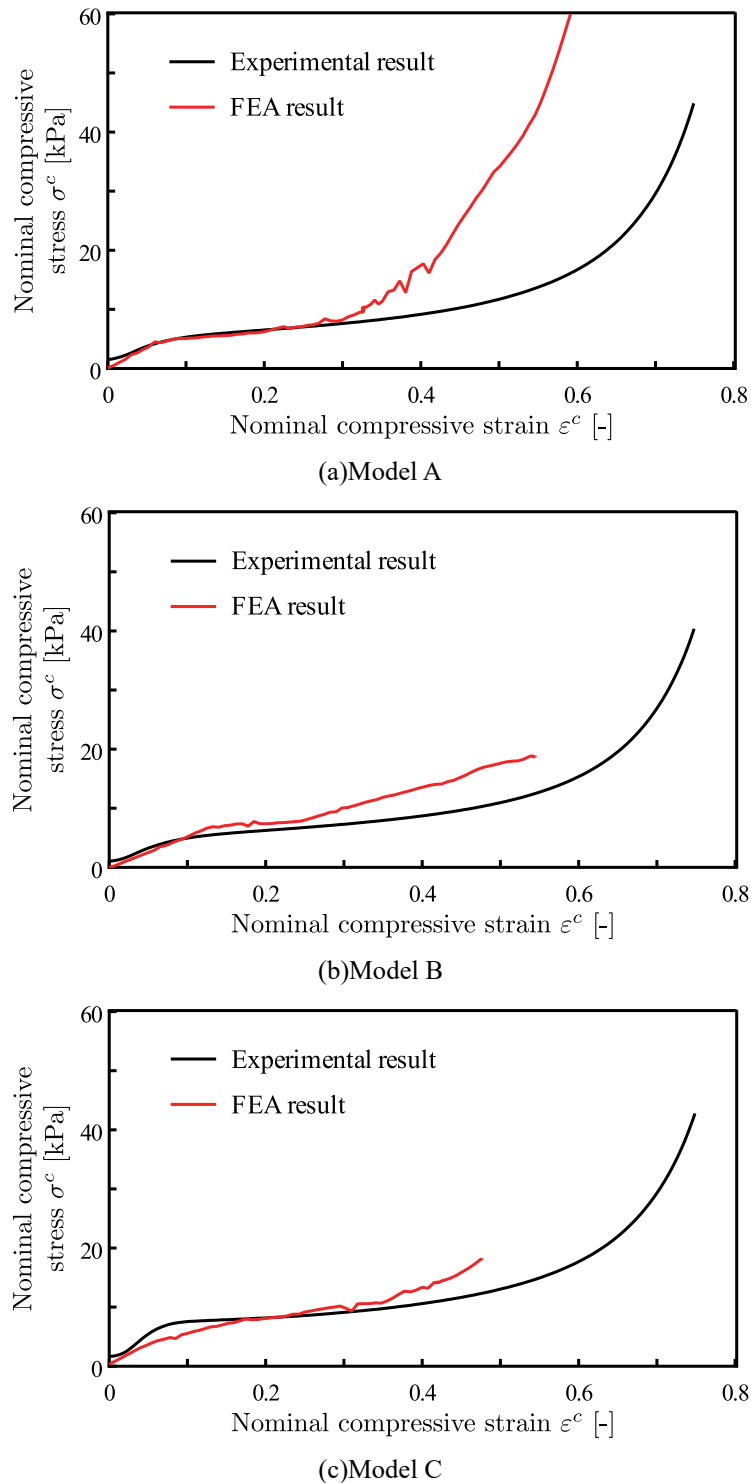


Figure 9. The experimental and FEA results in the relations between the macroscopic compressive stress and strain

202 4. Discussions

203 The compressive response of Polyurethane foam geometries was simulated using FE methods
 204 and compared with experiments. Foam specimens were scanned using X-ray CT and analysed to
 205 obtain geometries for FE simulations. Simulation results were in good agreement with experiments
 206 up to 0.3 strain. The finite element model over-predicted stresses, beyond that strain. Three different
 207 specimens were scanned and modelled to ensure repeatability of results.

208 Elastic buckling appears to be one of the dominant deformation mechanisms. The finite element
209 simulation results seem to have captured the strut deformation behaviour in agreement to relevant
210 literature [1]. Similar deformation mechanisms have been captured with virtually generated cell
211 structures such as Kelvin's cells [2,4,8,9,11,12,23,29] or Voronoi polyhedrons [24,27,30,31]. Previous
212 models based on X-ray CT scanned foam structures were mostly limited to small strains (up to 5.31%)
213 [43–45].

214 Hexahedron dominant meshing was used for the large deformation analyses. Struts in foam
215 materials can be long and narrow. Euler-Bernoulli beams have been widely employed for analytical
216 calculations [2,4] and numerical simulations [8,9,24,27,29–31]. However whilst beam models might be
217 beneficial in reducing complexity and calculation time, they might also add stiffness to the structure
218 and result in higher stress predictions by comparison to the experiment values. Hexahedron meshes
219 in large deformation problems, have been used for simplified geometries [11,12]. The presented
220 smoothing method and hexahedron dominant meshing are recommended for the complex X-ray
221 scanned geometries.

222 The foam struts at the lateral specimen boundary were unconstrained. Similarly to other studies
223 [31–33,36,43], compressive loads were applied in the model, by using rigid plates. Contact was defined
224 between the foam specimen and the rigid plates. The modelled specimens were smaller than those
225 used for experiments. However the boundary conditions seem to have been sufficient in capturing the
226 strut behaviour for strains up to 0.3. Due to the high porosity of the foam, the effect of surrounding
227 material at the boundary might have been effectively negligible for up to the strain of interest. However
228 more sophisticated boundary conditions might be required for achieving better accuracy beyond 0.3
229 strain, or for lower porosity foams. Surrounding cell structures could affect the computed region with
230 bending moments, forces, or contacts between the struts, particularly as the foam densifies. These
231 effects could potentially be taken into account by considering periodic boundary conditions [11,12].
232 However, this type of boundary condition requires the geometry in the model to be periodic and
233 therefore might be more difficult to apply in models of stochastic foam geometries.

234 The finite element model over-predicted stresses, beyond 0.3 strain. Figure 10 shows an example
235 of the deformed modelled specimen at 0.3 strain. As the specimen is compressed, struts that were
236 initially away from the boundary, might then deform and come into contact with the loading plates at
237 the boundary of the specimen. This could cause an increase in the stress response. Arguably this could
238 also occur during experiments however the model size is considerably smaller than the specimen
239 size in experiments, therefore the effect of these interactions would be more pronounced in the finite
240 element model simulation. A mitigating approach could be to selectively enable contacts between the
241 loading platens and parts of the foam, i.e. applying contacts only to the nodes on the boundary of the
242 foam rather than the whole specimen. Increasing the model domain size could also improve results.
243 However a larger model would also increase computational cost. A damage model was not included
244 in this study. Potentially the inclusion of a damage model could improve the accuracy at higher strains.

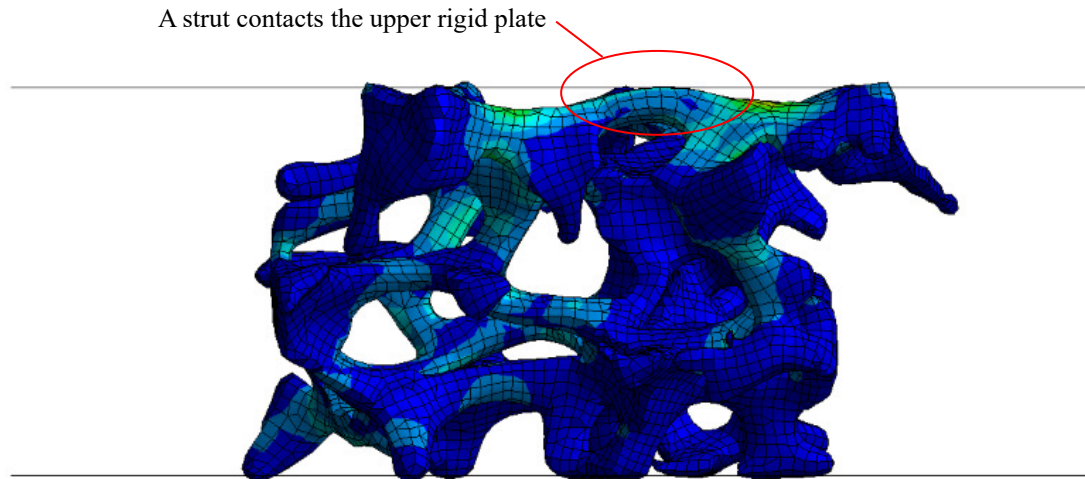


Figure 10. Deformed shape of the model A at the strain of 0.30, when a strut contacts the upper rigid wall

245 Analysing the models up to the densification region using implicit FE methods, with the periodic
246 boundary conditions or with larger domains remains a challenge. Additionally investigating the effect
247 of strut length and cross-section on the buckling behaviour of struts, and the effect of the cell size
248 variation on the linearity of the stress-strain response could inform manufacturing processes for future
249 products. These would be the topics of future work.

250 5. Conclusions

251 Polyurethane foam specimens, intended for automotive seat pads, were scanned using X-ray
252 computed tomography. The scans were converted to 3D CAD models and used to simulate uniaxial
253 compression test using the finite element method. The methodology for the scanning and the analyses
254 was described, and the analysis results were compared with experiments. All three numerical models
255 sufficiently captured the material behaviour in the linear elastic and plateau region of the stress-strain
256 curve. The conclusions for this study are summarised below:

- 257 • The investigated foams were scanned by X-ray computed tomography and their structures were
258 captured in 2D cross-section images.
- 259 • The observed cross-section images were converted to 3D CAD models using Image J and
260 Autodesk, Inc software products. The smoothed CAD models were analysed with commercial
261 FEA software (Ansys).
- 262 • Foam specimens were experimentally tested under uniaxial compression.
- 263 • Specimen deformations were analysed by the implicit finite element method with the hexahedron
264 and tetrahedron mixed meshing.
- 265 • The mechanical behaviour of foam specimens under compressive loading was sufficiently
266 captured at 0.25 nominal strain and within reasonable error margin.

267 The presented method was successfully used to analyse foam structures and provided a tool
268 in understanding the mechanism of compressive deformations in polyurethane foams. Commercial
269 CAD products and open source software were used for creating a solid mesh for FE analysis from
270 X-ray scans. The chosen approach was perhaps more efficient by comparison to alternative specialised
271 software at a higher cost or in-house development of custom tools. The dependence of the foam
272 macroscopic mechanical behaviour on microstructural features can now be further investigated to
273 inform manufacturing processes for future polyurethane foam products.

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275 formal analysis, M.I.; investigation, M.I.; writing–original draft preparation, M.I.; writing–review and editing,

276 M.I., R.G., P.S., B.S. and N.M.; supervision, P.S., B.S. and N.M.; project administration, N.M. All authors have read
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282 Abbreviations

283 The following abbreviations are used in this manuscript:

284 CT Computed tomography
285 FEA Finite element analysis
286 PUF Polyurethane foam

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