

# The cyclicity of job search effort in matching models

M. Alper Çenesiz<sup>a,b</sup> and Luís Guimarães<sup>b,c</sup>

<sup>a</sup>Nottingham Business School, Nottingham, NG1 4FQ, UK

<sup>b</sup>cef.up—FEP, Porto, 4200-464, Portugal; email: l.guimaraes@qub.ac.uk

<sup>c</sup>Queen's Management School, Queen's University Belfast, Belfast, BT9 5EE, UK

## Abstract

The canonical matching model is the workhorse model of the labour market but lacks a proper amplification mechanism for productivity shocks. One way to amplify the effects of shocks is to allow workers to endogenously adjust their job search effort: as search effort is procyclical in the canonical model, volatilities increase. Yet, the empirical literature points against procyclical search effort, raising doubts of how acyclical (or countercyclical) search effort can coincide with volatile labour market variables in matching models. We show that they can coincide in a model with procyclical value of leisure and alternating-offer wage bargaining.

**JEL classifications:** E24, E32, J22, J64

## 1. Introduction

The cyclicity of job search effort can be the key to understanding unemployment fluctuations. Suppose that the economy is expanding and job search effort is procyclical. Because workers are more intensively searching for jobs, vacancies are filled faster, implying a steeper fall in unemployment than would occur otherwise. On the contrary, if job search effort is countercyclical, it dampens the fluctuations in unemployment, preventing it from rising further in recessions.

The canonical matching model (e.g., [Pissarides, 2000](#), Chapter 5) predicts procyclical job search effort: because the returns to search, determined by the wage and job-finding probability, are procyclical, workers exert more search effort in good times than in bad times. Therefore, endogenizing job search effort in matching models increases volatilities, which overcomes the well-known [Shimer's \(2005\)](#) critique (e.g., [Gomme and Lkhagvasuren, 2015](#)). Yet, the empirical literature is far from agreeing that job search effort is procyclical; in fact, the bulk of the literature points against it. This poses the question: can a matching model simultaneously generate acyclical (or countercyclical) job search effort and highly volatile labour market variables? In this article, we show that the answer

is yes if wages are set a la [Hall and Milgrom \(2008\)](#)—but still flexible—and the value of leisure is procyclical.<sup>1</sup>

With the availability of new data sources on the search behaviour of unemployed workers, a growing empirical literature emerged trying to assess the cyclicity of job search effort in US data. [Shimer \(2004\)](#) uses the Current Population Survey (CPS) data and treats the number of job search methods in the CPS as a proxy for job search effort. He concludes that job search effort is countercyclical. Using data from the American Time Use Survey (ATUS), [DeLoach and Kurt \(2013\)](#) conclude that job search effort is acyclical due to two counteracting forces. The procyclical job-finding probability contributes to procyclical job search effort. But wealth effects contribute to countercyclical search effort, neutralizing the effect of the procyclical job-finding probability. [Tumen \(2014\)](#) corrects for composition effects in the estimates in Shimer and instead finds procyclical job search effort. [Gomme and Lkhagvasuren \(2015\)](#) merge the CPS with the ATUS data and document that the job search effort of short-term unemployed is procyclical; yet, they also document that the cyclicity of average search effort is statistically insignificant, suggesting that search effort is acyclical. [Mukoyama et al. \(2018\)](#) extend the analysis in Shimer by combining the CPS with the ATUS data. They document that job search effort is countercyclical and, in contrast with Tumen, their results hold after controlling for composition effects. [Leyva \(2018\)](#) uses the ATUS data and documents results similar to DeLoach and Kurt. In Leyva, two counteracting forces render search effort acyclical (or somewhat countercyclical): the effect of countercyclical value of a job offsets the effect of the procyclical job-finding probability. [Leduc and Liu \(2020\)](#) assume that job search effort is negatively related with the median duration of unemployment and, naturally, find procyclical job search effort. Importantly, most of these papers (all except [Shimer, 2004](#); [Tumen, 2014](#); [Leduc and Liu, 2020](#)) report that unemployed workers increased job search effort at the onset of the Great Recession, pointing to countercyclical job search effort.

Though indirectly, two other papers suggest that job search effort is countercyclical. [Faberman and Kudlyak \(2019\)](#) find that workers tend to search less in areas with tighter labour markets, which suggests that job search effort is countercyclical. [Hornstein and Kudlyak \(2017\)](#) conclude that job search effort is countercyclical if the elasticity of the matching function with respect to vacancies is in the range of one-third to one. This range covers most of the empirically plausible range that [Petrongolo and Pissarides \(2001\)](#) report for the elasticity of the matching function.

Although there is not yet a consensus, most of the empirical literature points against procyclical job search effort. But why do workers reduce job search effort when its returns are higher? A number of researchers suggest that it is because the value of leisure is procyclical (due to large income and wealth effects), which increases the cost of

1 Setting wages as in [Hall and Milgrom \(2008\)](#) typically introduces wage rigidity in matching models ([Hall and Milgrom, 2008](#); [Kehoe et al., 2020](#)). Furthermore, there are other forms to lower wage flexibility in matching models. For example, [Hall \(2005\)](#) proposes an extreme version with exogenous full wage rigidity. And, in a distinct approach, [Martin and Wang \(2018\)](#) propose a matching model in which workers' effort choice affects productivity and firms post wages to endogenously lower wage flexibility. Yet, as we detail below, we do not rely on wage rigidity to simultaneously obtain acyclical (or countercyclical) job search effort and volatile labour market variables. As the value of leisure is procyclical, wages set a la Hall and Milgrom become flexible in our setup.

forgone leisure when the returns to job search effort are higher.<sup>2</sup> Faberman and Kudlyak (2019) find that those with the lowest returns to search tend to exert more job search effort, suggesting that income effects dominate substitution effects in job search effort. Leyva (2018) argues that his evidence of countercyclical value of a job may result from income effects. DeLoach and Kurt (2013) document that the elasticity of job search effort with respect to wealth is negative. And Mukoyama *et al.* (2018) document that search effort is negatively correlated with several measures of aggregate wealth including housing and stock prices. Finally, in a goods-search context, Nevo and Wong (2019) report that, during the Great Recession, households increased certain shopping activities (e.g., purchases on sale and increased coupon usage rate) even though the estimated returns of those activities were lower. Given the significant reduction in income and wealth during the Great Recession, they relate income and wealth effects with the significant fall in the value of leisure.

In this article, we assess whether a matching model can simultaneously generate acyclical job search effort and highly volatile labour market variables if the value of leisure is procyclical. To this end, we set up two frameworks. We start with a simple framework in which the value of leisure is *exogenously* procyclical. Then, we *endogenize* the value of leisure.

In Section 2, we extend the canonical matching model by assuming that both working and job search effort deprive workers of their leisure and that the value of leisure is an exogenous function of productivity with a given elasticity. We show that the model cannot simultaneously generate acyclical (or countercyclical) job search effort and volatile labour market tightness. Two variables with opposite effects drive job search effort: labour market tightness (that determines substitution effects of leisure for expected income) and the value of leisure. In the canonical model, both variables have the same weight. Thus, given that labour market tightness is empirically much more volatile than the value of leisure (Pissarides, 2009; Chodorow-Reich and Karabarbounis, 2016), in a model that matches these empirical volatilities, volatile labour market tightness must coincide with procyclical job search effort. For this reason, we deviate from the core assumptions of the canonical matching model. In particular, we study standard variants of the matching model: the addition of fixed matching costs and alternating-offer wage bargaining (AOB; Hall and Milgrom, 2008; Pissarides, 2009; Christiano *et al.*, 2016; Ljungqvist and Sargent, 2017; Petrosky-Nadeau and Zhang, 2021).

We find that adding fixed matching costs (as in Pissarides, 2009) lowers the relative weight of labour market tightness in the dynamics of job search effort. But, unless we use an unrealistic calibration, this change alone is not enough for job search effort to be acyclical and labour market tightness to be highly procyclical as in data. To achieve that, we find that we must replace Nash bargaining with alternating-offer wage bargaining (henceforth, AOB) as in Hall and Milgrom (2008). This change also lowers the (relative) weight of labour market tightness in the dynamics of job search effort. And, under an extensive set of

- 2 Both wealth and income effects give rise to procyclical value of leisure if leisure is a normal good. Wealth effects give rise to procyclical value of leisure because housing, stocks, and other forms of wealth are procyclical; therefore, workers may feel richer and prefer to enjoy more leisure. Income effects can also give rise to procyclical value of leisure because of consumption pooling within households: for example, because wages and employment are procyclical, unemployed members in the household enjoy more consumption and, thus, also want more leisure.

calibrations, the variant with AOB generates very elastic labour market tightness (in most cases, much more than in data) and acyclical job search effort (especially if the value of leisure is highly elastic). Importantly, even though AOB directly lowers the elasticity of wages, our results do not counterfactually rely on rigid wages for new hires (Pissarides, 2009; Haefke *et al.*, 2013) because the elasticity of the value of leisure increases that of wages. In sum, a matching model with AOB overcomes Shimer's (2005) critique without depending on rigid wages and procyclical job search effort.

In Section 3, we readapt the model to offer microfoundations for the procyclicality of the value of leisure and, more importantly, to fully account for the general equilibrium effects. To this end, we make two main assumptions. First, we assume that workers share consumption within a large and representative household (e.g., Merz, 1995; Pissarides, 2000, Chapter 3.4; Hall and Milgrom, 2008; Chodorow-Reich and Karabarbounis, 2016; Christiano *et al.*, 2016; Leduc and Liu, 2020; Petrosky-Nadeau and Zhang, 2021). This conveniently allows us to study the role of income effects in the model: as income tends to increase in expansions, all workers, irrespective of their employment status, enjoy more consumption. Then, as leisure is a normal good, its value becomes endogenously procyclical and unemployed workers have less incentives to search for jobs. Second, we assume a constant relative risk aversion utility specification, which allows us to influence the elasticity of the value of leisure by controlling the degree of risk aversion.

Endogenizing the value of leisure reduces the volatility in the model because of general equilibrium feedback between labour market tightness and the value of leisure. But endogenizing the value of leisure improves the dynamics of the matching model with AOB. Under our benchmark calibration and a large set of other calibrations, the matching model generates elastic labour market variables, endogenously procyclical value of leisure, and endogenously acyclical (slightly countercyclical) job search effort without relying on real wage rigidity. Furthermore, the respective elasticities are close to their empirical counterparts.

Mukoyama *et al.* (2018) also attempt to combine elastic labour market tightness and acyclical (or countercyclical) job search effort. But our objectives and approaches differ substantially. They build a model with a generalized matching function including a degree of substitution between search effort and labour market tightness. Then, they set the degree of substitution *exogenously* to obtain countercyclical search effort. This allows them to show that countercyclical search effort reduces volatilities in matching models. Instead, we employ an ordinary matching function (in which search effort and labour market tightness are complements) and obtain acyclical or countercyclical search effort as an *endogenous* implication of standard changes to the matching model and elastic value of leisure.

## 2. Matching models embedding an exogenous value of leisure

In this section, we do a steady-state analysis of three variants of matching models. Our objective is to assess whether a model with matching frictions can simultaneously satisfy a number of goals. First, the model when hit by exogenous productivity shocks must generate highly volatile labour market tightness without relying on procyclical job search effort (e.g., Shimer, 2005; Mukoyama *et al.*, 2018). Second, the model must not satisfy these goals by means of real wage rigidity: to be consistent with the evidence in Haefke *et al.* (2013), wages of new hires must move about one-to-one with productivity (see also the survey by Pissarides, 2009).

The canonical matching model predicts procyclical job search effort because returns to search effort are procyclical. But this prediction contrasts with the findings in most of the empirical literature that point against procyclical search effort. We conjecture that introducing a procyclical value of leisure into matching models may reverse their prediction of procyclical value of leisure: if searching for jobs reduces leisure and leisure is more valuable when the returns to search are higher (and vice versa), then job search effort may become acyclical or countercyclical. To test this, we simply assume that the value of leisure is an exogenous function of productivity as in [Chodorow-Reich and Karabarbounis \(2016, Section 7\)](#).<sup>3</sup> Then, we check if the conditions that render acyclical or countercyclical search effort are in line with the conditions that satisfy all our other goals set above, particularly volatile labour market tightness and non-rigid real wages.

In this section, we test the implications of procyclical value of leisure in three variants of matching models. The first variant we analyse is the canonical model. In this variant, despite the negative effect of the exogenous value of leisure, search effort remains highly procyclical if the model matches the empirical volatilities of labour market tightness and the value of leisure. To this end, we add a second variant with fixed matching costs as in [Pissarides \(2009\)](#). Though this modification helps, alone it is not enough for the model to generate volatile labour market tightness jointly with acyclical job search effort. Then, in a third variant, we replace Nash bargaining with AOB as in [Hall and Milgrom \(2008\)](#). The relevance of the AOB variant is more than quantitative: it allows for volatile labour market tightness and acyclical job search effort, simultaneously.

## 2.1 The canonical model

Before we delve into more complex setups, it is useful to review the basics and start with the canonical matching model extended with endogenous job search effort ([Pissarides, 2000](#), Chapter 5) set in discrete time (as in, e.g., [Ljungqvist and Sargent, 2017](#)).

**2.1.1 Matching function** In all the models that we consider, workers and firms meet subject to matching frictions. A constant returns to scale matching function determines the number of matches in the economy,  $m$ . This matching function has an elasticity of  $0 < \eta < 1$  with respect to vacancies,  $v$ , and an elasticity of  $1 - \eta$  with respect to the unemployment rate,  $u$ , and the average job search effort in the economy,  $\bar{e}$ . These assumptions imply that  $\bar{e}f(\bar{e}, \theta) = \theta\mu(\bar{e}, \theta)$ , where  $\theta \equiv \frac{v}{u}$  is the labour market tightness,  $f(\bar{e}, \theta)$  is the job-finding probability per unit of search effort, and  $\mu(\bar{e}, \theta)$  is the vacancy-filling probability.

**2.1.2 Firms** Each firm employs at most one worker. To match with a worker, a firm must open a vacancy and pay a cost  $\kappa$  per period while waiting for a match. Once matched, the firm waits one period to start production. Then, the firm produces  $z$  units of output and pays the wage  $w$  to the worker every period until the match is exogenously destroyed, which occurs with probability  $\delta$  per period. Using  $J$  to denote the value of a matched firm and  $V$  to denote the value of an unmatched firm, we write the value functions as

$$J = z - w + \beta[\delta V + (1 - \delta)J], \quad (1)$$

3 This distinguishes the models in this section from those in Section 3. In that section, we assume a large household and curvature in the utility of consumption to deliver a countercyclical marginal value of consumption and, thus, an endogenously procyclical value of leisure.

$$V = -\kappa + \beta[(1 - \mu(\bar{e}, \theta))V + \mu(\bar{e}, \theta)J], \quad (2)$$

where we use  $\beta$  to denote the discount factor. As standard in the matching frictions literature, we assume that a free-entry condition holds,  $V = 0$ , implying

$$J = \frac{\kappa}{\beta\mu(\bar{e}, \theta)}. \quad (3)$$

**2.1.3 Workers** When unemployed, a worker receives unemployment benefits and enjoys leisure. We assume that unemployment benefits are denominated in goods units while leisure is denominated in time units. This difference in units requires us to employ a relative price (of leisure),  $l(z)$ , which we refer to as the value of leisure. If the value of leisure is constant, then this model is similar to [Pissarides \(2000, Chapter 5\)](#); but, we will mostly work with a procyclical value of leisure, in line with the evidence in [Chodorow-Reich and Karabarbounis \(2016\)](#).

The leisure enjoyed by an unemployed worker is the difference  $\chi - \psi e^\zeta / \zeta$ , where  $e$  is search effort of the worker,  $\zeta > 1$  measures the sensitivity of leisure to search effort, and  $\chi > 0$  and  $\psi > 0$  are both scale parameters.

Using  $W$  and  $U$  to denote the value from employment and unemployment, we write the value functions as

$$W = w + \beta[\delta U + (1 - \delta)W], \quad (4)$$

$$U = b + \left(\chi - \frac{\psi}{\zeta} e^\zeta\right)l(z) + \beta[(1 - ef(\bar{e}, \theta))U + ef(\bar{e}, \theta)W], \quad (5)$$

where  $b$  is the unemployment benefits. Taking average job search effort as given, each worker optimally chooses search effort according to

$$\psi e^{\zeta-1}l(z) = \beta f(\bar{e}, \theta)(W - U). \quad (6)$$

That is, in equilibrium, the marginal cost of search effort equals the *net* present discounted value from employment.

**2.1.4 Wage bargaining** The standard assumption is to assume that workers and firms split the match surplus according to Nash bargaining: after matching, the worker and the firm bargain over wages such that the bargained wage maximizes the Nash product,

$$w = \operatorname{argmax}(W - U)^\phi J^{1-\phi}, \quad (7)$$

where the parameter  $0 < \phi < 1$  measures the worker's bargaining power. This maximization problem implies

$$W - U = \frac{\phi}{1 - \phi}J, \quad (8)$$

which after a few derivations and using the equations above implies the wage equation:

$$w = \phi(z + e\theta\kappa/\bar{e}) + (1 - \phi)(b + (\chi - \psi e^\zeta/\zeta)l(z)). \quad (9)$$

**2.1.5 Elasticity of job search effort** In equilibrium, because all workers are equal, individual search effort,  $e$ , equals average search effort,  $\bar{e}$ . Using this equilibrium condition,

together with the relationship between  $f(\bar{v}, \theta)$  and  $\mu(\bar{v}, \theta)$  and Equations (3), (6), and (8) implies

$$\psi e^{\zeta} = \left( \frac{\phi \kappa}{1 - \phi} \right) \frac{\theta}{l(z)}. \tag{10}$$

Converting this equation into elasticities yields

$$\zeta \varepsilon_e = \varepsilon_\theta - \varepsilon_l, \tag{11}$$

where  $\varepsilon_x := d \log x / d \log z$  denotes the elasticity of the variable  $x$  with respect to productivity,  $z$ . Thus Equation (11) reads that the elasticity of job search effort is proportional to the difference between the elasticity of labour market tightness and the elasticity of the value of leisure. This proportionality has important upshots. The first is well known: if the value of leisure is constant,  $\varepsilon_l = 0$ , job search effort must be procyclical (e.g., Gomme and Lkhagvasuren, 2015). The intuition is simple: in times of high productivity, the returns to search effort are higher because of the direct effect of productivity on wages [see Equation (9)] and because firms open more vacancies, which tightens the labour market, increasing both wages and the job-finding probability per unit of search effort,  $f(\bar{v}, \theta)$ . Secondly, when  $\zeta \rightarrow \infty$ , the elasticity of job search effort  $\varepsilon_e$  approaches zero. This is because a large  $\zeta$  induces a large drop in leisure in response to elevated effort.

Furthermore, Equation (11) reveals that a procyclical value of leisure,  $\varepsilon_l > 0$ , reduces the elasticity of job search effort. During times of high productivity, the positive value of leisure makes searching for jobs more costly and, thereby, counteracts the higher returns to search effort. And, most importantly in the context of the article, Equation (11) entails that the standard matching model cannot generate acyclical job search effort unless the model fails its target for  $\varepsilon_\theta$ . This is because in data,  $\varepsilon_\theta \approx 7.56$  (Pissarides, 2009) and  $\varepsilon_l \approx 1.13$  (Chodorow-Reich and Karabarbounis, 2016), implying  $\varepsilon_e \approx 6.43/\zeta$ .<sup>4</sup> Thus, to attain the objectives we set up in the introduction, we must deviate from some of the core assumptions of the standard model.

## 2.2 Fixed matching costs

Following Pissarides (2009), in this variant, a firm’s hiring cost is the sum of a fixed component,  $H$ , paid after matching (and before bargaining) and a per-period component,  $\kappa$ , paid till the vacancy is filled. The value of an unmatched firm,  $V$ , becomes then

$$V = -\kappa - \mu(\bar{v}, \theta)H + \beta[(1 - \mu(\bar{v}, \theta))V + \mu(\bar{v}, \theta)J], \tag{12}$$

which changes the free-entry condition ( $V = 0$ ) to

$$J = \frac{\kappa}{\beta \mu(\bar{v}, \theta)} + \frac{H}{\beta}. \tag{13}$$

4 Chodorow-Reich and Karabarbounis (2016) do not report the elasticity of the forgone value of non-working time expressed in units of consumption, which in our notation refers to  $\chi/l(z)$ . Yet, they estimate four time series for  $\chi/l(z)$ , depending on the underlying utility function. Of those four time series we choose those for Cobb–Douglas utility and no fixed costs of working since in this case the estimated average  $\chi/l(z)$  (0.7) is very close to typical calibrations in the matching literature after Hall and Milgrom (2008). Following the steps in Chodorow-Reich and Karabarbounis to transform the variables and compute elasticities and assuming that  $\chi$  is constant, we find that  $\varepsilon_l = 1.13$  with 95% confidence interval of [0.9, 1.37].

The new free-entry condition exposes that if total hiring costs are kept the same, adding the fixed cost component reduces the effect of labour market tightness on job creation. Thus, *ceteris paribus*, after a positive productivity shock, more firms open vacancies the higher is the weight on  $H$ .

**2.2.1 Elasticity of job search effort** The change in the free-entry condition implies that we must rewrite the equation for job search effort in equilibrium:

$$\psi e^{\tilde{\zeta}} = \frac{\phi}{1 - \phi} \left( \kappa + \mu(e, \theta)H \right) \frac{\theta}{l(z)}. \quad (14)$$

Converting this equation into elasticities yields

$$\left( \zeta - \frac{H\mu(e, \theta)(1 - \eta)}{\kappa + H\mu(e, \theta)} \right) \varepsilon_e = \left( 1 - \frac{H\mu(e, \theta)(1 - \eta)}{\kappa + H\mu(e, \theta)} \right) \varepsilon_\theta - \varepsilon_l, \quad (15)$$

For  $H = 0$ , Equation (15) reduces to Equation (11) ( $\zeta \varepsilon_e = \varepsilon_\theta - \varepsilon_l$ ). For the other extreme,  $\kappa = 0$ , Equation (15) becomes  $(\zeta + \eta - 1)\varepsilon_e = \eta\varepsilon_\theta - \varepsilon_l$ . Thus, the relative effect of labour market tightness on search effort, in terms of elasticity values, depends on the weight of fixed matching costs,  $H$ , in total hiring costs,  $\kappa + \mu(\bar{e}, \theta)H$ . The interval for this effect is  $[\eta, 1]$ . The interim conclusion is that lower values of  $\kappa$  and  $\eta$  reduce, *ceteris paribus*, the cyclicity of job search effort. Yet, for given empirical values of  $\varepsilon_\theta$  and  $\varepsilon_l$ , the value of  $\eta$  required to generate countercyclical job search effort ( $\varepsilon_e < 0$ ) in the case of  $\kappa = 0$  is excessively low compared to values typically calibrated (0.15 versus 0.3–0.5) following the survey by Petrongolo and Pissarides (2001) of the empirical estimates of the elasticity of the job-finding probability with respect to labour market tightness.<sup>5</sup>

### 2.3 AOB

Following Hall and Milgrom (2008), in this variant, a matched firm and worker bargain over wages by taking turns in making wage offers. Thus, the threat point is not to break the match and receive outside values as in Nash bargaining ( $U$  and  $V$  in the worker's and firm's case, respectively) and is instead the value of making a counter-offer. If the two parties fail to agree on a wage during a bargaining round, the firm incurs a cost of delay,  $\alpha > 0$ , while the worker receives unemployment benefits and enjoys leisure without searching for jobs,  $b + \gamma l(z)$ . Furthermore, there is a probability  $s$  that the match exogenously breaks after each bargaining round and we assume that the first wage offer is made by the firm, denoted by  $w^f$ .

- 5 In a model with exogenous job search effort,  $\eta$  is the direct counterpart of the empirical estimates of the elasticity of job finding probability with respect to labour market tightness. This does not hold in a model with endogenous search effort because search effort is a function of labour market tightness. To find the direct counterpart of those estimates, one must, thus, calculate the elasticity of the job-finding probability,  $ef(e, \theta)$ , with respect to  $\theta$ , to identify reasonable values of  $\eta$ . Yet, we do not pursue this alternative because in a model with endogenous value of leisure as developed in Section 3,  $\eta$  does not determine alone the elasticity of  $ef(e, \theta)$  with respect to  $\theta$ ; it also depends on the parameters governing the response of  $l(z)$  to shocks. Furthermore, it would be counterproductive to assume a low  $\eta$  in Section 3 as it would excessively reduce the elasticity of  $ef(e, \theta)$  relative to the empirical counterparts as we increase the relative risk aversion coefficient of the representative household,  $\gamma$ .

Then a worker is indifferent between accepting  $w^f$  and making an alternative offer  $w^w$  if

$$W(w^f) = b + \chi l(z) + \beta[(1 - s)W(w^w) + sU]. \tag{16}$$

In the last expression, the left-hand side is the value of employment receiving the offered wage  $w^f$  and the right-hand side is the value of making the counter-offer  $w^w$ .

Similarly, a firm is indifferent between accepting the worker's offer  $w^w$  and making a new offer of  $w^f$  if

$$J(w^w) = -\alpha + \beta(1 - s)J(w^f). \tag{17}$$

The left-hand side is the value of the firm paying the wage  $w^w$  and the right-hand side is the value of making the counter-offer. Because the free entry condition,  $V = 0$ , still holds, the term  $\beta sV$  is dropped from the right-hand side of Equation (17).

In equilibrium, the firm's initial offer is such that the worker is indifferent between accepting it and making a counter-offer. In such a case, we assume that the worker accepts the firm's offer. Therefore, in equilibrium,  $\alpha$  is not paid and the wage equals  $w^f$ . To simplify matters in this section, we follow Ljungqvist and Sargent (2017) and assume that  $s = \delta$ . Then the wage is

$$w = w^f = \frac{b + \chi l(z) + \beta(1 - \delta)(z + \alpha)}{1 + \beta(1 - \delta)}. \tag{18}$$

Interestingly, if  $s = \delta$ , the wage is unresponsive to labour market tightness.<sup>6</sup>

**2.3.1 Elasticities** After some elementary algebra, we obtain the following equations describing the behaviour of job search effort and labour market tightness:

$$\psi e^{\zeta} l(z) = \frac{\beta(1 - \delta)(z + \alpha - b - \chi l(z))\mu(e, \theta)\theta}{(r + (1 - \zeta^{-1})\mu(e, \theta)\theta)(1 + \beta(1 - \delta))}, \tag{19}$$

$$\kappa/\mu(e, \theta) + H = \frac{z - b - \chi l(z) - \beta(1 - \delta)\alpha}{r(1 + \beta(1 - \delta))}, \tag{20}$$

where  $r = 1/\beta + \delta$ . Differentiating these two last equations yields

$$(\zeta - (1 - \eta)(\Psi_1 - \Psi_2))\varepsilon_e = (\eta\Psi_1 + (1 - \eta)\Psi_2)\varepsilon_\theta - \varepsilon_l, \tag{21}$$

where  $\Psi_1 := \frac{r\psi e^{\zeta} l(1 + \beta(1 - \delta))}{\beta(1 - \delta)\theta}$  and  $\Psi_2 := \frac{r^2 \kappa(1 + \beta(1 - \delta))}{\Psi_1}$ . Thus, if  $\varepsilon_\theta$  and  $\varepsilon_l$  are as in data,  $\eta\Psi_1 + (1 - \eta)\Psi_2$  must be approximately 1/8; by calibrating the model, we find that under various calibrations of the model, that target is attainable.

We also obtain the elasticity of labour market tightness:

$$\frac{(\zeta - 1)(\mu(\bar{e}, \theta)\theta + r)}{\mu(\bar{e}, \theta)}\varepsilon_\theta = \left(\frac{r}{\Psi_1} + \Psi_3\right)z - \left(\frac{r}{\Psi_1}(z + \alpha - b) + \Psi_3\chi l\right)\varepsilon_l \tag{22}$$

where  $\Psi_3 := \frac{(\zeta - 1)\mu(\bar{e}, \theta)\theta + (\zeta + \eta - 1)r}{(1 + \beta(1 - \delta))r\kappa(1 - \eta)}$ .

Because the coefficients in Equations (21) and (22) are relatively complex functions of the set of parameters, we proceed our analysis with numerical methods.

**2.3.2 Calibration** We calibrate the model to monthly US data and summarize our benchmark calibration in Table 1. We normalize  $z = 1$  and  $l(z) = 1$ . We target an annual

6 In Section 3, we study how  $s \neq \delta$  affects our results.

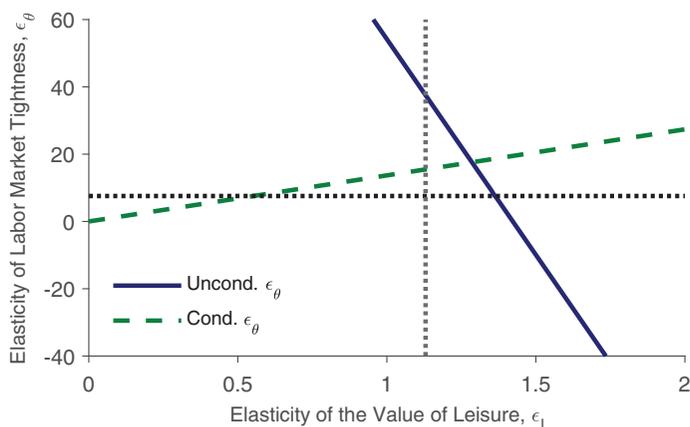
**Table 1.** Benchmark calibration.

Discount factor	$\beta = 0.996$
Rate of job destruction	$\delta = 0.036$
Leisure scalar	$\chi = 0.7$
Convexity of search effort disutility	$\zeta = 2$
Productivity	$z = 1$
Value of leisure	$l(z) = 1$
Matching function elasticity	$\eta = 0.5$
Variable hiring costs	$\kappa = 0.129w^f$
Fixed hiring costs	$H = 1.65w^f$
Unemployment benefits	$b = 0.06$

discount rate of 4.91%, implying that  $\beta = 0.996$ . Drawing on [Shimer's \(2012\)](#) measurement, we set the employment exit probability,  $\delta$ , to 3.6%. Based on the estimates in [Chodorow-Reich and Karabarbounis \(2016\)](#), we set  $b = 0.06$ .<sup>7</sup> We also follow [Chodorow-Reich and Karabarbounis \(2016\)](#) to set  $\chi$ . Their estimates range from 0.41 to 0.9, depending on the utility function that they assume. Among their estimates, we choose  $\chi = 0.7$  as our benchmark, which is very close to the typical calibration in the search and matching literature following [Hall and Milgrom \(2008\)](#). We also choose a quadratic disutility of search effort,  $\zeta = 2$ , which is consistent with the empirical literature (see e.g., [Gomme and Lkhagvasuren, 2015](#) and references therein). Following [Silva and Toledo \(2009, 2013\)](#), we assume that  $H$  corresponds to training costs and calibrate hiring costs as  $\kappa = 0.129w^f$  and  $H = 1.65w^f$ . This calibration implies that the bulk of hiring costs are fixed, which agrees with the estimates in [Christiano et al. \(2016\)](#). The matching function is Cobb–Douglas and takes the form  $m_0(eu)^{1-\eta}v^\eta$ , where  $m_0$  denotes matching efficiency. In line with the search and matching literature (in particular the evidence in [Petrongolo and Pissarides, 2001](#)), we set  $\eta = 0.5$ . Finally, we use  $m_0$  together with  $\alpha$  and  $\psi$  to impose that the (i) unemployment rate,  $u$ , equals 5.7%, (ii) job search effort,  $e$ , is normalized to 1 and (iii) and labour market tightness,  $\theta$ , equals 0.72 ([Pissarides, 2009](#)). This calibration strategy agrees with Hall and Milgrom, who also use  $m_0$  and  $\alpha$  as free parameters.

**2.3.3 Results** We want to test whether the AOB variant of the model with the exogenous value of leisure can generate a volatile labour market tightness jointly with acyclical or countercyclical search effort. Thus the two key variables are  $\varepsilon_\theta$  and  $\varepsilon_e$  while  $\varepsilon_l$  is exogenous. We first set  $\varepsilon_e = 0$  in [Equation \(21\)](#) and get  $\varepsilon_\theta$  as a function of  $\varepsilon_l$ . This gives us the elasticity of labour market tightness conditional on acyclical job search effort. The result is depicted by the green dashed line in [Fig. 1](#). The conditional elasticity of labour market tightness is upward sloping because labour market tightness and the value of leisure have opposing effects on the response of job search effort. For search effort to be acyclical, labour market tightness must be more procyclical when the value of leisure is more procyclical. By the same token, a point below (above) the green dashed line implies that job search effort is countercyclical (procyclical) at that point.

7 [Chodorow-Reich and Karabarbounis \(2016\)](#) obtain  $b = 0.06$  by considering that unemployment benefits amount to 30% of after-tax marginal product, that only 40% of unemployed workers actually receive benefits, and by accounting for take-up costs and other adjustments.



**Fig. 1.** Exogenous  $\varepsilon_l$ —baseline results.

*Note:* The figure depicts the baseline results in the variant with exogenous value of leisure and AOB. The green dashed line plots  $\varepsilon_\theta$  conditional on  $\varepsilon_\theta = 0$  (Equation 21). The blue solid line plots  $\varepsilon_\theta$  as implied by Equation (22). The dotted lines show the empirical counterparts of  $\varepsilon_l$  and  $\varepsilon_\theta$  (1.13 and 7.56, respectively).

The *unconditional* elasticity of labour market tightness, governed by Equation (22), is depicted by the blue solid line in Fig. 1. This line is downward sloping for two reasons. First, an increase in  $\varepsilon_l$  reduces the equilibrium response of job search effort, which increases hiring congestion externalities. Secondly, an increase in  $\varepsilon_l$  makes the wage more responsive to productivity (see Equation 18). Both work to reduce incentives to hire, implying a less elastic labour market tightness.

The empirical counterparts of  $\varepsilon_l$  and  $\varepsilon_\theta$  (1.13 and 7.56, respectively) are depicted by the two dotted lines in Fig. 1. If the value of leisure is unresponsive to productivity (point zero on the horizontal axis), the elasticity of labour market tightness far overshoots its empirical counterpart, and job search effort is procyclical (as the blue line is above the green line). If the value of leisure responds to productivity as in the data,  $\varepsilon_l \approx 1.13$ , then labour market tightness is still much more volatile than its empirical counterpart and continues to imply procyclical job search effort. But if the elasticity of the value of leisure is closer to its upper bound, Fig. 1 and Table 2 (right panel) show that acyclical and countercyclical job search effort coincide with an elasticity of labour market tightness that is very close to its empirical counterpart; In sum, we obtain two results. First, acyclical and countercyclical job search effort can coincide with volatile labour markets. Secondly, if the matching model with exogenous value of leisure and AOB has a volatility problem, it seems of excessive volatility.

This second result may look surprising in light of the unemployment volatility puzzle (Shimer, 2005) and the analysis in Chodorow-Reich and Karabarbounis (2016). Chodorow-Reich and Karabarbounis show that a procyclical opportunity cost of employment starkly reduces volatilities in models with matching frictions, including the model with AOB calibrated as in Hall and Milgrom (2008). Yet, Hall and Milgrom calibrate their model such that the unemployment volatility matches its empirical counterpart. Thus, given that a procyclical opportunity cost of employment reduces volatilities, volatilities in the model must be lower than in data. This result, however, does not exclude the possibility that a matching model with AOB calibrated differently might generate highly volatile

**Table 2.** Exogenous value of leisure

	$\varepsilon_l = 1.13$			$\varepsilon_l = 0$			$\varepsilon_l = 1.37$		
	$\varepsilon_\theta$	$\varepsilon_e$	$\varepsilon_w$	$\varepsilon_\theta$	$\varepsilon_e$	$\varepsilon_w$	$\varepsilon_\theta$	$\varepsilon_e$	$\varepsilon_w$
US data	7.56	$\leq 0$	1	7.56	$\leq 0$	1	7.56	$\leq 0$	1
Benchmark	37.51	0.82	0.96	182.35	6.82	0.53	6.75	-0.45	1.05
$\kappa = 0.4w^f$	12.07	0.06	0.97	60.65	3.18	0.53	1.76	-0.60	1.06
$\chi/l(z) = 0.41$	96.76	2.65	0.78	181.40	6.05	0.53	78.78	1.92	0.83
$\zeta = 4$	36.89	0.23	0.96	177.88	2.46	0.53	6.94	-0.25	1.05
$b = 0.15$	37.85	0.91	0.96	183.99	7.25	0.53	6.82	-0.43	1.05
$H = 0$	35.25	0.74	0.90	171.51	6.40	0.49	6.30	-0.47	0.99

Note: This table reports the elasticities of labour market tightness,  $\varepsilon_\theta$ , job search effort,  $\varepsilon_e$ , and wages,  $\varepsilon_w$  generated by the matching model with AOB and exogenous elasticity of the value of leisure,  $\varepsilon_l$ , under several calibrations. Each panel contrasts the results of the model for three values of  $\varepsilon_l$  with the empirical counterparts. The empirical counterpart of  $\varepsilon_l$  is 1.13 (left panel).  $\varepsilon_l = 1.37$  (right panel) is the upper bound.

labour market tightness even if the value of leisure is procyclical. In fact, our benchmark calibration shows that it can.

We find that there are three key parameters affecting volatility of labour market tightness in a matching model with AOB:  $\kappa$ ,  $\chi$ , and  $s$ . In particular, increasing any of these parameters reduces volatility in the model. The impact of the first two parameters is presented in Table 2, while the impact of  $s$  is studied in Section 3. Table 2 shows that if  $\kappa$  is about three times larger than in the benchmark (close to the value assumed in Pissarides, 2009 but much higher than in Hall and Milgrom, 2008 and Hagedorn and Manovskii, 2008), then the model delivers acyclical job search effort and  $\varepsilon_\theta$  is much closer to its empirical counterpart when  $\varepsilon_l = 1.13$ . This results from the higher elasticity of hiring costs to labour market tightness: hiring costs fluctuate more with  $\theta$  when  $\kappa$  is higher, which discourages opening vacancies in good times, lowering the volatility in the model.

Table 2 also shows that if  $\chi$  is very low (equal to the lowest estimate in Chodorow-Reich and Karabarbounis, 2016), then volatilities are greatly magnified. A low  $\chi$  implies that the wage becomes less sensitive to changes in the value of leisure (see Equation 18). As wages are less elastic, firms open more vacancies when productivity rises even if  $\varepsilon_l$  is high.<sup>8</sup>

Relative to benchmark, different values of  $\zeta$ ,  $b$ , and  $H$  do not markedly change volatilities in the model, irrespective of  $\varepsilon_l$ . Increasing  $\zeta$  essentially lowers the elasticity of job search effort, contributing to acyclical job search effort. Reducing  $b$  and  $H$  tend to lower the volatility in the model but not significantly. This result stems from our calibration strategy. As we assume that  $\alpha$  is a free parameter targeting hiring costs to satisfy Equation (20) in steady-state, changes in  $H$  and  $b$  are essentially accommodated by changes in  $\alpha$ . Put differently, different values of  $H$  and  $b$  essentially redistribute the fixed components in

8 In the scenarios in which Chodorow-Reich and Karabarbounis (2016) estimate that  $\chi/l(z) = 0.41$ , the implied average  $\varepsilon_l$  can be as high as 1.6. Yet, even in the region of 1.6, the volatility of labour market tightness far exceeds its empirical counterpart.

Equation (20) and barely affect the components that change with the cycle.<sup>9</sup> Yet, the restriction that  $\alpha > 0$  might be compromised if  $H$  and  $b$  are large.

In sum, this section generalizes the neat analysis of [Ljungqvist and Sargent \(2017\)](#). We find that, even if the value of leisure is procyclical, the unemployment volatility puzzle ceases to exist if Nash bargaining is replaced with AOB and the model is calibrated in a standard manner. Importantly, even though AOB directly lowers wage flexibility, the matching model does not rely on rigid wages to increase volatilities, agreeing with the evidence in [Pissarides \(2009\)](#) and [Haefke et al. \(2013\)](#). Most importantly, we find that under a standard calibration of the model, it is possible to simultaneously obtain *highly elastic labour market tightness, acyclical or countercyclical job search effort, elastic value of leisure, and elastic wages*. The combination of these four factors is, to the best of our knowledge, new to the literature.

### 3. Model with endogenous value of leisure

In the previous section, we study the implications of an exogenously procyclical value of leisure,  $l(z)$ , for the elasticity of job search effort and labour market tightness. Although this assumption renders insightful results, its abstraction from the ways in which  $l(z)$  varies with other endogenous variables can have quantitative importance. For example, if  $1/l(z)$  is (interpreted as) the marginal utility of consumption, then a tighter labour market should lead, *ceteris paribus*, to a higher increase in  $l(z)$  than otherwise because workers enjoy higher wages and job-finding probabilities. This, in turn, might attenuate the excessive volatility of labour market tightness that we found in the previous section, bringing the model closer to data.

In this section, we endogenize the value of leisure by assuming that a large household pools consumption among its members as in [Merz \(1995\)](#). If the household's utility function is concave in consumption and leisure is a normal good, then workers would like to enjoy more leisure in times of high consumption; there is an income effect. As a result, the procyclicality of labour income implies procyclical value of leisure. And, depending on the curvature of the utility function, a procyclical value of leisure may imply acyclical (or countercyclical) job search effort.

We do not claim that consumption pooling is a realistic assumption. Yet, it provides enormous convenience and simplification for various purposes ([Pissarides, 2000](#), Chapter 3; [Hall and Milgrom, 2008](#); [Chodorow-Reich and Karabarbounis, 2016](#); [Christiano et al., 2016](#); [Leduc and Liu, 2020](#); [Petrosky-Nadeau and Zhang, 2021](#)). In the current setup, this assumption allows us to transparently answer the following question: can a model with an endogenous value of leisure generate highly volatile labour market variables and acyclical job search effort?

9 To see this more clearly, we rearrange [Equation \(20\)](#):

$$\frac{z - \chi l(z)}{r(1 + \beta(1 - \delta))} - \kappa/\mu(\bar{e}, \theta) = H + \frac{b + \beta(1 - \delta)\alpha}{r(1 + \beta(1 - \delta))},$$

### 3.1 Representative household

The representative household derives utility from consumption and disutility from exerting effort in job search and from working in firms. The household chooses consumption,  $c_t$ , to maximize its expected utility,

$$E_t \sum_{j=t}^{\infty} \beta^{j-t} \left( \frac{c_j^{1-\gamma} - 1}{1-\gamma} - \frac{\psi}{\zeta} e_j^{\zeta} u_j - \chi n_j \right),$$

subject to its budget constraint,

$$c_t \leq w_t n_t + d_t,$$

where  $E_t$  denotes the expectation operator conditional on time  $t$  information set,  $n_t = 1 - u_t$  is the employment rate, and  $d_t$  is the profits from owning the firms. The parameter  $\gamma \geq 0$  is the coefficient of relative risk aversion and measures the curvature of the utility of consumption and, thus, of the value of leisure:

$$l_t(z) = c_t^{\gamma}. \quad (23)$$

The optimal choice of effort is made by each worker according to [Equation \(6\)](#).<sup>10</sup> The employment rate evolves according to

$$n_{t+1} = (1 - \delta)n_t + m_t. \quad (24)$$

### 3.2 Other building blocks

The rest of the model is as specified in Section 2. We focus on the AOB variant (Section 2.3) because it proved to be apt for generating volatile labour market tightness and acyclical job search effort. The current setup requires a few modifications. First, we add time subscripts to the variables. Secondly, we replaced  $t + 1$  variables by their expected value when appropriate. Thirdly, we treat labour productivity as a variable following a first-order autoregressive process. Fourthly, we replace the constant discount factor  $\beta$  with the effective discount factor:

$$\Lambda_{t,t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma}. \quad (25)$$

Finally, we add the resource constraint to close the model,

$$y_t = c_t + v_t(\kappa + \mu(e_t, \theta_t)H), \quad (26)$$

in which output,  $y_t$ , is split between consumption and total hiring costs.<sup>11</sup>

### 3.3 Results

We study shocks to aggregate productivity in the log-linear version of the model, assuming an autocorrelation coefficient of 0.98. As our benchmark we continue to calibrate the

10 Given that we assume that the utility of consumption and disutility of job search effort are additively separable, the choice of the representative household and of each worker delivers the same optimal job search effort.

11 The [Supplementary Appendix](#) presents the dynamic system of equations characterizing the equilibrium.

**Table 3.** Endogenous value of leisure

	$\varepsilon_\theta$	$\varepsilon_e$	$\varepsilon_w$	$\varepsilon_l$
US data	7.56	$\leq 0$	1	1.13
Benchmark	9.65	-0.18	0.96	1.16
$\kappa = 0.4w^f$	7.09	-0.08	0.95	1.09
$\chi/l(z) = 0.41$	19.35	-0.06	0.86	1.24
$\zeta = 4$	9.90	-0.14	0.96	1.16
$b = 0.15$	10.12	-0.05	0.96	1.16
$H = 0$	10.10	0.11	0.91	1.19
$s = 0.1$	7.01	-0.04	1.00	1.13
$\gamma = 1.1$	7.26	-0.34	1.00	1.24

Note: This table reports the elasticities of labour market tightness,  $\varepsilon_\theta$ , job search effort,  $\varepsilon_e$ , wages,  $\varepsilon_w$ , and value of leisure,  $\varepsilon_l$ , generated by the matching model with AOB and endogenous  $\varepsilon_l$ , under several calibrations.

model according to Table 1 (except for  $l(z)$ ) and to target the steady-state unemployment rate, job search effort, and labour market tightness as detailed in Section 2.3. But, differently from Section 2.3, we no longer assume that  $s = \delta$  for two reasons. First, as argued by Hall and Milgrom (2008), it seems more likely that  $s \geq \delta$  as the conditions during bargaining should be more fragile than during the employment relationship. Secondly, using Bayesian methods to estimate a New Keynesian model with matching frictions and AOB, Christiano *et al.* (2016) find that  $s$  is slightly larger than  $\delta$ . Therefore, we choose  $s = 0.05$ . Finally, we assume that  $\gamma = 1$  to obtain the standard log-utility specification.

We display the elasticities of labour market tightness, job search effort, value of leisure, and wages in Table 3.<sup>12</sup> Contrasting the models with endogenous and exogenous value of leisure (the latter presented in Section 2.3), an important result emerges: volatilities are smaller in the model with endogenous value of leisure. For example, our benchmark calibration implies an elasticity of the value of leisure of 1.16, which is close to its empirical counterpart of 1.13 targeted in the first block in Table 2; yet, the corresponding elasticity of labour market tightness is much lower (9.65 versus 37.51). Part of this can be explained by the fact that the shock in Section 2.3 is permanent, whereas in this section it is temporary. Another part is explained by  $s = 0.05 > \delta$  (9.65 versus 11.04). But the main reason is general-equilibrium feedback between the value of leisure and labour market tightness. The value of leisure,  $l(z)$ , depends on consumption (see Equation 23), which depends on income net of total hiring costs (see Equation 26). This income depends on labour productivity,  $z$ , but also on labour market tightness,  $\theta$ : when  $z$  rises, firms open more vacancies until the free-entry condition is satisfied; this, in turn, leads to more income (through higher employment) net of total hiring costs and, necessarily, even more consumption. Thus, in the model of this section,  $l(z)$  depends positively on  $z$  both directly and indirectly through  $\theta$ . This contrasts with the models in the previous section that assumed that  $l(z)$  is uncorrelated with  $\theta$ , focusing only on the direct effect of  $z$  on  $l(z)$  and muting an important propagation channel. This implies that if  $\theta$  is as volatile as in Section 2.3, then  $l(z)$  has to be more volatile, which increases the volatility of wages and, in turn, makes  $\theta$  less volatile.

12 The elasticities are based on the theoretical moments of the model assuming an Hodrick–Prescott filter of 100,000.

**Table 4.** Endogenous value of leisure—model with Nash bargaining

	$\varepsilon_\theta$	$\varepsilon_e$	$\varepsilon_w$	$\varepsilon_l$
US data	7.56	$\leq 0$	1	1.13
Benchmark	1.71	-0.11	1.02	1.09
$\kappa = 0.4w^f$	1.52	-0.10	1.02	1.09
$\chi/l(z) = 0.41$	1.96	-0.02	1.01	1.10
$\zeta = 4$	1.61	-0.06	1.02	1.09
$b = 0.15$	1.71	-0.11	1.02	1.09
$H = 0$	0.82	-0.10	0.95	1.02
$\gamma = 1.1$	1.34	-0.30	1.03	1.19

Note: This table reports the elasticities of labour market tightness,  $\varepsilon_\theta$ , job search effort,  $\varepsilon_e$ , wages,  $\varepsilon_w$ , and value of leisure,  $\varepsilon_l$ , generated by the matching model with Nash bargaining and endogenous  $\varepsilon_l$ , under several calibrations.

Because endogenous  $l(z)$  reduces volatilities, it helps to bring the volatilities generated by the matching model closer to their empirical counterparts. Our benchmark calibration delivers highly procyclical labour market tightness but only slightly above data (9.65 versus 7.56) together with an elasticity of the value of leisure also very close to data (1.16 versus 1.13) and acyclical job search effort. Furthermore, as emphasized in the previous section, despite the fact that AOB directly implies real-wage rigidity, the elastic value of leisure implies an elasticity of the wage of new hires close to one. Thus, in a matching model with wages set as in [Hall and Milgrom \(2008\)](#), acyclical (slightly countercyclical) job search effort emerge together with (i) procyclical wages, (ii) log-utility preferences, and (iii) volatile labour market tightness.

Our sensitivity analysis of the model with endogenous value of leisure confirms that  $\zeta$ ,  $b$ , and  $H$  barely affect the cyclical dynamics. It also confirms that higher values of  $\kappa$  reduce volatility. Yet, even though we experiment with a large value for  $\kappa$ , the model still delivers  $\varepsilon_\theta$ ,  $\varepsilon_l$ , and  $\varepsilon_w$  that are close to their data counterparts together with acyclical job search effort. Furthermore, [Table 3](#) confirms that reducing the steady-state value of  $\chi/l(z)$  can significantly enhance  $\varepsilon_\theta$  by reducing slightly  $\varepsilon_w$ .

As detailed above, we choose  $s = 0.05$ , which is slightly above  $\delta = 0.036$ . Yet, our calibrated  $s$  is smaller than in [Hall and Milgrom \(2008\)](#) who impose  $s \approx 4\delta$  to target the observed volatility of unemployment and in [Petrosky-Nadeau and Zhang \(2021\)](#) who also assume a relatively large number,  $s = 0.1 \approx 3\delta$ . Although these two calibrations are not based on direct evidence on  $s$ , we also consider the case of  $s = 0.1$  in our robustness checks and find that it clearly lowers  $\varepsilon_\theta$  but without severely compromising the results of the model. Finally, we add a robustness check to  $\gamma$ . Following [Boppart and Krusell \(2020\)](#), we experiment with  $\gamma$  slightly exceeding unity such that income effects on hours exceed substitution effects to be consistent with the downward trend of hours worked that they document. Our experiment with  $\gamma = 1.1$  shows that a higher  $\gamma$  reduces both  $\varepsilon_\theta$  and  $\varepsilon_e$  by raising  $\varepsilon_l$ . But, once again, the matching model with endogenous value of leisure performs remarkably well.

In [Table 4](#), we reproduce our experiments but instead of using AOB, we use Nash bargaining.<sup>13</sup> This exercise offers a number of results. (i) The volatility of labour market

13 To calibrate the variant with Nash bargaining, we still follow [Table 1](#). The major distinction in terms of calibration is that we use the workers' bargaining power,  $\phi$ , together with  $m_0$  and  $\psi$  to target the unemployment rate, job search effort, and labour market tightness in the steady state.

tightness is much lower with Nash bargaining than with AOB. (ii) Assuming a low  $H$  has small effects on volatilities in the variant with Nash bargaining, which suggests that fixed matching costs do not significantly amplify volatilities if agents are risk averse, contrasting with the findings in [Pissarides \(2009\)](#). (iii) Assuming a low value of  $\chi$  to increase the match surplus has little to no effect on volatility if  $\gamma = 1$ ; the reason is that a higher  $\chi$  also makes the Nash bargained wage more responsive to the curvature of the utility function (see [Equation 9](#)), implying that (*ceteris paribus*) workers demand higher wages to forgo their leisure; this result also contrast with the findings in [Hagedorn and Manovskii \(2008\)](#) that reducing the match surplus increases volatilities in the matching model. (iv) Finally, and most importantly for our purposes, acyclical job search effort always coincides with acyclical labour market tightness.

#### 4. Conclusion

This article (to the best of our knowledge) is the first showing that a matching model can *simultaneously* and *endogenously* generate a number of empirical patterns observed in the US economy: (i) acyclical (or countercyclical) job search effort; (ii) highly procyclical labour market tightness; (iii) procyclical value of leisure; and (iv) procyclical real wages of new hires. For a matching model to simultaneously generate these patterns is remarkable given that these models are known to generate volatilities of labour market variables that fall short of their empirical counterparts ([Shimer, 2005](#)) and we rule out early solutions to this volatility puzzle-like real-wage rigidity. Furthermore, [Chodorow-Reich and Karabarbounis \(2016\)](#) and [Mukoyama et al. \(2018\)](#) show that procyclical value of leisure and countercyclical job search effort exacerbate the volatility puzzle.

Importantly, our proposed solution relies on standard adjustments to the matching model. In particular, we show that the matching model with AOB as in [Hall and Milgrom \(2008\)](#) and endogenous value of leisure calibrated in a standard manner can concur with the empirical patterns.

As a caveat, our article concentrates solely on the US economy. Nonetheless, we have three reasons. First, we lack the empirical evidence needed to calibrate some of our model's parameters and calibration targets. Secondly, to study other economies, we might need to adapt our model. For example, if we were to model a typical European economy, we should consider different labour market institutions and, likely, an open economy framework.<sup>14</sup> Thirdly, and most importantly, almost all the empirical evidence about the cyclicity of job search effort uses US data. Therefore, there is a need for empirical research that would allow us to better understand the European and other labour markets.

In light of our results, it may be tempting and convenient to abstract from endogenous fluctuations in job search effort as standard in the literature. Yet, irrespective of the calibration of a matching model with constant job search effort, we also find (not reported) that volatilities are very low if the value of leisure is procyclical and we assume Nash bargaining. Put differently, it is crucial to assume AOB if the value of leisure is procyclical. Furthermore, assuming acyclical job search effort or building a model that endogenously generates it are clearly two distinct approaches. If a model embeds exogenous job search effort, then we should bear in mind that implicit in the setup and calibrations, there is a

14 See [Krogh \(2016\)](#) for a discussion of the unemployment volatility puzzle (particularly the consequences of wage rigidity in matching models) in the context of closed and small open economies.

rational expectations response of job search effort that may disagree with the empirical counterpart.

## Supplementary material

[Supplementary material](#) is available online at the OUP website.

1. [Online Appendix](#).

## Funding

This work was supported by the European Regional Development Fund through COMPETE 2020 Programa Operacional Competitividade e Internacionalização and by Portuguese public funds through Fundação para a Ciência e a Tecnologia (FCT)—Projects: IF/01569/2012/CP0155/CT0001 and POCI-01-0145-FEDER-006890. Luís Guimarães also thanks FCT's financial support—Project: 2020.00714.CEECIND.

## Acknowledgements

We are grateful for the comments and suggestions of the two referees, which substantially improved our article. We also thank seminar participants at the Bank of Estonia, King's College London, the Queen's University Belfast, the University of Bath, the University of Lisbon, the University of Nottingham Ningbo, and the University of Porto for helpful comments. This article replaces an earlier draft entitled: 'Income Effects and the Cyclicalities of Job Search Effort'.

## References

- Boppart, T. and Krusell, P. (2020) Labor supply in the past, present, and future: a Balanced-Growth perspective, *Journal of Political Economy*, **128**, 118–57.
- Chodorow-Reich, G. and Karabarbounis, L. (2016) The cyclicalities of the opportunity cost of employment, *Journal of Political Economy*, **124**, 1563–618.
- Christiano, L.J., Eichenbaum, M.S., and Trabandt, M. (2016) Unemployment and business cycles, *Econometrica*, **84**, 1523–69.
- DeLoach, S.B. and Kurt, M. (2013) Discouraging workers: estimating the impacts of macroeconomic shocks on the search intensity of the unemployed, *Journal of Labor Research*, **34**, 433–54.
- Faberman, R.J. and Kudlyak, M. (2019) The intensity of job search and search duration, *American Economic Journal: Macroeconomics*, **11**, 327–57.
- Gomme, P. and Lkhagvasuren, D. (2015) Worker search effort as an amplification mechanism, *Journal of Monetary Economics*, **75**, 106–22.
- Haefke, C., Sonntag, M. and van Rens, T. (2013) Wage rigidity and job creation, *Journal of Monetary Economics*, **60**, 887–99.
- Hagedorn, M. and Manovskii, I. (2008) The cyclical behavior of equilibrium unemployment and vacancies revisited, *American Economic Review*, **98**, 1692–706.
- Hall, R.E. (2005) Employment fluctuations with equilibrium wage stickiness, *American Economic Review*, **95**, 50–65.
- Hall, R.E. and Milgrom, P.R. (2008) The limited influence of unemployment on the wage bargain, *American Economic Review*, **98**, 1653–74.
- Hornstein, A. and Kudlyak, M. (2017) Estimating matching efficiency with variable search effort. 2017 Meeting Papers 881, Society for Economic Dynamics.
- Kehoe, P.J., Lopez, P., Midrigan, V., and Pastorino, E. (2020) Credit frictions in the great recession. National Bureau of Economic Research Working Paper No. 28201, National Bureau of Economic Research.

- Krogh, T.S. (2016) Real wage rigidity and the unemployment volatility puzzle in small open economies, *Oxford Economic Papers*, 68, 131–51.
- Leduc, S. and Liu, Z. (2020) The weak job recovery in a macro model of search and recruiting intensity, *American Economic Journal: Macroeconomics*, 12, 310–43.
- Leyva, G. (2018) Against all odds: job search during the great recession. Banco de México Working Papers No. 2018–13, Banco de México.
- Ljungqvist, L. and Sargent, T.J. (2017) The fundamental surplus, *American Economic Review*, 107, 2630–65.
- Martin, C. and Wang, B. (2018) Endogenous real wage rigidity in a search frictions model, *Oxford Economic Papers*, 70, 1016–35.
- Merz, M. (1995) Search in the labor market and the real business cycle, *Journal of Monetary Economics*, 36, 269–300.
- Mukoyama, T., Patterson, C., and Şahin, A. (2018) Job search behavior over the business cycle, *American Economic Journal: Macroeconomics*, 10, 190–215.
- Nevo, A. and Wong, A. (2019) The elasticity of substitution between time and market goods: evidence from the great recession, *International Economic Review*, 60, 25–51.
- Petrongolo, B. and Pissarides, C.A. (2001) Looking into the black box: a survey of the matching function, *Journal of Economic Literature*, 39, 390–431.
- Petrosky-Nadeau, N. and Zhang, L. (2021) Unemployment crises, *Journal of Monetary Economics*, 117, 335–53.
- Pissarides, C.A. (2000) *Equilibrium Unemployment Theory*, MIT Press, Cambridge.
- Pissarides, C.A. (2009) The unemployment volatility puzzle: is wage stickiness the answer? *Econometrica*, 77, 1339–1369.
- Shimer, R. (2004) Search intensity. *Mimeo*, University of Chicago, Chicago, IL.
- Shimer, R. (2005) The cyclical behavior of equilibrium unemployment and vacancies, *American Economic Review*, 95, 25–49.
- Shimer, R. (2012) Reassessing the ins and outs of unemployment, *Review of Economic Dynamics*, 15, 127–48.
- Silva, J.I. and Toledo, M. (2009) Labor turnover costs and the cyclical behavior of vacancies and unemployment, *Macroeconomic Dynamics*, 13, 76–96.
- Silva, J.I. and Toledo, M. (2013) The unemployment volatility puzzle: the role of matching costs revisited, *Economic Inquiry*, 51, 836–43.
- Tumen, S. (2014) Is search intensity countercyclical? *Unpublished*.