Competitive Elections, Incumbency Advantage, and Accountability

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May 5, 2021

Abstract

I present a model of repeated electoral competition between two parties. A part of the electorate votes retrospectively and considers the amount of rent-seeking by the incumbent party, while the prospective voters follow probabilistic party preferences when casting their votes. I show that it is possible to distinguish the effects of incumbency advantage and electoral punishment on the minimum level of rent-seeking consistent with equilibrium. As long as there is electoral punishment for excessive rent-seeking, a larger incumbency advantage increases accountability by decreasing the minimum amount of rentseeking consistent with equilibrium. The reason for this is that the larger the incumbency advantage is, the more important the result of the next election for all future election outcomes is. Consequently, the incumbent party is willing to give up more rent-seeking opportunities to improve its electoral prospects. Increased accountability due to a larger share of retrospective voters hurts the political selection aspect of elections because it enables the incumbent party to win without the support of the majority of the prospective voters.

JEL: D72, Keywords: Incumbency Advantage, Accountability, Competitive Elections, Probabilistic voting, Rent-seeking

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1 Introduction

There is a widely held belief that competitive elections are necessary for a wellfunctioning democracy. This belief goes back at least to Schumpeter (1942). Nonetheless, there have been surprisingly few attempts to define precisely what requirements must be fulfilled to make an election competitive. Brunell and Clarke (2012, 125) give a good description of a widespread understanding which equates competitiveness with small margins of victory, noting that "academics, journalists, and other commentators constantly extol the virtues of competitive elections while bemoaning the fact that there are not enough closely fought contests for important offices." Buchler (2007) suggests defining competitive elections as elections with a close to 50% chance for each side to win. However, political competition is not an end in itself. Just as competitive markets are desirable because they increase welfare, competitive elections are desirable because they increase they increase of the interest of the electorate.

In addition to the prospective elements of electoral competition, there are retrospective elements and the need for measures of competitiveness that capture them. One such measure is provided by electoral punishment, defined here as the loss in the likelihood of re-election when the incumbent party does not fulfill a performance standard set by the electorate. I demonstrate this measure's usefulness in a model of repeated political competition between two parties in which the performance standard takes the form of a maximum amount of acceptable rent-seeking. I show that the amount of rent-seeking which the voters have to accept is decreasing in electoral punishment and, more surprisingly and in contrast to the common view that close elections are desirable, the size of the incumbency advantage. The measure of the incumbency advantage applied here is the difference in probability of victory between running as the incumbent party and running as the opposition party. This measure is, by construction, identical for both parties and captures the causal impact of incumbency on the election outcomes.

Because a larger incumbency advantage leads to less rent-seeking, elections with a close to 50% chance of an opposition victory lead, ceteris paribus, to less accountability than elections with a sizable incumbency advantage. The reason is that the larger the incumbency advantage is, the larger the influence of the next election on all future election outcomes and thus the price the parties are competing for. Therefore, with a larger incumbency advantage, the incumbent party is willing to give up more rent-seeking opportunities to improve its electoral prospects, and the electorate can impose a more stringent performance standard.

The second part of the electorate is formed by prospective voters. Each prospective voter has preferences in favor of one of the two parties and votes accordingly. These preferences are repeatedly determined by a random process before each election and can be thought of as depending on the policy platforms or the perceived quality of the parties' candidates. Because of the random element in the voting decisions of prospective voters, the setup constitutes a repeated probabilistic voting model.

Because in the most plausible equilibrium the parties always comply with the rentseeking threshold and retrospective voters reward the incumbent party with their support, the incumbent party can be re-elected although the majority of the prospective voters prefers the opposition party. The probability that this happens is increasing in the share of retrospective voters. Therefore, a larger share of retrospective voters not only increases accountability but is likely to come at a cost in other dimensions of politics, especially if the two groups of voters differ only in their voting strategies and have similar preferences regarding policies and candidates.¹ Interestingly, if the reduced influence of prospective voters leads to a slowdown in the introduction of new ideas and, especially, new political leaders, this does constitute a reduction in competition in the sense described by Schumpeter (1942) whose dynamic conception of political competition is closely related to his conception of economic competition.

The following three cases demonstrate the relevance of the results for comparative political economics. The first case considers mature democracies, which usually combine a considerable amount of incumbency advantage with a credible opposition that allows for electoral punishment at the ballot box and corresponds with a considerable share of retrospective voters, possibly in combination with some additional electoral bias in favor of the incumbent party.

I discuss Germany as one such example. Since the first postwar election in 1949, only candidates of the two major parties CDU and SPD have been elected German chancellor, and just as in the model discussed here, the two parties repeatedly compete for this office.² Moreover, there is evidence for a sizable incumbency advantage:

¹Strictly speaking, the model discussed here does not allow for the two groups of voters to have similar preferences regarding policies and candidates because here the utility of retrospective voters depends solely on the amount of rent-seeking.

²CDU is short for the Christian Democratic Union of Germany, considered to be center-right.

In 15 out of 18 elections, the incumbent chancellor remained in office after the election. This was not due to a party-specific advantage for CDU candidates, as seen from the fact that of the five elections with an SPD chancellor as the incumbent candidate, the SPD won four.³ While this large incumbency advantage brought stability and accountability, there might have been a certain lack of new policy ideas. This is especially true for the period before the first SPD chancellorship after the election in 1969 and is consistent with the implication of the model that a large degree of accountability can come at the cost of the reduced influence of prospective voters. German economic policy provides one example. Keynesian ideas, for better or worse, arrived comparatively late in German economic policymaking, appearing only after the SPD became part of the government in 1966 for the first time. Keynesian economics had already achieved a strong position in German academia much earlier (Hagemann 2017). A list of German elections and their results is provided in Appendix A.⁴

The second case involves countries with reasonably fair and competitive elections but a low incumbency advantage or even a disadvantage and, moderate degrees of electoral punishment. This corresponds to a small share of retrospective voters in combination with an electoral bias for the opposition. Central and Eastern European countries after the restoration of democracy provide some examples for this. Roberts (2008) considers 34 free elections between 1992 and 2006 in 10 different countries in Central and Eastern Europe and finds what he calls "hyperaccountability". However, in the end this "hyperaccountability" amounts to none or little accountability with respect to rent-seeking. While Roberts finds that better economic performance and lower unemployment in particular were rewarded in elections, he also reports that the expectations were so high that only approximately 30% of the parties in the government before an election remained in government after the election. This evidence suggests that the voters were looking for parties that could successfully handle the economy and, especially, the labor market but were mostly unable to find such par-

SPD is short for the Social Democratic Party of Germany, considered to be center-left. The Federal Chancellor of the Federal Republic of Germany is the head of government. The chancellor's role is comparable to that of the prime minister in most other parliamentary democracies.

³Moreover, the incumbency advantage seems party-specific, not person-specific. The CDU and SPD replaced their chancellors with another party member between general elections three times; only Kurt Georg Kiesinger lost his chancellorship in an election following such a replacement.

⁴For an up to date introduction to the German political system see Marschall (2018). An alternative in English is provided by Schmidt (2003).

ties. Because the voters used past performance as indicative of future performance, this form of economic voting should be interpreted as prospective. It has the consequence of making elections an ineffective tool for holding politicians accountable for corruption. Alternatively, as Roberts (2008, p. 533) puts it, "if incumbents know they will lose, then they may decide to enrich themselves when in power".

The third case combines an even larger incumbency advantage with low or nonexistent electoral punishment and thus, at best, minimal accountability. There are many examples of this combination because it naturally occurs in countries where elections are routinely rigged in favor of the current government and large parts of the press are controlled by it. Russia under President Putin currently provides the most prominent example.

Related literature Important early contributions that argue for the empirical importance of retrospective voting include Key (1966) and Fiorina (1981). Formal models of electoral accountability were first developed by Barro (1973) and Ferejohn (1986). In this literature, politicians are the agents, and voters are their principals. The model presented here is in line with the focus on controlling behavior in office, specifically rent-seeking and effort provision by the politicians, in these early models. Except for Klingelhöfer (2015), the idea of having two parties that alternate in office has been neglected. However, such a setup seems a natural choice for analyzing, for example, elections in the United States with its two dominating parties. Instead, the focus has been on individual politicians who lose office forever when they lose an election.

An influential critique of the focus on punishing past behavior in models of political accountability is provided by Fearon (1999), who points out the difficulty of holding politicians accountable for past actions in an election if the challenger is not perceived as an equally attractive candidate by the electorate. In this view, rational voters will always vote in favor of the candidate or party that would maximize their future utility if elected. These voters consider past behavior only in so far as it is informative about the expected future performance of the incumbent. Fearon's contribution resulted in an increased focus on the selection of politicians instead of the previous focus on their behavior in office. However, Ashworth, Bueno de Mesquita, and Friedenberg (2017) show that while there can be a trade-off between selecting good-types and holding politicians accountable for past actions, up to a point, both aims can be

achieved simultaneously. Moreover, as Feddersen and Sandroni (2006) point out, the probability that the decision of a single voter is pivotal is negligible in national elections. In the model presented here, the electorate is represented by a continuum of voters, and, just as in real-world national elections, no voter ever casts a decisive vote. Although the retrospective voters know that they are never pivotal, they are nonetheless strategic in the sense that they optimize over their performance standard.

The number of models that combine prospective and retrospective voting motives is small.⁵ Klingelhöfer (2015) shows that when accountability and policy determination are analyzed separately, and there is also uncertainty over the preferences of voters, important interdependencies between the two dimensions will be overlooked. Specifically, because indifferent voters vote for the incumbent party as long as it limits its rent-seeking, the opposition party will try to differentiate itself on the policy dimension in order to have a chance to win the election. This effect cannot be shown in any purely forward- or backward-looking model of policy determination. Van Weelden (2013) shows how the electorate can use the policy dimension to increase accountability. To achieve this, a representative voter increases the punishment effect of not re-electing an incumbent by replacing her with a politician from the opposite political spectrum and, in this way, decrease rent-seeking. While voters suffer from more polarized equilibrium policies, they are nonetheless better off because the increase in accountability has a larger effect on their welfare. However, in contrast to the model presented here, neither of these models allows for different groups of voters with different voting motives. Instead, they allow all voters to combine retrospective and prospective voting motives. While rationality requires voters to vote prospectively whenever they are not indifferent between the candidates, competition on the policy dimension ensures that this is regularly the case.

The empirical existence of incumbency advantage in many advanced economies is well documented, especially in the United States. Eggers (2017) provides a short and up-to-date review of the relevant literature. Klašnja and Titiunik (2017) provide a convincing attempt to explain incumbency disadvantage in Brazilian mayoral elections. Besides empirical evidence, they also provide a model in which the lack of accountability leads to incumbency disadvantage. More specifically, in their model,

⁵Persson and Tabellini (2000), an influential textbook in theoretical political economics, treats accountability and policy determination in separate chapters without considering any possible interaction.

weak accountability is due to insufficient control over politicians by the parties that nominate them. Therefore, low accountability reduces the incumbency advantage, the opposite direction of causality than that in the model presented here. Klašnja and Titiunik's results are nonetheless complementary to the ones provided here because both directions of causality could be relevant simultaneously.

Outline of the paper In Section 2, the basic model of political accountability is presented. After deriving the most stringent constant performance standard consistent with compliance by the parties in equilibrium, I show that this performance standard is also consistent with noncompliance by both parties. I derive two additional, less stringent constant performance standards that are more likely to lead to compliance and therefore lead to more plausible equilibria. The qualitative comparative static results discussed in the introduction are shown to be independent of the exact equilibrium selection. In Section 3, an explicit model of the electorate is provided, and the conditional re-election probabilities of Section 2 are endogenized. Here, it is shown that greater accountability due to a larger number of retrospective voters comes at a cost because the incumbent party can win against an opposition party that is preferred by the prospective voters. At the end of Section 3, the examples from the introduction are reconsidered before the paper ends with a Conclusion.

2 The model

There are two parties, A and B. The incumbent party in period $t \ge 0$ is denoted by $I_t \in \{A, B\}$, while the opposition party is denoted by $O_t = \{A, B\} \setminus I_t$.⁶ In each period $t \ge 0$, the incumbent party I_t chooses the amount r_t of rent-seeking. The opposition party cannot engage in any rent-seeking until it wins an election and becomes the new incumbent party. The maximum amount of rent-seeking is normalized to 1, and thus, $r_t \in [0, 1]$ in all t.⁷ An election takes place after the amount of rent-seeking r_t is observed by the representative retrospective voter (RRV). The RRV decides whether

⁶The identity of the incumbent party in the first period plays no role in the following analysis. It can be thought of as being drawn by nature from an unspecified probability distribution.

⁷A more general framework would allow for benefits of office and shirking separately from the rent-seeking aspect. I focus on rent-seeking throughout. This does not lead to any loss of generality because the rents can be thought of as the net benefits of office after the costs of effort. Providing less than maximum effort is a form of shirking by the parties.

to vote for the incumbent party in period t ($v_t = 1$) or not ($v_t = 0$). If $v_t = 1$ and party J = A, B is the incumbent party in period t, it wins with probability $\alpha_J \in [0, 1]$. In expectation, party A is the (weakly) more popular party, conditional on the support of the RRV. Let $\Delta \alpha \equiv \alpha_A - \alpha_B \geq 0$ denote this difference in popularity between the parties. If $v_t = 0$, the incumbent party wins only with probability $\alpha_J - EP$. In other words, $EP \in [0, \alpha_B]$ constitutes the size of the *electoral punishment* for losing the support of the *RRV*. The reason why the *RRV* only partly determines the election outcome is the existence of prospective voters whose voting decisions are discussed in detail in Section 3. There, an explicit model of the voting stage is added and α_A, α_B and EP are derived from parameters describing the composition of the electorate.

The winner of the election becomes the next incumbent party I_{t+1} , and the new period t+1 begins with the incumbent party I_{t+1} choosing its amount of rent-seeking. Thus, every period t constitutes one stage of an infinite horizon multi-stage game in which the elections constitute chance moves by nature that determine the binary state variable I_t .⁸ All actions, including the chance move determining the election outcome, are immediately observed by both parties and the *RRV*. In other words, the players have perfect information.

Both parties J = A, B maximize their expected discounted value from rent-seeking activities:

$$U_J = E_0 \sum_{t=0}^{\infty} \beta^t 1_J(I_t) r_t,$$
 (1)

where $\beta \in (0, 1)$ is the discount factor. $1_J(I_t)$ is an indicator function that takes a value of 1 in each period t in which party J is in office and a value of 0 in each period in which party J is not in office. Thus, the parties profit only from rent-seeking while they are in office. The *RRV* aims to minimize the value of the expected discounted rent-seeking activities of the incumbent parties:

$$U_{RRV} = E_0 \sum_{t=0}^{\infty} \beta^t (1 - r_t).$$
 (2)

These two payoff functions imply that in every period, a total payoff of 1 is divided between the incumbent party and the electorate. In other words, the game is constant-

⁸The chance moves themselves are, conditional on the voting decisions of the RRV, *i.i.d.* across states and periods. The action dependence of the change moves distinguishes the class of game discussed here from a typical setup in the otherwise structurally related legislative bargaining literature. For a recent example of the latter, see Baron (2019).

 $\mathrm{sum.}^9$

2.1 Strategies

The history of the game at the beginning of period t consists of the information of the identity of the previous incumbent parties, their chosen level of rent-seeking, and the past voting decisions of the RRV. It is denoted by $h_t = ((I_0, r_0, v_0), ..., (I_{t-1}, r_{t-1}, v_{t-1}))$ in all periods $t \ge 1$, while the history in period t = 0 is $h_0 = \emptyset$.

Only pure strategies are considered. The strategies of the parties and the RRV determine their moves in every period t. These moves can, in principle, depend on the entire previous history h_t of the game as well as, in the case of the RRV, the identity of the incumbent party and its rent-seeking in period t:

 $s_J: r_t(h_t) \in [0, 1]$ in periods t in which $J = I_t$ with J = A, B, $s_{RRV}: v_t(h_t, I_t, r_t) \in \{0, 1\}$ in each t.

2.2 Solving the model

The equilibrium concept applied throughout is subgame perfection with some additional refinements for equilibrium selection. While the setup does not constitute an infinitely repeated game, the single deviation principle applies to all multi-stage games. Consequently, the following analysis proceeds in a fashion similar to an analysis in the familiar infinitely repeated game setting.

2.2.1 Some useful results

Let $q_{J,s}^{I}$ $(q_{J,s}^{O})$ denote the conditional probability of party J = A, B being the incumbent party in period t + s after being the incumbent (opposition) party in period t. To simplify the notation, I use $q_{J} \equiv q_{J,1}^{I}$ to denote the probability of re-election for party J if s = 1. This probability is, for example, given by α_{J} if both parties always comply. Throughout the following analysis, K always denotes the party competing with party J.

⁹The constant-sum assumption simplifies the analysis and the notation. It is not meant to suggest that rent-seeking has no undesirable effects aside from the redistribution of resources.

Lemma 1 Consider the case of constant re-election probabilities q_J with J = A, B. If party J is the incumbent (opposition) party in period t, the probability that party J is in office in period t + s, conditional on being the incumbent party in period t, is:

$$q_{J,s}^{I} = 1 - q_{K,s}^{O} = \begin{cases} \frac{1 - q_{K}}{2 - q_{J} - q_{K}} + (q_{J} + q_{K} - 1)^{s} \cdot \frac{1 - q_{J}}{2 - q_{J} - q_{K}} & \text{if } q_{J} + q_{K} < 2\\ 1 & \text{if } q_{J} = q_{K} = 1. \end{cases}$$
(3)

The probability that party J = A, B is in office in period t + s, conditional on being the opposition party in period t, is:

$$q_{J,s}^{O} = 1 - q_{K,s}^{I} = \begin{cases} \frac{1 - q_{K}}{2 - q_{J} - q_{K}} - (q_{J} + q_{K} - 1)^{s} \cdot \frac{1 - q_{K}}{2 - q_{J} - q_{K}} & \text{if } q_{J} + q_{K} < 2\\ 0 & \text{if } q_{J} = q_{K} = 1. \end{cases}$$

$$(4)$$

Proof. See Appendix B.

To capture the causal impact of incumbency on the election outcomes, I define the incumbency advantage as the difference in probability of victory between running as the incumbent party and running as the opposition party. This effect is, by construction, the same for both parties, and given by $q_J + q_K - 1$.¹⁰ If $q_J + q_K < 2$, $q_{J,s}^I$ converges to:

$$LS_J = \frac{1 - q_K}{2 - q_J - q_K}.$$
 (5)

Consequently, LS_J is the long-term share of periods in office for party J.

Let ρ_J denote a constant level of rent-seeking of party J whenever it is in office, and q_J and q_K the constant re-election probabilities of the two parties. The corresponding expected present discounted payoffs of party J as the incumbent and as the opposition party are denoted by $V_J^I(q_J, q_K, \rho_J)$ and $V_J^O(q_J, q_K, \rho_J)$, respectively. With the help of Lemma 1, these expected present discounted payoffs and their difference

¹⁰Because $q_J + q_K - 1 = \frac{q_J - (1 - q_J) + q_K - (1 - q_K)}{2}$, this measure of the incumbency advantage equals the unweighted average of the difference in probability of a victory of the incumbent party compared to a victory of the opposition party when either party is in power. For $q_J \neq q_K$, this measure is different from the long-term difference in the likelihood of the incumbent party winning compared to the opposition party winning an election. The latter is given by the average difference in probability of winning weighted by the expected long-term share of periods in office: $\frac{(1-q_K)(2q_J-1)+(1-q_J)(2q_K-1)}{2-q_J-q_K}$. Because this weighted average mixes up the effects of the advantages of incumbency with the effects of the differences in popularity of the two parties, it does not measure the causal effect of incumbency on election outcomes.

 $\Delta V_J(q_J, q_K, \rho_J) \equiv V_J^I(q_J, q_K, \rho_J) - V_J^O(q_J, q_K, \rho_J) \text{ can be calculated:}$

Corollary 1

$$V_J^I(q_J, q_K, \rho_J) = \frac{1}{1 - \beta} \cdot \frac{1 - \beta \cdot q_K}{1 - \beta (q_J + q_K - 1)} \cdot \rho_J,$$
(6)

$$V_J^O(q_J, q_K, \rho_J) = \frac{1}{1 - \beta} \cdot \frac{\beta(1 - q_K)}{1 - \beta(q_J + q_K - 1)} \cdot \rho_J,$$
(7)

$$\Delta V_J(q_J, q_K, \rho_J) = \frac{\rho_J}{1 - \beta (q_J + q_K - 1)}.$$
(8)

Proof. See Appendix B.

Corollary 1 implies that if there is no incumbency advantage $(q_J + q_K - 1 = 0)$, the difference in expected discounted payoffs is equal to the rent payment the incumbent party receives in the current period. If there is an incumbency advantage $(q_J + q_K - 1 > 0)$, the difference is larger than ρ_J because being the incumbent party also increases the likelihood of holding office in all future periods. If there is an incumbency disadvantage $(q_J + q_K - 1 < 0)$, the difference is smaller than ρ_J because being the incumbent party decreases the likelihood of holding office in the next period.¹¹ Because both parties enjoy the same incumbency advantage whenever they are in office, it follows that if both parties engage in the same constant level of rent-seeking ρ , then $\Delta V_J(q_J, q_K, \rho) = \Delta V_K(q_K, q_J, \rho)$.

2.2.2 Equilibria with constant rent-seeking thresholds

Following the literature going back to Barro (1973) and Ferejohn (1986), the voting decision of the RRV follows a performance standard for the incumbent party. Specifically, this performance standard is the largest amount of rent-seeking \bar{r}_t in period t for which the RRV casts her vote in favor of the incumbent party I_t . It turns out that such thresholds are only consistent with compliance if they are identical for both parties. Moreover, it is shown that such constant thresholds by the RRV allow for both the smallest and the largest possible present discounted level of rent-seeking by the parties consistent with any equilibrium of the game. Let \bar{r}_J with J = A, B denote

¹¹To be precise, it decreases the probability of being in office in all future periods for which s - t is odd and increases the probability for all future periods for which s - t is even. The future periods for which s - t is odd are, on average, one period closer to t and therefore less heavily discounted. Moreover, the influence of being the incumbent party in period t on being the incumbent party in period s is larger for the periods which are closer to the present.

the potentially party-specific thresholds, s_J with J = A, B denote the strategies of the parties, and s_{RRV} denote the strategy of the RRV.

Constant threshold with compliance equilibrium

Definition 1 Constant threshold with compliance strategy profile (CTCSP): $\bar{r}_J \in [0,1]$ with J = A, B, $s_J : r_t = \bar{r}_J$ in each period t in which $J = I_t$ with J = A, B, $s_{RRV} : v_t = \begin{cases} 1 & \text{if } r_t \leq \bar{r}_{I_t} \\ 0 & \text{if } r_t > \bar{r}_{I_t} \end{cases}$ in each period t.

Because both parties always comply in the CTCSP, in each period in which party J = A, B is in office, the re-election probability is α_J , and the level of rent-seeking is \bar{r}_J . Let $IA \equiv \alpha_J + \alpha_K - 1$ denote the corresponding incumbency advantage.

Proposition 1 A CTCSP constitutes a subgame perfect Constant threshold with compliance equilibrium (CTCE) iff $\bar{r} = \bar{r}_A = \bar{r}_B \in [\bar{r}^*, 1]$ with $\bar{r}^* = \frac{1-\beta \cdot IA}{1-\beta(IA-EP)}$.

Proof. See Appendix B.

The proof is an application of the single deviation principle and shows that for all thresholds below \bar{r}^* , the avoidance of electoral punishment does not provide enough compensation for the reducing rent-seeking to the threshold level.¹² Moreover, the voting decision is only optimal if the *RRV* is indifferent between the two parties. This implies that the thresholds of both parties must be identical in any *CTCE*. Corollary 1 and some algebra lead to the next proposition.

Proposition 2 In the CTCE with threshold \bar{r}^* , both parties achieve the lowest expected discounted payoff consistent with equilibrium, and the RRV achieves the largest expected discounted payoff consistent with equilibrium.

Proof. See Appendix B.

In other words, the CTCE with threshold \bar{r}^* leads to the maximum possible level of accountability. Moreover, it is obvious that the CTCE with the degenerate

¹²The single-deviation principle states that to establish a subgame perfect equilibrium in a multistage game in which the overall payoffs are the discounted sums of uniformly bounded per-period payoffs, such as the one discussed here, it is sufficient to show that no single-deviation in any subgame can make the deviating player better off. For a formal statement and proof of the single-deviation principle, see Fudenberg and Tirole (1991).

threshold $\bar{r} = 1$ leads to the lowest possible expected discounted payoff of 0 for the *RRV*. In this case, the incumbent party has the unconditional support of the *RRV* and there is no accountability for rent-seeking. In addition, the thresholds $\bar{r} \in (\bar{r}^*, 1)$ allow for any intermediate level of accountability between the two extremes of maximal accountability and no accountability at all. Consequently, the focus on constant rent-seeking allows for all present discounted payoffs of the *RRV* that are consistent with any equilibrium, and constant rent-seeking puts no restrictions on the equilibrium degree of accountability.

While it is often not explicitly stated, in the literature on political accountability that has emerged since Barro (1973) and Ferejohn (1986), it is commonly assumed that the electorate or a representative voter can optimize over the threshold for reelection and their preferred equilibrium is played. This makes finding the best possible equilibrium for the voters the de facto equilibrium refinement. In line with this literature, the CTCE with threshold \bar{r}^* provides a natural starting point for the analysis. However, it turns out that for an interval of intermediate values of the constant threshold \bar{r} , both compliance and noncompliance by the parties are consistent with a subgame perfect equilibrium.

Equilibria with noncompliance I first discuss an equilibrium with noncompliance in which the parties never comply and, subsequently, a more sophisticated equilibrium in which the parties never comply along the equilibrium path but use the threat of a *CTCE* to enforce noncompliance.

Definition 2 Constant threshold with noncompliance strategy profile (CTNSP): $\bar{r}_J \in [0,1)$ with J = A, B, $s_J : r_t = 1$ in each period t in which $J = I_t$ with J = A, B, $s_{RRV} : v_t = \begin{cases} 1 \text{ if } r_t \leq \bar{r}_{I_t} \\ 0 \text{ if } r_t > \bar{r}_{I_t} \end{cases}$ in each period t.

Here, the degenerate threshold $\bar{r}_J = 1$ is ruled out because voting unconditionally for the incumbent party by the *RRV* is inconsistent with noncompliance. Because neither party ever complies in the *CTNSP*, the re-election probability is $\alpha_J - EP$, and the level of rent-seeking is always maximal. While $IA \equiv \alpha_J + \alpha_K - 1$ is defined as before, the incumbency advantage in the *CTNSP* is given by $IA - 2 \cdot EP$. **Proposition 3** A CTNSP constitutes a subgame perfect Constant threshold with noncompliance equilibrium (CTNE) if either EP > 0 and $\bar{r}_J \in [0, \bar{r}^{**}]$ for J = A, Bwith $\bar{r}^{**} = \frac{1-\beta(IA-EP)}{1-\beta(IA-2\cdot EP)}$; or EP = 0 and $\bar{r}_J \in [0, 1)$ for J = A, B.

Proof. See Appendix B.

The proof of Proposition 3 is another application of the single deviation principle. It shows that for all thresholds above \bar{r}^{**} , the additional re-election chances provide so much compensation for the reduced level of rent-seeking that they are inconsistent with noncompliance even if the competing party does not comply. Thus, while \bar{r}^* provides a lower bound for compliance, \bar{r}^{**} provides an upper bound for noncompliance.

An incumbent party expects to return to office sooner after losing an election if the other party does not comply. Therefore, it is more attractive to play noncompliance if the other party does the same, and compliance with the performance standard is a strategic complement in the sense of Bulow, Geanakoplos, and Klemperer (1985). Strategic complements constitute a well-known cause of multiple equilibria. Therefore, it is not surprising that $\bar{r}^{**} > \bar{r}^*$ as long as there is electoral punishment (EP > 0) and \bar{r}^{**} exists. Within the interval $\bar{r} \in [\bar{r}^*, \bar{r}^{**}]$, both parties never complying and both parties always complying are both consistent with equilibrium. This is an important consequence of the setup with two parties that interchange in holding office.

Compliance as punishment for compliance Because a party is better off if the other party complies whenever in office, the question arises if the parties can use strategies that incentivize the competing party to comply. While always playing comply instead of noncompliance itself is a first step in this direction, even stronger incentives can be provided by punishing the other party by complying oneself after the other party has deviated from noncompliance.¹³ Specifically, the threat of a CTCEin which both parties play compliance forever is used to sustain an equilibrium with maximum rent-seeking in every period. The parties are colluding in their noncom-

 $^{^{13}}$ Usually, one talks about players who are punished for noncompliance rather than for compliance, and, of course, they are punished by the RRV for noncompliance. However, in a noncompliance equilibrium, compliance with the RRV can be interpreted as breaking an implicit or explicit agreement between the two competing parties. This is very similar to the well-known repeated oligopoly games with collusion in which firms attempt to sustain prices higher than the equilibrium prices of the one-shot game.

pliance with the threshold and punish each other for deviations from this collusion. With the *CTCE* as *punishment phase* for deviating incumbent parties, it is possible to sustain maximum rent-seeking in the *normal phase* of the game even for thresholds $\bar{r} > \bar{r}^{**}$ for which noncompliance is not sustainable as a *CTNE*.¹⁴

Definition 3 Sophisticated constant threshold with noncompliance strategy profile (SCTNSP):

 $\bar{r}_{J} \in [0,1) \text{ with } J = A, B,$ $s_{J} : r_{t} = \begin{cases} 1 \text{ if } r_{s} > \bar{r} \text{ in each period } s < t \\ \bar{r}_{J} \text{ if } r_{s} \leq \bar{r} \text{ in any period } s < t \end{cases} \text{ in each period } t \text{ in which } J = I_{t} \text{ with } J = A, B.$ $s_{RRV} : v_{t} = \begin{cases} 1 \text{ if } r_{t} \leq \bar{r}_{I_{t}} \\ 0 \text{ if } r_{t} > \bar{r}_{I_{t}} \end{cases} \text{ in each period } t.$

In the normal phase of the *SCTNSP*, the re-election probability is $\alpha_J - EP$, and the level of rent-seeking is 1. In the *punishment phase*, which is only reached if one of the parties deviates and complies with its threshold, both parties always comply, the re-election probability is α_J , and the level of rent-seeking is \bar{r}_J in each period in which party J is in office.

Proposition 4 A SCTNSP constitutes a subgame perfect Sophisticated constant threshold with noncompliance equilibrium (SCTNE) if EP > 0 and $\bar{r} = \bar{r}_A = \bar{r}_B \in [\bar{r}^*, \bar{r}^{***}]$, with $\bar{r}^* = \frac{1-\beta \cdot IA}{1-\beta(IA-EP)}$ and $\bar{r}^{***} = \frac{2-\beta(IA+1-\Delta\alpha-2\cdot EP)}{2-\beta(IA+1-\Delta\alpha)} \cdot \frac{1-\beta \cdot IA}{1-\beta(IA-2\cdot EP)}$.

Proof. See Appendix B. ■

The upper bound for a threshold consistent with the SCTNE is provided by the largest threshold consistent with the weakly more popular party A playing noncompliance in equilibrium. This upper bound is lower for party A because the less popular party is less often in office when both parties are always complying than when neither party ever complies.¹⁵ Consequently, for identical thresholds, moving from both parties never complying to both parties always complying constitutes a more severe

¹⁴Essentially, the parties play grim trigger strategies that are well-known from other applications. However, here, we have a somewhat unusual setup with the parties not moving simultaneously and a third player, the RRV. More sophisticated carrot and stick strategies in the spirit of Abreu (1986) are inconsistent with equilibrium because if only one party is expected to comply with its threshold, the RRV always votes for this party in order to decrease rent-seeking.

¹⁵This follow from $\frac{\partial LS_B}{\partial q_A} + \frac{\partial LS_B}{\partial q_B} = \frac{q_B - q_A}{(2 - q_A - q_B)^2} \leq 0$, and the fact that electoral punishment for noncompliance is the same for both parties.

punishment for the less popular party than for the more popular one. It is easy to verify that $\bar{r}^{***} \geq \bar{r}^{**}$. This result also follows directly from the construction of the equilibrium with a harder punishment following the deviation in the *SCTNSP*.¹⁶

2.2.3 The optimal constant threshold for the representative retrospective voter

If the performance standard is not due to social norms but determined by the electorate, a rational RRV chooses the most stringent threshold for which she expects compliance by the parties. According to Proposition 1, compliance of both parties is only feasible if the constant threshold is independent of the identity of the incumbent party. Consequently, only such identical thresholds are discussed here. Moreover, because accountability has already been shown to be inconsistent with EP = 0, the following discussion is restricted to the case EP > 0. The CTNE is the only equilibrium consistent with any constant threshold $\bar{r} < \bar{r}^*$ because the additional chance of re-election is not sufficient to induce compliance by an incumbent party, independent of the strategy of the other party. As we have learned in Propositions 1-3, if $\bar{r} \in [\bar{r}^*, \bar{r}^{***}]$, either both parties complying or neither party complying with the threshold are both consistent with equilibrium. While for $\bar{r} \in [\bar{r}^*, \bar{r}^{**}]$ noncompliance is optimal if the other party always plays noncompliance, and, consequently, all three types of equilibrium discussed in Propositions 1-3 are consistent with any constant threshold within this interval, for $\bar{r} \in (\bar{r}^{**}, \bar{r}^{***}]$, noncompliance by incumbent parties is only sustainable in a SCTNE. For $\bar{r} > \bar{r}^{***}$, only the CTCE constitutes an equilibrium. The question arises as to whether there is some plausible equilibrium refinement to decide which equilibrium is played if $\bar{r} \in [\bar{r}^*, \bar{r}^{***}]$.

A forward induction argument Within the interval $\bar{r} \in [\bar{r}^*, \bar{r}^{**})$, the *CTCE* is not robust against the following forward induction argument: Assume the incumbent party deviates and does not comply in the period(s) immediately before it loses office. Even without communication between the parties, the new incumbent party

¹⁶The inequality holds strictly as long as $\alpha_A < 1$. If $\alpha_A = 1$, in the *SCTNSP* party *A* never loses office again after deviating and complying once, and then, after reaching the punishment subgame, complying forever. In the *CTNE* with threshold r^{**} , the incumbent party is, by the construction of r^{**} , always indifferent between compliance and noncompliance. Therefore, if $\alpha_A = 1$ and the threshold is r^{**} , complying forever and therefore staying in office forever must have the same expected discounted payoff as the equilibrium strategy of never complying. Therefore, $\alpha_A = 1$ implies that $r^{**} = r^{***}$.

has reason to believe that this was not a single incidence of deviation. On the contrary, it seems plausible to interpret the deviation as also communicating a message along the following lines: Our party is, from now on, engaging in the maximum amount of rent-seeking. Given the threshold $\bar{r} \in [\bar{r}^*, \bar{r}^{**})$, it is rational for you to do the same. In other words, we just started playing CTNE.¹⁷ A similar argument applies for the whole interval $\bar{r} \in [\bar{r}^*, \bar{r}^{***})$ if noncompliance is interpreted as moving to a SCTNE and further noncompliance is only expected in return for noncompliance. However, for $\bar{r} \in [\bar{r}^{**}, \bar{r}^{***})$, the forward induction argument is less convincing because the SCTNE uses a CTCE as punishment phase. If this punishment phase is ever reached, the same forward induction argument that was used to rule out the CTCE will apply again, and this, in turn, would cast doubts on the credibility of the punishment. However, without a credible punishment phase, the SCTNE does not constitute a credible alternative to the CTCE. In other words, it can be argued that the punishment phase of the SCTNE is not renegotiation proof.¹⁸

In summary, for a constant threshold $\bar{r} \in [\bar{r}^*, \bar{r}^{**})$, and perhaps even for $\bar{r} \in [\bar{r}^*, \bar{r}^{***})$, it seems reasonable to expect that after an incumbent party does not comply with the performance threshold once, both parties engage in the maximum amount of rent-seeking forever. Loosely speaking, the parties switch from a *CTCE* to a *CTNE* or a *SCTNE*. However, if the incumbent party of the first period foresees this, it deviates from a *CTCE* right away. Therefore, either a *CTNE* or a *SCTNE* is likely to be implemented from the first period, and the best equilibrium from the *RRV*'s perspective, the *CTCE* with threshold \bar{r}^* , is not played. While a *CTCE* is the expected equilibrium if the *RRV* implements a constant performance standard, the threshold chosen by the *RRV* to avoid noncompliance would be \bar{r}^{**} (or perhaps \bar{r}^{***}) but not the most stringent \bar{r}^* . If a *CTCE* is played, the incumbent party in each period *t* is re-elected with probability α_{I_t} and, therefore, $IA \equiv \alpha_J + \alpha_K - 1$ constitutes the incumbency advantage.

Next, we discuss whether there are other strategies for the RRV that can achieve rent-seeking at level \bar{r}^* in a way that is robust against the forward induction argument.

¹⁷While this argument invokes a form of forward induction, I am unaware of any other application of forward induction to infinite stage games. The concept of forward induction is explained in, for example, Mas-Colell, Whinston, and Green (1995).

¹⁸Moreover, in the SCTNE, the action of an incumbent depends on the play in previous periods. However, an incumbent party is in a structurally identical subgame whenever it decides about rentseeking. Often, attention is restricted to stationary strategies. In other words, it is assumed that the decisions in structurally identical subgames are identical.

2.2.4 An equilibrium with maximum electoral punishment

The Constant threshold with compliance and eternal punishment strategy profile (CTCEPSP) can be described with two different phases of the game. In the normal phase, both parties are said to have normal standing, and incumbent parties are supported by the RRV as long as they do not engage in rent-seeking above their threshold. In other words, the normal phase is very similar to the CTCSP. However, in contrast to the CTCSP, the game enters a different phase after excessive rent-seeking by an incumbent party occurs once. In this punishment phase, the party that engaged in excessive rent-seeking is called the party without standing, while the other party is the party in good standing. Independent of the amount of rent-seeking, the RRV now always supports the party in good standing. Consequently, in the punishment phase, both parties always engage in maximum rent-seeking. Once the punishment phase is reached, the game stays there forever. Parties with normal standing in period t are contained in the set N_t , while W_t denotes the party without standing and G_t denotes the party in good standing in period t if the game reaches the punishment phase. A formal description of the CTCEPSP is given in the following definition.

Definition 4 Constant threshold with compliance and eternal punishment strategy profile (CTCEPSP):

$$\begin{split} & N_{0} = \{A, B\}, W_{0} = \emptyset, G_{0} = \emptyset \\ & \bar{r}_{J} \in [0, 1] \text{ with } J = A, B, \\ & \bar{r}_{t} = \begin{cases} & \bar{r}_{I_{t}} \quad \text{if } I_{t} \in N_{t} \\ & \emptyset \quad \text{if } I_{t} \notin N_{t} \end{cases} \quad \text{in each period } t, \\ & \emptyset \quad \text{if } I_{t-1} \in N_{t-1} \wedge r_{t-1} \geq \bar{r}_{t-1} \\ & \emptyset \quad \text{if } I_{t-1} \in N_{t-1} \wedge r_{t-1} > \bar{r}_{t-1} \\ & \emptyset \quad \text{if } N_{t-1} = \emptyset \end{cases} \\ & W_{t} = \begin{cases} & W_{t-1} \quad \text{if } W_{t-1} \neq \emptyset \\ & I_{t-1} \quad \text{if } I_{t-1} \in N_{t-1} \wedge r_{t-1} > \bar{r}_{t-1} \\ & \emptyset \quad \text{otherwise} \end{cases} \\ & W_{t} = \begin{cases} & G_{t-1} \quad \text{if } G_{t-1} \neq \emptyset \\ & O_{t-1} \quad \text{if } I_{t-1} \in N_{t-1} \wedge r_{t-1} > \bar{r}_{t-1} \\ & \emptyset \quad \text{otherwise} \end{cases} \\ & s_{J} : r_{t} = \begin{cases} & \bar{r}_{t} \quad \text{if } I_{t} \in N_{t} \\ & 1 \quad \text{if } I_{t} \notin N_{t} \end{cases} \quad \text{in each period } t \text{ in which } J = I_{t} \text{ with } J = A, B, \end{cases} \end{split}$$

$$s_{RRV}: v_t = \begin{cases} 1 & \text{if } I_t \in N_t \ \land \ r_t \leq \bar{r}_{I_t} \\ 0 & \text{if } I_t \in N_t \ \land \ r_t > \bar{r}_{I_t} \\ 0 & \text{if } I_t = W_t \\ 1 & \text{if } I_t = G_t \end{cases} \quad in \ each \ period \ t.$$

In the normal phase of the CTCEPSP, both parties always comply, the reelection probability is α_J , and rent-seeking is \bar{r}_J in each period in which party J = A, B is in office. In the punishment phase, the party in good standing is re-elected with probability α_{G_t} , the party without standing is re-elected with probability $\alpha_{W_t} - EP$, and rent-seeking is maximal in every period.

Proposition 5 A CTCEPSP constitutes a subgame perfect Constant threshold with compliance and eternal punishment equilibrium (CTCEPE) iff $\bar{r} = \bar{r}_A = \bar{r}_B \in [\bar{r}^*, 1]$ with $\bar{r}^* = \frac{1-\beta \cdot IA}{1-\beta (IA-EP)}$.

Proof. See Appendix B.

In a *CTCEPE*, always playing compliance as incumbent party constitutes a best response independently of the strategy of the other party. Without noncompliance as a possible reaction to noncompliance by the other party, the possibility of equilibria with both parties colluding to play noncompliance does not arise, and the forward induction argument that made the *CTCE* with threshold \bar{r}^* implausible does not apply. For a sophisticated *RRV*, the *CTCEPE* with $\bar{r} = \bar{r}^*$ seems to be a way to implement the most stringent achievable constant rent-seeking threshold \bar{r}^* without allowing the parties to collude on playing noncompliance.¹⁹

However, while the CTCEPE is immune to collusion by the parties, it is problematic for a different reason. Losing the support of the RRV unconditionally and forever is the hardest punishment a party can face with respect to its future electoral prospects. For the RRV, implementing this form of punishment comes at the cost of losing any control over future rent-seeking. Therefore, once the punishment phase is reached, the RRV would prefer to switch to an equilibrium with at least some accountability and less than maximum rent-seeking. For example, once the punishment phase is reached, the RRV could announce she is re-electing incumbent parties that stick to a threshold slightly above \bar{r}^{***} . Such a threshold would make switching back to compliance optimal for any incumbent party, while the RRV would have no

¹⁹The RRV might prefer to use a slightly larger threshold than \bar{r}^* to avoid indifference between compliance and noncompliance by the incumbent party.

reason to renege on her announcement. This casts doubts on the credibility of the punishment in any CTCEPE. The possibility of renegotiation with the parties constitutes a problem not just for the credibility of a CTCEPE but for any punishment that lasts longer than just one period. However, if only punishments that last one period are credible, the RRV has no possibility of stopping the parties from colluding with each other to play noncompliance against stringent thresholds, and it seems impossible for the RRV to implement threshold \bar{r}^* in a credible way. Consequently, a CTCE with threshold \bar{r}^{**} remains the most plausible equilibrium.

The comparative statics for the incumbency advantage and the electoral punishment are qualitatively the same for all thresholds derived, as will be shown in the next Section. Consequently, the most important qualitative results are robust to the details of the equilibrium selection discussed here and in the previous subsection.

2.2.5 Comparative statics

Because there is no accountability for EP = 0, here it is assumed that EP > 0. First, the results for the most stringent threshold \bar{r}^* are derived. The most interesting result is the effect of incumbency advantage on the minimal sustainable amount of constant equilibrium rent-seeking \bar{r}^* derived in Proposition 1:

$$\frac{\partial \bar{r}^*}{\partial IA} = -\frac{\beta^2 \cdot EP}{\left(1 - \beta (IA - EP)\right)^2} < 0.$$
(9)

A larger incumbency advantage increases the maximum level of accountability when there is electoral punishment. The reason for this is that the more likely the respective incumbent party is to be re-elected in future periods, the more important it is to win the next election. As a consequence, the incumbent party is willing to forgo a larger amount of rent-seeking in return for avoiding electoral punishment, and a lower rentseeking threshold is consistent with equilibrium.

Sustainable minimal rents also decrease in EP and β :

$$\frac{\partial \bar{r}^*}{\partial EP} = -\frac{\beta (1 - \beta \cdot IA)}{\left(1 - \beta (IA - EP)\right)^2} < 0, \tag{10}$$

$$\frac{\partial \bar{r}^*}{\partial \beta} = -\frac{EP}{\left(1 - \beta (IA - EP)\right)^2} < 0. \tag{11}$$

These two results are far less surprising than the effect of the incumbency advantage.

While electoral punishment is a necessary condition for the incumbency advantage to influence accountability, the size of the incumbency advantage also provides an upper bound for the electoral punishment because $EP \leq \alpha_B \leq \frac{\alpha_A + \alpha_B}{2} = \frac{IA+1}{2}$. The lowest possible value of \bar{r}^* is reached when IA = EP = 1 and $\bar{r}^* = 1 - \beta$.

Qualitatively, the effects of changes of the incumbency advantage and the electoral punishment are the same for the thresholds \bar{r}^{**} and \bar{r}^{***} derived in Propositions 3 and 4.

$$\frac{\partial \bar{r}^{**}}{\partial IA} = \frac{-\beta^2 \cdot EP}{(1 - \beta (IA - 2EP))^2} < 0, \tag{12}$$

$$\frac{\partial \bar{r}^{***}}{\partial IA} = \frac{-2\beta^2 \cdot EP \left(1 - \beta(1 - \Delta\alpha)\right) \left(3 - 2\beta(IA - EP) - \beta(1 - \Delta\alpha)\right)}{\left(2 - \beta(IA + 1 - \Delta\alpha)\right)^2 \left(1 - \beta(IA - 2 \cdot EP)\right)^2} < 0, \quad (13)$$

$$\frac{\partial \bar{r}^{**}}{\partial EP} = -\beta \frac{1 - \beta \cdot IA}{(1 - \beta (IA - 2EP))^2} < 0, \tag{14}$$

$$\frac{\partial \bar{r}^{***}}{\partial EP} = -\frac{2\beta \left(1 - \beta \cdot IA\right) \left(1 - \beta (I - \Delta \alpha)\right)}{\left(2 - \beta (IA + 1 - \Delta \alpha)\right) \left(1 - \beta (IA - 2 \cdot EP)\right)^2} < 0.$$
(15)

The relationship between \bar{r}^{**} and β is, as expected, a negative one:

$$\frac{\partial \bar{r}^{**}}{\partial \beta} = -\frac{EP}{1 - \beta (IA - 2 \cdot EP)^2} < 0, \tag{16}$$

The effect of an increase of β on the threshold \bar{r}^{***} is always negative for small values of β but can, depending on the other parameters, become positive for large ones:²⁰

$$\frac{\partial \bar{r}^{***}}{\partial \beta} = \frac{4 \cdot EP}{\left(2 - \beta (IA + 1 - \Delta \alpha)\right)^2} \cdot \frac{1 - \beta \cdot IA}{1 - \beta (IA - 2 \cdot EP)}$$

$$- \frac{2 - \beta (IA + 1 - \Delta \alpha - 2 \cdot EP)}{2 - \beta (IA + 1 - \Delta \alpha)} \cdot \frac{2 \cdot EP}{\left(1 - \beta (IA - 2 \cdot EP)\right)^2}$$
(17)

 \bar{r}^{***} is the only threshold that depends on the difference in popularity:

$$\frac{\partial \bar{r}^{***}}{\partial \Delta \alpha} = -\frac{2\beta^2 \cdot EP(1-\beta \cdot IA)}{\left(2-\beta (IA+1-\Delta \alpha)\right)^2 \left(1-\beta (IA-2 \cdot EP)\right)} < 0.$$
(18)

²⁰To give an example, for the values $\Delta \alpha = 0.2$ and EP = 0.3, if IA = 0.2, then $\partial \bar{r}^{***}/\partial \beta$ becomes positive for values $\beta > 0.7143$, but if IA = 0.8, $\partial \bar{r}^{***}/\partial \beta$ remains negative for all values of β .

2.2.6 Stability

The discount factor β does not necessarily constitute a measure of pure time preference and therefore impatience. One possible interpretation is that β , at least partly, reflects the possibility of a severe exogenous shock to the political system. The resulting changes to the rules of the game can potentially make past rent-seeking irrelevant. If the probability that this occurs before any given period t is given by $1 - \beta^s$, and the pure time preference is reflected in a discount factor β^p , then $\beta = \beta^p \cdot \beta^s$.²¹ Section 2.2.5 shows that a smaller discount factor β leads to less accountability. Therefore, if less political stability is reflected in a smaller β^s , this leads, ceteris paribus, to less accountability.

3 The electorate

The model analyzed so far allows for different specifications of the voters and their motives and, with some adjustments, is probably consistent with political systems in which accountability is achieved by means other than elections.²² The following explicit introduction of prospective voter who cast their ballots according to their probabilistic party preferences shows that increased accountability can come at a cost. A party can win an election even if it is less popular among the prospective voters.²³ At the end of this Section, the three examples from the introduction are reconsidered.

3.1 The game

As in Section 2, the amount of rent-seeking to engage in when in office are the only decisions made by the two parties and the parties' objective remains maximizing the dynamic payoff function (1). Instead of a single RRV, the electorate now consists of a continuum of voters, and the incumbent party wins the election if it achieves a share of at least half of the votes. There are two distinct types of voters. All

²¹This approach follows Klingelhöfer (2017).

²²Obviously, this can only be the case for political systems in which politicians are more likely to lose office if their accomplishments are not sufficient to reach some form of performance standard.

²³The source of these preferences is not made explicit in the model, but they can, for example, depend on the policy platforms, the candidates, the influence of the media or the success of the election campaigns of the parties. Most likely, they are due to a combination of all these and other factors.

retrospective voters have the same objective as the RRV in Section 2, given by the dynamic payoff function (2). The prospective voters have preferences over the party in office that are stochastically determined in every period. $\gamma_r \in [0, 0.5]$ denotes the share of retrospective voters and $\gamma_p = 1 - \gamma_r$ the share of prospective voters in the electorate.²⁴

The voting decisions of the retrospective voters Because the electorate consists of a continuum of voters, no single vote is ever pivotal for an election outcome. Nonetheless, all voters are assumed to cast their votes as if they were pivotal. Therefore, all retrospective voters cast identical votes, and the RRV introduced in Section 2 is representative of the retrospective group of voters.

The voting decisions of the prospective voters The share of voters for the incumbent party among the prospective voters is determined in each period when nature draws their share ι_t . The outcome of this random process becomes only observable after the election in period t has taken place. The *PDF* and *CDF* of these independent and identically distributed random variables ι_t depend only on the party J = A, B in office:

$$f(\iota|J) = \begin{cases} 0 & \text{if } \iota < 0 \\ 1 + b_J(2\iota - 1) & \text{if } 0 \le \iota \le 1 \\ 0 & \text{if } \iota > 1 \end{cases}, \ F(\iota|J) = \begin{cases} 0 & \text{if } \iota < 0 \\ \iota + b_J(\iota^2 - \iota) & \text{if } 0 \le \iota \le 1 \\ 1 & \text{if } \iota > 1 \end{cases}$$
(19)

Here, $b_J \in [-1, 1]$ is the bias for or against (if negative) the incumbent party when party J is in office. If there is no bias, $b_J = 0$, and ι is uniformly distributed within the interval [0, 1]. $b_J > 0$ leads to a *preference-driven* advantage for the incumbent party. This preference-driven advantage can be divided into two separate effects. While $b_I \equiv \frac{b_A+b_B}{2}$ denotes the average electoral bias for incumbent party independent of their identity, $\Delta b \equiv b_A - b_B \geq 0$ reflects the difference in popularity between the two parties.²⁵

²⁴If $\gamma_r \ge 0.5$, the retrospective voters alone decide each election. Therefore, any values of $\gamma_r > 0.5$ leads to the same equilibria as $\gamma_r = 0.5$.

²⁵The assumption $\Delta b \ge 0$ ensures consistency with the assumption $\alpha_A > \alpha_B$ made in Section 2.

The probabilities of re-election If the incumbent party has the support of the RRV in period t, it wins iff the sum of the retrospective voters and its supporters among the prospective voters constitute a majority. This is the case iff $\gamma_r + \iota_t \cdot \gamma_p \ge 0.5$ or:

$$\iota_t \ge \frac{0.5 - \gamma_r}{\gamma_p} = \frac{1 - 2\gamma_r}{2(1 - \gamma_r)}$$

Given the distribution of ι_t , the probability that this inequality holds and the incumbent party J = A, B wins with the support of the RRV is:

$$\alpha_J = 1 - F\left(\frac{1 - 2\gamma_r}{2(1 - \gamma_r)}|J\right) = \frac{1}{2(1 - \gamma_r)} + \frac{b_J\left(1 - 2\gamma_r\right)}{4\left(1 - \gamma_r\right)^2}.$$
 (20)

The probability of victory is 1 if $\gamma_r = 0.5$ and the support of the retrospective voters alone decides the election.

If the incumbent party does not have the support of the RRV in period t, it wins iff $\iota_t \cdot \gamma_p \ge 0.5$ or:

$$\iota_t \ge \frac{1}{2\gamma_p} = \frac{1}{2(1-\gamma_r)}$$

Given the distribution of ι_t , the probability that this inequality holds and the incumbent party J = A, B with without the support of the RRV is:

$$\alpha_J - EP = 1 - F\left(\frac{1}{2(1-\gamma_r)}|J\right) = 1 - \frac{1}{2(1-\gamma_r)} + \frac{b_J\left(1-2\gamma_r\right)}{4\left(1-\gamma_r\right)^2}.$$
 (21)

Combining Equations (20) and (21) we find that:

$$EP = \frac{\gamma_r}{1 - \gamma_r}.$$
(22)

Naturally, an incumbent party faces electoral punishment only when the share of retrospective voters is positive.²⁶

3.2 Electorate and equilibrium

Given the values for α_A , α_B , and EP that correspond to the fundamental parameters γ_r , b_I and Δb describing the electorate, the retrospective voters, represented by the

²⁶Because electoral punishment depends only on the share of retrospective voters, estimates of their share in the electorate could provide a proxy for electoral punishment in applied work.

RRV, and the parties and face the same incentives and constraints as in Section 2. The prospective voters vote for their preferred party and are not considered to be players in the formal analysis of the election game. As discussed in 2.2.3, the most plausible outcome is a CTCE, and consequently, for the rest of this section, it is assumed that a CTCE is played. Accordingly, the incumbency advantage, the difference in popularity between the parties, and a party's long-term share of periods in office are given by:

$$IA = \frac{\gamma_r}{1 - \gamma_r} + \frac{b_I \left(1 - 2\gamma_r\right)}{2 \left(1 - \gamma_r\right)^2},\tag{23}$$

$$\Delta \alpha = \frac{\Delta b (1 - 2\gamma_r)}{4 \left(1 - \gamma_r\right)^2},\tag{24}$$

$$LS_J = \frac{2(1 - \gamma_r) - b_K}{4(1 - \gamma_r) - b_J - b_K}.$$
(25)

Equation (23) shows that the equilibrium voting decisions of the retrospective voters lead to an accountability-driven incumbency advantage.²⁷ Iff $b_I > 0$, there is additional preference-driven incumbency advantage that is independent of any popularity difference Δb between the parties.²⁸

Using Equations (22), (23), and (24), it is now possible to restate the rent-seeking thresholds in terms of the fundamental parameters of the model and derive the comparative statics with respect to them:

$$\bar{r}^* = \frac{1 - \beta \cdot \left(\frac{\gamma_r}{1 - \gamma_r} + \frac{b_I (1 - 2\gamma_r)}{2(1 - \gamma_r)^2}\right)}{1 - \beta \cdot \left(\frac{b_I (1 - 2\gamma_r)}{2(1 - \gamma_r)^2}\right)},\tag{26}$$

$$\bar{r}^{**} = \frac{1 - \beta \cdot \left(\frac{b_I(1-2\gamma_r)}{2(1-\gamma_r)^2}\right)}{1 - \beta \cdot \left(\frac{b_I(1-2\gamma_r)}{2(1-\gamma_r)^2} - \frac{\gamma_r}{1-\gamma_r}\right)},\tag{27}$$

$$\bar{r}^{***} = \frac{2 - \beta \left(\frac{1 - 2\gamma_r}{1 - \gamma_r} + \frac{b_I (1 - 2\gamma_r)}{2(1 - \gamma_r)^2} - \frac{\Delta b(1 - 2\gamma_r)}{4(1 - \gamma_r)^2}\right)}{2 - \beta \left(\frac{1}{1 - \gamma_r} + \frac{b_I (1 - 2\gamma_r)}{2(1 - \gamma_r)^2} - \frac{\Delta b(1 - 2\gamma_r)}{4(1 - \gamma_r)^2}\right)} \cdot \frac{1 - \beta \left(\frac{\gamma_r}{1 - \gamma_r} + \frac{b_I (1 - 2\gamma_r)}{2(1 - \gamma_r)^2}\right)}{1 - \beta \left(\frac{b_I (1 - 2\gamma_r)}{2(1 - \gamma_r)^2} - \frac{\gamma_r}{1 - \gamma_r}\right)}.$$
(28)

²⁷That some degree incumbency advantage is a natural feature of models of political accountability even when there are no structural asymmetries between incumbents and opposition has already been pointed out by Austen-Smith and Banks (1989).

²⁸If there is no preference-driven incumbency effect ($b_I = 0$), then $IA = EP = \frac{\gamma_r}{1 - \gamma_r}$. This equality is due to the fact that the incumbency advantage and the electoral punishment are both given by the difference in probability between winning with and without the support of the retrospective voters. A closer inspection of \bar{r}^* , \bar{r}^{**} and \bar{r}^{***} reveals that the degree of accountability is mostly driven by the parameters β and γ_r and less so by b_I (and Δb). This is not surprising because β determines how important future results are for the players and γ_r is the only determinant of the electoral punishment and, as Equation (23) shows, the major determinant of the incumbency advantage, especially whenever γ_r is large.

It is insightful to calculate the effects of the fundamental parameters on the incumbency advantage, the electoral punishment, and the difference in popularity. They can be used to calculate the comparative statics with respect to the fundamental parameters γ_r , b_I and Δb via the chain rule, what clarifies the channels by which the fundamental parameters effect accountability. The nonzero partial derivatives of IA, EP, and $\Delta \alpha$ with respect to γ_r , b_I and Δb are:

$$\frac{\partial IA}{\partial \gamma_r} = \frac{1}{\left(1 - \gamma_r\right)^2} - b_I \frac{\gamma_r}{\left(1 - \gamma_r\right)^3} \ge 0, \ \frac{\partial IA}{\partial b_I} = \frac{1 - 2\gamma_r}{2\left(1 - \gamma_r\right)^2} \ge 0, \tag{29}$$

$$\frac{\partial EP}{\partial \gamma_r} = \frac{1}{(1 - \gamma_r)^2} > 0, \tag{30}$$

$$\frac{\partial \Delta \alpha}{\partial \gamma_r} = -\frac{\Delta b \cdot \gamma_r}{2\left(1 - \gamma_r\right)^3} \le 0, \ \frac{\partial \Delta \alpha}{\partial \Delta b} = \frac{1 - 2\gamma_r}{4\left(\gamma_r - 1\right)^2} \ge 0.$$
(31)

Using the results derived in Section 2.2.5, straightforward chain rule reasoning implies that an increase in b_I , the bias in favor of the incumbent party, increases accountability because it increases the size of the incumbency advantage while EP and $\Delta \alpha$ remain unaffected. Consequently, all three thresholds decrease in b_I . An increase in the share of retrospective voters increases both the incumbency advantage and the electoral punishment and, therefore, also decreases the thresholds \bar{r}^* and \bar{r}^{**} . \bar{r}^{***} is the only threshold that depends on $\Delta \alpha$. Therefore, in the case of \bar{r}^{***} the impact of a chance in γ_r is less straightforward because a larger share of retrospective voters decreases the difference in popularity between the parties, and this, in turn, has a positive effect on \bar{r}^{***} . However, this is not sufficient to overturn the negative relationship between \bar{r}^{***} and γ_r , as is confirmed in the following Proposition 6 that summarizes the nonzero comparative statics of the thresholds with respect to the fundamental parameters γ_r , b_I and Δb .

Proposition 6 $\frac{\partial \bar{r}^*}{\partial \gamma_r} < 0$; $\frac{\partial \bar{r}^{**}}{\partial b_I} \le 0$; $\frac{\partial \bar{r}^{***}}{\partial \gamma_r} < 0$; $\frac{\partial \bar{r}^{***}}{\partial b_I} \le 0$; $\frac{\partial \bar{r}^{***}}{\partial b_I} \le 0$; $\frac{\partial \bar{r}^{***}}{\partial b_I} \le 0$; $\frac{\partial \bar{r}^{***}}{\partial \Delta b} \le 0$. All inequalities hold strictly for $\gamma_r \in (0, 0.5)$.

Proof. See Appendix B. ■

3.3 The costs of accountability

Along the equilibrium path of a CTCE, only the incumbent party can win without the support of the majority of the prospective voters. The probability that this happens in an election in which party J is the incumbent is:

$$\int_{\frac{1-2\gamma_r}{2(1-\gamma_r)}}^{0.5} f(\iota|J)d\iota = \frac{\gamma_r}{2(1-\gamma_r)} \left(1 - b_J \frac{\gamma_r}{2(1-\gamma_r)}\right).$$
(32)

Thus, the probability that an incumbent party is re-elected despite the majority of prospective voters preferring the opposition party is increasing in the share of retrospective voters and is decreasing in the bias in favor of the incumbent party. Consequently, the larger accountably provided by a larger share of retrospective voters does come at the cost of making the election results less representative of voters' preferences. For the case of $\gamma_r < 0.5$, we can calculate the long-term average difference in support of the winning party and the losing party among the prospective voters. I refer to this difference as the average support margin (ASM).²⁹

Proposition 7 If $\gamma_r < 0.5$, the average support margin in favor of the winning party among the prospective voters is given by:

$$ASM = 0.5 - \frac{\gamma_r^2}{2(1-\gamma_r)^2} + \frac{2\gamma_r^3}{3(1-\gamma_r)^3} \cdot \frac{(b_A + b_B)(1-\gamma_r) - b_A \cdot b_B}{4(1-\gamma_r) - b_A - b_B} \le 0.5.$$
(33)

Proof. See Appendix B.

A change in the share of retrospective voters changes the size of the average support margin:

$$\frac{\partial ASM}{\partial \gamma_r} = -\frac{\gamma_r}{(1-\gamma_r)^3} + \frac{2\gamma_r^2}{(1-\gamma_r)^4} \cdot \frac{(b_A+b_B)(1-\gamma_r) - b_A \cdot b_B}{4(1-\gamma_r) - b_A - b_B} + \frac{2\gamma_r^3}{3(1-\gamma_r)^3} \cdot \frac{(b_A-b_B)^2}{(4(1-\gamma_r) - b_A - b_B)^2}$$
(34)

This derivative has the expected negative sign for small and moderately large shares of retrospective voters $\gamma_r > 0$, and whenever the difference in the expected popularity between the parties is small enough (if $b_A = b_B$, then $\frac{\partial ASM}{\partial \gamma_r}$ is never positive). However, $\partial ASM/\partial \gamma_r$ becomes positive for $\gamma_r > 0.4753$ if at least one party J = A, B is re-elected with very high probability (due to a large b_J) while the other

²⁹If $\gamma_r = 0.5$, the party that holds office in the first period stays in office forever.

party is sufficiently less popular whenever in office.³⁰ Inspecting Equation (33) shows that the existence of retrospective voters always leads to a lower ASM compared to the case without any retrospective voters. Therefore, the existence of a small interval of combinations of γ_r , b_A and b_B that results in $\partial ASM/\partial \gamma_r > 0$ does not overturn the insight that the accountability that is only possible with some degree of retrospective voting comes at a cost on the prospective dimension of politics. The exact welfare consequences of the reduced influence of the prospective voters on elections cannot be shown without assumptions about the distribution of preferences of the prospective voters. This is beyond the scope of this paper.

3.4 The three examples reconsidered

In this section, the three examples from the introduction are reconsidered. Equations (26), (27), and (28) show that a large share of retrospective voters γ_r is sufficient to generate the case of mature democracies with large electoral punishment, large incumbency advantage and large degrees of accountability for any values of b_I (and Δb). Moreover, small values of γ_r can generate low electoral punishment in combination with a small incumbency advantage. This can explain the lack of accountability in Central and Eastern European countries after the restoration of democracy. Moreover, in combination with a negative b_I , a small γ_R is consistent with a tiny incumbency advantage or even disadvantage, as can be seen from Equation (23). Thus, while small b_I is helpful but not necessary to generate low levels of accountability, it can explain the empirically observed incumbency disadvantage in some countries.

The third case combines an even larger incumbency advantage with low or nonexistent electoral punishment. While it is possible to generate a reasonable large incumbency advantage of 0.5 even with $\gamma_r = 0$ and therefore no electoral punishment with values of b_I close enough to 1, the microfoundations provided here do not fit the story of rigged elections well. They should therefore not be interpreted as an adequate model of Russia under Putin.

³⁰The intuition behind this result is that for large values of γ_r , Equation (20) implies that even the party that is less popular among prospective voters is likely to stay in office for several periods after being elected once. However, in the periods following a surprise victory of the less popular party until it loses office again, it is, in expectations, less popular among the prospective voters than the more popular party would be. Although a further increase in γ_r from an already large value increases the average number of periods the less popular party spends in office before losing an election, it decreases its chances to win office in the first place sufficiently to increase ASM. This effect plays no role if $b_A = b_B$ and both parties are equally popular.

4 Conclusion

I show that close elections lead to less accountability compared to elections with a sizable incumbency advantage. The reason is that the larger the incumbency advantage is, the larger the influence of the next election on all future election outcomes. This larger influence, in turn, makes every election outcome more consequential. A considerable amount of electoral punishment and a large incumbency advantage are not only compatible with each other but naturally coincide if there is a sufficient share of retrospective voters who hold the incumbent party accountable for excessive rent-seeking. Because retrospective voters are not the only source of incumbency advantage, empirical research should distinguish between these two causes of electoral accountability.

The fact that two parties repeatedly compete and, therefore, can regain office results in the possibility of equilibria with and without compliance for the same performance standard. Consequently, the strictest performance standard consistent with compliance is unlikely to be observed because the parties can collude to play noncompliance whenever they encounter it. Foreseeing this, the voters choose a less stringent performance standard that always leads to compliance in equilibrium.

The probabilistic voting model presented here allows for two different groups of voters that use different voting strategies. In future research, this framework can be extended to additional groups and more voting strategies. For example, the introduction of partisan voters could allow for a detailed analysis of the effects of gerrymandering, the practice of manipulating district boundaries to improve the electoral prospects of a party, which is common in the US. While an enormous body of literature is concerned with gerrymandering, its effects on the incentives of incumbent politicians have so far not received much attention.³¹ However, electoral incentives matter for incumbent politicians as much as equilibrium re-election probabilities do. Consequently, a party that has the opportunity to engage in gerrymandering is likely to have other objectives in addition to increasing the number of districts it is likely to win.

Acknowledgment I would like to thank a very encouraging referee, Rongrong Sun; Florian Biermann; Stephanie Chan and the participants of the Henan-Soul Workshop on

³¹For a recent overview of the literature on gerrymandering, see Holden (2016).

Distributive Justice; Markets and Development 2018; the 2018 Australasian Public Choice Conference in Melbourne; a seminar at RWTH Aachen University; EAI2019 in Xiamen; the Econometric Society Australasian Meeting 2019 in Perth; The Tokyo Conference on Economics of Institutions and Organizations 2019 at the University of Tokyo and seminar participants Xi'an Jiatong Liverpool University and at TU Dortmund for their helpful comments and suggestions. Parts of this research were accomplished during stays at Zeppelin University and RWTH Aachen University. I would like to thank both institutions for their hospitality.

Declarations:

Funding: Not applicable

Conflicts of interest: Not applicable

Availability of data and material: Not applicable

Code availability: Not applicable

Authors' contributions: Single authored

Appendix

Appendix A - German federal elections and chancellors since 1949

Table 1 contains the results of the German federal elections since 1949. The chancellor is elected by the German parliament (Bundestag) whose members are elected in a general election. While the chancellor is not elected directly, her position is the most important outcome of German federal elections, and the party of the chancellor is usually considered to be the winner of the election. The early election of 1983 is problematic because it was held shortly after the FDP, a smaller party that was previously the coalition partner of the SPD, but switched partners and instead formed a coalition with the CDU. This resulted in chancellor Helmuth Schmidt (SPD) losing the office to Helmuth Kohl (CDU) without a general election taking place. Because this happened only a few months before the early election of 1983, at least from the perspective of retrospective voters, the SPD, rather than the CDU, should be seen as the incumbent party in 1983. However, this has only a marginal effect on the incumbency advantage in the sample: With the SPD as the incumbent party in 1983, the incumbent party loses the chancellorship after 4 of 18 elections, still less than 1/4in total, and the SPD wins 4 out of 6 elections in which it provides the incumbent chancellor. Moreover, 1982 has been the only time that the chancellorship switched between SPD and CDU between general elections.

" Insert Table 1 here"

Vear of election	Party affiliation and name of Chancelor	Party affiliation and name of Chancellor	Insumbant party vistory?
rear or election	immediately before the election	immediately after the election	incumbent party victory?
1949	None - first general election after war	CDU Konrad Adenauer	No incumbent party
1953	CDU Adenauer	CDU Konrad Adenauer	yes
1957	CDU Adenauer	CDU Konrad Adenauer	yes
1961	CDU Adenauer	CDU Konrad Adenauer	yes
1965	CDU Ludwig Erhard	CDU Ludwig Erhard	yes
1969	CDU Kurt Georg Kiesinger	SPD Willy Brandt	no
1972	SPD Willy Brandt	SPD Willy Brandt	yes
1976	SPD Helmut Schmidt	SPD Helmut Schmidt	yes
1980	SPD Helmut Schmidt	SPD Helmut Schmidt	yes
1983	CDU Helmut Kohl	CDU Helmut Kohl	Formally yes. However, Kohl (CDU) had only become chancellor a few month before the election when he succeeded Schmidt (SPD). Consequently, retrospective voters could only evaluate the tenure of Schmidt since 1980.
1987	CDU Helmut Kohl	CDU Helmut Kohl	yes
1990	CDU Helmut Kohl	CDU Helmut Kohl	yes
1994	CDU Helmut Kohl	CDU Helmut Kohl	yes
1998	CDU Helmut Kohl	SPD Gerhard Schröder	no
2002	SPD Gerhard Schröder	SPD Gerhard Schröder	yes
2005	SPD Gerhard Schröder	CDU Angela Merkel	no
2009	CDU Angela Merkel	CDU Angela Merkel	yes
2013	CDU Angela Merkel	CDU Angela Merkel	yes
2017	CDU Angela Merkel	CDU Angela Merkel	yes

Table 1 - German election results since 1949

Appendix B - Proofs

Proof Lemma 1. Let $p_{J,t}$ denote the probability that party J is in office in period t. The probability $p_{J,t+1}$ of party J being in office in period t+1 given a probability $p_{J,t}$ of being in office in period t is:

$$p_{J,t+1} = q_J \cdot p_{J,t} + (1 - q_K)(1 - p_{J,t}) = (q_J + q_K - 1)p_{J,t} + (1 - q_K).$$

If $q_J + q_K < 2$, solving forward gives:

$$p_{J,t+s} = \frac{1 - q_K}{2 - q_J - q_K} + (q_K + q_J - 1)^s \cdot \left(p_{J,t} - \frac{1 - q_K}{2 - q_J - q_K} \right) \text{ for all } s \ge t.$$

If party J is the incumbent party in period t, $p_{J,t} = 1$; if party J is the opposition party in period t, $p_{J,t} = 0$. Substituting accordingly for $p_{J,t}$ leads to $q_{J,s}^{I}$ and $q_{J,s}^{O}$ for the case $q_{J} + q_{K} < 2$ in Equations (3) and (4). If $q_{J} = q_{K} = 1$, the incumbent party in period t is also the incumbent party in period s.

Proof Corollary 1. If $q_J + q_K < 2$:

$$\begin{split} V_J^I(q_J, q_K, \rho_J) &= \sum_{s=0}^{\infty} \beta^s \cdot q_{J,s}^I \cdot \rho_J \\ &= \left(\frac{1 - q_K}{(1 - \beta)(2 - q_J - q_K)} + \frac{1}{1 - \beta(q_J + q_K - 1)} \cdot \frac{1 - q_J}{2 - q_J - q_K} \right) \rho_J \\ &= \frac{1}{1 - \beta} \cdot \frac{1 - \beta \cdot q_K}{1 - \beta(q_J + q_K - 1)} \cdot \rho_J, \\ V_J^O(q_J, q_K, \rho_J) &= \sum_{s=0}^{\infty} \beta^s \cdot q_{J,s}^O \cdot \rho_J \\ &= \left(\frac{1 - q_K}{(1 - \beta)(2 - q_J - q_K)} - \frac{1}{1 - \beta(q_J + q_K - 1)} \cdot \frac{1 - q_K}{2 - q_J - q_K} \right) \rho_J \\ &= \frac{1}{1 - \beta} \cdot \frac{\beta(1 - q_K)}{1 - \beta(q_J + q_K - 1)} \cdot \rho_J. \end{split}$$

If $q_J = q_K = 1$:

$$V_J^I(1,1,\rho_J) = \sum_{s=0}^{\infty} \beta^s \cdot \rho_J = \frac{\rho_J}{1-\beta} = \frac{1}{1-\beta} \cdot \frac{1-\beta \cdot q_K}{1-\beta(q_J+q_K-1)} \cdot \rho_J,$$
$$V_J^O(1,1,\rho_J) = 0 = \frac{1}{1-\beta} \cdot \frac{\beta(1-q_K)}{1-\beta(q_J+q_K-1)} \cdot \rho_J.$$

For any $0 \leq q_J, q_K \leq 1$:

$$\begin{split} \Delta V_J(q_J, q_K, \rho_J) &\equiv V_J^I(q_J, q_K, \rho_J) - V_J^O(q_J, q_K, \rho_J) \\ &= \frac{1}{1 - \beta} \cdot \frac{1 - \beta \cdot q_K}{1 - \beta(q_J + q_K - 1)} \cdot \rho_J - \frac{1}{1 - \beta} \cdot \frac{\beta(1 - q_K)}{1 - \beta(q_J + q_K - 1)} \cdot \rho_J \\ &= \frac{\rho_J}{1 - \beta(q_J + q_K - 1)}. \end{split}$$

Proof Proposition 1. The single deviation principle applies because the multi-stage game under consideration has overall payoffs that are discounted sums of uniformly bounded per-period payoffs. Given the strategy profile, a deviation to a level of rent-seeking strictly below the threshold makes the incumbent party worse off in the current period without changing expected future payoffs. An incumbent party J = A, B that deviates once by engaging in rent-seeking above the threshold has an expected discounted future payoff of, at most:

$$1 + \beta \left((\alpha_J - EP) V_J^I(\alpha_J, \alpha_K, \bar{r}_J) + (1 - (\alpha_J - EP)) V_J^O(\alpha_J, \alpha_K, \bar{r}_J) \right).$$
(35)

 $V_J^I(\alpha_J, \alpha_K, \bar{r}_J)$, the expected discounted payoff from engaging in rent-seeking at the threshold level, can be restated recursively as:

$$V_J^I(\alpha_J, \alpha_K, \bar{r}_J) = \bar{r}_J + \beta \left(\alpha_J V_J^I(\alpha_J, \alpha_K, \bar{r}_J) + (1 - \alpha_J) V_J^O(\alpha_J, \alpha_K, \bar{r}_J) \right).$$
(36)

It is a best response for an incumbent party J to always comply with the threshold amount \bar{r}_J as long as (36) is not smaller than (35). Using Corollary 1 and the definition of IA, this condition can be restated as:

$$1 - \bar{r}_J \le \beta \cdot EP \cdot \frac{\bar{r}_J}{1 - \beta \cdot IA}.$$
(37)

Equation (37) holds with equality for J = A, B iff $\bar{r}_J = \bar{r}^*$:

$$\bar{r}^* = \frac{1 - \beta \cdot IA}{1 - \beta (IA - EP)}.$$
(38)

Because the left-hand side of Equation (37) decreases in \bar{r}_J while its right-hand side increases in \bar{r}_J , condition (37) holds for all $\bar{r}_J \geq \bar{r}^*$. The strategy of the *RRV* is a best response iff both parties engage in the same constant level of rent-seeking \bar{r} whenever they are in office, and therefore the vote of the *RRV* does not affect her expected payoff. Combining the conditions for optimality of the rent-seeking and the voting condition, it follows that any *CTCSP* with a constant threshold $\bar{r} \in [\bar{r}^*, 1]$ for both parties constitutes a subgame perfect *CTCE*.

Proof Proposition 2. By engaging in the maximum amount of rent-seeking whenever in office, party J = A, B can guarantee itself an expected discounted payoff of at least $V_J^I(\alpha_J - EP, \alpha_K, 1)$ in periods in which it is the incumbent party, and of at least $V_J^O(\alpha_J - EP, \alpha_K, 1)$ in periods in which it is the opposition party. Therefore, no lower expected discounted payoffs for the incumbent and the opposition parties are consistent with any equilibrium. It remains to be shown that $V_J^I(\alpha_J - EP, \alpha_K, 1)$ and $V_J^O(\alpha_J - EP, \alpha_K, 1)$ are identical to the expected discounted payoffs in the *CTCE* with threshold \bar{r}^* :

$$V_J^I(\alpha_J - EP, \alpha_K, 1) = \frac{1}{1 - \beta} \cdot \frac{1 - \beta \alpha_K}{1 - \beta (\alpha_J - EP + \alpha_K - 1)}$$
$$= \frac{1}{1 - \beta} \cdot \frac{1 - \beta \alpha_K}{1 - \beta (\alpha_J + \alpha_K - 1)} \cdot \frac{1 - \beta (\alpha_J + \alpha_K - 1)}{1 - \beta (\alpha_J + \alpha_K - 1 - EP)} = V_J^I(\alpha_J, \alpha_K, \bar{r}^*),$$

$$V_J^O(\alpha_J - EP, \alpha_K, 1) = \frac{\beta}{1 - \beta} \cdot \frac{1 - \alpha_K}{1 - \beta(\alpha_J - EP + \alpha_K - 1)}$$
$$= \frac{\beta}{1 - \beta} \cdot \frac{1 - \alpha_K}{1 - \beta(\alpha_J + \alpha_K - 1)} \cdot \frac{1 - \beta(\alpha_J + \alpha_K - 1)}{1 - \beta(\alpha_J + \alpha_K - 1 - EP)} = V_J^O(\alpha_J, \alpha_K, \bar{r}^*).$$

Because both parties expect the lowest possible expected discounted rent-payments consistent with equilibrium and the game is constant-sum, it follows that in the CTCE with threshold \bar{r}^* , the RRV achieves the largest expected discounted payoff consistent with equilibrium.

Proof Proposition 3. As in Proposition 1, the single-deviation principle is applied.

Not complying but engaging in any level of rent-seeking below the maximum amount of 1 decreases the expected discounted payoff of an incumbent party. An incumbent party J = A, B that deviates once and complies with the threshold has an expected discounted future payoff of, at most:

$$\bar{r}_J + \beta \cdot \alpha_J V_J^I(\alpha_J - EP, \alpha_K - EP, 1) + \beta (1 - \alpha_J) V_J^O(\alpha_J - EP, \alpha_K - EP, 1).$$
(39)

 $V_J^I(\alpha_J - EP, \alpha_K - EP, 1)$, the expected discounted value from not deviating and therefore engaging in maximum amount of rent-seeking, can be rewritten recursively as:

$$V_J^I(\alpha_J - EP, \alpha_K - EP, 1)$$

$$= 1 + \beta(\alpha_J - EP)V_J^I(\alpha_J - EP, \alpha_K - EP, 1)$$

$$+ \beta(1 - (\alpha_J - EP))V_J^O(\alpha_J - EP, \alpha_K - EP, 1).$$

$$(40)$$

It is a best response for an incumbent party J never to comply with the threshold amount \bar{r} as long as (40) is not smaller than (39). Using Corollary 1 and the definition of IA, this condition becomes:

$$1 - \bar{r}_J \ge \beta \cdot EP \cdot \frac{1}{1 - \beta (IA - 2 \cdot EP)}.$$
(41)

Equation (41) holds with equality for J = A, B iff $\bar{r}_J = \bar{r}^{**}$:

$$\bar{r}^{**} = \frac{1 - \beta(IA - EP)}{1 - \beta(IA - 2 \cdot EP)}.$$
(42)

Because the left-hand side of Equation (41) decreases in \bar{r}_J , \bar{r}^{**} is the largest threshold consistent with noncompliance. If EP > 0, then $\bar{r}^{**} < 1$ and maximum rent-seeking is optimal for party J = A, B iff $\bar{r}_J \in [0, \bar{r}^{**}]$. If EP = 0, then $\bar{r}^{**} = 1$ and maximum rent-seeking is optimal for party J for any $\bar{r}_J \in [0, 1)$. The strategy of the retrospective voter is a best response because the level of rent-seeking is maximal in every period regardless of how she votes.

Proof Proposition 4. Once more, the single deviation principle is applied. The subgame reached after compliance by one of the parties is structurally identical to the CTCE. Therefore, to be consistent with subgame perfection, the same conditions

as those in the CTCE apply, and the threshold must be identical for both parties and not smaller than \bar{r}^* . If EP = 0, $\bar{r}^* = 1$, and the subgame beginning after one of the two parties complies once does not constitute a CTCE for any threshold $\bar{r} < 1$. Therefore, the case of EP = 0 is inconsistent with a SCTNE. In the normal phase, an incumbent party that deviates and restricts its rent-seeking to the threshold \bar{r}_J has an expected payoff of $V_J^I(\alpha_J, \alpha_K, \bar{r}_J)$, the same as in the CTCE with threshold \bar{r}_J . If the incumbent party stays on the equilibrium path in the normal phase of the game, its expected present discounted payoff is $V_J^I(\alpha_J - EP, \alpha_K - EP, 1)$. Therefore, a SCTNSP can constitute a subgame perfect SCTNE only for thresholds \bar{r}_J for which:

$$V_J^I(\alpha_J - EP, \alpha_K - EP, 1) \ge V_J^I(\alpha_J, \alpha_K, \bar{r}_J) \text{ for } J = A, B.$$
(43)

Let \bar{r}_J^{***} be implicitly defined by $V_J^I(\alpha_J - EP, \alpha_K - EP, 1) = V_J^I(\alpha_J, \alpha_K, \bar{r}_J^{***})$. In Equation (43), the left-hand side is a constant while the right-hand side increases in \bar{r}_J^{***} . Therefore, if $\bar{r}_J^{***} < 1$, then \bar{r}_J^{***} constitutes the largest threshold for which it is optimal for party J to play noncompliance and engage in maximum rent-seeking. Solving for \bar{r}_J^{***} gives:

$$\bar{r}_{J}^{***} = \frac{\frac{1-\beta(\alpha_{K}-EP)}{1-\beta(\alpha_{J}-EP+\alpha_{K}-EP-1)}}{\frac{1-\beta\alpha_{K}}{1-\beta(\alpha_{J}+\alpha_{K}-1)}} = \frac{1-\beta(\alpha_{K}-EP)}{1-\beta\alpha_{K}} \cdot \frac{1-\beta(\alpha_{J}+\alpha_{K}-1)}{1-\beta(\alpha_{J}+\alpha_{K}-1-2\cdot EP)}.$$

It follows that $\bar{r}_A^{***} = \bar{r}_B^{***}$ iff $\alpha_A = \alpha_B$ and $\bar{r}_A^{***} < \bar{r}_B^{***}$ iff $\alpha_A > \alpha_B$. Consequently,

$$\bar{r}^{***} = \bar{r}_A^{***} = \frac{1 - \beta(\alpha_B - EP)}{1 - \beta\alpha_B} \cdot \frac{1 - \beta(\alpha_A + \alpha_B - 1)}{1 - \beta(\alpha_A + \alpha_B - 1 - 2 \cdot EP)}$$
$$= \frac{2 - \beta(IA + 1 - \Delta\alpha - 2 \cdot EP)}{2 - \beta(IA + 1 - \Delta\alpha)} \cdot \frac{1 - \beta \cdot IA}{1 - \beta(IA - 2 \cdot EP)}$$

provides the upper bound for an identical threshold for both parties that is consistent with noncompliance in the normal phase of a SCTNE. Because EP > 0 in any SCTNE, it follows that $\bar{r}^{***} < 1$.

Proof Proposition 5. Again, the single deviation principle is applied. The strategy of the RRV does not influence her payoff because both parties engage in the same level of rent-seeking iff $\bar{r}_A = \bar{r}_B$. This is also true in the *punishment phase* where the level of rent-seeking increases to the maximum level of 1. Consequently, the voting decisions of the RRV constitute a best response, while different thresholds for the

parties would not be consistent with equilibrium.

 $V_J^I(\alpha_J - EP, \alpha_K, 1)$ is the expected discounted payoff of an incumbent party that deviates in the normal phase of the CTCEPSP by engaging in the maximum amount of rent-seeking and enters the punishment phase as a party without standing. $V_J^I(\alpha_J, \alpha_K, \bar{r}^*)$ is the expected discounted payoff of an incumbent party from complying with the threshold \bar{r}^* . In the proof of Proposition 2, it is shown that $V_J^I(\alpha_J - EP, \alpha_K, 1) = V_J^I(\alpha_J, \alpha_K, \bar{r}^*)$. The equality of the expected discounted payoffs implies that \bar{r}^* is the lowest threshold consistent with an CTCEPE. In the punishment phase of the game, maximum rent-seeking is optimal for both parties because rent-seeking does not influence their expected future payoffs.

Proof Proposition 6.

$$\begin{split} \frac{\partial t^*}{\partial \gamma_r} &= \frac{\partial t^*}{\partial IA} \cdot \frac{\partial IA}{\partial \gamma_r} + \frac{\partial t^*}{\partial EP} \cdot \frac{\partial EP}{\partial \gamma_r} \\ &= -\frac{4\beta^2(1-\gamma_r)^3\gamma_r}{(2(1-\gamma_r)^2 - \beta \cdot b_I(1-2\gamma_r))^2} \cdot \left(\frac{1}{(1-\gamma_r)^2} - b_I\frac{\gamma_r}{(1-\gamma_r)^3}\right) \\ &- \frac{2\beta(1-\gamma_r)^2(2(1-\gamma_r)^2 - 2\beta(1-2\gamma_r))^2}{(2(1-\gamma_r)^2 - \beta \cdot b_I(1-2\gamma_r))^2} \cdot \frac{1}{(1-\gamma_r)^2} < 0, \\ \frac{\partial t^*}{\partial t_I} &= \frac{\partial t^*}{\partial IA} \cdot \frac{\partial IA}{\partial b_I} = -\frac{4\beta^2(1-\gamma_r)^3\gamma_r}{(2(1-\gamma_r)^2 - \beta \cdot b_I(1-2\gamma_r))^2} \cdot \frac{(1-2\gamma_r)}{(2(1-\gamma_r)^2)} \leq 0, \\ \frac{\partial t^{**}}{\partial \gamma_r} &= \frac{\partial t^{**}}{\partial IA} \cdot \frac{\partial IA}{\partial \gamma_r} + \frac{\partial t^{**}}{\partial EP} \cdot \frac{\partial EP}{\partial \gamma_r} \\ &= -\frac{\beta^2 \cdot \frac{1-\gamma_r}{(1-\gamma_r)^2 - \beta \cdot b_I(1-2\gamma_r))^2} \cdot \left(\frac{1}{(1-\gamma_r)^2} - b_I\frac{\gamma_r}{(1-\gamma_r)^3}\right) \\ &- \beta \frac{1-\beta(\frac{b_I(1-2\gamma_r)}{(2(1-\gamma_r)^2 - \frac{1-\gamma_r}{1-\gamma_r})})^2 \cdot \left(\frac{1}{(1-\gamma_r)^2} - b_I\frac{\gamma_r}{(1-\gamma_r)^3}\right) \\ &- \beta \frac{1-\beta(\frac{b_I(1-2\gamma_r)}{(2(1-\gamma_r)^2 - \frac{1-\gamma_r}{1-\gamma_r})})^2 \cdot \frac{1}{(1-\gamma_r)^2} < 0, \\ \frac{\partial t^{***}}{\partial b_I} &= \frac{\partial t^{***}}{\partial EP} \cdot \frac{\partial EP}{\partial t_I} + \frac{\partial t^{***}}{\partial t_I - \frac{1-\gamma_r}{(1-\gamma_r)^2 - \frac{1-\gamma_r}{1-\gamma_r})} \cdot \frac{1}{(1-\gamma_r)^2} \cdot \frac{1}{(1-\gamma_r)^2} < 0, \\ \frac{\partial t^{***}}{\partial \gamma_r} &= \frac{\partial t^{***}}{\partial EP} \cdot \frac{\partial EP}{\partial t_I} + \frac{\partial t^{***}}{\partial t_I - \frac{1-\gamma_r}{(1-\gamma_r)^2 - \frac{1-\gamma_r}{1-\gamma_r})}} \cdot \frac{1}{(1-\gamma_r)^2} \cdot \frac{1}{(1-\gamma_r)^2} \cdot \frac{1}{(1-\gamma_r)^2} + \frac{1}{(1-\gamma_r)^2} \cdot \frac{1}{(1-\gamma_r)^2} - \frac{1-\gamma_r}{(1-\gamma_r)^2} + \frac{1}{(1-\gamma_r)^2} - \frac{1-\gamma_r}{1-\gamma_r})} \cdot \frac{1}{(1-\gamma_r)^2} \cdot \frac{1}{(1-\gamma_r)^2} \cdot \frac{1}{(1-\gamma_r)^2} + \frac$$

$$= -\frac{2\beta^2 \frac{\gamma_r}{1-\gamma_r} \left(1 - \beta \left(\frac{\gamma_r}{1-\gamma_r} + \frac{b_I (1-2\gamma_r)}{2(1-\gamma_r)^2}\right)\right)}{\left(1 - \beta \left(\frac{b_I (1-2\gamma_r)}{2(1-\gamma_r)^2} - \frac{\gamma_r}{1-\gamma_r}\right)\right) \left(2 - \beta \left(\frac{\gamma_r}{1-\gamma_r} + \frac{b_I (1-2\gamma_r)}{2(1-\gamma_r)^2} + 1 - \frac{\Delta b (1-2\gamma_r)}{4(1-\gamma_r)^2}\right)\right)^2} \frac{1-2\gamma_r}{4(1-\gamma_r)^2} \le 0.$$

Terms that contain partial derivatives that are 0, e.g. $\frac{\partial EP}{\partial b_I}$, are not stated. All the inequalities hold strictly for $\gamma_r \in (0, 0.5)$. **Proof Proposition 7.**

$$\begin{split} ASM &= \sum_{J=A,B} LS_J \cdot \left(\int_{\frac{1-2\gamma_r}{2(1-\gamma_r)}}^1 (2\iota-1)f(\iota|J)d\iota + \int_0^{\frac{1-2\gamma_r}{2(1-\gamma_r)}} (1-2\iota)f(\iota|J)d\iota \right) \\ &= \sum_{J=A,B} LS_J \cdot \left(\int_{\frac{1-2\gamma_r}{2(1-\gamma_r)}}^1 (2\iota-1)(1+b_J(2\iota-1)d\iota + \int_0^{\frac{1-2\gamma_r}{2(1-\gamma_r)}} (1-2\iota)(1+b_J(2\iota-1))d\iota \right) \\ &= \sum_{J=A,B} LS_J \cdot \left(0.5 - \frac{\gamma_r^2}{2(1-\gamma_r)^2} + \frac{\gamma_r^3}{3(1-\gamma_r)^3} \cdot b_J \right) \\ &= 0.5 - \frac{\gamma_r^2}{2(1-\gamma_r)^2} + \frac{2\gamma_r^3}{3(1-\gamma_r)^3} \cdot \frac{(b_A+b_B)(1-\gamma_r) - b_A \cdot b_B}{4(1-\gamma_r) - b_A - b_B} \le 0.5. \end{split}$$

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