A Non-Iterative Reasoning Algorithm for Fuzzy Cognitive Maps based on Type 2 Fuzzy Sets

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ABSTRACT

A Fuzzy Cognitive Map (FCM) is a causal knowledge graph connecting concepts using directional and weighted connections making it an effective approach for reasoning and decision making. However, the modelling and reasoning capabilities of a conventional FCM for real world problems in the presence of uncertain data is limited as it relies on Type 1 Fuzzy Sets (T1FSs). In this work, we extend the capability of FCMs for capturing greater uncertainties in the interrelations of the modelled concepts by introducing a new reasoning algorithm that uses Type 2 Fuzzy Sets based on z slices for the modelling of uncertain weights connecting FCM's concepts. These Type 2 Fuzzy Sets are generated using interval valued data from surveyed participants and aggregated using the Interval Agreement Approach method. Our algorithm performs late defuzzification of the FCM's values at the end of the reasoning process, preserving the uncertainty in values for as long as possible. The proposed algorithm is applied to the evaluation of the performance of modules of an undergraduate Mathematical programme. The results obtained show a greater correlation to domain experts' subjective knowledge on the modules' performance than both a corresponding FCM with weights modelled using T1FS and a statistical method currently used for evaluating the modules' performance. Sensitivity analysis conducted demonstrates that the new algorithm effectively preserves the propagation of uncertainty captured from input data.

Keywords: Fuzzy Cognitive Map (FCM), reasoning algorithm, Interval Agreement Approach (IAA), Type 2 Fuzzy Sets (T2FSs), sensitivity analysis

1. Introduction

Fuzzy Cognitive Maps (FCMs) are fuzzy directed graphs that represent information about causal relations among interrelated concepts of a modelled system. They were introduced by Bart Kosko [1] as an extension of Cognitive Maps [2] that introduced fuzziness in causal relations and concepts for modeling real world systems. The rationale for this was the existence of data uncertainties in assigning the values of concepts and causal relations, which could be handled more effectively using fuzzy sets. In the last few decades, there has been a noticeable research trend towards applying FCMs in various domain areas [3]. The literature reveals the success of FCMs in reasoning and modelling in Engineering [4], [5], Medicine [6], Business [7], Software Engineering [8] and Politics [9]. FCMs have gained momentum in these fields due to their simplicity and strong mathematical structure that enhances their prediction, analysis and reasoning capabilities.

Although the effectiveness of conventional FCMs were demonstrated in prior work, they had some drawbacks regarding their reasoning and modelling abilities. Conventional FCMs cannot be effectively used in applications requiring a high level of uncertainty and/or where relations within the modelled domain are nonlinear and/or non-monotonic. Typically, FCMs rely on crisp values generated by defuzzification of the fuzzy sets that represent weights between concepts. In this way, an FCMs' ability to represent and control knowledge with high randomness and uncertainties between the concepts is hindered. Moreover, conventional FCMs cannot handle more than one relation between the concepts, and it cannot model a grey domain environment (an environment with multi-meaning). To overcome these shortcomings, several extensions to conventional FCMs have been proposed, which are classified into three categories, based on the types of drawbacks they aim to overcome [10]: 1) overcoming the drawbacks of modelling uncertainty and handling more relationships between the concepts, 2) dynamicity and 3) drawbacks related to a rule-based knowledge representation.

Most of these proposed extensions focused on improving the representation of the weight associated with causal relations between FCM concepts as the weights play a crucial role for knowledge representation and uncertainty propagations while reasoning. For example, Intuitionistic fuzzy sets (IFSs) [11] were used to model causal edges of a FCM in [12, 13] and thus the iterative reasoning algorithm was modified to be compatible with the introduced IFSs. In IFSs, the concept of hesitancy was modelled by assigning not just a degree of belonging of an element to an IFS, but the degree of the element's non-belongingness to the set as well. Thus, Intuitionistic Fuzzy Cognitive Maps (iFCMs) were introduced to capture the hesitancy of experts in modelling of interrelations between the concepts using *If-Then* rules. Although, the iFCM was less affected by missing input data compared to the conventional FCM, incorporating IFSs made the reasoning process for each iteration much more complex.

A Fuzzy Grey Cognitive Map (FGCM), proposed in [14], was based on Grey Systems to represent domains with a high level of uncertainty, which included limited and incomplete data. In the FGCM, the weights of causal links are represented by grey intervals, which enhance the capability of the FGCM to represent the uncertain relationships of the domain. Grey Intervals measure both the intensity of existing causal relationships between two concepts and absent relations between any two concepts with partially or completely unknown intensity. Although the FGCM is adapted for handling uncertainty using the grey intervals, it is incapable of modelling dynamic and nonlinear relations. The conventional FCM was extended to the Triangular Fuzzy Cognitive Map in [15] by representing uncertain interrelations among the concepts using triangular fuzzy numbers. As a further step offering additional expressiveness and flexibility in representing values of concepts and weights of causal relationships, the approach proposed in [16] represented the concepts and edges of the FCM using intervals. In this approach, the membership degrees of an element's belonging to the concept and the edge's weight were given as intervals whose widths were understood as a measure of uncertainty. In [17], edges in FCMs were modelled using fuzzy *If-Then* rules that considered standard semantics of fuzzy sets, where the qualitative representation of the knowledge about causal relations was represented using T1FSs.

Although the former extensions of FCMs have achieved noticeable success in reasoning in the presence of imprecise or missed information, the weights of their causal relations rely on T1FSs, which limit their ability to capture higher levels of uncertainties associated with real world applications. This deficiency of the T1FS is due to its crisp membership function that hinders the modelling ability of the T1FS in domains that involve a high level of uncertainty. This shortcoming of the T1FS led to the extension of T1FSs through the introduction of a higher level of fuzzy sets, where the grade of the membership is uncertain.

Type 2 Fuzzy Sets (T2FSs) are fuzzy sets that represent the membership's grades of elements, called the secondary membership grades, using T1FSs. The T2FSs were defined to handle higher levels of uncertainty that were present in many real-world problems [18] and to handle uncertain membership functions. Thus, T2FSs can model a high level of uncertainty, where determining the precise membership of the fuzzy set is difficult or even impossible. Although T2FSs are shown to outperform T1FSs in modelling uncertainties, the complexities of computation with the T2FSs are increased due to the existence of an additional two dimensions required for the T1FS membership grades. To overcome these complexities, some novel representations of T2FSs were proposed; for example, Interval Type 2 Fuzzy Sets (IT2FSs) were introduced in [18], where domains of fuzzy sets were intervals. IT2FSs were used in different applications, for example, to model the words and hence capture more linguistic uncertainties in [19], and [20], or to design a fuzzy fault detection filter for Markov jump systems in [21] and [22].

Although the success of IT2FSs in the former applications was acknowledged, as they offered a wide scope for capturing and accurately representing input uncertainties, several studies have suggested the necessity of moving to an alternative representation of T2FS, where the third dimension can be exploited to capture more uncertainties. In that respect, zT2FS were defined to reduce complexities of representation and calculations for general T2FSs [23] to fully utilise their third dimension. Whilst in IT2FSs the degree of the secondary membership grade of each element is fixed to be 1 in the third dimension, in zT2FSs it is a value between 0 and 1, inclusively. The approach based on the zT2FS supports the smooth transition from a fuzzy logic system relying on IT2FSs to a fuzzy logic system relying on zT2FSs. zT2FSs were used in various applications, for example, in [24] and [25], where the use of T1FSs and IT2FSs was not possible due to a high level of uncertainty; however, it was well captured by zT2FSs.

As demonstrated, the weights of the casual relations in FCMs are crucial for the propagation of uncertainty and T2FSs and their representations, such as IT2FSs and zT2FSs, are more capable than T1FSs for capturing more uncertainties and hence enhancing the decision modelling. Some methods were proposed to enhance FCMs by introducing T2FSs to the modelling of their edges' weights. For example, in [26], two Interval Type 2 Fuzzy Cognitive Maps (IT2FCMs) for a flight control system were proposed, one for stabilizing the attitude and altitude dynamics and one for tracking trajectories. Weights of the causal relations between the concepts were represented by IT2FSs to capture the inter uncertainty of the experts' opinions about the assigned weights. Subsequently, these fuzzy weights were defuzzified to scalar values to be used in the FCM's reasoning algorithm. A comparison between the IT2FCM and the FCM which relied on T1FSs weights (the T1FCM), showed that the IT2FCM performed better than the T1FCM in the presence of noise and imperfect conditions, as the IT2FSs capture more uncertainties compared to the T1FSs.

In [27], a methodology for modelling weights of a FCM edges using zT2FSs was proposed, creating a zT2FCM for an autism diagnosis in toddlers. The proposed zT2FCM had 20 concepts that had casual relations, with one decision concept that predicted the autistic disorder. Weights were generated using the Interval Agreement Approach (IAA) [28] to capture high level uncertainties in the presence of imprecise interval valued data acquired from different doctors in a hospital. The results presented were more accurate and consistent with doctors' opinions compared with those of a conventional FCM presented in [29], using the same data.

In both these studies, the reasoning method applied in the proposed IT2FCM and zT2FCM, respectively, was iterative, where the value of a decision concept was calculated using an arithmetic function and then defuzzified into a scalar value in each iteration. This suggests a loss of information captured in the weights pertaining to the Type 2 Footprint of Uncertainties [18] and secondary membership values (z slices), that can no longer propagate to influence the selected decision concept.

The research presented in this paper is motivated by two important complex issues identified from the previous works. Firstly, representation of weights of FCM's edges using a particular type of fuzzy set is crucial for the knowledge capture, modelling and its propagation through the FCM. Initially, the weights were represented using T1FSs. However, to overcome some of the identified drawbacks when using this modelling tool, several other approaches were proposed, such as zT2FSs. Second, the reasoning algorithm used in all the previous work was iterative and required defuzzification of the weights in the FCMs that could inevitably have led to the loss of some information.

Therefore, this research proposes an extension of FCMs and their reasoning algorithm to overcome the drawbacks of modelling and reasoning in the presence of uncertainty in relationships between the concepts. The main contributions of this research are as follows:

1) Interval Agreement Approach (IAA) is applied to aggregate both intra and inter uncertainty of experts to model the FCM's weights using zT2FSs that are kept in the reasoning algorithm in their fuzzy set forms until the end of the reasoning process.

2) A new non-iterative reasoning algorithm is developed that uses zT2FSs rather than their defuzzified values; the output value of a decision concept is only defuzzified at the very end of the reasoning process. This late defuzzification reduces the chance of losing information and uncertainties captured in the modelled system compared to using the conventional reasoning process.

3) The accuracy of the proposed reasoning algorithm is examined using the case study of a real-world problem of evaluating the performance of 30 modules taught on an undergraduate Mathematical programme. A novel FCM is generated for this problem by collecting experts' opinions. Results obtained by applying the new reasoning algorithm are compared with those generated by a method based on statistics, currently used in the Department of Mathematics and Applied Sciences under consideration, a standard FCM with weights represented using T1FSs and subjective opinions of lecturers in the Department. The comparison is performed using a statistical measure of correlation.

4) Sensitivity analysis carried out provides new insights into the impact of an FCM structure on uncertainty propagation. Experiments demonstrate that the new reasoning algorithm preserves the propagation of uncertainty across weighted connections in the FCM during the reasoning process.

The rest of this paper is organised as follows. Section 2 provides an essential background about the main concepts related to this paper, including FCMs, zT2FSs and IAA that generates z slices. Section 3 presents the new proposed reasoning algorithm. Section 4 describes a case study related to the evaluation of module performance using different methods and reasoning algorithms, including the reasoning algorithm proposed. Section 5 presents results obtained using these different methods and their comparison. The sensitivity analysis is presented in Section 6. Finally, Section 7 includes the conclusion and future work.

2. Background

In this section, we present background concepts relevant to the proposed reasoning algorithm, including FCMs, zT2FSs and IAA.

2.1. FCM

An FCM's structure comprises of a set of nodes, a set of weighted and directed edges (links) between the nodes [1]. The nodes represent the main concepts of the modelled system and the weighted edges represent the causal relations among the nodes. The mathematical structure of the FCM, as shown in Fig. 1, consists of m concepts $C_i,\ i = 1,2,\dots,m$, linked by the weighted and directed edges e_{ij}, where each edge e_{ij} has weight $W_{i,j}$ of a causal relation from concept j which affects concept i, $i = 1,\dots,m$ and $j = 1,\dots,m-1$.

All weights $W_{i,j}$ can be arranged in a connection matrix as follows:

$$\begin{bmatrix} W_{1,1} & W_{1,2} & \dots & W_{1,m-1} \\ W_{2,1} & W_{2,2} & \dots & W_{2,m-1} \\ & & \dots & \\ W_{m,1} & W_{m,2} & \dots & W_{m,m-1} \end{bmatrix} \quad (1)$$

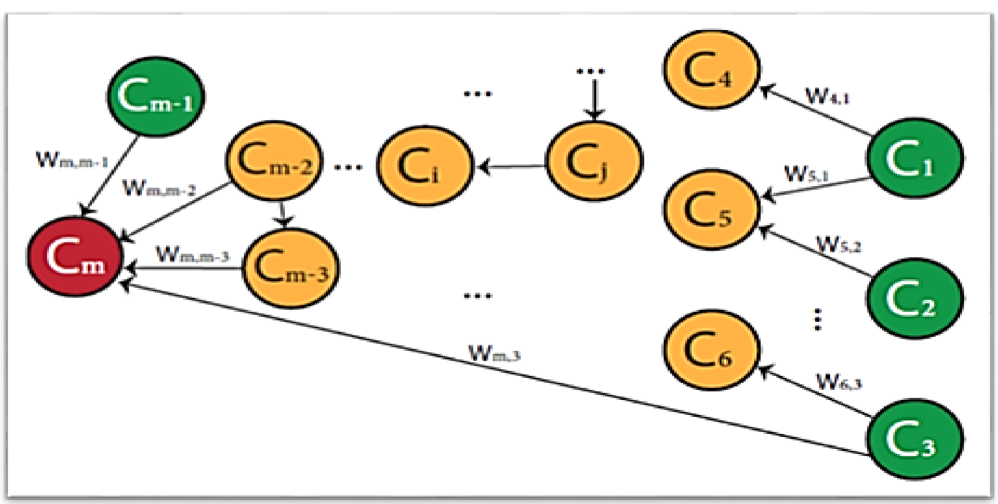

Fig. 1. Structure of the FCM

The concepts are classified into the following groups:

- Input, representing concepts that impact other concepts, but are not influenced by other concepts;
- Intermediate, representing concepts that impact other concepts and are influenced by other concepts;
- Decision, representing the concept that is only influenced by other concepts, but which does not impact other concepts and represents the output of the modelled system.

When determining FCM's weight $W_{i,j}$, the following rules apply:

- If C_j is an input concept, then $W_{1,j}, W_{2,j}, \dots, W_{m,j}$ are weights from input concept j to concepts $C_1, C_2, \dots, C_m$, respectively, and $W_{j,1} = W_{j,2} = \dots = W_{j,m} = 0$; this means that there is no impact on input concept C_j .
- If concept C_j does not affect concept C_i then $W_{i,j} = 0$.
- For all $i = 1,2, \dots, m$, $W_{i,i} = 0$, as concept C_i does not affect itself.

The process of designing an FCM for a specific problem is tackled by experts who have experience on the aspects of the modelled system alongside related historical data [30]. The experts determine the number of concepts and the causal relations among them. After the construction of the FCM and inputting initial values to the concepts, the concepts of the FCM interact with each other to produce the value of the decision concept C_m, which determines the required decision. The following iterative reasoning algorithm [4] is used:

$$C_i^{(k+1)} = f\left(C_i^{(k)} + \sum_{\substack{j=1 \\ i \neq j}}^{m-1} C_j^{(k)} * W_{i,j} \right) \tag{2}$$

where $C_i^{(k+1)}$ is the value of the concept C_i at time $k+1$, $C_i^{(k)}$ is the value of concept C_i at time k of the iterative reasoning process, $W_{i,j}$ is the weight of the edge from the concept C_j to the concept C_i and f is the sigmoid threshold function which is calculated as follows:

$$f(x) = \frac{1}{1 + e^{-\lambda x}} \tag{3}$$

where $1 \leq \lambda \leq 5$ is typically determined empirically and $f(x)$ has a value between 0 and 1. The iterative reasoning process reaches an equilibrium state when the values of the concepts stop changing. Different variations of the original reasoning algorithm and threshold functions have been proposed [4].

FCMs have been extensively developed and applied due to their simplicity in construction and their flexibility that allow them to add and/or remove concepts when required to enhance the model representation and obtain the outputs values [31]. A method for determining four hyperparameters for construction of a FCM, including: 1) the FCM ‘s number of concepts, 2) the window size for the moving-window technique for using available data, 3) the number of epochs used for training and 4) the learning training rate, was proposed in [32]. In order to decrease the error that was accumulated in iterations, an adaptive FCM for forecasting was proposed in [33], where causal interrelationships between concepts were changing over time depending on the state of the concept.

2.2. zT2FSs

Fuzzy Sets theory provides various methods and tools for dealing with uncertainties and imprecisions. The literature reveals that there is a progression in the development of fuzzy sets since it was proposed in 1965 (known as a T1FS). Since then, different types of FSs for modelling various types of uncertainty and imprecisions have been defined, such as T1FSs and T2FSs.

T1FS is a set which is characterised by a membership function that has grade values between 0 and 1. An extension to a T1FS is a T2FS characterized by a fuzzy membership function, where the membership value for each element of the set is itself a T1FS set in [0, 1], enabling T2FSs to capture and handle uncertainties about the degree of membership of an element in the fuzzy set. The existence of this third dimension, i.e., membership degrees of the membership function of a T2FS complicates the process of determining parameters of the T2FSs, as they are characterised by a three-dimensional fuzzy membership function. To reduce this difficulty, different representations of T2FSs were introduced, including IT2FSs and zT2FSs.

An IT2FS provides the membership grade *u* within an interval *J*, where the third dimension is fixed to 1.

Given the universe of discourse E, the IT2FS $\tilde{A}$ is defined in three dimensional space as a set of ordered pairs as follows:

$$\tilde{A} = \left\{ \left((x, u_i (x)), 1 \right) \mid x \in E, u_i (x) \in J_x, \quad J_x \subseteq [0,1] \right\} \tag{4}$$

A zT2FS is generated by slicing the third dimension z of a T2FS into a vertical slice Z_i (referred to as a z slice), each at level z_i. Each slice is an IT2FS where the amplitude of the third dimension is z_i, rather than 1. Formally, given the universe of discourse E, the zT2FS is a collection of an infinite number of z slices Z_i, where:

$$Z_i = \left\{ \left((x, u_i(x)), z_i \right) \mid x \in E, \quad u_i(x) \in J_x, \quad J_x \subseteq [0,1] \right\} \tag{5}$$

and hence

$$zT2FS = \int_{0 \leq i \leq I} Z_i, I \rightarrow \infty \tag{6}$$

In case of a discrete and finite universe of discourse, this is expressed as:

$$zT2FS = \sum_{0 \leq i \leq \mathrm{I}} Z_i \tag{7}$$

Note that $\int$ and $\sum$ both represent operations of union on the zT2FS, for continuous and discrete cases of slices Z_i, respectively. The work in [23], provides further definitions of the z slices operations.

2.3. IAA

Zadeh’s concept of Computing With Words (CWW) [34] led to the conclusion that there was a need to operate with a fuzzy set able to capture semantic uncertainties of words. Advantages of using T2FSs with CWW have been extended to capture the word’s uncertainties inherent in peoples’ opinions collected through interval valued surveys. Various techniques have been developed for accurately capturing people’s opinions about words and concepts using interval survey tools to create a fuzzy-based model. The rationale for the design of an interval based survey is to capture uncertainties of the responses using interval valued data. The study in [35] emphasised the potential significance of collecting the responses in the form of intervals as they were efficient in handling of uncertainties that might be inherent in individual responses. These uncertainties may arise from a variety of factors, including different levels of knowledge or information available to the respondent, inherent randomness, variability or vagueness in the answer due to different points of view or context. Miller et al. [36] proposed an approach for using interval-valued data obtained from a survey to construct zT2FSs. This approach, named IAA, captures both uncertainties in fuzzy sets, intra and inter uncertainties, without losing any information either by reprocessing data or by removing outliers. Here, the inter uncertainty refers to the opinion variation between a group of experts and intra uncertainty refers to the variation that a given expert has in his/her opinions over time and in different contexts. The resulting zT2FS includes all the information obtained from the data and, hence, it is well suited for reasoning and decision-making. Further, the proposed approach provides a distinction between intra and inter uncertainties represented in two different axes compared to other proposed methods, as in [37] [38], where all the uncertainties are captured in the Footprint of Uncertainty and the third dimension is ignored [39]. IAA reduces the need for pre-assuming the type of membership function during the creation of the model. Instead, it generates the desired fuzzy sets from the available data. Consequently, the resultant fuzzy set captures more uncertainties and can include more information [40]. The literature review reveals that several applications have used IAA as an approach to accurately model uncertainties that were captured as interval valued data and reflected the opinions and perceptions of survey participants, for example, [41] and [42].

The application of IAA for generating zT2FSs from intervals of survey's responses includes the following two phases [28]:

Phase 1. Intra response uncertainty of each participant surveyed N times is represented using a T1FS. Each T1FS is generated as the union of all the participant's response intervals with membership $y_1 = \frac{1}{N}$, union of all 2-tuple intersections of the response intervals with associated membership degree $y_2 = \frac{2}{N}$, union of all the 3-tuple intersections of response intervals with associated membership degree $y_3 = \frac{3}{N}$, generalising to the union of i-tuple intersections with the membership degree $y_i = \frac{i}{N}$, where $i \in \{1,2, \dots, N\}$ and N is the number of surveys [28]. Hence, the generated T1FS, A, resulting from this phase has membership function $u(A)$ obtained as union of intervals associated with membership degrees $y_i, i = 1, 2, \dots N$.

Phase 2. The T1FSs, created in Phase 1 for all surveyed participants, are aggregated based on the level of agreement among each tuple of the surveyed participants, generating a zT2FS. A similar process to Phase 1 is used to generate z slices [28]. Note that the number of slices is equivalent to the number of participants P. Each slice $Z_j, j \in \{1,2, \dots, P\}$ has a membership degree $z_j = j/P$ on z axis. It is worth noticing that z_j represents the level of agreement among P participants. The resulting zT2FS represents both intra and inter responses in y and z axes.

After the former phases, the fuzzy agreement model Z is generated as follows:

$$Z = \bigcup_{j=1}^{P} Z_j \tag{8}$$

For further details about using IAA method for generating z slices from crisp interval valued data, where the intervals have endpoints obtained from survey responses, the reader is referred to [28].

3. Non- Iterative Type 2 Fuzzy Reasoning Algorithm with Late Defuzzification (NILD)

This research proposes a new reasoning algorithm for zT2FCMs where the weights of the edges among the concepts are represented by zT2FSs. This new reasoning algorithm is called Non-Iterative Type 2 Fuzzy Reasoning Algorithm with Late Defuzzification (NILD). The weights are obtained using IAA described in Section 2.3. It is worth noting that the structure of zT2FCM used here is the same as the structure of FCM mentioned in Section 2.1 (see Fig.1). Furthermore, the same symbols C_i, $i = 1, \dots, m$ are used to represent concepts and their values. The concepts' values, which are initially singletons, are special cases of T2FSs where both primary and secondary membership functions equal to 1, as there is no uncertainty in their initial values. These singleton values may be extended to fuzzy sets during the process of reasoning, as will be shown later in this section. In a given relation from C_j to C_i, the value of C_j (which may be crisp or a T2FS) is used to determine the value of C_i after it is affected by C_j relation with weight $W_{i,j}$. The decision concept is C_m and its initial value is set to zero.

Considering the structure of zT2FCM illustrated in Fig. 1, NILD comprises three phases to accommodate the use of zT2FSs for representing the values of concepts and weights of the zT2FCM, as follows:

Phase 1. The values of all causally linked concepts, with the exception of decision concept C_m, are evaluated by considering the concepts' input values and zT2FSs weights between the concepts. Reasoning starts from input concepts C_j, $j = 1 \ldots m-1$, to the affected concept C_i, $i = 1, \ldots, m$. The reasoning is carried out by calculating pre- and post values of concept C_i as follows:

$$C_i^{(post)} = C_i^{(pre)} + \sum_{\substack{j=1 \\ j \neq i}}^{(m-1)} \left(C_j \star W_{i,j} \right) \quad (9)$$

where $C_i^{(pre)}$ is the pre value of concept C_i before it is affected by concepts C_j and $C_i^{(post)}$ is the post value of concept C_i after it is affected by concepts C_j, $j = 1, \ldots, m-1$, $j \neq i$. Note that $\sum$ indicates the aggregation (union) considering the impact of all causal concepts $j = 1, \ldots,$ $m-1$, $j \neq i$ on affected concept i. The union operator used in NILD is defined as follows:

Let two zT2FSs, A and B, be given, where each of them is generated using IAA. The number of slices of A and B is dependent on the number of the surveyed' participants and the number of survey iterations. For the purpose of this definition, let us assume that A and B are generated using IAA based on responses in the form of intervals from three participants surveyed twice. Hence, A and B can be represented as presented in Table 1 and the union $(A + B)$ is defined as presented in Table 2.

Table 1

T2FSs A and B

Slice of set A	Level of the slice	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = \frac{1}{3}$	$[a_1, b_1]$	$[c_1, d_1]$
Z_2	$z_2 = \frac{2}{3}$	$[e_1, f_1]$	$[g_1, h_1]$
Z_3	$z_3 = 1$	$[i_1, j_1]$	$[k_1, l_1]$

Slice of set B	Level of the slice	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = \frac{1}{3}$	$[a_2, b_2]$	$[c_2, d_2]$
Z_2	$z_2 = \frac{2}{3}$	$[e_2, f_2]$	$[g_1, h_2]$
Z_3	$z_3 = 1$	$[i_2, j_2]$	$[k_2, l_2]$

Table 2

T2FS A+B

Slice	Level of the slice	$\boldsymbol{y_1 = 0.5}$	$\boldsymbol{y_2 = 1}$
$\boldsymbol{Z_1}$	$z_1 = \frac{1}{3}$	$[a_1, b_1] \cup [a_2, b_2]$	$[c_1, d_1] \cup [c_2, d_2]$
$\boldsymbol{Z_2}$	$z_2 = \frac{2}{3}$	$[e_1, f_1] \cup [e_2, f_2]$	$[g_1, h_1] \cup [g_2, h_2]$
$\boldsymbol{Z_3}$	$z_3 = 1$	$[i_1, j_1] \cup [i_2, j_2]$	$[k_1, l_1] \cup [k_2, l_2]$

A new operator denoted by $\star$ is defined to represent the compatibility of causal node value A with causal weight B, which is also a zT2FS, as presented in Table 3.

Table 3

T2FS $A \star B$

Slice	Level of the slice	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = 1/3$	$[a_1, b_1] \cap [a_2, b_2]$	$[c_1, d_1] \cap [c_2, d_2]$
Z_2	$z_2 = 2/3$	$[e_1, f_1] \cap [e_2, f_2]$	$[g_1, h_1] \cap [g_2, h_2]$
Z_3	$z_3 = 1$	$[i_1, j_1] \cap [i_2, j_2]$	$[k_1, l_1] \cap [k_2, l_2]$

When C_i is an input concept, then $C_i^{(post)} = C_i^{(pre)}$, because $W_{i,j} = 0$ for all $j = 1 \dots m - 1$ (see the definition of input concept given in Section 2.1).

To find $C_i{}^{(post)}$ in (9), the following steps are performed:

Step 1: Find $C_j \star W_{ij}$ (find the impact of causal node C_j on node C_i where the weight between the two nodes is W_{ij})

For $j = 1, \dots, m - 1$, where C_j affects C_i

If C_j is singleton Then

If $C_j \in W_{ij}$ Then

(this means that value C_j belongs to any x-interval of W_{ij} slices, see Fig. 2 (a))

$S = (C_j \star W_{ij})$

(where the resultant S is a zT2FS that includes those intervals of W_{ij} that include C_j)

Else $S = 0$ (see Fig. 2 (b))

End if

Else If C_j is zT2FS Then

$S = C_j \star W_{ij}$ (S is calculated as defined in Table 3 when both C_j and W_{ij} are zT2FSs)

End If

End for

Step 2: Find $C_i^{(pre)} + S$ (Find the value of node, $C_i^{(post)}$, considering its value before and after the impact of other nodes. Note that + indicates the union, i.e. the aggregation of impacts).

This step depends on the resultant zT2FS S calculated in Step 1, as considered in the following two cases:

Case 1: S has a slice where $y = 1$ and $z = 1$

If $C_i^{(pre)}$ belongs to x-interval of S where $y = 1$ and $z = 1$ Then (see Fig. 2 (c))

$S = C_i^{(pre)} + S$

$C_i^{(post)} = S$

Else x-interval of S where $y = 1$ and $z = 1$ is extended to include $C_i^{(pre)}$ as follows:

Let us assume that x-interval of S where $y = 1$ and $z = 1$ is an interval (a, b):

If $C_i^{(pre)} < a$ Then

x-interval of S becomes $S = (C_i^{(pre)}, b)$ (see Fig. 2 (d))

Else If $C_i^{(pre)} > b$ Then

x-interval of S becomes $S = (a, C_i^{(pre)})$

End If

$C_i^{(post)} = C_i^{(pre)} + S$

End If

Note that in Case 1, $C_i^{(post)}$ becomes zT2FS.

Case 2: S does not have a slice where $y = 1$ and $z = 1$

Add to S a slice $((C_i^{(pre)}, C_i^{(pre)}), 1, 1)$

(this means to add a singleton $C_i^{(pre)}$ with y = 1 and z = 1)

$C_i^{(post)} = C_i^{(pre)} + S$, (where $C_i^{(post)}$ is a zT2FS, and $C_i^{(pre)} + S$ is calculated as the union of two zT2FSs, as defined in Table 2)

The former steps, Step 1 and Step 2, show that after C_j impacts C_i, the value of $C_i^{(post)}$ is either a singleton (crisp), or a zT2FS. Note that in (9) when $C_i^{(pre)}$ is crisp and $\sum_{\substack{j=1 \\ j \neq i}}^{(m-1)} (C_j \star W_{i,j}) = 0$, $C_i^{(post)}$ becomes crisp also.

In Phase 1, the post values of all concepts except the decision concept are calculated.

Phase 2. The result of the reasoning algorithm in Phase 1 is used to determine the value of the decision concept C_m as follows (note that the initial value of $C_m = 0$). The value of the decision concept C_m is determined based on the weights W_{mi} of the edges from the concepts C_i, $i = 1,2, \dots, m-1$, with values $C_i^{(post)}$ determined in Phase 1, to decision concept C_m. Therefore, it is calculated as:

$$C_m^{(post)} = \underbrace{C_m^{(pre)}}_{=0} + \sum_{i=1}^{(m-1)} \left(C_i^{(post)} \star W_{mi} \right) \quad (10)$$

where $\left(C_i^{(post)} \star W_{mi} \right)$ is calculated as in Phase 1, Step 1 and $\sum$ of z2TFSs is defined in Table 2. Therefore, $C_m^{(post)}$ is also a zT2FS.

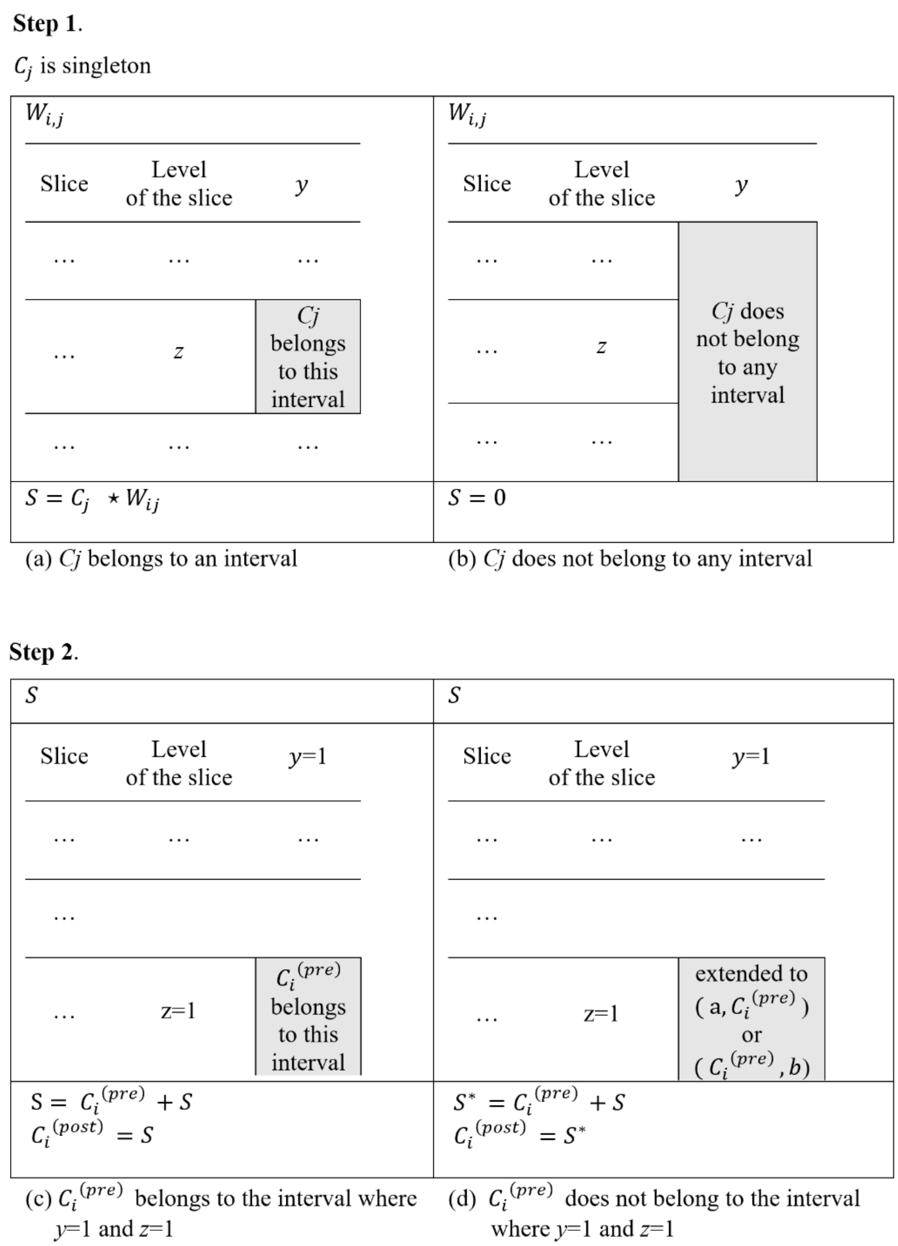

Fig. 2. Phase 1 of NILD algorithm

Phase 3. The zT2FS that represents the post value of C_m, $C_m^{(post)}$, is defuzzified at the end of the reasoning by using the centroid defuzzification method for z slices as presented in [23]; hence

$$C_{C_m^{(post)}} = \sum_{i=1}^{N} z_i * C_{Z_i} \quad (11)$$

where $C_{C_m^{(post)}}$ is the centroid of $C_m^{(post)}$ and C_{Z_i} is the centroid of each Z_i slice in $C_m^{(post)}$.

It can be noted that by applying Phase 1 and Phase 2 of NILD, the uncertainty of both concepts and weights is captured by postponing any defuzzification until the end of the reasoning. This is the novelty of this algorithm compared to the conventional reasoning of FCMs where the values of weights and concepts are crisp due to their early defuzzification, and, therefore, most of the information captured in zT2FSs may be lost. Therefore, the NILD reasoning algorithm supports preservation and propagation of information and input uncertainties, which affect the value of the decision outcome, till the end of the reasoning process to.

4. Evaluation of NILD Effectiveness

To evaluate the effectiveness of the proposed NILD algorithm in processing uncertainties in human decision making, a real-world problem of evaluating a module performance (MP) was considered. In every academic institution, the MP is a very important indicator that influences students' progression on a course. In most institutions, the decision about the MP relies on simple statistics of the modules, such as marks average and marks' standard deviation. This may not capture the importance and causal influence of different factors affecting the MP as well as account for the subjective decision makers (lecturers) points of view related to it. FCMs have the potential to capture the interplay of these factors.

We conducted experiments using real data of 30 mathematical modules offered in the Department of Mathematics and Applied Sciences (MASC) at Middle East College (MEC), Oman. MP results were obtained by using different methods for evaluating MPs, including:

1) the Student Information System (SIS) and the Traffic Light System (TLS), currently used at MEC to calculate MPs, based on a statistical approach,
2) a zT2FCM and NILD algorithm proposed; we explain how the data on the 30 modules were collected and used to construct the zT2FCM and how the NILD algorithm was applied for reasoning and generating the MPs for the modules,

3) a T1FCM constructed using the collected data with the iterative reasoning algorithm given in (2),
4) using MEC's lecturers' subjective opinions of the MP of the selected modules.

4.1. Student Information System and Traffic Light System in MEC

In MEC, the results that students achieved in each of their taught modules at the end of each semester were recorded in the SIS and the following statistical summaries were calculated: CW - the total result for the coursework, ESE – the total end semester examination result, MP - the average of both CW and ESE, SD - standard deviation of the results, PP - pass percentage, and ATT - the attendance of students in each module. The MPs were evaluated using the TLS that relied on the statistical attributes PP and SD only to appraise the performance of the module. The TLS classified MP into three colour codes: Green, Amber and Red using the ranges for PP and SD, as given in Table 4.

Table 4

TLS colour code

Colour	PP	SD	MP	S_1
Green	$\geq 90\%$	$8 - 12$	$66.6 \leq \text{MP} \leq 100$	0.833
Amber	$80\% - 89\%$	$5 - 8$ or $12 - 16$	$33.33 < \text{MP} < 66.6$	0.5
Red	$< 80\%$	< 5 or > 16	$0 \leq \text{MP} \leq 33.33$	0.1667

As per the practice at MEC, the colour code assignment of a module was based on the results falling within both the PP and SD ranges of the colour. In case the result did not fall in both ranges of the colour, the SD was considered to assign the performance colour code to the module. For example, if a module had PP of 72% and SD of 27.49, then both conditions were satisfied, and the performance classification of the module was Red. However, if a module had a PP of 81.25% and SD of 16.55, then the performance colour code of the module was still classed as Red, due to its SD value, rather than Amber.

To compare the results obtained from the TLS with results obtained by using the zT2FCM and NILD with the T1FCM using the reasoning method given in (2), the scale of the MP score, from 0 to 100, was split into three equal length intervals for the three possible colour codes. The colour of each module was mapped into a corresponding interval as shown in Table 4. Centroid S_1 of each interval was obtained as the midpoint of the interval.

For testing purposes, data was collected from the SIS for 30 randomly selected modules including the modules' ATT, CW and ESE. After collecting data, the modules' colours that reflected MPs were determined using the TLS. They were mapped into the corresponding intervals and their centroids S_1 were obtained as shown in Table 11.

4.2. Construction of MPFCM - a zT2FCM for evaluating module performance

4.2.1. Nodes

The zT2FCM for evaluating module performance, named Module Performance FCM – MPFCM, was constructed as follows. The process of constructing MPFCM started with determining the nodes that represented the main concepts required for evaluating MP. Thus, three lecturers from MEC discussed and agreed on factors that impacted MP. Consequently, these factors were used as concepts in the MPFCM. The resulting concepts were identified as follows: ATT - attendance, CW – total coursework result and ESE – total end semester examination result, as causal concepts, and MP as the decision concept. The lecturers then decided on how the concepts casually interrelated within the MPFCM based on their domain experiences. In this way, the MPFCM, which consisted of four nodes that represented the identified concepts and four edges representing the interrelations among these concepts, was constructed (see Fig. 3).

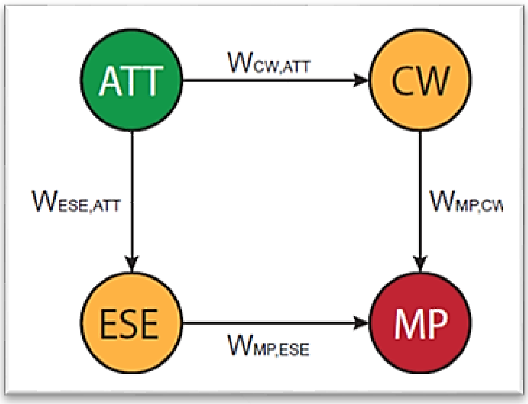

Fig. 3. *MPFCM*

4.2.2. Weights

To determine the weights of the interrelation edges among the four identified concepts, an interval-valued survey of four questions was designed, one for each of the four interrelations among the MPFCM's concepts. Each question, given in Appendix A, was related to the identified edge among each two concepts. The survey was administered, where for each question, the three lecturers were asked to represent their opinions about the impact of causal concept C_j to affected concept C_i. For instance, in Question 1, they were asked how ATT impacted ESE and their replies were used to determine the weight $W_{ESE,ATT}$. Lecturers answered each question by drawing an ellipse on a Likert scale ranging from 0 to 100. This ellipse represented their uncertainty about the weight of the edge between the two concepts. An example of answering a question on the impact of Attendance on the Total End Semester Examination result is shown in Fig. 4, where the response of one of the lecturers was given as interval [30, 65].

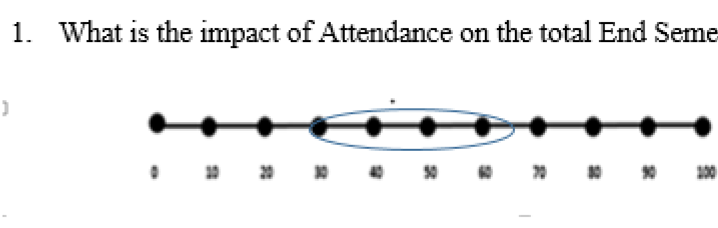

Fig. 4. Example of responses on a Likert scale

The survey was repeated after a period of four weeks when the same lecturers were surveyed again to capture their intra uncertainty about their assigned weights among the concepts. The literature reveals that repeatedly surveyed people may provide differing responses as they may have better information or details about the issue or increased/decreased uncertainty resulting in differing ellipse endpoints (intervals) derived from their responses [28], [36]. As such, it was expected that the lecturers might have provided differing evaluations of the weights among the concepts.

The obtained intervals for each question were extracted from the survey and aggregated to capture intra and inter uncertainties of the lecturers in assigning the weight between each two causally linked concepts in MPFCM. The intervals were aggregated using IAA as detailed in Section 2.3, when $P = 3$ and $N = 2$, as three lecturers were surveyed twice. Hence, the generated weights between each of two concepts were represented as zT2FSs. The weights among the concepts in the defined MPFCM structure (see Fig. 3), namely $W_{\mathrm{CW},ATT}$, $W_{\mathrm{ESE},ATT}$, $W_{MP,CW}$ and $W_{MP,ESE}$ as zT2FSs were obtained by using IAA, and are presented in Table 5.

Table 5

Weights of MPFCM

Weight	Slice	Level	$y_1 = 0.5$	$y_2 = 1$
$W_{ESE,ATT}$	Z_1	$z_1 = 1/3$	[0.15,0.75]	[0.15,0.35] ∪ [0.40,0.75]
	Z_2	$z_2 = 2/3$	[0.37,0.72]	[0.40,0.70]
	Z_3	$z_3 = 1$	[0.38,0.45]	ϕ
$W_{CW,ATT}$	Z_1	$z_1 = 1/3$	[0.05, 0.42] ∪ [0.47, 0.82]	[0.12, 0.40] ∪ [0.50, 0.80]
	Z_2	$z_2 = 2/3$	[0.20, 0.40]	[0.28. 0.38]
	Z_3	$z_3 = 1$	ϕ	ϕ
$W_{MP,CW}$	Z_1	$z_1 = 1/3$	[0.33, 0.88]	[0.40, 0.80]
	Z_2	$z_2 = 2/3$	[0.38, 0.75]	[0.45, 0.65]
	Z_3	$z_3 = 1$	[0.60, 0.72]	[0.60, 0.65]
$W_{MP,ESE}$	Z_1	$z_1 = 1/3$	[0.38, 0.90]	[0.55, 0.80]
	Z_2	$z_2 = 2/3$	[0.50,0.85]	[0.65, 0.75]
	Z_3	$z_3 = 1$	[0.58, 0.77]	[0.68, 0.72]

After constructing MPFCM, the data collected from the SIS for the 30 selected modules were used as initial values of the concepts of MPFCM. The new NILD reasoning algorithm was applied with the data of each of the 30 modules to calculate the values of the decision concept MP. The decision concept values were in the form of zT2FSs, which were defuzzified as given in formula (11), to obtain their centroids S_2, (see Table 11).

4.2.3. Example of reasoning in MPFCM

In this section, an example of calculating MP for one of the modules using the new reasoning NILD algorithm is presented. Data collected from the SIS for Module 22 were as follows: $CW = 0.796$, $ESE = 0.72$ and $ATT = 0.81$.

Note that these values represented the pre values of the MPFCM's concepts CW, ESE and ATT, respectively. Using the MPFCM shown in Fig. 3 and the NILD phases detailed in Section 3, we obtained post values $ATT^{(post)}, CW^{(post)}, ESE^{(post)},$ and MP as follows:

$ATT^{(post)} = ATT^{(pre)} = 0.81$, as ATT was the input concept that was not affected by other concepts (Phase 1 of NILD algorithm) and, thus, its post value was equal to its pre value.

$CW^{(post)} = CW^{(pre)} + (ATT^{(post)} \star W_{CW,ATT})$, where $CW^{(pre)} = 0.796$. As $ATT^{(post)} \in W_{CW,ATT}$, i.e., belonged to Z_1 slice, with degree $y_1 = 0.5$, based on Phase 1, Step 1 of NILD, $(ATT^{(post)} \star W_{CW,ATT})$ became the zT2FS, as given in Table 6. As $(ATT^{(post)} \star W_{CW,ATT})$ does not have a slice where $y = 1$ and $z = 1$, using Step 2, Case 2 in Phase 1, slice ((0.796, 0.796), 1, 1,) was added and $CW^{(post)}$ became zT2FS as presented in Table 7.

$ESE^{(post)}$ was calculated as follows:

$ESE^{(post)} = ESE^{(pre)} + (ATT^{(post)} \star W_{ESE,ATT})$

As $ATT^{(post)} \notin W_{ESE,ATT}$ based on Phase 1, Step 1 of the NILD algorithm,

$ATT^{(post)} \star W_{ESE,ATT} = 0$ and, therefore, $ESE^{(post)} = 0.72 + 0 = 0.72$.

Table 6

$ATT^{(post)} \star W_{CW,ATT}$

Slice	Level	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = 1/3$	$[0.47, 0.82]$	ϕ
Z_2	$z_2 = 2/3$	ϕ	ϕ
Z_3	$z_3 = 1$	ϕ	ϕ

Table 7

$CW^{(post)}$

Slice	Level	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = 1/3$	$[0.47, 0.82]$	ϕ
Z_2	$z_2 = 2/3$	ϕ	ϕ
Z_3	$z_3 = 1$	ϕ	$[0.796, 0.796]$

Following Phase 2 of the reasoning algorithm, MP of Module 22 was calculated as follows:

$$MP^{(post)} = MP^{(pre)} + \left((CW^{(post)} \star W_{MP,CW}) + (ESE^{(post)} \star W_{MP,ESE})\right)$$

where $MP^{(pre)} = 0$.

By following Phase 1, Step 1 of the reasoning algorithm, the z slices of $\left(CW^{(post)} \star W_{(MP,CW)}\right)$ and $\left(ESE^{(post)} \star W_{(MP,ESE)}\right)$ were obtained as presented in Table 8 and Table 9, respectively. Following Phase 2 and Phase 3 of the reasoning algorithm, MP was calculated and then defuzzified to a crisp value using centroid of z slices, as presented in Table 10.

Hence, $MP^{(post)} = 0.6869$.

Therefore, the performance of the Module 22 is MP = 68.69%

Table 8

$T2FS\ CW^{(post)} \star W_{MP,CW}$

Slice	Level	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = 1/3$	[0.47,0.82]	ϕ
Z_2	$z_2 = 2/3$	ϕ	ϕ
Z_3	$z_3 = 1$	ϕ	ϕ

Table 9

$T2FS\ ESE^{(post)} \star W_{MP,ESE}$

Slice	Level	$y_1 = 0.5$	$y_2 = 1$
Z_1	$z_1 = 1/3$	[0.38,0.9]	[0.55, 0.8]
Z_2	$z_2 = 2/3$	[0.5,0.85]	[0.65, 0.75]
Z_3	$z_3 = 1$	[0.58, 0.77]	[0.68, 0.72]

Table 10

MP

Slice	Level of the slice	$y_1 = 0.5$	$y_2 = 1$	Centroid of the slice	Overall centroid
Z_1	$z_1 = \frac{1}{3}$	[0.38,0.9]	[0.55,0.8]	$\frac{1}{3}/0.6633$	0.6869
Z_2	$z_2 = \frac{2}{3}$	[0.5,0.85]	[0.65,0.75]	$\frac{2}{3}/0.6917$	
Z_3	$z_3 = 1$	[0.58,0.77]	[0.68,0.72]	$1/0.6917$	

4.3. T1FCM

As mentioned earlier, to test the effectiveness of the proposed reasoning algorithm for zT2FCMs, it was compared with a T1FCM using the standard iterative reasoning algorithm given in (2). Therefore, a T1FCM was created for evaluating MP of modules and then compared with MPFCM. This T1FCM had the same structure as the MPFCM (see Fig. 3). It consisted of the same four concepts and four causal relations. However, the weights were represented by T1FSs which were defuzzified to crisp values that can be used in the reasoning algorithm given in (2). To obtain the weights of the T1FCM, the intervals obtained from the second iteration of the original survey were used to generate T1FSs, as it was assumed that by this time lecturers had a better understanding about the survey's context. The obtained response intervals of the three lecturers for each causal relation were aggregated using IAA. A T1FS for each causal relation was determined in such a way that the intersection of all three response intervals had a membership degree of 1, the union of intersections of each 2 response intervals had a membership degree of 2/3 and the union of all three response intervals had a membership degree of 1/3. The created T1FS for each causal relation modelled the agreement between the lecturers and represented its weight. The derived weights were defuzzified to obtain crisp weights and by doing this, the T1FCM was prepared for reasoning using the iterative reasoning algorithm given in (2). The T1FCM used the concept values obtained from the SIS for all the modules that had been considered for reasoning in MPFCM. The iterative reasoning algorithm was run in eight iterations, i.e., until the value of MP converged. The value S_3 of decision concept MP in the T1FCM was obtained as a singleton between 0 and 1, for each of the 30 modules, as shown in Table 11.

4.4. Lecturers' opinions about the modules' performance

As there is no ground truth data for determining MP, to evaluate the effectiveness of the MPFCM, and hence demonstrate the effectiveness of the NILD algorithm proposed for zT2FCMs, the MPs generated by the methods using MPFCM, T1FCM and the TLS were compared with the MPs evaluated subjectively by lecturers. For this reason, an interval-valued survey designed for generating the zT2FSs (see Appendix B) was used to survey the same three lecturers from MEC to represent their opinions based on experiences, about the performance of the 30 modules under consideration, given the module's statistics: MP, SD and ATT. As in the former survey, the lecturers represented their opinions by drawing an ellipse on a Likert scale in the range from 0 to 100. After collecting the opinions from the lecturers, given as a percentage, they were converted into values between 0 and 1 to make them suitable for being represented as fuzzy sets. After extracting the intervals from each lecturer, IAA was applied to create a fuzzy agreement model of the three lecturers, as explained in Section 2.3. After applying IAA on the lecturers' opinions for each module, the centroid S_4 was calculated by defuzzifying the resulting fuzzy set for each module, as presented in Table 11. Therefore, in this experiment the lecturers acted as both the experts who designed MPFCM and who evaluated the performance of each of the modules based on the module's results. The rationale for this was to determine to what extent the newly proposed zT2FCM's reasoning algorithm NILD produced outputs that corresponded to that of the decision makers and to validate its ability to mimic human reasoning within this application domain.

5. Comparison of the results

The lecturers' evaluations of MPs were used as the benchmark for comparison between the other methods used to evaluate MPs, namely MPFCM, T1FCM and the TLS. The rationale of this was to determine which of the methods used had the highest correlation with the lecturers (decision makers) opinions, and hence was more capable of processing uncertainties in the human reasoning process. The correlation of the results obtained from each of the compared methods was calculated using the coefficient of Pearson correlation (ρ) . The value of ρ reflects the strength/association of the relation between two variables as follows; $0 < \rho < 0.3$ represents a small correlation, $0.3 \leq \rho < 0.5$ represents medium correlation and $0.5 \leq \rho \leq 1$ represents high correlation [43]. For this purpose, the defuzzified outputs (centroids) S_1, S_2, S_3 and S_4 of the 30 modules were obtained for the four methods, namely TLS, MPFCM, T1FCM and lecturers' outputs, respectively, as shown in Table 11. The lecturers' evaluation outputs S_4 were used as the benchmark for the comparison.

Table 11

Defuzzified outputs of the methods applied

Module	S_1 TLS	S_2 MPFCM	S_3 T1FCM	S_4 Lecturers
1	0.1667	0.4363	0.8636	0.6773
2	0.1667	0.4513	0.8592	0.6793
3	0.1667	0.6697	0.8584	0.6708
4	0.1667	0.2728	0.8584	0.5774
5	0.1667	0.6350	0.8585	0.6421
6	0.1667	0.1083	0.8576	0.6165
7	0.1667	0.4431	0.8582	0.7151
8	0.5000	0.4514	0.8596	0.7938
9	0.1667	0.2906	0.8584	0.5571
10	0.1667	0.2808	0.8588	0.7021
11	0.5000	0.2875	0.8586	0.7395
12	0.1667	0.2371	0.8576	0.5529
13	0.1667	0.1622	0.8599	0.5349
14	0.1667	0.4364	0.8587	0.6813
15	0.5000	0.4365	0.8587	0.6976
16	0.1667	0.2875	0.8570	0.8038
17	0.1667	0.2783	0.8582	0.7751
18	0.1667	0.6089	0.8583	0.6920
19	0.1667	0.2850	0.8583	0.4592
20	0.1667	0.4142	0.8579	0.5869
21	0.5000	0.2783	0.8582	0.6448
22	0.1667	0.6869	0.8575	0.7573
23	0.1667	0.2783	0.8585	0.6132
24	0.1667	0.2783	0.8578	0.5882
25	0.1667	0.4879	0.8574	0.6064
26	0.1667	0.6350	0.8577	0.6372
27	0.5000	0.2808	0.8582	0.6508
28	0.1667	0.1828	0.8571	0.6361
29	0.5000	0.4514	0.8586	0.6361
30	0.1667	0.2906	0.8578	0.5412

Pearson correlation ρ between outputs S_4 and outputs S_1, S_2, and S_3 of the TLS, MPFCM and T1FCM results, respectively, were calculated, as presented in Table 12.

Table 12

Correlation results between lecturers' evaluations and the results of the TLS, MPFCM and T1FCM

$\rho_{(S_4,S_1)}$	$\rho_{(S_4,S_2)}$	$\rho_{(S_4,S_3)}$
0.28	0.34	0.08

The results indicated that the correlation between the lecturers' evaluations and MPFCM was moderate with a value of 0.34, which was higher than the correlation between the lecturers and the TLS and T1FCM with ρ values of 0.28 and 0.08, respectively. The ρ values show that the proposed MPFCM had a greater agreement with the lecturers' evaluations due to a higher correlation with their decisions as compared to the other methods. Accordingly, we concluded that the zT2FCM with NILD algorithm had a greater agreement with human evaluations compared to the conventional FCM (T1FCM) and the statistical approach (the TLS).

To further investigate the ability of NILD in preserving and propagating uncertainties derived from the changes in uncertainty of participants' responses, a sensitivity analysis was conducted as discussed in Section 6.

6. Sensitivity Analysis

There are papers reported in the literature that have analysed propagation of uncertainty within models in general. For example, [44] compared six methods for uncertainty propagation used to estimate the distribution of model outputs, assuming that input parameters had specific probability distributions. Furthermore, [45] considered propagation of uncertainty modelled as intervals. Here authors provided a theoretical study on the relation between the widths of the input intervals and the output interval, proposing operations on interval-valued data. However, to the best of our knowledge, there are no papers that focus on uncertainty propagations in FCMs.

We decided to conduct a practical examination on uncertainty propagation in FCMs to obtain insights on how NILD algorithm propagates uncertainty in weights. To better understand how a change in the weights of a zT2FCM affects the value of the decision concept when using NILD algorithm, we performed sensitivity analysis by changing the uncertainty of each weight in MPFCM by δ, where δ = 0.01, 0.025, 0.05, 0.07, and 0.1. This meant that the response intervals for each question from each lecturer was extended by the above values of δ and then aggregated to produce the corresponding weights in the form of zT2FSs using IAA, as described in Section 2.3. This emulated increasing uncertainty of each weight as the widths of the response intervals were increased [35],[36]. These experiments aimed to analyse if there was a relationship between uncertainty changes of MPFCM's weights and the output value of the decision concept MP. Hence the weights represented the independent variables, and the decision concept represented the dependent variable.

Table 13

Output values of MP after changing the uncertainty in $W_{MP,CW}$ by δ

Module	Output of MP with original values of weights	Output of MP when the uncertainty of $W_{MP,CW}$ was changed by $\delta = 0.01$	Output of MP when the uncertainty of $W_{MP,CW}$ was changed by $\delta = 0.025$	Output of MP when the uncertainty of $W_{MP,CW}$ was changed by $\delta = 0.05$	Output of MP when the uncertainty of $W_{MP,CW}$ was changed by $\delta = 0.07$	Output of MP when the uncertainty of $W_{MP,CW}$ was changed by $\delta = 0.1$
1	0.4363	0.4350	0.4340	0.4305	0.4294	0.6027
2	0.4513	0.4510	0.4420	0.4425	0.4425	0.4277
3	0.6697	0.6680	0.6670	0.6638	0.6077	0.6027
4	0.2727	0.2720	0.2720	0.6027	0.6027	0.6027
5	0.6350	0.6330	0.6420	0.6055	0.6077	0.6027
6	0.1105	0.1105	0.1105	0.1105	0.3483	0.3483
7	0.4430	0.4427	0.4544	0.4305	0.4294	0.4277
8	0.4513	0.4511	0.4508	0.4508	0.4425	0.4425
9	0.2905	0.6080	0.6069	0.6055	0.6044	0.6027
10	0.2808	0.2800	0.2784	0.6055	0.6044	0.6027
11	0.2875	0.2870	0.2866	0.2775	0.2738	0.2722
12	0.2393	0.2390	0.2393	0.2393	0.2393	0.2393
13	0.1622	0.1622	0.1622	0.2722	0.2722	0.6027
14	0.4363	0.4355	0.4340	0.4305	0.6077	0.6027
15	0.4363	0.4355	0.4340	0.6111	0.6077	0.6027
16	0.2952	0.2952	0.2952	0.2952	0.2997	0.2952
17	0.2783	0.2725	0.2763	0.2750	0.6044	0.6027
18	0.6088	0.6080	0.6069	0.6055	0.6044	0.6027
19	0.2850	0.2847	0.2844	0.2844	0.2844	0.6027
20	0.4141	0.4122	0.4094	0.4052	0.4019	0.6027
21	0.2783	0.2775	0.2763	0.2750	0.6044	0.6027
22	0.6869	0.6869	0.6869	0.6869	0.6869	0.6869
23	0.2783	0.2775	0.6069	0.6055	0.6044	0.6027
24	0.2783	0.2775	0.6069	0.6055	0.6044	0.6027
25	0.5068	0.5068	0.5068	0.5068	0.5068	0.5068
26	0.6350	0.6330	0.6423	0.6144	0.6077	0.6027
27	0.2808	0.2800	0.2784	0.6055	0.6044	0.6027
28	0.1827	0.1833	0.1827	0.1827	0.1827	0.4287
29	0.4513	0.4511	0.4508	0.4425	0.4425	0.4302
30	0.2905	0.2897	0.2886	0.6055	0.6044	0.6027

The method developed for conducting the sensitivity analysis of MP for each module included the following steps:

Step 1: The uncertainty of response intervals of each question, given by the lecturers, were changed by extending interval boundaries by δ, while keeping the centres of the intervals the same.

Step 2: The intervals produced in Step 1 were aggregated using IAA to generate the zT2FS for each MPFCM weight.

Step 3: The value of the decision concept MP for each module was determined by considering one changed weight while keeping the remaining weights the same and using NILD algorithm.

Step 4: Steps 1, 2 and 3 were repeated for all the MPFCM weights.

For example, the output values of the decision concept MP for each module after changing the uncertainty of the weight $W_{MP,CW}$ by corresponding δ values are presented in Table 13.

Based on the analysis of the obtained results, the following conclusions were made:

1) The change of uncertainty of $W_{ESE,ATT}$ by $\delta = 0.01$ and 0.025 did not affect the value of MP. Changing its uncertainty by $\delta = 0.05, 0.07$ and 0.1 changed the value of MP only for a small number of modules (6 out of 30). The Pearson correlation coefficient between the original values of MP and its values after changing the uncertainty of $W_{ESE,ATT}$ was 1 when $\delta = 0.01$ and 0.025, which represented a strong association. It started declining slightly when $\delta = 0.05$, 0.07 and 0.1, from 0.97 to 0.9, as these changes affected a small number of modules.

2) The change of uncertainty of $W_{CW,ATT}$ by $\delta = 0.01$ affected the value of MP for some modules. With higher increases of δ, from 0.025 to 0.1, the values of MP were changed for almost 90% of the modules. The Pearson correlation coefficient between the original values of MP and its values after changing the uncertainty of $W_{CW,ATT}$ was 0.98 when $\delta = 0.01$ and this started declining when δ was increased; however, this still reflected a high association.

Although the weights $W_{ESE,ATT}$ and $W_{CW,ATT}$ linked the input concept and the intermediate concepts, it could be observed that the impact of changing uncertainty of $W_{CW,ATT}$ was a fraction higher than the impact when $W_{ESE,ATT}$ was changed. The reason for this could be that the level of agreement (overlapped intervals) in $W_{ESE,ATT}$ was a fraction higher than the agreement in $W_{CW,ATT}$ (see Table 5). Hence, the influence of increasing uncertainty of $W_{ESE,ATT}$ was smaller.

3) The change of uncertainty of $W_{MP,CW}$ by $\delta = 0.01$ and 0.025 slightly affected the value of MP for some modules. However, by increasing δ further, there was a noticeable change in MP values for most modules. The Pearson correlation coefficient between the original values of MP and its values after changing the uncertainty of $W_{MP,CW}$ was 0.93, when $\delta = 0.01$. This declined by increasing δ to reach 0.32, when $\delta = 0.1$. Hence, the correlation between the values declined from a high (0.93) to a medium (0.32) correlation.

4) The change of uncertainty of $W_{MP,ESE}$ affected the value of MP for most modules. By increasing δ, the Pearson correlation coefficient between the original values of MP and its values after the changes declined from 0.91 to 0.69. Thus, it could be concluded that this association was still high.

However, the impact of changing uncertainty of $W_{MP,CW}$ was higher than the impact when uncertainty of $W_{MP,ESE}$ was changed. This could be explained as follows. There was more uncertainty originally in the agreement in $W_{MP,CW}$ than in $W_{MP,ESE}$ (as widths of the intervals in $W_{MP,CW}$ were higher than in $W_{MP,ESE}$). Hence, increasing the uncertainty of $W_{MP,CW}$ led to more overlaps with other intervals generated from intersection and union of the concept CW (CW $^{(post)}$) and weight $W_{MP,CW}$. Consequently, there was a greater effect on the value of MP.

5) The impact of changing uncertainty of weights of direct edges to MP, such as $W_{MP,CW}$ and $W_{MP,ESE}$, was higher than the impact of changing the uncertainty in the weights of edges between the input and intermediate concepts, such as weights $W_{ESE,ATT}$ and $W_{CW,ATT}$. However, the effect of changing uncertainty of $W_{CW,ATT}$ (where there was less agreement in its zT2FS and, therefore, more uncertainty) was combined with the effect of changing uncertainty of $W_{MP,CW}$, hence the value of MP was seen to be affected more.

We also calculated the coefficient of determination R^2 [46], for determining the level to which outputs were affected by changes in input uncertainties, based on linear regression. The coefficient of determination R^2 was calculated between the original values of MP, when there were no changes in the MPFCM weights, and its values when there was a change of δ in a weight, as presented in Table 14. The value of R^2 was used to determine the percentage of modules that were not affected by changing the uncertainty of a specific weight. Consequently, the percentage of the affected modules was $(1 - R^2)$. For example, R^2 between the original values of MP and its values when there was a change in the weight $W_{CW,ATT}$ by δ = 0.05, was 0.845, as presented in Table 14. This indicated that with a change in uncertainty of weight $W_{CW,ATT}$ by 5%, MP of 84.5% of the 30 modules (around 25 modules) remained the same, while it was changed for 15.5% modules (5 modules).

Table 14

Values of R^2

δ	$W_{ESE,ATT}$	$W_{CW,ATT}$	$W_{MP,CW}$	$W_{MP,ESE}$
0.01	1	0.958	0.869	0.824
0.025	1	0.908	0.659	0.74
0.05	0.952	0.845	0.369	0.54
0.07	0.789	0.747	0.227	0.53
0.1	0.814	0.747	0.105	0.476

From Table 14, we can observe the following:

1) As δ increased, R^2 decreased, i.e., $1 - R^2$ increased. Therefore, as the uncertainty of the weights increased, the number of modules with changes in MP values increased too.

2) When the uncertainty of $W_{ESE,ATT}$ and $W_{CW,ATT}$ was changed by δ, from 0.01 to 0.1 (this represented a change of uncertainty from 1% to 10%), there was a slight change in the output value of MP, but the regression was still high; the values of R^2 decreased from 1 to 0.814 and from 0.958 to 0.747, when these uncertainties in $W_{ESE,ATT}$ and $W_{CW,ATT}$ were applied, respectively. Therefore, we can conclude that MP was less sensitive to changes in uncertainty of the weights $W_{ESE,ATT}$ and $W_{CW,ATT}$.

3) By increasing uncertainty of $W_{MP,CW}$ and $W_{MP,ESE}$, there was a considerable change in the output values of MP. The values of R^2 dropped from 0.869 to 0.105 and from 0.824 to 0.476, when uncertainty changes in $W_{MP,CW}$ and $W_{MP,ESE}$ were increased from 1% to 10%, respectively. Hence,

we can conclude that the value of MP was more sensitive to the changes in uncertainty of the weights $W_{MP,CW}$ and $W_{MP,ESE}$.

The previous observations showed that the proposed NILD algorithm propagated well the uncertainty in weights and the decision concept values obtained were sensitive to changes in uncertainty of FCM's weights. Greater sensitivity to uncertainty changes in weights of a direct edge to the decision concept was observed. It was also observed that a considerable change in the value of the decision concept could occur when the cause concept was intermediate. In this case, the intermediate concept was also affected by other concepts and uncertainty was propagated between other affected concepts and then finally to the decision concept.

7. Conclusion and Future Work

This paper proposes a new approach to reasoning in FCMs that uses zT2FSs to represent weights of edges between concepts. The zT2FSs are produced by applying IAA to interval valued data. The new NILD algorithm is developed for FCMs with weights represented using zT2FSs. New operations in the proposed reasoning algorithm are defined in such a way as to make the reasoning compatible with zT2FSs. The proposed reasoning algorithm preserves the captured uncertainties throughout the causal reasoning process by delaying the defuzzification to the end of the process. This makes the new reasoning algorithm more robust in comparison to the conventional iterative reasoning approaches where uncertainties of information may be lost due to early defuzzification.

To evaluate the effectiveness of the proposed reasoning algorithm, a real-world problem of evaluating module performance was considered. Experiments were conducted using real data about module performance obtained from a higher education institution in the Middle East and used to construct the zT2FCM for evaluating module performance. The results obtained by applying the new reasoning algorithm demonstrated the ability of the algorithm to evaluate module performance that were more correlated to experts' decisions. This demonstrates the ability of the zT2FCM using the NILD reasoning algorithm to better mimic human reasoning in the presence of intra and inter uncertainties in the opinions of domain subjects, when compared to an FCM with a standard iterative reasoning method. To further validate NILD and analyse its ability to propagate input uncertainties within the FCMs structure and its impact on the decision concept, sensitivity analysis was conducted. It was observed that both changes in uncertainties of zT2FS based weighted edges from intermediate concepts to the decision concept and from input to intermediate concepts impacted the value of the decision concept to different degrees. The results demonstrated that the NILD algorithm enabled a propagation of uncertainty which affected outcome decisions.

Future work on the proposed zT2FCM with NILD algorithm could be carried out in different directions such as representing concepts' values using zT2FSs and applying zT2FCMs with the NILD algorithm in different domains.

Appendix A. Questions to determine the weight of the interrelation among the concepts in MPFCM

1. What is the impact of Attendance on the End semester result?
2. What is the impact of Attendance on Course work result?
3. What is the impact of Course work result on the review of Module performance?
4. What is the impact of End semester result on the review of Module performance?

Appendix B. A question to determine the module performance based on teachers experiences

How do you evaluate the module performance, if the result summary of a module at the end of the semester is given as follows:

1. End Semester Examination result
2. Course Work result and
3. Attendance percentage.

References

[1] B. Kosko, “Fuzzy cognitive maps,” International Journal of Man-Machine Studies., vol. 24, no. 1, pp. 65–75, 1986, doi: 10.1016/S0020-7373(86)80040-2.

[2] R. Axelrod, Structure of Decision: The Cognitive Maps of Political Elites. Princeton University Press, 2015.

[3] E. I. Papageorgiou, "Review study on fuzzy cognitive maps and their applications during the last decade," 2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011), 2011, pp. 828-835, doi: 10.1109/FUZZY.2011.6007670.

[4] C. D. Stylios and P. P. Groumpos, “Modeling Complex Systems Using Fuzzy Cognitive Maps,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, Cybern. Part ASystems Humans., vol. 34, no. 1, pp. 155–162, Jan. 2004, doi: 10.1109/TSMCA.2003.818878.

[5] X. Gao, X. Pan, X. Liu, W. Pedrycz, and Z. Wang, “Modeling of the ship steady turning motion based on multiblock of fuzzy cognitive maps,” Applied Ocean Research., 110, p.102604, 2021

[6] C. D. Stylios, V. C. Georgopoulos, G. A. Malandraki, and S. Chouliara, “Fuzzy cognitive map architectures for medical decision support systems,” Applied Soft Computing., 2007, vol. 8, no. 3, pp. 1243-1251, 2008.

[7] P. Hajek and O. Prochazka, “Interval-Valued fuzzy cognitive maps for supporting business decisions,” in 2016 IEEE International Conference on Fuzzy Systems, FUZZ-IEEE 2016, Nov. 2016, pp. 531–536, doi: 10.1109/FUZZ-IEEE.2016.7737732.

[8] C. D. Stylios and P. P. Groumpos, “Fuzzy Cognitive Maps in modeling supervisory control systems,” Journal of Intelligent & Fuzzy Systems., vol. 8, no. 1, pp. 83–98, Jan. 2000.

[9] A. S. Andreou, N. H. Mateou, and G. A. Zombanakis, “Soft computing for crisis management and political decision making: The use of genetically evolved fuzzy cognitive maps,” Applied Soft Computing., vol. 9, no. 3, pp. 194–210, Feb. 2005, doi: 10.1007/s00500-004-0344-0.

[10] E. I. Papageorgiou, “Learning algorithms for fuzzy cognitive maps—a review study,” IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)., vol. 42, no. 2, pp. 150–163, 2012.

[11] K. T. Atanassov, “Intuitionistic Fuzzy Sets,” International Journal Bioautomation., 20 (S1), S1-S6, 2016.

[12] D. K. Iakovidis and E. Papageorgiou, “Intuitionistic fuzzy cognitive maps for medical decision making,” IEEE Transactions on Information Technology in Biomedicine, vol. 15, no. 1, pp. 100–107, Jan. 2011, doi: 10.1109/TITB.2010.2093603.

[13] E. I. Papageorgiou and D. K. Iakovidis, “Intuitionistic fuzzy cognitive maps,” IEEE Transactions on Fuzzy Systems., vol. 21, no. 2, pp. 342–354, 2013, doi: 10.1109/TFUZZ.2012.2214224.

[14] J. L. Salmeron, “Modelling grey uncertainty with fuzzy grey cognitive maps,” Expert Systems with Applications., vol. 37, no. 12, pp. 7581–7588, Dec. 2010, doi: 10.1016/j.eswa.2010.04.085.

[15] E. Yesil, M. F. Dodurka, and L. Urbas, “Triangular fuzzy number representation of relations in Fuzzy Cognitive Maps,” in IEEE International Conference on Fuzzy Systems, Sep. 2014, pp. 1021–1028, doi: 10.1109/FUZZ-IEEE.2014.6891653.

[16] P. Hajek and W. Froelich, “Integrating TOPSIS with interval-valued intuitionistic fuzzy cognitive maps for effective group decision making,” Information Science. (Ny)., vol. 485, pp. 394–412, Jun. 2019, doi: 10.1016/j.ins.2019.02.035.

[17] P. Zdanowicz and D. Petrovic, "New Mechanisms for Reasoning and Impacts Accumulation for Rule-Based Fuzzy Cognitive Maps," IEEE Transactions on Fuzzy Systems., vol. 26, no. 2, pp. 543-555, April 2018, doi: 10.1109/TFUZZ.2017.2686363.

[18] J. M. Mendel and R. I. B. John, “Type-2 fuzzy sets made simple,” IEEE Transactions on Fuzzy Systems., vol. 10, no. 2, pp. 117–127, Apr. 2002, doi: 10.1109/91.995115.

[19] F. Liu and J. M. Mendel, “An interval approach to fuzzistics for interval type-2 fuzzy sets,” 2007 IEEE International Fuzzy Systems Conference, 2007.

[20] S. Coupland, J. M. Mendel, and D. Wu, “Enhanced interval approach for encoding words into interval type-2 fuzzy sets and convergence of the word fous,” International Conference on Fuzzy Systems, 2010.

[21] P. Cheng, H. Wang, V. Stojanovic, S. He, K. Shi, X. Luan, F. Liu, and C. Sun, “Asynchronous fault detection observer for 2-D markov jump systems,” IEEE Transactions on Cybernetics., pp. 1–12, 2021.

[22] X. Zhang, H. Wang, V. Stojanovic, P. Cheng, S. He, X. Luan, and F. Liu, “Asynchronous fault detection for interval type-2 fuzzy nonhomogeneous higher-level Markov jump systems with uncertain transition probabilities,” IEEE Transactions on Fuzzy Systems., pp. 1–1, 2021.

[23] C. Wagner and H. Hagras, “Toward general type-2 fuzzy logic systems based on zSlices,” IEEE Transaction on Fuzzy Syst., vol. 18, no. 4, pp. 637–660, 2010, doi: 10.1109/TFUZZ.2010.2045386.

[24] H. Hassani, J. Zarei, M. M. Arefi, and R. Razavi-Far, “Zslices-based general type-2 fuzzy fusion of support vector machines with application to Bearing Fault Detection,” IEEE Transactions on Industrial Electronics., vol. 64, no. 9, pp. 7210–7217, 2017.

[25] J. Adams and H. Hagras, “A type-2 fuzzy logic approach to explainable AI for regulatory compliance, fair customer outcomes and market stability in the global financial sector,” 2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 2020.

[26] A. Amirkhani, M. Shirzadeh, and T. Kumbasar, “Interval type-2 fuzzy cognitive map-based flight control system for Quadcopters,” International Journal of Fuzzy Systems., vol. 22, no. 8, pp. 2504–2520, 2020.

[27] A. Al Farsi, F. Doctor, D. Petrovic, S. Chandran, and C. Karyotis, “Interval valued data enhanced fuzzy cognitive maps: Torwards an appraoch for Autism deduction in Toddlers,” IEEE International Conference on Fuzzy Systems (FUZ-IEEE), Aug. 2017, doi: 10.1109/FUZZ-IEEE.2017.8015702.

[28] C. Wagner, S. Miller, J. M. Garibaldi, D. T. Anderson, and T. C. Havens, “From Interval-Valued Data to General Type-2 Fuzzy Sets,” IEEE Transactions on Fuzzy Systems., vol. 23, no. 2, pp. 248–269, Apr. 2015, doi: 10.1109/TFUZZ.2014.2310734.

[29] A. Kannappan, A. Tamilarasi, and E. I. Papageorgiou, “Analyzing the performance of fuzzy cognitive maps with non-linear hebbian learning algorithm in predicting autistic disorder” Expert Systems with Applications., vol. 38, no. 3, pp. 1282–1292, Mar. 2011, doi: 10.1016/j.eswa.2010.06.069.

[30] E. I. Papageorgiou and J. L. Salmeron, “A review of fuzzy cognitive maps research during the last decade,” IEEE Transactions on Fuzzy Systems., vol. 21, no. 1, pp. 66–79, 2013, doi: 10.1109/TFUZZ.2012.2201727.

[31] S. A. Gray, E. Zanre, and S. R. J. Gray, “Fuzzy cognitive maps as representations of mental models and group beliefs,” In Fuzzy cognitive maps for applied sciences and engineering, Springer vol. 54, pp. 29–48, Jan. 2014.

[32] A. Jastrzebska, G. Nápoles, W. Homenda, and K. Vanhoof, "Fuzzy Cognitive Map-Driven Comprehensive Time-Series Classification," IEEE Transactions on Cybernetics., doi:10.1109/TCYB.2021.3133597.

[33] Y. Wang, F. Yu, W. Homenda, W. Pedrycz, Y. Tang, A. Jastrzebska, and F. Li, "The Trend-Fuzzy-Granulation-Based Adaptive Fuzzy Cognitive Map for Long-Term Time Series Forecasting," IEEE Transactions on Fuzzy Systems., doi: 10.1109/TFUZZ.2022.3169624.

[34] L. A. Zadeh, “The concept of a linguistic variable and its application to approximate reasoning-I,” Information Sciences. (Ny)., vol. 8, no. 3, pp. 199–249, Jan. 1975, doi: 10.1016/0020-0255(75)90036-5.
[35] Z. Ellerby, C. Wagner, and S. B. Broomell, “Capturing richer information: On establishing the validity of an interval-valued survey response mode,” Behavior Research Methods., vol. 54, no. 3, pp. 1240–1262, 2021.
[36] S. Miller, C. Wagner, J. M. Garibaldi and S. Appleby, "Constructing General Type-2 fuzzy sets from interval-valued data," 2012 IEEE International Conference on Fuzzy Systems, 2012, pp. 1-8, doi: 10.1109/FUZZ-IEEE.2012.6251221.
[37] J. M. Mendel, “Computing with words and its relationships with fuzzistics,” Information Sciences (Ny)., vol. 177, no. 4, pp. 988–1006, Feb. 2007, doi: 10.1016/j.ins.2006.06.008.
[38] F. Liu and J. M. Mendel, "An Interval Approach to Fuzzistics for Interval Type-2 Fuzzy Sets," 2007 IEEE International Fuzzy Systems Conference, 2007, pp. 1-6, doi: 10.1109/FUZZY.2007.4295508.
[39] J. Mendel, Uncertain rule-based fuzzy systems: Introduction and new directions. USA: Prentice Hall, 2001.
[40] G. J. Klir and T. A. Folger, Fuzzy Sets, Uncertainty and Information. Prentice-Hall International, Inc., 1988.
[41] J. Navarro, C. Wagner, U. Aickelin, L. Green, and R. Ashford, “Exploring differences in interpretation of words essential in medical expert-patient communication,” in 2016 IEEE International Conference on Fuzzy Systems, FUZZ-IEEE 2016, Nov. 2016, pp. 2157–2164, doi: 10.1109/FUZZ-IEEE.2016.7737959.
[42] J. Navarro, C. Wagner, U. Aickelin, L. Green and R. Ashford, "Measuring agreement on linguistic expressions in medical treatment scenarios," 2016 IEEE Symposium Series on Computational Intelligence (SSCI), 2016, pp. 1-8, doi: 10.1109/SSCI.2016.7849895.
[43] “Pearson Product-Moment Correlation - When you should run this test, the range of values the coefficient can take and how to measure strength of association.” https://statistics.laerd.com/statistical-guides/pearson-correlation-coefficient-statistical-guide.php (accessed Feb. 16, 2021).
[44] S. Mohammadi and S. Cremaschi, “Efficiency of uncertainty propagation methods for estimating output moments,” Computer Aided Chemical Engineering., pp. 487–492, 2019.

[45] T. da Cruz Asmus, G. Pereira Dimuro, B. Bedregal, J. A. Sanz, R. Mesiar, and H. Bustince, “Towards interval uncertainty propagation control in Bivariate Aggregation Processes and the introduction of width-limited interval-valued overlap functions,” Fuzzy Sets and Systems., vol. 441, pp. 130–168, 2022.
[46] “Linear Regression Analysis in SPSS Statistics - Procedure, assumptions and reporting the output.” https://statistics.laerd.com/spss-tutorials/linear-regression-using-spss-statistics.php (accessed Jan. 24, 2021