

# Blockchain-Based Secure and Efficient Secret Image Sharing with Outsourcing Computation in Wireless Networks

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**Abstract**—Secret Image Sharing (SIS) is the technology that shares any given secret image by generating and distributing  $n$  shadow images in the way that any subset of  $k$  shadow images can restore the secret image. However, in the existing SIS schemes, the shadow images will be easily tampered and corrupted during the communication, which will pose serious security issues. Recently, blockchain has emerged as a promising paradigm in the field of data communication and information security. To securely communicate and effectively protect the secret image data in wireless networks, we propose a Blockchain-based Secure and Efficient Secret Image Sharing (BC-SEISIS) scheme with outsourcing computation in wireless networks. In the proposed BC-SEISIS scheme, the shadow images are encrypted and stored in the blockchain to prevent them from being tampered and corrupted. The identity authentication-enabled smart contract is deployed to achieve the  $(k, n)$  threshold for secret image restoring. Furthermore, to reduce the computational burden of smart contract and users, an efficient outsourcing computation method is designed to outsource the restoring task, which is securely implemented by agent miners in the encryption domain. Theoretical analysis and extensive experiments demonstrate that the BC-SEISIS scheme can achieve desirable communication security and high computational efficiency in the wireless networks.

**Index Terms**—Blockchain, Secret image sharing, Wireless networks, Outsourcing computation.

## I. INTRODUCTION

WITH the rapid development of wireless communication technology, the broadcast nature of wireless medium makes multimedia information in wireless communication

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very vulnerable to various security threats such as message modification, tampering, and corruption in wireless networks [1]–[3]. Thus, it is necessary to guarantee the security of any given secret image in the wireless networks. To this end, Secret Image Sharing (SIS) is the technology that shares the secret image by generating  $n$  shadow images in the way that any subset of  $k$  shadow images can restore the secret image [4]–[13]. The  $(k, n)$ -SIS scheme was first proposed by Thien and Lin [4], which is based on the theory of polynomial. In this scheme, every  $k$  secret pixels are used as  $k$  coefficients of a  $(k-1)$ -degree polynomial  $f(x)$ . Then, a dealer can produce  $n$  shadow pixels by computing  $f(x_i)$ , where  $x_i$  is an integer and  $i \in [1, n]$ . After repeating the above process until all pixels of the secret image have been processed,  $n$  shadow images are generated and then transmitted to  $n$  corresponding participants on networks. By Lagrange’s interpolation algorithm, any  $k$  shadow images can jointly restore the secret image, but  $(k-1)$  or fewer shadow images cannot. Therefore, the  $(k, n)$ -SIS scheme can be deemed as a threshold-based cryptography scheme.

Although  $(k-1)$  or fewer shadows cannot disclose the information of the secret image, it is very likely that these shadows would be tampered or corrupted in the wireless networks, which will cause the original secret image not to be correctly restored. Thus, it is an urgent demand for SIS schemes to prevent the shadow images from being tampered and/or corrupted in the wireless networks [3], [9]–[13].

To alleviate the above issue, some SIS schemes based on image steganography have been proposed [6], [7], [14]–[16]. Generally, each generated shadow image is hidden into a cover image by imperceptibly modifying the cover image before transmitting in networks. As a result, the risk of tampering and corrupting shadow images can be reduced significantly. It is notable that, although the hidden shadow images are hard to detect by human eyes, it is likely that these shadows can be detected by the powerful steganalysis tools [17]–[19].

As a decentralized data structure, blockchain can effectively address a lot of security issues of the traditional centralized data structures, and thus it has been widely employed in a variety of fields, such as finance [20], [21], healthcare [22]–[24], entertainment [25], [26], supply chain [22], [27], and transportation [28]–[30]. It has become one of promising paradigms in the field of data communication and information security, since it cannot only prevent secret data from being tampered and/or corrupted, but also address the issues of

malicious codes and behaviors [27], [31]–[35].

Motivated by the great potential of blockchain, to securely communicate and effectively protect the secret image data distributed on the networks, we propose a Blockchain-based Secure and Efficient Secret Image Sharing (BC-SEISIS) with outsourcing computation in wireless networks. In the proposed BC-SEISIS scheme, the shadow images are first generated from a given secret image, and then they are encrypted by Fully Homomorphic Encryption (FHE) algorithm and stored in the blockchain to prevent them from being tampered and corrupted during the wireless communication. In the stage of secret image restoring, the identity authentication-enabled smart contract is designed and deployed to achieve the  $(k, n)$  threshold of SIS for secret image restoring. Furthermore, to reduce the computational burden of smart contract and users, an efficient outsourcing computation method is designed to outsource a considerable part of secret image restoring task, which is securely implemented by agent miners in the encryption domain. Theoretical analyses and extensive experiments demonstrate that the BC-SEISIS scheme not only shows strong resistance to data tampering and corruption, but also has high computational efficiency. Our contributions are summarized as follows:

**1) The novel SIS scheme is proposed based on the blockchain and FHE algorithm.** To the best of our knowledge, this is the first work that employs the blockchain and FHE algorithm in the field of SIS. Since the encrypted information stored on blockchain is hard to tamper, the proposed BC-SEISIS scheme not only protects the secret image from leaking, but also prevents the shadow images from being tampered with and corrupted during the wireless communication.

**2) The proposed scheme supports the automatic identification of participant identity and triggering of restoring task.** To facilitate the secret image restoring stage, by the smart contract, the proposed scheme automatically identifies the participants who apply for the restoring process, and outsources the restoring tasks after receiving enough authorizations. Moreover, the Automatic Identity Authentication (AutoIDAuth)-enabled smart contract is designed to realize the  $(k, n)$  threshold of SIS.

**3) The FHE-based outsourcing computation method is designed to achieve secret image restoring in the encryption domain.** In the BC-SEISIS scheme, the shadow images are encrypted with the FHE algorithm before storing on the blockchain. Owing to the property of the FHE algorithm, the FHE-based outsourcing computation method can outsource the polynomial computation in the encryption domain by miners in the blockchain, which can reduce the computation burden for the smart contract and users significantly.

The remainder of this paper is structured as follows. Section II presents the related work. Section III describes the proposed BC-SEISIS scheme. Section IV theoretically analyzes the security of the system. Section V presents and analyzes the experimental results. Conclusions are given in Section VI.

## II. RELATED WORK

First, we review the typical SIS schemes. Then, as the proposed BC-SEISIS scheme is highly related to the blockchain

technologies including Inter Planetary File System (IPFS) and smart contract, we also introduce these blockchain technologies.

### A. Secret Image Sharing

Naor and Shamir [36] first proposed the secret sharing scheme in 1979. In essence, instead of directly communicating the original secret data on the network, this secret sharing scheme generates a set of random-like data, called as shares or shadows, from given secret data, and then distributes them to the participants to ensure that each participant has one shadow. In this scheme, the secret data elements are hidden into the constant coefficient of a  $(k - 1)$ -degree polynomial  $f(x)$ . Then, someone can generate  $n$  shadows by computing  $f(x_i)$ , where  $x_i$  is a real number and  $x_i \in [0, p - 1]$ ,  $i \in [1, n]$ . After repeating the above process for every  $k$  data elements of the secret image,  $n$  shadows are generated and then sent to  $n$  corresponding participants on networks. By Lagrange's interpolation algorithm, any  $k$  shadows can jointly restore the secret image, but no information of secret image can be revealed by  $(k - 1)$  or fewer shadow images. Therefore, this scheme can be deemed as a  $(k, n)$  threshold SIS scheme. Thien and Lin [4] extended Shamir's scheme for image data, and hid every  $k$  secret pixels into all the  $k$  coefficients of the polynomial. As a result, the sizes of shadows are decreased to  $\frac{1}{k}$  of that of the original secret image.

Subsequently, many SIS schemes were proposed in the past decades. They focus on using different hiding methods to improve the efficiency of sharing [37]–[39], studying the degree of dependence on trusted third-party [40], and the flexibility and robustness of restoring images [38], [41]. After carefully reviewing these references, the comprehensive comparison between the proposed scheme and the related SIS schemes are demonstrated in Table I. Note that, although all the above SIS schemes can protect the secret image from being obtained by the others without enough shadows, they cannot prevent the shadow images from being tampered and corrupted in communication networks, which will cause the secret image cannot be exactly restored.

To alleviate the above issue, some SIS schemes have been proposed based on image steganography to hide each generated shadow image into a cover image before distribution [6], [7], [14], [15]. Lin *et al.* [6] first combined steganography and SIS technologies to enhance the security of shadow images during the communication. They hid the shadow images into common meaningful cover images imperceptibility. Lin *et al.* [14] proposed a SIS scheme using invertible steganography technology, in which both the cover images and shadow images can be exactly restored. Although these steganography-based SIS schemes can reduce the possibility of shadow images being attacked to some extent, it is very likely that the existence of shadow images hidden in the covers can be exposed by the statistical feature-based steganalysis methods [18], [19], thereby compromising the security of secret communication. Also, some other SIS schemes have proposed based on the integrity verification technology, which can verify the integrity of shadow images to determine whether they have been tampered or not [14], [15].

TABLE I

COMPARISONS BETWEEN THE RELATED SECRET SHARING SCHEMES AND THE PROPOSED SCHEME. (VG: VISUAL CRYPTOGRAPHY, RG: RANDOM GRID, HE: HOMOMORPHIC ENCRYPTION)

	[38]	[39]	[40]	[41]	The proposed method
Security model	N/A	N/A	N/A	Secure	Semi-honest
Pixel expansion	Yes	Minimum	No	No	No
Hiding method	VC	RG	RG	VC	HE
Code book needed	Yes	Yes	No	No	No
Trusted third-part needed	Yes	Yes	No	Yes	Yes
Tamper resistance	No	No	No	No	Yes
Recovery type	Visible	Recognizable	Recognizable	Recognizable	Lossless

In summary, the steganography-based SIS schemes and the integrity verification-based SIS schemes cannot effectively prevent the shadow images from being tampered and corrupted during the wireless communication, which will pose serious security vulnerabilities.

### B. Blockchain Technologies

**Blockchain:** In the literature [27], [31]–[35], [42]–[44], blockchain is usually regarded as a decentralized and distributed data ledger. By using blockchain technology, new information is added to a block and is made available to all the users or nodes in a distributed network. As the maintainers of blockchain, the miners are the network nodes mainly responsible for producing new blocks via the Proof of Work (PoW) mechanism [45]. The PoW mechanism also improves the cost of different malicious attacks and the security level of the blockchain system [46]. Although all the network nodes can observe the data stored on the blockchain, it is very difficult for them to tamper and corrupt the data unless someone breaks 51% network nodes at the same time.

Recently, the blockchain technology has gained increasing attention by integrating its potential benefits into IoT systems [47]–[52]. The researchers explored blockchain technology to improve the security and efficiency of IoT systems. For example, Zuo *et al.* [47] proposed a new cooperative Mobile Edge Computing (MEC) blockchain computation offloading scheme. The cooperative approach can serve for more IoT devices with offloading computations compared with noncooperative schemes. The non-trustworthy MEC verification scheme [49] was proposed for blockchain IoT system, which can allocate computing resources to IoT nodes more reasonably. In [50], the delay-limited mining task based on PoW was formulated as a non-cooperative game, and the Continuous Relaxation and Greedy Rounding (CRGR)-based alternating iterative algorithm was proposed to efficiently achieve optimal delay-limited computation offloading for all users.

The above approaches have demonstrated the security and effectiveness of blockchain techniques in wireless network environment, which inspire us to propose the blockchain-based secure and efficient secret image sharing with outsourcing computation in wireless networks. Compared with the traditional secret sharing schemes without blockchain [41], [53]–[57], our scheme has the following unique advantages in terms of the security performance. (1) Anti-tampering ability to verify the validity of the data [58]–[60], (2) the transparency of allowing nodes in the network to obtain information [61],

[62], and (3) the interactivity of multi-party security cooperation without a third-party organization [63]–[65]. Therefore, this paper proposes a novel secure and efficient SIS scheme based on the blockchain.

**IPFS:** Motivated by the decentralized characteristic of blockchain, a peer-to-peer distributed file system, *i.e.*, Inter Planetary File System (IPFS), has been proposed [66]–[68]. Different from the traditional distributed file systems that depend on a centralized server for file management and storage, IPFS stores and shares the file data among different network nodes without the need of a central server [68]. The IPFS can offer a high-through storage model by using the Distributed Hash Tables (DHT).

Since the encrypted shadow images files produced by the BC-SEIS scheme usually have large sizes, directly storing these large sized files on the blockchain will cause large storage burden and high latency for the blockchain. Thus, instead of storing these image files directly on the blockchain, in the BC-SEIS scheme, we store the image files in the IPFS, and then send the hash strings (the file’s addresses) returned by the IPFS to the blockchain. If someone is allowed to access an image file, he computes its address at first and then downloads the file from the IPFS according to this address.

**Smart Contract:** The smart contract has been widely used and studied with the development of blockchain. In 1994, Szabo [69] proposed the concept of “smart contract” and defined a smart contract as a computerized transaction protocol that supports the implementation of the contract’s terms. Smart contracts are usually deployed on the blockchain in the form of codes, and they are executed in Ethereum Virtual Machine (EVM) [28], which is a platform of conducting smart contracts on the blockchain. Each smart contract is assigned a unique address on the blockchain, and it can be invoked by being sent transactions. Generally, the smart contract runs on every network node independently and automatically in a predefined manner [28], [70]–[72]. The use of smart contracts can reduce the transaction cost for participants and the occurrence of abnormal and malicious actions on the blockchain. Due to the above advantages of a smart contract, in the proposed BC-SEIS scheme, we design the AutoIDAuth-enabled smart contract to realize the  $(k, n)$  threshold of SIS.

### III. THE PROPOSED BC-SEIS SCHEME

In this section, we introduce the proposed BC-SEIS scheme in detail. In Section III-A, we introduce the general framework of BC-SEIS scheme. Section III-B describes the

image sharing process. Section III-C elaborates on the image restoring process.

#### A. The Framework of Proposed BC-SEISIS scheme

1) *Security model*: We considered the semi-honest model throughout the paper. Specifically, any number of participants with fewer than  $k$  would conspire with external users in the wireless network to attempt to obtain, tamper or corrupt the image shares of others. Moreover, we assume that attackers cannot compromise the security of the blockchain network. In other words, the attackers do not have the ability to hold most of the computing power and network resources in the blockchain, since most of the nodes in the blockchain network are assumed to be honest and reliable. In addition, we introduce the BFV homomorphic encryption algorithm to ensure the secure outsourcing of secret image restoring. According to the illustration in [72]–[74], this encryption algorithm has superior performances in the efficiency and feasibility, and it is also considered to be secure enough.

2) *Roles*: To facilitate the introduction of the BC-SEISIS scheme, we first list the main roles, which are given as follows.

**Dealer**: The *Dealer* is the owner of secret image, who takes charge of generating and distributing a set of shadow images and an encryption key to the group of *Participants*.

**Participants**: In the secret image sharing stage, each *participant* receives one shadow image and the encryption key from the *Dealer*. Then, he encrypts the shadow image by the key, and uploads the encrypted shadow to the blockchain. In the stage of secret image restoring, each *Participant* decides whether to authorize the access of his shadow upon the request of *Applicant*.

**Applicant**: The *Applicant* is one of the *Participants* who intends to restore the secret image by the designed AutoIDAuth-enabled smart contract.

**Agent Miners**: The *Agent Miners* are the network nodes on the blockchain. They are responsible for the computation of polynomials in encryption domain for the secret image restoring.

3) *Trust setup phase*: Assigns a mutual-trust party (the *dealer*), who calculates  $n$  secret shares according to the original secret image and distributes them to  $n$  *participants* securely, so that any  $k$  or more *participants* who share their image shares can easily recover the original secret, but any group that only knows  $k - 1$  or less shares cannot recover the secret.

4) *Framework*: Then, we introduce the framework, as shown in 1. It contains the secret image sharing stage and secret image restoring stage. The main notations used in the proposed scheme are listed in TABLE II.

**Secret image sharing stage**: Given a secret image  $SI$ , the *Dealer* first generates  $n$  shadow images  $\{S_i | 1 \leq i \leq n\}$ , and distributes the shadows and the private key of FHE algorithm denoted as  $(S_i, sk)$  to  $n$  *Participants* in a secure way. Subsequently, the *Participants* encrypt their own shadows by the FHE algorithm. Finally, the *Participants* store the encrypted shadow images  $S'_i$  to IPFS, and then upload the address of encrypted file returned from IPFS to the blockchain.

TABLE II  
THE NOTATIONS IN THE PROPOSED SCHEME

Notation	Description
$k, n$	The threshold value of $(k, n)$ -SIS
$SI$	The original secret image
$S_i$	The $i$ -th shadow image
$sk, pk$	The private key and public key of FHE algorithm
$S'_i$	The $i$ -th encrypted shadow image
$w \times h$	The size of the original secret image
$a_i$	The $i$ -th coefficient of the polynomial
$N$	The number of pixels divided into each batch
$v_i^j$	The $j$ -th vector set segmented from the $i$ -th shadow image
$len$	The number of vectors in $v_i^j$
$S'_i(x)$	The encrypted shadow image
$Ln(x)$	The Lagrange interpolation polynomial
$X_i$	The encrypted form of the random number $x_i$
$A_{tj}$	The encrypted coefficients of polynomial

**Secret image restoring stage**: First, the *Applicant* sends the request of image restoring with his identity information to the designed AutoIDAuth-enabled smart contract when he intends to restore the secret image. Then, the smart contract verifies the identity, and asks for the authorizations of the access of the *Participants*' shadow images. Once the smart contract collects  $k$  authorizations, it outsources a part of secret image restoring task, *i.e.*, the computation of polynomials in encryption domain on the blockchain. Subsequently, some *Agent Miners* compute the polynomials in encryption domain, and then send the coefficients of polynomials to the smart contract. Finally, the smart contract verifies these coefficients and sends them to the *Applicant*, and the *Applicant* computes the secret image by these coefficients.

In the following, we elaborate on the two stages of the framework of proposed BC-SEISIS scheme.

#### B. Secret Image Sharing Stage

The stage of a secret image sharing consists of three main steps: shadow image generation, shadow image encryption, and shadow image uploading. Each step is detailed as follows.

**Step (1) Shadow image generation**: To generate the shadow images from a given secret image  $SI$  with the size of  $w \times h$ , like the existing polynomial-based SIS schemes, we also adopt the polynomial  $(k - 1)$ -degree polynomial, which is defined as follows.

**Definition 1.** Given two positive integers  $k$  and  $n$ , where  $2 \leq k \leq n$ , and we get a  $(k - 1)$ -degree polynomial as follows:

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_{k-1}x^{k-1} \cdot \text{mod} \cdot p \quad (1)$$

Where,  $(a_1, a_2, \dots, a_{k-1})$  are the coefficients of the polynomial and  $p$  is a prime number.

By using the values of every  $k$  secret pixels as the  $k$  coefficients of the polynomial, the *Dealer* inputs a set of  $n$  random numbers, *i.e.*,  $(x_1, x_2, \dots, x_n)$ , no more than  $p$  into the  $(k - 1)$ -degree polynomial to compute the corresponding  $n$  shadow pixels  $\{f(x_i) | 1 \leq i \leq n\}$  by

$$f(x_i) = a_0 + a_1x_i + a_2x_i^2 + \dots + a_{k-1}x_i^{k-1} \cdot \text{mod} \cdot p \quad (2)$$

Then, the *Dealer* repeats the above process until all the pixels of the secret image have been traversed. Consequently,

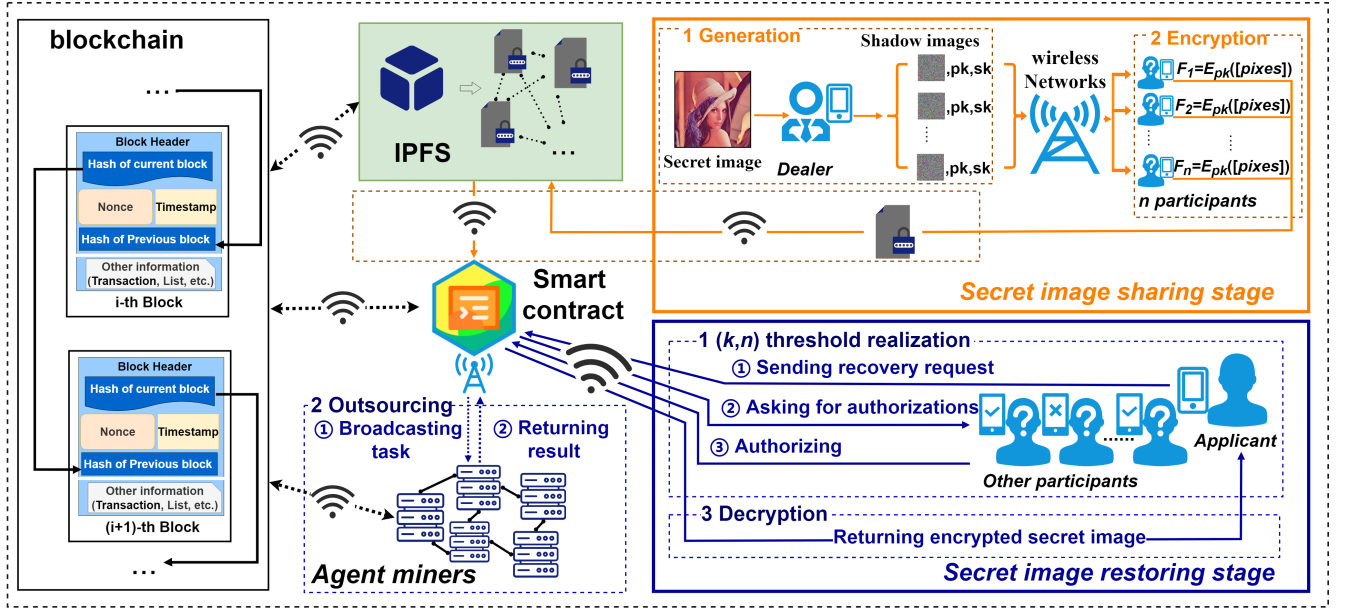


Fig. 1. The framework of proposed BC-SEIS scheme

$n$  shadow images are obtained, and the sizes of shadow images are  $\frac{1}{k}$  of that of the original secret one. Note that all the  $n$  random numbers  $\{x_i | 1 \leq i \leq n\}$  are publicly available.

In the existing polynomial-based SIS schemes, the prime number  $p$  is usually set as 251. Thus, for each pixel within the range of  $[251, 255]$  in the secret image, it should be truncated to 250 or be expanded to two pixels before sharing. Consequently, the secret image will be distorted or be expanded significantly. To avoid the above issues, like Yang *et al.* SIS scheme [13], we set the prime number  $p$  as 257 to generate the shadow images. If a very few pixels of generated shadow images exceed 255, we only slightly modify these pixels of original secret image to ensure that all pixels of generated shadows are within the range of  $[0, 255]$ .

**Step (2) Shadow image encryption:** To enable the outsourcing computation of secret image restoring, we adopt the FHE to encrypt those shadow images before uploading them to the blockchain, since FHE algorithm allows secure computations on encrypted data without decrypting it. Compared with other FHE algorithms [73]–[76], BFV algorithm [77] has superior performances in the aspects of efficiency and feasibility due to its smaller relinearization key [77]–[79]. Thus, we choose BFV as the FHE algorithm in the proposed BC-SEIS scheme for shadow image encryption.

It is notable that, in the restoring stage of the traditional SIS schemes, the *Applicant* generally uses only one pixel from each of  $k$  shadow images to compute the coefficients of  $(k - 1)$ -degree polynomial by Lagrange's interpolation algorithm. Those  $k$  coefficients are the  $k$  pixels of the original secret image. The *Applicant* repeats the above process until all secret pixels are computed. However, the restoring process is quite time-consuming, especially in the encrypted domain. To efficiently encrypt and decrypt the secret image, a batch encryption strategy is designed to encrypt the generated shadow

images before uploading to the blockchain.

As shown in 2, instead of encrypting the pixels one by one, the *Participants* first divide the pixels of each shadow image  $S_i$  to form a set of vectors  $\{v_i^j | 1 \leq j \leq len\}$ , represented by

$$\left\{ \begin{array}{l} v_i^1 = \{S_i(1), S_i(2), \dots, S_i(N)\} \\ v_i^2 = \{S_i(N + 1), S_i(N + 2), \dots, S_i(2N)\} \\ \dots \\ v_i^j = \{S_i(N \times (j - 1) + 1), S_i(N \times (j - 1) + 2) \dots \\ \dots, S_i(N \times j)\} \\ \dots \\ v_i^{len} = \{S_i(N \times (len - 1) + 1), \\ S_i(N \times (len - 1) + 2), \dots, S_i(w \times h)\} \end{array} \right. \quad (3)$$

Where the value of  $len$  refers to the number of vectors in this set. Then, those vectors are encrypted sequentially by BFV algorithm as  $\{v_i^j | 1 \leq j \leq len\}$ . Finally, the set of encrypted vectors are packed as an encrypted shadow image file  $S_i^t$ .

By the above batch encryption strategy, a batch of pixels can be encrypted by one encryption operation, and these encrypted pixels can be also decrypted by a single decryption operation. Consequently, the encryption and decryption can be efficiently implemented, and thus a lot of computing time can be saved. That is also proven by the experimental results given in Sections V-C and V-D.

**Step (3) Shadow image uploading:** It is worth noting that directly storing these large sized encrypted files on the blockchain will cause large storage burden and high latency for the blockchain. Thus, the *Participants* upload each encrypted file to the IPFS, and the IPFS returns a unique hash string and uses it as the file's address. Then, this address is sent to the blockchain.

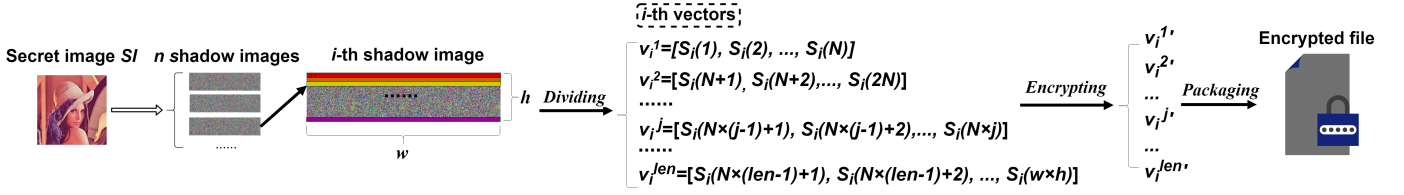


Fig. 2. The process of batch encryption.

### C. Secret Image Restoring Stage

In the stage of secret image restoring, there are three main steps:  $(k, n)$  threshold realization, encrypted domain polynomial computation, and secret image decryption. To restore the original secret image, we design the AutoIDAuth-enabled smart contract for realizing the  $(k, n)$  threshold of SIS and the FHE-based outsourcing computing method for implementing the polynomial computation in encryption domain.

**Step (1)  $(k, n)$  threshold realization:** As introduced in Section II-B, the smart contract is essentially a computing independently and automatically on every node of the network protocol deployed on the blockchain in form of codes. It runs in a predefined manner, thereby decreasing the transaction cost for participants and the occurrence of abnormal and malicious actions on the blockchain. Due to the advantages of the smart contract, we design the AutoIDAuth-enabled smart contract to realize the  $(k, n)$  threshold of SIS by authenticating the identities of Participants. To realize the  $(k, n)$  threshold of SIS, the AutoIDAuth-enabled smart contract works as follows.

When the *Applicant* intends to restore a secret image, he is required to send the restoring request with his identity and gas reward to the smart contract by the HTTP SSL protocol. Then, the smart contract authenticates the identity of *Applicant*. If the authentication is passed, the smart contract broadcasts the *Applicant's* restoring request to the  $n$  *Participants* to ask for their authorizations for access of the encrypted shadow image files.

The *Participants* who agree to authorize will send their identity information to the smart contract, and the smart contract will authenticate the *Participants'* identities. If the number of *Participants'* authorizations is larger than  $k$ , the smart contract will automatically trigger the following process, *i.e.*, the encrypted domain polynomial computation; Otherwise, the *Applicant* will be informed that the original secret image cannot be restored for the lack of enough *Participants'* authorizations.

**Step (2) Encrypted domain polynomial computation:** As mentioned in the previous step, once the smart contract receives at least  $k$  *Participants'* authorizations, it will trigger a part of secret image restoring task, *i.e.*, encrypted domain polynomial computation. Note that it is infeasible for the smart contract to directly implement the computation task, since the computation will cause large computing burden and high latency for the smart contract. Thus, the FHE-based outsourcing computing method is proposed to outsource the polynomial computation task to the *Agent Miners* of blockchain. In this method, BFV [73] is selected as the FHE

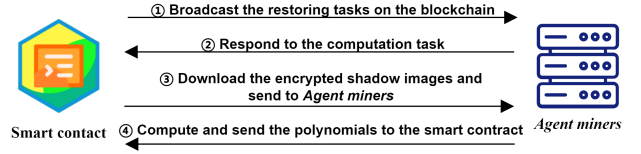


Fig. 3. The illustration of outsourcing process, in which the smart contract outsources the task of secret image computation to the *Agent miners*.

algorithm to encrypt the generated shadow images, since BFV has superior performances in the aspects of efficiency and feasibility [73], [77], as mentioned in Section III-B. The outsourcing process is described as follows.

As shown in the 3, after collecting more than  $k$  *Participants'* authorizations, the smart contract broadcasts the task of encrypted domain polynomial computation to the *Agent miners* on the blockchain. Then, the *Agent miners* respond to the task. By the  $k$  *Participants'* authorizations, the smart contract downloads the corresponding encrypted shadow image files from IPFS, and sends them to  $m$  randomly selected *Agent miners* for implementing this task. Afterward, these *Agent miners* compute the polynomials in encrypted domain separately, and send the coefficients of computed polynomials to the smart contract. Finally, the smart contract compares all the computed polynomials to decide whether to keep the computed result. Specifically, if all the computed results are the same, the smart contract will keep the coefficients of computed polynomials and pay the gas reward to the *Agent miners*; Otherwise, it will broadcast the computation task again. That ensures the computation process is implemented correctly and securely. The computation of polynomials in encrypted domain is elaborated as follow.

Suppose an *Agent miner* receives  $k$  encrypted shadow images  $\{S_i^j | 1 \leq i \leq n\}$  from the smart contract. According to the FHE and Lagrange Interpolation algorithms, the encrypted secret image  $SI'$  can be restored from these shadows by computing a set of  $(k-1)$ -degree polynomials.

Recall that every  $N$  pixels of each shadow image are transformed to a vector, and the vector is encrypted before uploading the shadow image. Consequently, for  $i$ -th shadow image, a set of encrypted vectors are generated and packaged  $\{v_i^j | 1 \leq i \leq k, 1 \leq j \leq len\}$ , where  $len$  means the number of encrypted vectors. By using all the  $j$ -th encrypted vectors  $\{v_i^j | 1 \leq i \leq k\}$  of the  $k$  encrypted shadow images, the corresponding  $(k-1)$ -degree polynomial  $Ln(x)$  can be computed

by using Lagrange Interpolation, represented by

$$Ln(x) = \sum_{i=1}^k v_i^{j'} \prod_{t=1, t \neq i}^k \frac{(x - X_t)}{(X_i - X_t)} \quad (4)$$

Where,  $X_i$  and  $X_t$  are the encrypted forms of the two numbers  $x_i$  and  $x_t$ , which are used in (1) to generate image shadows.

To facilitate the polynomials expression, we take  $m_i = \prod_{t=1, t \neq i}^k (X_i - X_t)$  and  $K = \prod_{i=1}^k m_i$ . Thus, (13) can be further represented by

$$Ln(x) = \sum_{i=1}^k \frac{v_i^{j'}}{m_i} \prod_{t=1, t \neq i}^k (x - X_t) \quad (5)$$

For the  $i$ -th basic polynomial  $\prod_{t=1, t \neq i}^k (x - X_t)$ , it can be expanded to a  $(k-1)$ -degree polynomial with the coefficients  $D_{it}$ ,  $1 \leq t < k$ , and thus  $Ln(x)$  can be expressed by

$$Ln(x) = \sum_{i=1}^k \frac{v_i^{j'}}{m_i} (D_{i0} + D_{i1}X^1 + \dots + D_{i,k-1}X^{k-1}) \quad (6)$$

In (6), every coefficient is divided by  $m_i$ . However, the division operation cannot be supported in BFV algorithm. To handle this issue, we introduce the  $K$ 's inverse element denoted as  $invK$  in the field  $F_p$ , and it satisfies:

$$K \cdot \otimes \cdot invK \cdot \text{mod} \cdot p = 1 \quad (7)$$

According to Fermat's little theorem, if  $p$  is a prime greater than an integer  $a$ ,  $a^{(p-1)} = 1(\text{mod}p)$  and  $invK$  is equal to  $K^{(p-2)}$ , where the fast exponentiation algorithm is used to calculate the value of  $invK$ . Then, both sides of (6) are multiplied by  $invK * K$  to eliminate the division operation by

$$\begin{aligned} Ln'(x) &= Ln(x) * K * invK \\ &= \sum_{i=1}^k \frac{v_i^{j'}}{m_i} * K * invK * \sum_{t=0}^{k-1} D_{it} * X^t (\text{mod} \cdot p) \\ &= \sum_{i=1}^k v_i^{j'} * K_i * invK * \sum_{t=0}^{k-1} D_{it} * X^t (\text{mod} \cdot p) \\ &= \sum_{i=1}^k C_{ij} \sum_{t=0}^{k-1} D_{it} * X^t (\text{mod} \cdot p) \end{aligned} \quad (8)$$

Where,  $C_{ij} = v_i^{j'} * K_i * invK$ , and  $K_i = \prod_{t=1, t \neq i}^k m_t$ , which is the product of all denominators except for  $m_i$ . We calculate the coefficient:  $A_{tj} = \sum_{i=1}^k C_{ij} * D_{it}$ , which is the coefficient of  $X^t$  when using all the  $j$ -th encrypted vectors to compute the polynomial.

Finally, after receiving the coefficient set  $\{A_{tj} | 1 \leq k \leq len, 0 \leq t < k\}$ , the *Applicant* decrypts the received coefficients as the vectors and transforms them to the pixel values, and then implements modulus operation on these values by  $p$  to compute the original secret image  $SI$ .

Note that, in the stage of secret image restoring, a large amount of computation resources are required in calculating the coefficients of base polynomials like  $\prod_{t=1, t \neq i}^k (x - X_t)$ . If we compute these base polynomials directly, the computational complexity is  $O(2^k)$ . The time consumption will increase

exponentially with the increase of  $k$ . To address this issue, based on the idea of recording and reusing the sum of terms of polynomials, we propose an improved computing strategy, called as prefix-sum, to reduce the complexity to  $O(k^2)$ .

Given a function of continuous multiplication  $g(x) = \prod_{i=1}^n (x + a_i)$ , by expanding the right side of function completely and merging the congeners, the result is given as follow:

$$\begin{aligned} g(x) &= x^n + \left( \sum_{i=1}^n a_{i1} \right) x^{n-1} + \dots \\ &\dots + \left( \sum_{i_1=1}^{n-t+1} a_{i_1} \sum_{i_2=i_1+1}^{n-t+2} a_{i_2} \dots \sum_{i_t=i_{t-1}+1}^n a_{i_t} \right) x^t \\ &+ \dots + \sum_{i=1}^n a_i \end{aligned} \quad (9)$$

We can define the values of  $D_i$  as the coefficients. The values of  $D_i$  can be represented by:

$$\left\{ \begin{aligned} D_0 &= 1 \\ D_1 &= a_1 + \dots + a_l + \dots + a_n (1 \leq l \leq n) \\ D_2 &= a_1 (a_2 + \dots + a_n) + \dots + a_l (a_{l+1} + \dots + a_n) \\ &\quad + \dots + a_{n-1} a_n (1 \leq l \leq n-1) \\ D_3 &= a_1 (a_2 (a_3 + \dots + a_n) + \dots + a_{n-1} a_n) \\ &\quad + a_2 (a_3 (a_4 + \dots + a_n) + \dots + a_{n-1} a_n) + \dots \\ &\quad + a_{n-2} a_{n-1} a_n \\ &\dots \end{aligned} \right. \quad (10)$$

To simplify the expression, we introduce  $T_{i,j}$  to represent the sum of the first  $j$  terms in  $D_i$ 's formula. The result is given as follows:

$$\left\{ \begin{aligned} D_0 &= 1 \\ D_1 &= a_1 + \dots + a_l + \dots + a_n (1 \leq l \leq n) \\ D_2 &= a_1 (T_{1,n} - T_{1,1}) + \dots + a_l (T_{1,n} - T_{1,l}) + \dots + \\ &\quad a_{n-1} (T_{1,n} - T_{1,n-1}) (1 \leq l \leq n-1) \\ D_3 &= a_1 (T_{2,n-1} - T_{2,1}) + \dots + a_l (T_{2,n-1} - T_{2,l}) + \dots + \\ &\quad a_{n-2} (T_{2,n-1} - T_{2,n-2}) (1 \leq l \leq n-2) \\ &\dots \\ &\dots \\ D_j &= \sum_{t=1}^{n-t+1} a_t (T_{t-1,n-t+2} - T_{t-1,t}) (l \geq 2) \\ &\dots \end{aligned} \right. \quad (11)$$

In the calculation process, recording and reusing  $T_{i,j}$  can effectively reduce the computation costs. If we have known the value of  $T_{i-1,l}$ , the complexity of calculation  $D_j$  is only  $O(n)$ .

**Step (3) Secret image decryption:** After the outsourcing computation, the smart contract receives the coefficients of computed polynomials. Then, the smart contract will transmit the computed polynomials to the *Applicant*. After receiving the computed polynomials, the *Applicant* decrypts the coefficients



as the vectors and transforms them to the pixel values, and then implements the modulus operation on these pixel values by  $p$  to restore the original secret image  $SI$ .

#### IV. SECURITY ANALYSES

In this section, the threshold security of the BC-SESI scheme is analyzed and then its resistance to some common attacks including tampering, corruption, and approximating attacks are discussed.

**Theorem 1:** Only when  $k$  or more *Participants* agree to authorize the access of their shadow images, the original secret image can be restored.

**Proof:** In the BC-SESI scheme, the designed AutoIDAuth-enabled smart contract acts as  $(k, n)$  threshold function of SIS. The smart contract requires to authenticate the identities of the *Applicant* and *Participants* before sending the encrypted shadows to the *Applicant*. When the number of *Participants* passing the authentication is less than  $k$ , the smart contract will not trigger the restoring task. In other words, the original secret image can be restored only when any  $k$  or more *Participants* agree to authorize the access of their shadow images. As a result, any  $k - 1$  or less authorizations cannot restore the secret images in our scheme.

Moreover, to restore the secret image, the BC-SESI outsources a considerable part of the secret image restoring task, *i.e.*, the polynomial computation in the encryption domain. The polynomial computation algorithm is stored as an executable file deployed on IPFS, and thus it cannot be modified due to the distributed characteristics of IPFS.

In addition, in the BC-SESI scheme, the original secret image is restored by the Lagrange interpolation algorithm. In the literature [4], [6], [36], the interpolation theorem states that one and only one  $(k - 1)$ -degree polynomial can pass through  $k$  given points at the same time in the two-dimensional space (the points refer to the image shadows in our scheme). A corollary is that any number of shadows fewer than  $k$  cannot correctly restore the desired  $(k - 1)$ -degree polynomial. In essence, there are countless  $(k - 1)$ -degree polynomials in two-dimensional space that pass through the points fewer than  $k$  simultaneously. Therefore, in the case of fewer than  $k$  shadow image, it is not possible to obtain the secret  $SI$  by Lagrange interpolation algorithm.

**Theorem 2:** Attackers cannot modify or obtain the information of the original images throughout the IPFS and blockchain environment.

**Proof:** In the semi-honest environment, the possible attacks are given as follows: (1) modifying the image shares before the restoring task, (2) keeping the image shares during the restoring stage and restoring the shadow image after the restoring task is completed. The proof of security against the two attacks is detailed as follows.

(1) By using the blockchain technology, new information is added to a block and can be used by all users or nodes in the distributed network. Due to the distributed and decentralized characteristics of the structure of blockchain data, each node of the network can save the shadow images stored on the blockchain in the encrypted form. However, since most of the

nodes in the blockchain network are assumed to be honest and reliable, it is difficult to modify these data, unless 51% of nodes are compromised simultaneously.

(2) The smart contract solves the authentication problem in the secret image sharing. Each participant will be assigned the unique authentication information, which makes the impersonation attack invalid. Even if the miners intercept the information, *i.e.*  $k$  encrypted shadow images  $\{s'_i | 1 \leq i \leq n\}$ , in the restoring calculation and use them for private calculation and restoring task, the miners can only obtain the coefficients of polynomial in the encrypted domain by:

$$Ln(x) = \sum_{i=1}^k v_i^{j'} \prod_{t=1, t \neq i}^k \frac{(x - X_t)}{(X_i - X_t)} \quad (12)$$

To separate the different coefficients, according to the Fermat's little theorem, we introduce the concept of inverse element, so that the formula is deduced to:

$$Ln'(x) = \sum_{i=1}^k C_{ij} \sum_{t=0}^{k-1} D_{it} * X^t \pmod{\cdot p} \quad (13)$$

By the above formula, the attacker may obtain the set of coefficients that are encrypted during the secret image sharing phase. Hence, without the decryption key  $sk$ , the attacker cannot obtain any useful information from these encrypted coefficients. Therefore, the attacker cannot restore the original image from these encrypted coefficients. In summary, the attackers cannot modify or obtain the information of the original images in the IPFS and blockchain environment.

**Theorem 3:** Approximating attack can be resisted effectively.

**Proof:** Approximating the coefficients is one of the most common attacks on polynomial-based SIS schemes. With the consideration of the correlation of neighboring pixels, someone can assume that  $a_0 \approx a_1 \approx a_2 \approx \dots \approx a_{(k-1)}$ . Hence,  $a_0 \approx f(1)/k$ , which means one of the coefficients is approximately obtained without any shadows and it will give a clue about the information of secret image  $SI$  without encryption. This drawback makes it possible to restore the secret image with the number of image shadows fewer than  $k$ .

However, the original secret image is partitioned into a set of parts beforehand, so as to transform the pixels into multi-dimensional vectors. That can not only significantly improve the calculation speed, but also effectively break the relationships between neighboring pixels. Since the relationships between adjacent pixels will not be kept, the approximating attacks can be resisted effectively. Therefore, the proposed scheme is secure against the approximation attack.

#### V. EXPERIMENTAL RESULT AND ANALYSES

To test the performance of the BC-SESI scheme, extensive experiments are implemented and the corresponding analyses are also given. All experiments are conducted on Win10 operating system with i7-9750H CPU @2.60GHz.



TABLE III  
IMPACTS OF  $K$  AND  $N$  ON RESTORING TIME CONSUMPTION AND ACCURACY

Value of $k$ in the threshold of SIS	Minimum $N$	Time consumption of computing a polynomial	Restoring accuracy
$k=2$	16,384	0.697s	100%
$k=3$	16,384	1.99s	100%
$k=4$	32,768	19.5s	100%
$k=5$	32,768	36.4s	100%
$k=6$	32,768	60.0s	100%

### A. Parameter Selection

In the proposed BC-SEISIS scheme, there are two key parameters, *i.e.*,  $k$  and  $N$ . Here,  $k$  means the minimum number of shadow images used for restoring the original secret one, and  $N$  means the number of pixels divided into each batch during the batch encryption for shadow images. In this section, we measure the impacts of the parameters on the performance of restoring the time consumption and accuracy of the proposed BC-SEISIS scheme.

In the BFV algorithm [77], the encryption process is usually accompanied by some noise, and both addition and multiplication will amplify the noise in the decryption result. In the process of computing a single polynomial, encrypting more pixels can lead to more noise in the encryption result. Thus, if  $N$  exceeds a certain value, the accuracy of image restoring will be affected. With the increase of  $N$ , the computation time of the restoring process increases significantly. Also, the parameter  $k$  determines the depth of the multiplication, and has impact on the restoring time consumption and accuracy. To find a good balance between the restoring efficiency and accuracy, it is necessary to test the impacts of the two parameters on the restoring time consumption and accuracy of the proposed BC-SEISIS scheme.

TABLE III shows the impacts of  $k$  and  $N$  on the restoring time consumption and accuracy. Minimum  $N$  means the minimum value of  $N$  that can ensure the secret image is 100% restored. From this table, to ensure that the restoring accuracy is maintained at a 100%,  $N$  is set as 16,384 when  $k=2$  or 3, and  $N$  is set as 32,768 when  $k \geq 4$ . Also, it is clear that the time consumption of computing a single polynomial is only less than 2 seconds when  $N=16,384$ . The time consumption increases significantly, when  $N=32768$ .

### B. Quality of Restored Secret Image

We present the results of secret images restored by the BC-SEISIS scheme. 4 shows an experimental result of the original version and the restored version of a  $256 \times 256$  image by the proposed BC-SEISIS scheme. In this experiment, a (4, 6)-threshold case of our scheme is used. 4(a) shows an image with the size of  $256 \times 256$ . 4(b)~(g) show the generated six shadow images, which are noisy images and their sizes are  $\frac{1}{4}$  of the original secret image. Finally, 4(h) is the restored secret image, which is produced from any four shadow ones.

The quality of the restored secret image by the proposed BC-SEISIS scheme is tested by different statistical metrics including Root Mean Square Error (RMSE) and Peak Signal to Noise Ratio (PSNR).

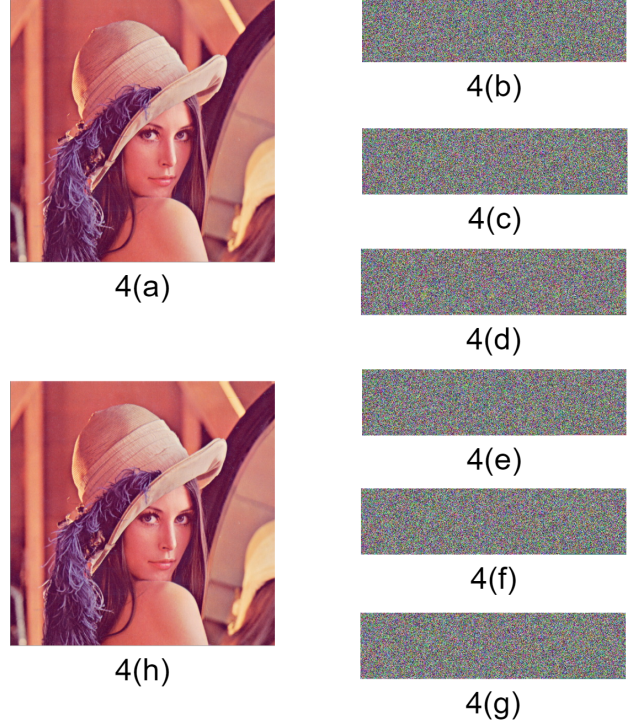


Fig. 4. The comparison between an original secret image and the restored one. (a) the original secret image, (b) (g) the generated shadow images, (h) the restored secret image.

**Definition 2** The value of RMSE describes the distinction between the original secret image and the restored one. RMSE is defined by

$$RMSE = \sqrt{\frac{1}{M \times N} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (S[i, j] - RI[i, j])^2} \quad (14)$$

where  $SI[i, j]$  and  $RI[i, j]$  are the  $(i, j)$ -th pixels of secret image and the restored image with the sizes of  $M \times N$ , respectively. Smaller RMSE value means higher quality of the restored secret image.

**Definition 3.** PSNR measures the similarity between the original secret image and the restored one. PSNR is defined by

$$PSNR(db) = 10 \times \log_{10} \left( \frac{255 \times 255}{RMSE} \right) \quad (15)$$

Where, a higher PSNR value means a higher quality of the restored secret images.

In the experiment, when 65536 pixels were restored, the RMSE value is only 0.0771 and the PSNR value reaches up to 59.2611, as shown TABLE IV. Thus, it is clear that there is

TABLE IV  
RMSE AND PSNR VALUES OF SECRET AND RESTORED IMAGES OF  
BC-SEISIS SCHEME

Number of pixels	RMSE	PSNR
65536	0.0771	59.2611

almost no difference between the original secret image and the restored image. Thus, the quality of the secret image restored by the BC-SEISIS is desirable.

### C. Validity of Batch Encryption Strategy on Efficiency

To improve the efficiency of generating shadow images, the batch encryption strategy is designed and adopted in Section III-B. The process of secret image restoring consists of multiple operation of computing polynomials by Lagrange's interpolation algorithm. In the traditional scheme, each coefficient of the polynomial is used to hide one pixel and each polynomial computation can restore  $k$  pixels. In the proposed BC-SEISIS scheme, batch encryption strategy can greatly enhance the restoring efficiency of secret image. It can hide  $N$  pixels into each coefficient and each polynomial computation can restore  $N \times k$  pixels simultaneously. In this section, we will conduct the experiments to compare the time consumption and the number of restored pixels per each polynomial computation by traditional encryption strategy and batch encryption strategy. In addition, we will finally compare the efficiency of the two strategies by observing the time consumption of restoring an image with the size of  $512 \times 512$ .

According to TABLE V, the restore efficiency of the batch encryption strategy is much higher than that of the traditional one, in which the pixels are encrypted one by one. For different values of  $k$ , different values of  $N$  are selected to keep the restoring accuracy at 100%. When  $k = 6$ , the restoring time per pixel consumed by the batch encryption scheme is nearly 800 times faster than that of the traditional encryption strategy. Since each polynomial computation can restore  $N \times k$  pixels, the time for restoring the whole image by using the batch encryption strategy is much less than that by the traditional strategy. It is obvious that the batch encryption strategy can greatly improve the efficiency of our scheme, especially when  $k$  equals to a large value.

### D. Validity of Prefix-Sum Strategy on Efficiency

As described in Section III, we introduce a *Prefix-Sum* strategy to efficiently compute the coefficients of continuous multiplication polynomials  $\prod_{(t=1, t \neq i)}^k (x - X_t)$ . The proposed *Prefix-Sum* strategy significantly reduce the computational complexity of our scheme from  $O(2^k)$  to  $O(k^2)$ . In the following part, we will compare the time consumption of each polynomial computation between the standard computation strategy and the proposed *Prefix-Sum* computation strategy.

As shown in 5, by comparing the efficiency of the two computation strategies, it is clear that the time consumption of the the standard computation strategy is slightly lower than that of *Prefix-Sum* computation strategy when  $k$  is 2

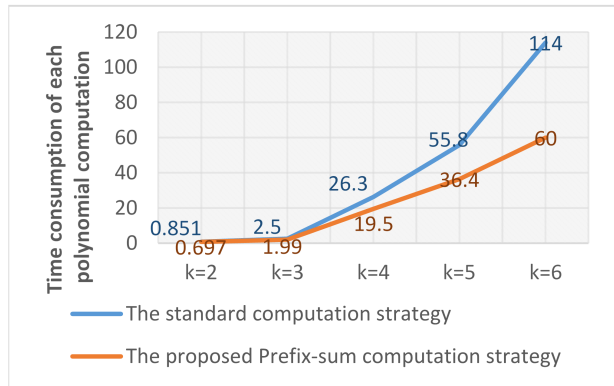


Fig. 5. Efficiency comparison between the standard computation strategy and the proposed *Prefix-Sum* computation strategy.

or 3. When  $k$  is greater than 3, it is obvious that the time consumption of the standard computation strategy grows much faster than that of the *Prefix-Sum* computation strategy. In summary, the proposed *Prefix-Sum* computation strategy can efficiently improve the efficiency of the proposed scheme, especially when  $k$  is equal to a large value.

### E. Time Saved in Users' Side

As described in Section III-C, the *Applicant* outsources the task of computing polynomials in the encrypted domain to the *Agent miners*. Then, the *Applicant* also should decrypt the computed polynomial coefficients as vectors and transforms them to the pixel values, and implement a modulus operation by  $p$  on the pixel values to restore the original secret image. Thus, we analyze the time saved at the users' side by the outsourcing computation.

In the following part, we will test the users' time consumption  $T_1$  with outsourcing computation, the time consumption  $T_2$  of restoring the secret image, and the saving rate of time consumption  $\alpha = \frac{(T_2 - T_1)}{T_2}$ . The time consumption is shown in TABLE VI. From this table, since the *Applicant* outsources the complex polynomial computation process, the saving rate of users' side can reach 62.7% to 74.9%, when the value of  $k$  ranges from 2 to 6; The outsourcing computing can effectively save rate of time consumption at the users' side at least 62.7%; In addition, with the increasing the value of  $k$ , the polynomial computation process becomes more complex, and the proportion of the modular operation in the total restoring task is reduced. Thus, the saving rate of the time consumption increases with the increase of  $k$ . In summary, it is proven that the outsourcing computing method can significantly saves users' computing resources.

### F. The probability of cracking the secret image

In this subsection, we text the probability of successfully cracking the secret image via random guess. Suppose an attacker cannot obtain the decryption key  $sk$ , which is protected well by the participants. According to (1) and (3) in Section III-B, the probabilities of successfully cracking the secret

TABLE V  
EFFICIENCY COMPARISON BETWEEN BATCH ENCRYPTION STRATEGY AND TRADITIONAL ENCRYPTION STRATEGY IN RESTORING STAGE

Schemes	Number of restored pixels	Consuming time	Restoring 512×512 picture
$k=2$	Traditional	2	0.678s
	Batch	16384	0.697s
$k=3$	Traditional	3	1.98s
	Batch	24576	1.99s
$k=4$	Traditional	4	19.6s
	Batch	65536	19.5s
$k=5$	Traditional	5	36.1s
	Batch	81920	36.4s
$k=6$	Traditional	6	59.6s
	Batch	98304	60.0s

TABLE VI  
TIME SAVED BY THE PROPOSED BC-SEIS SCHEME IN THE USERS' SIDE WITH DIFFERENT VALUES OF  $K$

	$T_1$ (ms)	$T_2$ (ms)	$\alpha$ (%)	the size of $SI$
$k=2$	0.081	0.217	62.7	$128 \times 256$
$k=3$	0.081	0.233	65.2	$128 \times 256$
$k=4$	0.163	0.502	67.5	$256 \times 256$
$k=5$	0.163	0.573	71.6	$256 \times 256$
$k=6$	0.163	0.649	74.9	$256 \times 256$

image hidden into the encrypted coefficients of polynomials can be computed by

$$P_c = \frac{1}{256^{k^2 \times len}} \quad (16)$$

Where  $k$  means the threshold value and  $len$  means the number of polynomials of the proposed BC-SEIS scheme. The computed  $P_c$  values with different values of  $k$  are listed in Table VII. It is clear that the probabilities of successfully cracking the secret image are quite small, which indicates the proposed scheme is secure enough.

## VI. CONCLUSION

In this paper, to securely communicate and effectively protect secret image data in wireless communication, we have presented the BC-SEIS scheme with outsourcing computation in wireless networks. In this scheme, the generated shadows are encrypted and stored in the blockchain to prevent them from being tampered and/or corrupted. The identity authentication-enabled smart contract is deployed on the blockchain to achieve the  $(k, n)$  threshold of SIS for secret image restoring. The FHE-based outsourcing computation method is designed to outsource the task of secret image restoring to reduce the computational burden of smart contract and users. The theoretical analyses and extensive experiments prove that the BC-SEIS scheme not only achieves desirable security, but also has high computational efficiency.

The proposed BC-SEIS scheme has been proven that it can effectively manage and protect the images distributed on the networks. Thus, it has great significance in practical applications. In the days and future of digital world, we plan to further reduce the computational burden of smart contract and users, and also attempt to improve the outsourcing computation method to completely outsource all the verification and computation operations of the SIS task in the wireless networks.

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TABLE VII  
THE PROBABILITIES OF SUCCESSFUL CRACKING SECRET IMAGE PROTECTED BY BC-SEISIS SCHEME

	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$
$P_c$	$1.84 \times 10^{-19}$	$7.92 \times 10^{-28}$	$1.84 \times 10^{-19}$	$1.21 \times 10^{-24}$	$7.92 \times 10^{-28}$



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Things, blockchain, and information security.

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