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Multi-attribute decision making on mitigating a collision of an autonomous vehicle on motorways

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1. Introduction

A considerable attention has been recently devoted to autonomous vehicles and their safety implications. The architecture of autonomous vehicles which comprised of a perception and a decision making system was reviewed in (Badue et al., 2021). The perception systems are typically focussed on tasks of static obstacles mapping, moving obstacles detection and tracking, road mapping, and so on, while the decision-making systems are responsible of route planning, path planning, behaviour selection, motion planning and control. One of the crucial problems of autonomous vehicles is safety. Consideration has been most often placed on urban environments and avoidance of obstacles (Geronimi et al., 2016). Automotive engineering based research in the area of obstacle avoidance included development of Automatic Emergency Braking (AEB) models (Harper et al., 2016). However, it is unrealistic to assume that all collisions can be prevented, and, therefore, attention must be focussed on what a vehicle can do when facing an unavoidable collision. Furthermore, autonomous vehicles on motorways have thus far been investigated scarcely. A model for safe lane-change manoeuvres on motorways was presented in (Cesari et al., 2017). The authors assessed a lane-change manoeuvre using a model predictive control approach where traffic predictions were described in scenarios.

In this paper, a novel problem of an autonomous vehicle in a complex motorway environment is considered, when a collision with a vehicle in front is imminent. The motorway is referred to as “a complex environment”, because all six vehicles ahead and behind the autonomous vehicle in all three lanes, including the lane where the autonomous vehicle is and both adjacent lanes are taken into account. A new methodology to select a lane into which the autonomous vehicle should manoeuvre into or stay put, considering the autonomous vehicle and all other six vehicles, ahead and behind in all three lanes, is proposed and investigated. It combines a new simulation model of the vehicles on the motorway and multi-attribute decision making (MADM) methods to recommend which lane the autonomous vehicle should manoeuvre into or stay put. Following on from the research presented in (Gilbert et al., 2018 and Pickering et al., 2018), the simulation model based on dynamic braking is developed, to calculate the potential impact velocities the autonomous vehicle may have when facing multiple collisions possibly in multiple lanes of the motorway. This simulation model can be considered to be an evolution of the current Adaptive Cruise Control and AEB systems. Multiple collisions in multiple lanes give rise to many options for the autonomously controlled vehicle to decide upon to avoid or mitigate collisions. Generally, MADM methods have been extensively applied to various management problems, when a selection of an alternative decision can be based on identified criteria, but scarcely to engineering problems, and, in particular, not to decision making of autonomous vehicles. To the best of the authors’ knowledge, they are only a few papers that applied MADM methods to autonomous vehicle problems (Chen et al., 2014 and Furda and Vlacic, 2010). However, both developed models considered different type of problems compared to the problem handled in this paper; they selected driving manoeuvres in simplified urban environments which included two and one lane and two and one vehicle, respectively. The driving manoeuvres are determined in such a way as to react to a deceleration of a vehicle ahead or vehicle stopped in the same lane, respectively.

The main novelties of the research presented in the paper are as follows: (1) three MADM methods including Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Analytic Hierarchy Process (AHP) and Analytical Network Process (ANP) are analysed and applied to a complex motorway environment to support the decision on which lane the autonomous vehicle should manoeuvre into or stay in, based on the severity of impacts with other vehicles; the motorway environment considered includes 6 vehicles (3 ahead and 3 behind the autonomous vehicle) in the lane of the autonomous vehicle and the adjacent lanes, (2) a novel simulator is developed to calculate, in real time, values of parameters identified to be relevant to the severity of collision between an autonomous vehicle and all other vehicles, including impact velocities of the collisions between the autonomous vehicle and the vehicles ahead and behind in all three lanes, the manoeuvre acceleration and time-to-collision; these values are used as inputs into the MADM methods, and (3) various simulation scenarios are run and analysed to provide insights into the effect of automotive vehicle parameters, e.g., braking and steering, and parameters of other vehicles in the same or adjacent lanes on the possible collision outcome and the recommended lane for manoeuvring.

The paper is organised as follows. Section 2 reviews research on collision avoidance of autonomous vehicles and applications of MADM methods to autonomous vehicles’ problems. Section 3 defines the research problem under consideration. Section 4 describes the simulator developed to calculate input criteria values for the MADM methods applied, while Section 5 gives details of the MADM methods applied to a benchmark scenario of an autonomous vehicle. Section 6 presents different simulation scenarios based on the benchmark and results obtained. Section 7 concludes the paper and discusses future work.

2. Literature review

Collision avoidance was considered in (Eidehall et al., 2007) where a Cartesian coordinate system was developed to calculate the positions of other vehicles and to assess whether a manoeuvre was dangerous by evaluating the traffic around the vehicle. However, this was a preventive model which assessed the risks of the vehicle's lane departures. Vehicle trajectories were modelled in (Ammoun and Nashashibi, 2009), first using a geometric approach, followed by a dynamic approach. The geometric approach required less estimation time due to lower computational effort requirements. The system calculated a time-to-collision to represent a collision risk. A framework for a semi-autonomous vehicle which calculated a best-case trajectory for a lane-change manoeuvre, with the aim of avoiding an imminent hazard ahead was proposed in (Anderson et al., 2010).

A combined steering and braking controller to avoid a hazard ahead was presented in (Hayashi et al., 2012). The avoidance system determined whether a braking only, or a steering and braking manoeuvre would result in collision avoidance. However, the mitigation control was limited as it was focused on avoidance only. If a collision was unavoidable, the system would act like an AEB system, and simply apply full braking to reduce the impact velocity. The steering manoeuvre assumed that full braking could be applied, without considering the vehicle dynamics limitations. Instead of just applying braking, such as in an AEB system, the combination of steering and braking opens up the possibility for further actions, i.e. to steer around the hazard ahead. The autonomous vehicle therefore has more possible actions it can take to reduce the risk of the potential collisions.

Multi-objective decision making models (MODM) generate a solution considering more than one objective. They have been scarcely applied to engineering problems (Chiandussi et al., 2012). In the area of autonomous vehicles, MODM models have been developed mainly for path planning in urban environments, for maritime and aerial vehicles and for problems such as overtaking of an autonomous vehicle. For example, an MODM model for avoiding a static obstacle and a low-speed dynamic obstacle vehicle in an urban environment was developed in (Chen et al., 2019); a multi objective path planning for aerial vehicles was proposed in (Wu et al., 2011) and for underwater vehicles in (Hu et al., 2020); a data driven reinforced learning model to support overtaking decision making was investigated in (Xu et al., 2019).

Contrary to MODM methods which generate solutions, MADM methods mathematically select or rank given alternatives based on selected criteria. Different MADM methods are proposed in the literature and advantages of their applications to different optimisation problems have been identified. However, MADM methods were scarcely applied to autonomous vehicles' problems. In (Chen et al., 2014), MADM methods were applied to select a manoeuvre for an autonomous vehicle in a much simplified dual carriageway environment with only three vehicles involved: the autonomous vehicle, a vehicle ahead and a vehicle aside. Two MADM methods were applied sequentially in two steps of the algorithm for selecting a driving manoeuvre: AHP for obtaining the weights of criteria used in selecting the manoeuvre and TOPSIS for ranking the alternative manoeuvres. Five manoeuvre alternatives were considered including Accelerate, Decelerate, Lane Keeping, ChangetoLeftLane and ChangetoRightLane. The criteria considered were based on the rule: the more distance between vehicles, the more safety provided. They included distances between the autonomous vehicle and the vehicles ahead and aside, distances between the autonomous vehicle and road lanes, speed limits and time to reach destination. One experiment was conducted to demonstrate the effectiveness of the approach. Furda and Vlacic (2010), applied an MADM method, the Simple Additive Weighting method, to select the most appropriate driving manoeuvre from the set of feasible ones. A utility function was used to evaluate the achievement of each attribute (criterion) for each of the feasible manoeuvre alternatives. A very simple example was used to demonstrate the application of the MADM method to the autonomous vehicles' ability to make appropriate driving decisions in city road traffic situations. The example included an autonomous vehicle and one stopped vehicle, and 6 possible manoeuvring alternatives: to pass the stopped vehicle with small or fast speed and small or large lateral distance or to wait behind the stopped vehicle at a small or a large distance.

3. Research problem

Motorways investigated in this research are multiple-carriageway roads, with controlled access, where vehicles travel at high speeds. In the UK, the maximum speed limit for these roads is 70mph (miles per hour), at which speed collisions can be fatal or result in a serious injury, as reported in (House of Commons Library, 2013). A typical three lane motorway is considered, with an autonomous vehicle driving in the middle lane whilst following another vehicle at a set distance, as presented in Figure 1. All three lanes are occupied by vehicles ahead of the autonomous vehicle, and vehicles behind the autonomous vehicle. It is assumed that the vehicle ahead in the same lane as the autonomous vehicle comes to a sudden stop, with the other vehicles ahead in the adjacent lanes decelerating as a reaction to the hazard in the middle lane. In this research, the problem under consideration is to select a lane in which the autonomous vehicle will need to stay in or to manoeuvre into which will lead to the smallest severity of the collision, considering the vehicles ahead and behind of the autonomous vehicle, in the same or adjacent lanes.

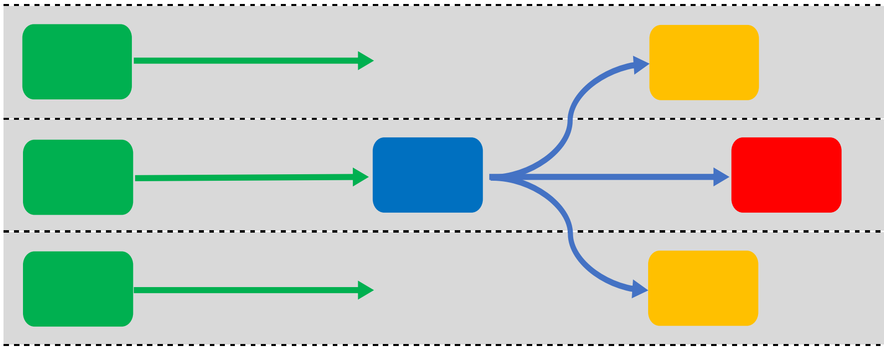

Figure 1. Autonomous vehicle and vehicles ahead and behind on a motorway

It is assumed that all vehicles can communicate their velocities, positions, and braking values to the autonomous vehicle via Vehicle-to-Vehicle (V2V) communication. The autonomous vehicle selects the lane to stay in or manoeuvre into based on evaluation of potential collisions with the vehicles ahead and behind in each lane.

Autonomous vehicles are expected to drastically improve road safety by removing human error. However, there is not yet a guarantee that 100% of accidents can be prevented, and so an autonomous driving decision making process does have a potential application.

4. Simulation

A novel simulator has been developed and implemented to calculate the severity of the potential collisions with all vehicles presented previously in Figure 1. It is accurate enough to provide useable results for the decision-making process and sensitivity analysis, while existing vehicle simulators, such as ADAMS Car and VI-Grade, are computationally heavy, requiring a significant amount of time to complete complex simulations.

The simulation is based on discrete-time Cartesian coordinates where the vehicle's position and velocity are known in any time sample. As discussed above, it is assumed that the vehicle in front is braking suddenly, thus the vehicles in the adjacent lanes are reacting by decelerating. The positions of the vehicles are calculated and compared in every time sample. The impact is signified when their separation distances are zero. The potential impact speeds are calculated by finding the time when collision occurs between the autonomous vehicle and all potential collision vehicles.

As all vehicles are decelerating, a dynamic braking equation given in (Rajamani, 2011) is used to calculate the deceleration of each vehicle as:

$$M\ddot{x} = F_{Long} - F_{Resistance} \tag{1}$$

where M is the vehicle mass, $\ddot{x}$ is the vehicle acceleration, F_{Long} denotes the forces acting on the vehicle to accelerate it, or decelerate by braking, and $F_{Resistance}$ is the total resistance forces acting on the vehicle including aerodynamic drag, rolling resistance and gradient of incline. $F_{Resistance}$ is described by the following formula:

$$F_{Resistance} = F_{aero} + R_x + Mg\sin(\theta) \quad (2)$$

$$F_{aero} = \frac{1}{2}\rho C_d A_F (V_x + V_{wind})^2 \quad (3)$$

where g is the gravitational constant, θ is the angle of incline, ρ is the density of air, C_d is the aerodynamic drag coefficient of the vehicle, A_F is the largest cross sectional area of the vehicle, V_x is the longitudinal speed, and V_{wind} is the headwind of the air the vehicle is driving through (assumed to be 0 in the simulation). A simplified equation for rolling resistance R_x is adapted from (Yin and Jin, 2013):

$$R_x = C_r Mg \quad (4)$$

where C_r is the coefficient of rolling resistance.

Formula (1) is applied to determine deceleration of the autonomous vehicle in the longitudinal direction, in order to calculate the severity of collision of the autonomous vehicle with the vehicle ahead. However, when the autonomous vehicle manoeuvres into an adjacent lane, its acceleration includes both acceleration in longitudinal and lateral directions. Its trajectory is modelled using a sinusoidal wave. The lateral acceleration a_y in the planned trajectory and the vehicle's velocity are taken into account to determine a longitudinal acceleration (braking value) a_x that can be applied without over-saturating the tyres and the vehicle losing control.

In order to calculate lateral acceleration a_y, the yaw rate limited by friction is determined first, as given in (Blundell and Harty, 2004):

$$\dot{\psi}_{friction} = \frac{\mu . g}{v} \quad (5)$$

where μ is the coefficient of friction, and v is the vehicle's forward velocity. Lateral acceleration a_y is then calculated using the following equation given in (Rajamani, 2011):

$$a_y = \ddot{y} + v_x \dot{\psi}_{friction} \quad (6)$$

where $\dot{\psi}_{friction}$ is given by Equation (5), $\ddot{y}$ is the double derivative of the planned trajectory, and v_x is the vehicle's maximum velocity taken to be the starting velocity.

Using the elliptical equation to describe the limits of acceleration in the longitudinal $a_{x.max}$ and lateral $a_{y.max}$ directions, as presented in Figure 2, braking value a_x can be determined using the following equations:

$$\frac{a_x^2}{a_{x.max}^2} + \frac{a_y^2}{a_{y.max}^2} = 1 \quad (7)$$

where:

$$a_y = a_{y.max}\sin(t) \quad (8)$$

$$a_x = a_{x.max}\cos(t) \quad (9)$$

in which t is the angle subtended by the vector described by a_x and a_y. With a_y and $a_{y.max}$ known, Equation (7) can be rearranged to find angle t. With $a_{x.max}$ also known, a_x can be determined as a maximum braking value for the lane change manoeuvre. This principle of using a ‘g-g’ diagram to plan vehicle speed was also utilised in (Kritayakirana and Gerdes, 2012).

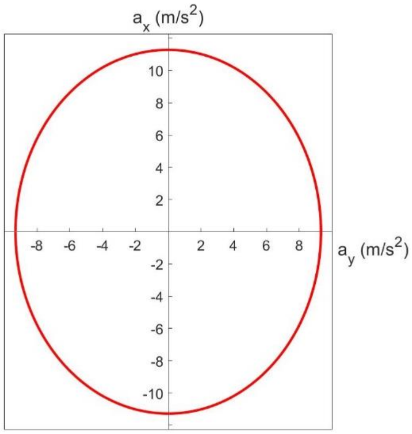

Figure2. Elliptical 'g-g' diagram

In order to determine the severity of the collisions, the simulator calculates the following parameters:

- relative velocities of the collisions between the autonomous vehicle impacting the vehicles ahead,
- relative velocities of the collisions between the autonomous vehicle impacting the vehicles behind,
- the manoeuvre accelerations (describing the severity of the manoeuvre of the autonomous vehicle into an adjacent lane),
- time-to-collision (a greater time giving longer to mitigate any potential collision).

The impact velocity of two vehicles is calculated as follows:

$$\Delta v = |v_1 - v_2| \tag{10}$$

where v_1 and v_2 are the velocities of the two colliding vehicles at the moment of impact. The manoeuvre acceleration is calculated using Equation (8), and the time-to-collision is a value determined by the simulator when the separation between the two colliding vehicles equals zero.

However, in the following situations, the simulator disqualifies a lane for the autonomous vehicle to manoeuvre into:

- it is determined that a collision occurs before the manoeuvre is complete,
- the required yaw rate to complete the manoeuvre is higher than the maximum yaw rate as limited by friction given in Equation (5),
- it is determined that the autonomous vehicle will roll over. Each wheel experiences a vertical tyre force (F_z) calculated throughout the planned manoeuvres. If that force equals zero, this signifies that the wheel is not in contact with the ground and has lifted, thereby suggesting the vehicle has rolled or is approaching a rollover (Doumiati et al., 2009); this is given by:

$$F_{Z_{fl}} = \frac{1}{2}M\left(\frac{l_r}{l}g - \frac{h}{l}a_x\right) - M\left(\frac{l_r}{l}g - \frac{h}{l}a_x\right)\frac{h}{d_f g}a_y \tag{11}$$
$$F_{Z_{fr}} = \frac{1}{2}M\left(\frac{l_r}{l}g - \frac{h}{l}a_x\right) + M\left(\frac{l_r}{l}g - \frac{h}{l}a_x\right)\frac{h}{d_f g}a_y$$
$$F_{Z_{rl}} = \frac{1}{2}M\left(\frac{l_f}{l}g + \frac{h}{l}a_x\right) - M\left(\frac{l_f}{l}g + \frac{h}{l}a_x\right)\frac{h}{d_r g}a_y$$
$$F_{Z_{rr}} = \frac{1}{2}M\left(\frac{l_f}{l}g + \frac{h}{l}a_x\right) + M\left(\frac{l_f}{l}g + \frac{h}{l}a_x\right)\frac{h}{d_r g}a_y$$

where a_x and a_y are the longitudinal and lateral accelerations, respectively, d_f and d_r are the track widths front and rear, respectively, h is the height of the CoM, l is the total wheelbase length, l_f and l_r are the distances from the CoM to the front and rear axles respectively, M is vehicle mass, subscripts fl, fr, rl, and rr refer to front left, front right, rear left and rear right wheels, respectively.

- the velocity of the autonomous vehicle for a steering manoeuvre into an adjacent lane in any time sample exceeds the skidding speed. The following static equation describes the safety of a manoeuvre by calculating the skidding speed (Kett, 1982):

$$Skidding\ Speed = \sqrt{\left\{g\ r\ \left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)\right\}} \tag{12}$$

where r is the radius of the turn (steady state) and θ is the bank angle of the road. The skidding speed is the speed at which the vehicle tyre will begin to skid for a given radius of turn, coefficient of friction and velocity.

5. MADM methods for selection of the lane for collision

5.1 MADM methods

MADM is based on comparing alternatives against selected criteria (attributes) using metrics by which the criteria are measured. An evaluation matrix is created, where each column represents an alternative and each row represents a criterion. It is common in decision problems that criteria are in conflict. One criterion may give the preferred alternative differently to another criterion. It is also possible that no alternative optimises all of the criteria simultaneously. Therefore, an alternative which achieves the most suitable trade-off among the criteria is to be found. MADM methods also allow for criteria to be weighted to reflect the importance of criterion in the decision making. This is modelled by creating the Priority Vector of weights.

Three well-known MADM methods investigated in this research are TOPSIS, AHP, and ANP. They were selected to be applied and compared due to their following characteristics: (a) they are very different in their nature, (b) they provide complete ranking of considered alternatives and (c) they are often applied for various problems, mainly management type, but not for the novel problem of autonomous vehicle on a motorway and selection of a vehicle on an adjacent or the same lane to collide with, that is considered in this paper.

TOPSIS (Hwang and Yoon, 1981) is based on a geometric assessment of each alternative which is compared to an artificially created 'ideal solution'. The ideal solution has the most preferable value of each criterion, which is typically a minimum or a maximum value. TOPSIS is easy to implement and can handle a large amount of data.

AHP reduces a complex multi-objective decision making, by using pairwise comparisons of criteria and alternatives to objectively rank the alternatives (Saaty, 1980). The comparisons are based on subjective and objective assessments. The comparisons are assessed for consistency, to ensure that even subjective comparisons are not biased. AHP utilises vector normalisation to remove the scale of criteria/alternatives from decision making. It selects or ranks the alternatives using scores obtained by multiplying the priority vector and the decision matrix. The higher the score, the better the alternative with respect to the considered criterion.

ANP is an evolution of AHP which introduces feedback into the decision making (Saaty, 1996). The idea is that the criteria are reassessed based on how much of an influence they have really had on the decision; in this way the criteria weights are re-tuned.

5.2 Benchmark scenario

A benchmark scenario is defined by setting the relevant parameters of the autonomous vehicle and vehicles ahead and behind as given in Table 1 and Table 2, respectively. The following time is the time required for the autonomous vehicle to reach the rear of the vehicle ahead. The benchmark parameters of the vehicles ahead and behind of the autonomous vehicle are set in such a way that a collision can occur both ahead and behind the autonomous vehicle in all available lanes. In the benchmark scenario, all vehicles in all lanes have identical parameters. The autonomous vehicle is initially following the vehicle ahead in Lane 2 (the middle lane) at a set speed and distance. Although the presented scenario is not arbitrarily defined, in the sense that the collision among the vehicles is imminent and unavoidable, it is worth noting that the proposed MADM methods can be applied to any practical and arbitrary situation. In addition to the benchmark scenario, various scenarios are defined and sensitivity analyses are carried out in Section 6, where different parameters of the benchmark scenario are varied.

Table 1. Autonomous vehicle benchmark parameters

Autonomous Vehicle Parameters	
Initial Velocity	$70mph$
Following Time	$1.4s$
Maximum Longitudinal Braking	$8m/s^2$
Maximum Lateral Acceleration	$8.5m/s^2$

Table 2. Vehicles ahead and behind benchmark parameters

Vehicles Ahead and Behind Parameters	Vehicle Ahead	Vehicle Behind
Mass	$2000kg$	$2000kg$
Initial Velocity	$70mph$	$70mph$
Headway Distance from Autonomous Vehicle	$12m$	$-20m$
Inputted Deceleration	$7m/s^2$	$5m/s^2$

The autonomous vehicle is in Lane 2 and can move to one lane adjacent to its current lane, allowing 3 possible lane options to manoeuvre: to stay in current Lane 2, or to move into adjacent Lane 1 or Lane 3.

Simulation of the benchmark scenario gives the results presented in Table 3. The benchmark demonstrates that the results obtained for Lane 1 and Lane 3 are identical. The velocities of vehicles in Lane 2 are higher than those in Lane 1 and Lane 3, except for the vehicle ahead which is 0m/s (i.e. the vehicle is at a full-stop). A collision occurs earlier if the autonomous vehicle stays in Lane 2, while it will have a greater amount of time to react if it moves into either Lane 1 or Lane 3. All lanes demonstrate similar manoeuvre accelerations and all lanes are open, signified by 1 as opposed to 0 where this signifies a disqualified or closed lane. Thus, all of the lanes are considered in the decision making. The outputs given in Table 3 are the inputs to the three MADM methods which recommend the lane for the autonomous vehicle to steer into.

Table 3. Benchmark scenario simulation results

	Lane 1	Lane 2	Lane 3
Vehicle Ahead Velocity (m/s)	9.119	0	9.119
Autonomous Vehicle Velocity Collision Ahead (m/s)	13.132	11.456	13.132
Autonomous Vehicle Velocity Collision Behind (m/s)	3.692	10.358	3.692
Vehicle Behind Velocity (m/s)	12.689	21.425	12.689
Manoeuvre Acceleration (m/s^2)	8.776	8.310	8.776
Time-To-Collision (s)	3.080	2.417	3.080
Lanes Open	1	1	1

It is worth noting that the simulations are run using MATLAB 2016a, on a $3.10GHz$ processor with $8GB$ RAM. The most time consuming task is calculation of dynamic braking of the vehicles. Calculating just the Autonomous Vehicle's behaviour takes nearly $1s$ (0.983s), while the simulation of dynamic braking of all other vehicles takes 4.801 s. The dynamic braking simulation does perform far more accurately compared to the linear braking approximation, because it involves more complex calculations. However, this time is long and not applicable in practice, where a decision must be made very quickly. However, if each vehicle calculates its own velocities and displacements, and communicates these values to other vehicles, the time required to simulate motorway vehicles can be much improved.

The numerical outputs from the simulator described in Section 4 and given in Table 3 for the benchmark scenario are used as criteria values for the MADM methods. They are used to decide which lane the autonomous vehicle should manoeuvre into based on all calculated collision severities. Interestingly, with 2 potential collisions in each lane, with the vehicle ahead and behind, it may happen that one lane involves both the most and least severe collisions.

5.3 Priority vector

All three methods consider weighted criteria by using priority vector *w*. The higher the weight, the more important the criterion. Of course, the weights have a strong impact on the output of the MADM methods. Sensitivity of the outputs on the weights is a very important research question, but outside the scope of this paper. While TOPSIS uses subjectively given criteria weights, AHP and ANP include a procedure for obtaining the weights based on a pairwise comparison (Saaty and Vargas, 2004). In this research, the criteria weights are calculated by applying this procedure, and then used in all the three MADM methods.

The procedure for obtaining the weights consists of the following steps:

1. Define subjectively the weights of each pairwise comparison of criteria in matrix *A*, using 1 to 9 scale, see Table 4. If the first criterion is more important than the second, 1 indicates the highest, while 9 indicates the lowest importance of the first criteria. If the second criteria is more important, the reciprocal comparison values are used, as demonstrated in Table 4. The grey shaded boxes show that when one criterion is compared against itself, the pairwise comparison is equal to 1. The criteria weights for the problem under consideration are determined subjectively as follow. The two impacts of the autonomous vehicle with vehicles ahead and behind are given the equal and highest importance 1. The manoeuvre acceleration is the next most important criterion. This is due to the fact that manoeuvre accelerations can lead to a potential injury risk to the vehicle occupant(s). The time-to-collision is given the lowest importance compared to other criteria as it does not indicate a risk to injury directly.

Table 4. Pairwise comparison of criteria given in matrix *A*

	Impact Velocity Ahead	Impact Velocity Behind	Manoeuvre Acceleration	Time-To-Collision
Impact Velocity Ahead	1	1	1/3	1/8
Impact Velocity Behind	1	1	1/3	1/8
Manoeuvre Acceleration	3	3	1	1/4
Time-To-Collision	8	8	4	1

Minor changes to the criteria weights can cause a change in the priority vector to be calculated at the end of the procedure and in the overall decision made.

2. The columns of matrix A are summed and the resulting vector is normalised using the following equation:

$$\widehat{a_{i\,j}} = \frac{a_{i\,j}}{\sum_{j=1}^{4} a_{i\,j}} \tag{13}$$

where $\widehat{a_{i\,j}}$ is the normalized criterion value and $\sum_{j=1}^{4} a_{i\,j}$ is the sum of all alternative values $a_{i\,j}$. The sum of all normalised criterion values $\sum \widehat{a_{i\,j}}$ is 1.

3. Priority vector *w* is determined by forming matrix *Aw* and the maximum Eigenvalue λ_{max} in the following equation:

$$Aw = \lambda_{max} w \tag{14}$$

Priority vector w is approximately determined by averaging the value of each row of the normalised matrix *A* which was shown to be a good approximation of the Eigenvalue λ_{max}.

4. Consistency Index ($C.I.$) is calculated to determine consistency of subjectively compared importance of the criteria using the following formula:

$$C.I. = \frac{\lambda_{max} - n}{n - 1} = \frac{4.0402 - 4}{4 - 1} = 0.0134 \tag{15}$$

where n is the number of criteria and λ_{max} is the Eigenvalue.

5. Consistency Ratio ($C.R.$) is calculated by dividing the consistency index by the corresponding empirically determined Random Consistency Index *R.I.* (Saaty and Vargas, 2012), given in Table 5.

Table 5. Random Consistency Index Numbers *R.I.*

N	1	2	3	4	5	6	7
R.I.	0	0	0.52	0.88	1.11	1.25	1.35

C.R. is calculated using the following equation.

$$C.R. = \frac{C.I.}{R.I.} = \frac{0.0134}{0.88} = 0.0152 \tag{16}$$

Resultant *C.R.* must be no greater than 10%; if C.R. is less than 10%, it allows for minor inconsistencies in the subjective ratings. In this scenario, *C.R.* = 1.52% which means that the consistency among pairwise criteria comparison is acceptable.

Using the pairwise comparisons and Equation (14), priority vector *w* with C.R.=1.52% is obtained as given in Table 6.

Table 6. Priority Vector

	Impact Velocity Ahead	Impact Velocity Behind	Manoeuvre Acceleration	Time-To-Collision
Priority Vector	0.3920	0.3920	0.1709	0.0452

5.4 TOPSIS

The TOPSIS method uses simulation results for each alternative lane and criteria given in Table 7.

Table 7. Decision matrix simulation results for the benchmark scenario

	Lane 1	Lane 2	Lane 3
Impact Velocity Ahead (m/s)	4.01	11.46	4.01
Impact Velocity Behind (m/s)	9.00	11.07	9.00
Manoeuvre Acceleration (m/s^2)	8.78	8.31	8.78
Time-To-Collision (s)	3.08	2.42	3.08

TOPSIS consists of the following steps:

1. Standardise the decision matrix. The decision matrix is the matrix of attribute values giving the values of alternatives for each criterion. The standardisation uses the following equation:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{3} a_{ij}^2}}, \quad i = 1, 2, 3, \quad j = 1, 2, 3, 4 \tag{17}$$

where a_{ij} is the criterion value of alternative i and criterion j, r_{ij} is the standardised value of each criterion value; the results are presented in Table 8.

Table 8. Standardised decision matrix

	Lane 1	Lane 2	Lane 3
Impact Velocity Ahead	0.190	0.541	0.190
Impact Velocity Behind	0.425	0.523	0.425
Manoeuvre Acceleration	0.588	0.556	0.588
Time-To-Collision	0.618	0.485	0.618

Typically, each criterion is standardised separately. However, to maintain the magnitude of both impact velocities, impact velocities of the vehicles ahead and behind are standardised together. This maintains the magnitude of these collisions, so that the collisions ahead can be compared with the collisions behind.

2. Calculate the weighted standardised decision matrix by multiplying the standardized criterion values r_{ij} by the corresponding weight priority vector w to obtain the weighted standardised value of each criterion t_{ij} , the results are presented in Table 9:

$$t_{ij} = w_j \cdot r_{ij},\ i = 1, 2, 3,\ j = 1, 2, 3, 4 \tag{18}$$

Table 9. TOPSIS weighted decision matrix

	Lane 1	Lane 2	Lane 3
Impact Velocity Ahead	0.074	0.212	0.074
Impact Velocity Behind	0.167	0.205	0.167
Manoeuvre Acceleration	0.100	0.095	0.100
Time-To-Collision	0.028	0.022	0.028

3. Create the artificial Ideal S^* and Negative Ideal S^- solutions from the available criterion values in the weighted standardised decision matrix formed in Step 2. The Ideal and Negative Ideal solutions are vectors created by selecting the most and least desirable values obtained for each criterion (Table 10).

Table 10. TOPSIS Ideal and Negative Ideal solutions

	Ideal Solution S^{*}	Negative Ideal Solution S^{-}
Impact Velocity Ahead	0.074	0.212
Impact Velocity Behind	0.167	0.205
Manoeuvre Acceleration	0.095	0.100
Time-To-Collision	0.028	0.022

4. Calculate the distance from the Ideal S^{*} and Negative Ideal S^{-} solutions for each of the weighted criterion values t_{ij}. The distance is calculated as the Euclidean distance, as proposed in (Yoon and Hwang, 1995):

$$S_i^{*} = \sqrt{\sum_{j=1}^{3}\left(t_{ij} - t_j^{*}\right)^2}, i = 1, 2, 3 \tag{19}$$

$$S_i^{-} = \sqrt{\sum_{j=1}^{3}\left(t_{ij} - t_j^{-}\right)^2}, i = 1, 2,3. \tag{20}$$

where t_j^{*} is the ideal criterion value for criterion j, and t_j^{-} is the negative ideal criterion value for criterion j. Therefore, $S_1^{*} = 0.005, S_2^{*} = 0.143, S_3^{*} = 0.005$ and $S_1^{-} = 0.143, S_2^{-} = 0.005, S_3^{-} = 0.143$.

5. Calculate the relative closeness C_i^{*} of each alternative *i* to the Ideal Solution (Yoon and Hwang, 1995):

$$C_i^{*} = \frac{S_i^{-}}{\left(S_i^{*} + S_i^{-}\right)}, \quad i = 1, 2, 3 \tag{21}$$

Therefore, $C_1^{*} = 0.964, C_2^{*} = 0.036, C_3^{*} = 0.964$.

6. The optimal alternatives are alternatives *i* = 1 and *i* = 3 with equal ranking number C_1^{*} and C_3^{*}, closest to 1. This means that these alternatives are geometrically closest to the ideal solution S^{*}. When two or more C_i^{*} are preferred, the default decision is to select the lowest lane number. i.e., Lane 1 is preferred over Lane 3. This is because the scenarios represent a UK motorway, in which Lane 1 represents the slowest lane of traffic and it is the closest to the emergency lane (hard shoulder), should emergency vehicles need to attend the scene of an accident. This default can be switched for countries driving on the right-hand side of the road. Therefore, it is assumed that the lane with the slowest traffic and closest to the emergency lane will be the safest when there are no other metrics to state otherwise.

Generally, in some scenarios, two lanes may have identical results. When there is no clear benefit to selecting one lane over another, a default decision is made, which is to select the lowest lane number,

5.5 AHP

The AHP method includes the following steps:

1. Normalize each alternative value in the decision matrix given in Table 7 against each criterion, using Equation (13). The results are presented in Table 11.

Table 11. Normalised decision matrix

	Lane 1	Lane 2	Lane 3
Impact Velocity Ahead	0.083	0.236	0.083
Impact Velocity Behind	0.185	0.228	0.185
Manoeuvre Acceleration	0.339	0.321	0.339
Time-To-Collision	0.305	0.389	0.305

2. Multiply the normalised decision matrix by the priority vector w to calculate the weighted normalised decision matrix, see Table 12. The decision matrix is normalised in the following way. The rows of the normalised decision matrix should sum to 1. To give the impact velocities with the vehicles ahead and the vehicles behind the equal weights, the corresponding 6 values associated with the collisions in the three lanes are normalised together, meaning the values of the two criteria will sum to 1. In the case when one of these criteria has much lower impact velocities than the other, their magnitudes must be maintained, as all collisions are considered equal.

Table 12. Weighted normalised decision matrix

	Lane 1	Lane 2	Lane 3
Impact Velocity Ahead	0.032	0.093	0.032
Impact Velocity Behind	0.073	0.089	0.073
Manoeuvre Acceleration	0.058	0.055	0.058
Time-To-Collision	0.014	0.018	0.014

3. Sum all criterion values for each alternative and normalise them to calculate the weighted rank vector, Table 13.

Table 13. Weighted rank matrix

	Lane 1	Lane 2	Lane 3
Rank	0.291	0.418	0.291

4. The alternative with the ranking number closest to 0 is selected (in this case, they are Lanes 1 and 3). The reason is that in this research problem, all the decision criteria have to be minimised except the time-to-collision which has to be maximised. The decision matrix for AHP takes this into account by normalising the reciprocal values for this criterion. As discussed earlier, the lane with the smallest number is selected, i.e. Lane 1.

5.6 ANP

The ANP method evolves from the AHP method by introducing pair-wise comparisons of alternatives and criteria values. It starts by repeating the steps of the AHP method to determine the weighted decision matrix in Steps 1 and 2. The next steps outline the ANP method.

1. Assess the influence of the criteria' values on the decision made with respect to the criteria, i.e. how much a criterion value of each criterion has influenced the rank value of each alternative, compared to all of the other criterion values. This is how the feedback of the ANP process is formed and used. The influence of the criterion value of each lane, given in Table 14, is determined by normalising the criteria values of each lane, using normalised decision matrix in Table 11. This assesses which criteria have or have not influenced the overall result.

Table 14. ANP influence of criteria values on the decision

	Lane 1	Lane 2	Lane 3
Impact Velocity Ahead	0.091	0.201	0.091
Impact Velocity Behind	0.203	0.194	0.203
Manoeuvre Acceleration	0.372	0.274	0.372
Time-To-Collision	0.335	0.331	0.335

Construct the Supermatrix (Saaty, 1996). The Supermatrix given in Table 15 is formed by inputting the clusters of criteria and alternatives. It includes the priority vector weights (yellow boxes), the weighted decision matrix (orange boxes) and the influences of alternative lanes on the decision (blue boxes). The columns of the Supermatrix must be normalised, so that the matrix will converge in the next step.

Table 15. Supermatrix

	Goal	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Lane 1	Lane 2	Lane 3
Goal	1							
Criterion 1	0.392	1				0.091	0.201	0.091
Criterion 2	0.392		1			0.203	0.194	0.203
Criterion 3	0.171			1		0.372	0.274	0.372
Criterion 4	0.045				1	0.335	0.331	0.335
Lane 1		0.032	0.073	0.058	0.014	1		
Lane 2		0.093	0.089	0.055	0.018		1	
Lane 3		0.032	0.073	0.058	0.014			1

2. Calculate the Limit Supermatrix by raising the Supermatrix to power $k + 1$. The value of k is the power to which the matrix converges in successive iterations, i.e., when all values in each row become identical to the values obtained in the previous iteration, see Table 16.

Table 16. Limit Supermatrix

	Goal	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Lane 1	Lane 2	Lane 3
Goal	0	0	0	0	0	0	0	0
Criterion 1	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070
Criterion 2	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075
Criterion 3	0.163	0.163	0.163	0.163	0.163	0.163	0.163	0.163
Criterion 4	0.549	0.549	0.549	0.549	0.549	0.549	0.549	0.549
Lane 1	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043
Lane 2	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056
Lane 3	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043

3. Normalise the columns of the Limit Supermatrix in their clusters. This gives the normalized rankings for the criteria and the normalized rankings for the alternatives, see Table 17 and Table 18, respectively.

Table 17. Normalised criteria weights

	Criterion 1	Criterion 2	Criterion 3	Criterion 4
Weight	0.082	0.087	0.190	0.641

Table 18. Normalised alternative ranks

	Lane 1	Lane 2	Lane 3
Rank	0.304	0.391	0.304

4. The alternatives with the ranking number closest to 0, are Lane 1 and Lane 3. Lane 1 is selected for the same reason as when AHP is applied; it is nearer to the emergency lane, if needed, compared to Lane 3.

Alternative ranking values for the benchmark scenario using TOPSIS, AHP and ANP methods are compared in Table 19.

Table 19. MADM rankings of the alternative lanes in the benchmark scenario

	Lane 1	Lane 2	Lane 3
TOPSIS	0.964103	0.035897	0.964103
AHP	0.290834642	0.418330717	0.290834642
ANP	0.304252996	0.391494008	0.304252996

The MADM rankings indicate that for the benchmark scenario all three MADM methods select either Lane 1 or 3 as the preferred lane. The most preferable option for TOPSIS is the rating closest to 1, as this is a geometric distance of the alternative lanes closest to the artificially created ideal solution. AHP and ANP are set up so the preferred alternative in the ranking is closest to 0. Two lanes, Lane 1 and Lane 3 have identical smallest ranking values. As discussed before, when there are two or more identical rank values, it is the lowest lane number, Lane 1, that is selected as this is the slowest lane and closest to the emergency lane.

6. Sensitivity analysis

Sensitivity analysis is carried out to test the impact different vehicles' parameters have on the MADM lane selection. In each scenario only 1 parameter is changed compared to the benchmark scenario and its impact on the lane selection is recorded. Impact of changing the maximum braking a_x, the maximum lateral acceleration a_y and CG (Centre of Gravity) height which affect the steering are analysed in Sections 6.1 to 6.3, respectively, using the simulation results of the autonomous vehicle. In Sections 6.4 to 6.11, only one parameter of a vehicle in Lane 3 is varied, including the mass of the vehicle ahead and behind, the initial velocity of the vehicle ahead and behind, the initial headway distance of the vehicle ahead and behind, and the deceleration (braking) of the vehicle ahead and behind.

The decisions made by the MADM methods in the following sections are assessed by the subject expert. The subject expert's decision is a human assessment of the simulation results, of what is thought to be the best lane selection. The decision from the MADM methods on the lane selection is compared to the subject expert's decision. In the event that two or more lanes have identical MADM ranking values, as explained before, a default decision is made to select the lowest lane number.

The results obtained are presented and discussed in the following sections.

6.1 Autonomous vehicle maximum longitudinal deceleration (braking)

The maximum longitudinal acceleration a_x is the braking the autonomous vehicle applies. With dynamic braking, this value is actually slightly higher with the resistance forces acting on the vehicle. However, for the steering manoeuvres this value cannot be exceeded, otherwise a loss in tyre grip will occur.

Simulation reveals that a lane-change manoeuvre will be disqualified when this braking value is set at $-1.2m/s^2$ or lower (i.e. closest to 0) due to an impact occurring before the lane-change manoeuvre is complete. However, the autonomous vehicle should stay in Lane 2 although a manoeuvre into Lanes 1 and 3 is possible, when the braking value is set at $-3.0m/s^2$ or lower, due to the low resultant braking value for the lane-change manoeuvre. The tyre saturation using Equations (7) to (9) set a braking value of $-1.77m/s^2$, when the maximum braking is set at $-3.0m/s^2$. It is important to note that even though the maximum longitudinal deceleration may be set, the resultant braking is determined by the tyre saturation, and so a lower braking value may be applied safely without over-saturating the tyres.

The maximum rate of deceleration for this simulation scenario is $-9.1m/s^2$, before Lanes 1 and 3 are disqualified for manoeuvre. This disqualification is due to the skidding speed given in Equation (12), with this value being lower than the autonomous vehicle's initial speed. However, the skidding speed requires the coefficient of friction to be increased to complete the lane-change manoeuvre. Increasing the available longitudinal distance to complete the lane-change manoeuvre would increase the skidding speed limit of the vehicle, as given by Equation (12), as it represents the low radius of curvature required to complete the manoeuvre. Therefore, Lanes 1 and 3 are disqualified for manoeuvre based on the skidding speed.

6.2 Autonomous vehicle maximum lateral acceleration (steering)

The benchmark autonomous vehicle braking value for this scenario is $-4.71m/s^2$, as calculated from the tyre saturation, Equations (7) to (9). The lowest value that the maximum lateral acceleration can be set to before Lanes 1 and 3 are disqualified due to a collision occurring before the lane-change manoeuvre is complete is $6.9m/s^2$. This leads to a collision with the vehicles ahead and the braking value for the lane-change manoeuvres at this maximum lateral acceleration is $-0.78m/s^2$.

The collision avoidance for the vehicles ahead in Lanes 1 and 3 is calculated based on the maximum lateral acceleration set to $12.3m/s^2$ or higher. It is observed that the higher this value, the more severe the collision accelerations for the vehicles behind. As the autonomous vehicle can decelerate quicker, the relative impact speeds between the autonomous vehicle and vehicles behind is greater. At lateral accelerations set to $7.2m/s^2$ or lower, there is a collision avoidance in Lanes 1 and 3 for the vehicles behind.

Results of the sensitivity analysis displayed in Figure 3 show that Lanes 1 and 3 have identical rank values throughout and they are preferred over Lane 2. With a maximum lateral acceleration of 6.9m/s^2 to 9.1m/s^2, the collisions ahead in Lanes 1 and 3 occur before the collision behind. With lateral accelerations of 9.2m/s^2 and greater, the available braking for the autonomous vehicle is greater. Therefore, the collisions with the vehicles behind in Lanes 1 and 3 occur before the collisions with the vehicles ahead. When the lateral acceleration values are set at 9.5m/s^2 or greater, the autonomous vehicle braking is sufficient to prevent a collision ahead. All the MADM methods considered select Lane 1 in every simulation run as the preferable lane. Thus, the range at which the steering manoeuvre is possible is found through undertaking the sensitivity analysis. It is concluded that as long as the steering manoeuvre is possible, it is preferable to remain in the current lane.

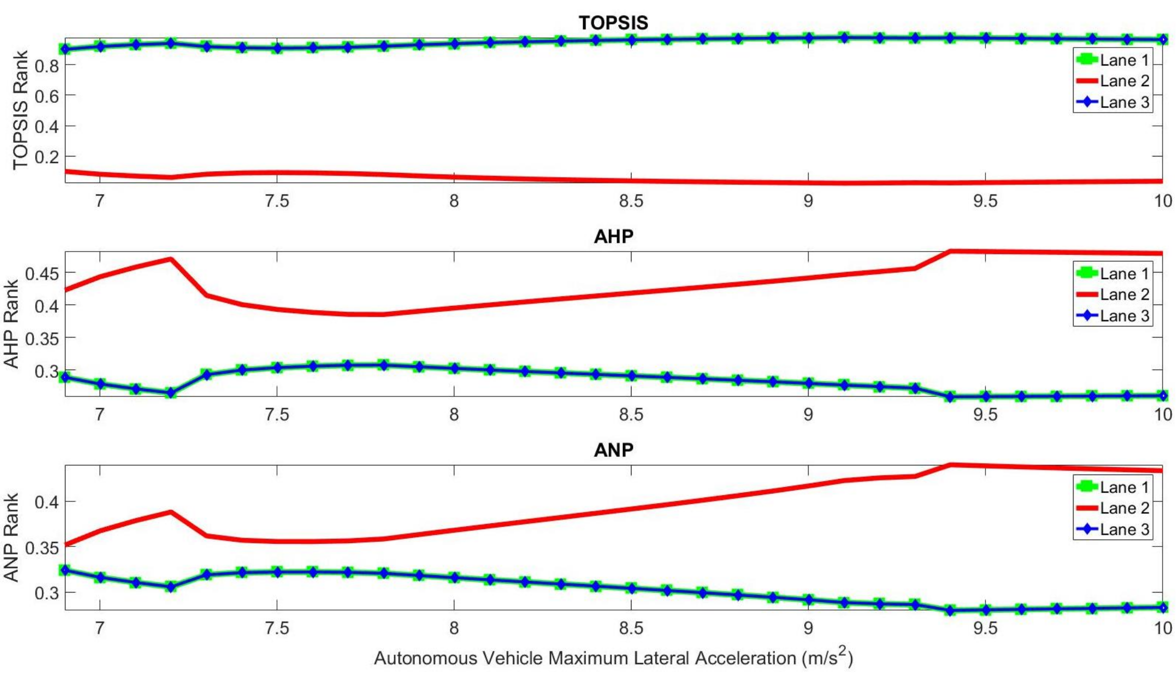

Figure 3. Impact of lateral manoeuvre acceleration of the autonomous vehicle

6.3 Autonomous Vehicle CG height

The CG height is used to determine if the autonomous vehicle will rollover during the lane-change manoeuvre. The sensitivity analysis determines if a rollover will occur and if needed, disqualifies the manoeuvre. It is calculated that a rollover results with a CG height set at 1.21m or higher. However, when examining the vertical wheel loads with the height set just below the rollover height, at 1.20m, one can see that, even though very low, there is a vertical load pushing down on all wheels; the Rear Left wheel reaches a minimum force of 0.000016kN) and the Front Left wheel reaches a minimum value of 0.35kN (Figure 4). This force prevents the vehicle from rollover.

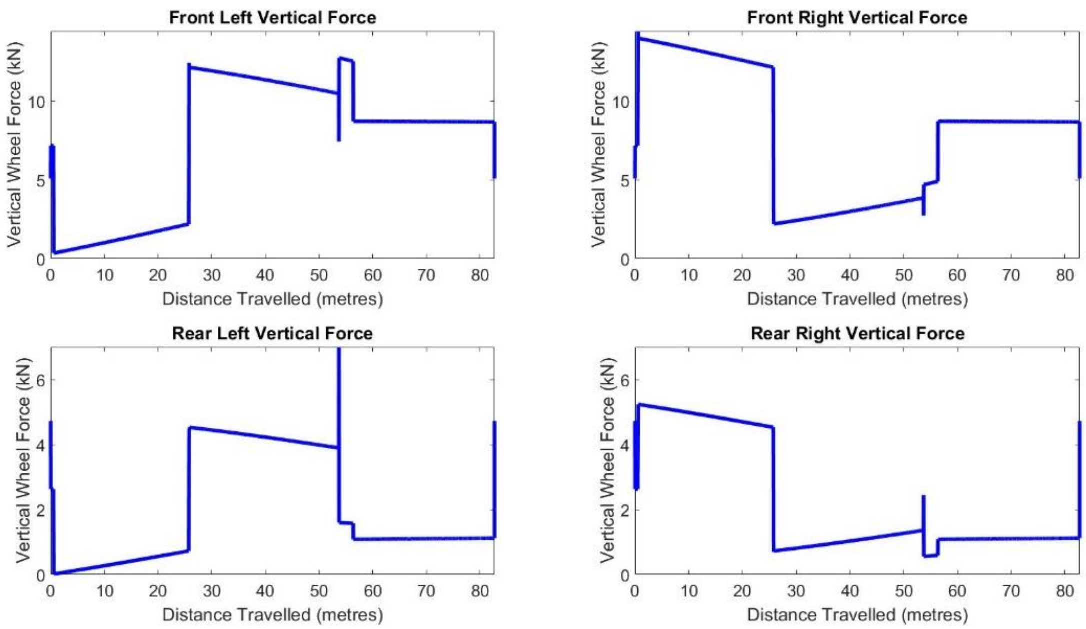

Figure 4. Vertical tyre force approaching rollover

Figure 4 suggests that the vertical wheel force limit should be set higher than 0kN, as a safety factor. When it is 0kN there is no vertical force pushing the tyre down onto the road. This means that a value of 0kN mathematically represents a tyre lifting from the ground. This value would need to be determined by vehicle dynamics testing. At 53.61m, the steering manoeuvre ends and then the autonomous vehicle can increase its braking from the value determined by the tyre saturation, given by to the maximum longitudinal braking, (see Equations (7) to (9)). Once the steering manoeuvre is complete, full braking is applied and both the wheels on the front axle have identical vertical forces to each other, as do the two rear wheels.

6.4 Vehicles ahead mass

In this scenario the mass of the vehicle ahead in Lane 3 is varied from 900kg to 4000kg, while the benchmark is 2000kg. Adjusting the mass affects the impact velocities as the vehicle mass affects the dynamic braking. Figure 5 displays the sensitivity analysis results when the mass of the vehicle ahead in Lane 3 is varied. All of the three MADM methods select the same lane in each simulation. It is observed that the decisions made by the MADM methods do not select a collision with the lighter vehicle. The preferred lane to steer into and have a collision is Lane 3 when the vehicle ahead is between 900kg and just below 2000kg. At 2000kg, the preferred lane to select for the collision is Lane 1. From 2100kg to 4000kg, the autonomous vehicle selects the larger vehicle in Lane 3 to collide with.

The rank values do not change drastically when the vehicle ahead's mass is changed from 900kg to 4000kg. A closer look at the simulation results reveals that for a mass of 900kg, the vehicle ahead has a velocity at the moment of impact of 9.91m/s, and the autonomous vehicle has a velocity of 14.42m/s. The simulation with a vehicle ahead mass of 4000kg shows that the vehicle ahead's impact velocity drops to 8.72m/s, and the autonomous vehicle is travelling at 12.51m/s. The larger mass vehicle does have an effective deceleration lower than the lighter mass vehicle. This allows for a greater stopping distance for the autonomous vehicle, and so the autonomous vehicle's preference is to select the collision with the larger mass vehicle in Lane 3.

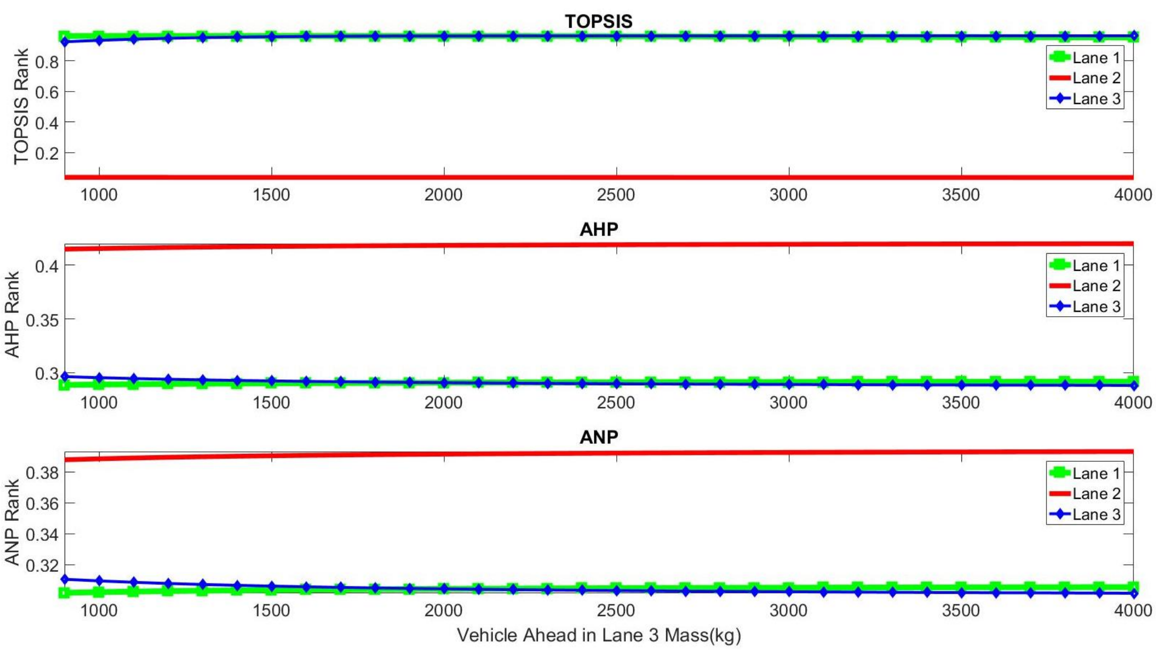

Figure 5. Varying the mass of the vehicle ahead in Lane 3.

6.5 Vehicles behind mass

The vehicles behind in Lane 3 are examined using the same range of mass as in the previous section. All of the three MADM methods select the same lane, and a similar trend to the results presented in the previous section is observed. The lane ranking values presented in Figure 6 give similar results across the range of masses simulated; the important difference being that in this situation, the autonomous vehicle selects the lighter mass vehicle.

The deceleration is again the influencing factor, as the lighter vehicle behind will reduce its impact velocity more than a heavier vehicle will. The lane ranking values for Lanes 1 and 3 are very close, but there is a preference to select the lane with the lighter mass vehicle. Comparing the observations in Section 6.3 with these results, it is concluded that the autonomous vehicle selects the lane with the larger mass vehicle ahead for itself to collide with and the lane with the lighter mass vehicle behind to be impacted into. For masses from 900kg to 1900kg of the vehicle behind in Lane 3, the preferred lane is Lane 3. However, for masses in the range of 2000kg to 4000kg, Lane 1 is preferred.

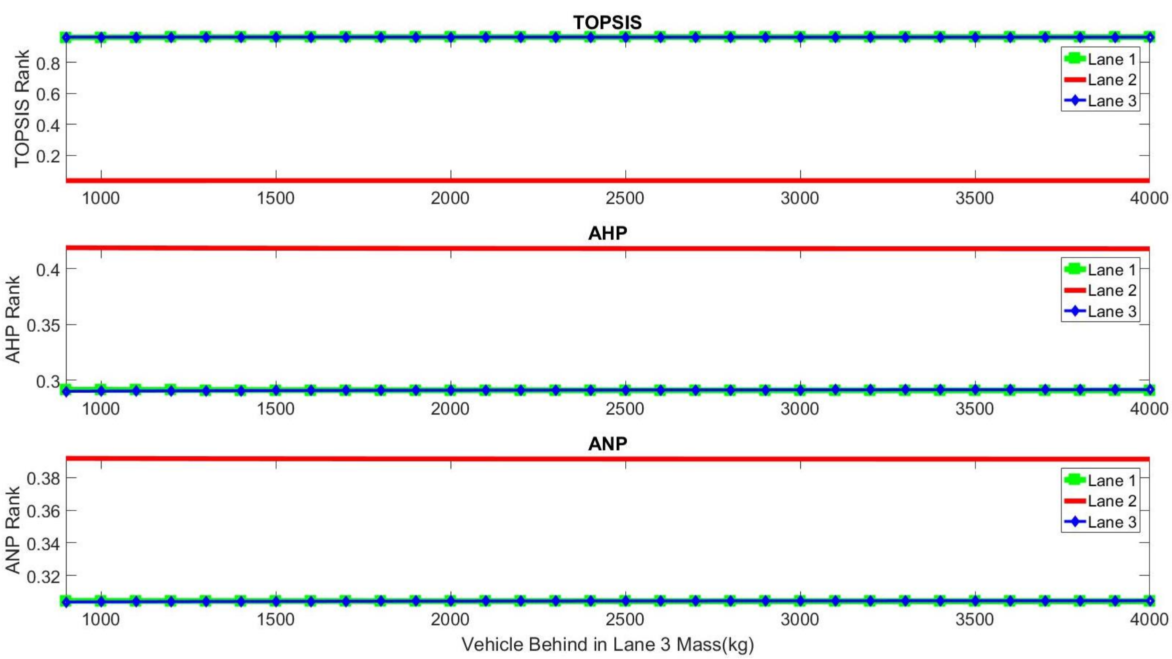

Figure 6. Varying the mass of the vehicle behind in Lane 3

6.6 Vehicle ahead initial velocity

If the initial velocity of a vehicle ahead in an adjacent lane is below 65mph, a collision will occur before the lane-changing manoeuvre is complete. If this velocity is lowered further, the initial headway distance must be increased from 12m, in order to enable a lane-change manoeuvre. Collision avoidance will occur if this initial velocity is set higher than 72mph for the vehicle ahead, as the 12m headway distance is sufficient to allow the autonomous vehicle to stop before a collision occurs.

Figure 7 shows the results obtained for varying initial velocity of the vehicle ahead in Lane 3. All MADM methods have selected the same lane in each simulation. It is important to note that only the initial velocity of the vehicle ahead in Lane 3 is varied, thus the initial velocities of the vehicles in Lanes 1 and 2 remained constant at 70mph. Lane 3 is the preferred choice when the initial velocity of the vehicle ahead in Lane 3 is higher than the autonomous vehicle's 70mph. Lane 1 is the preferred choice when the initial velocity is 70mph or lower in Lane 3. Furthermore, at 72.5mph, collision avoidance is achieved ahead in Lane 3.

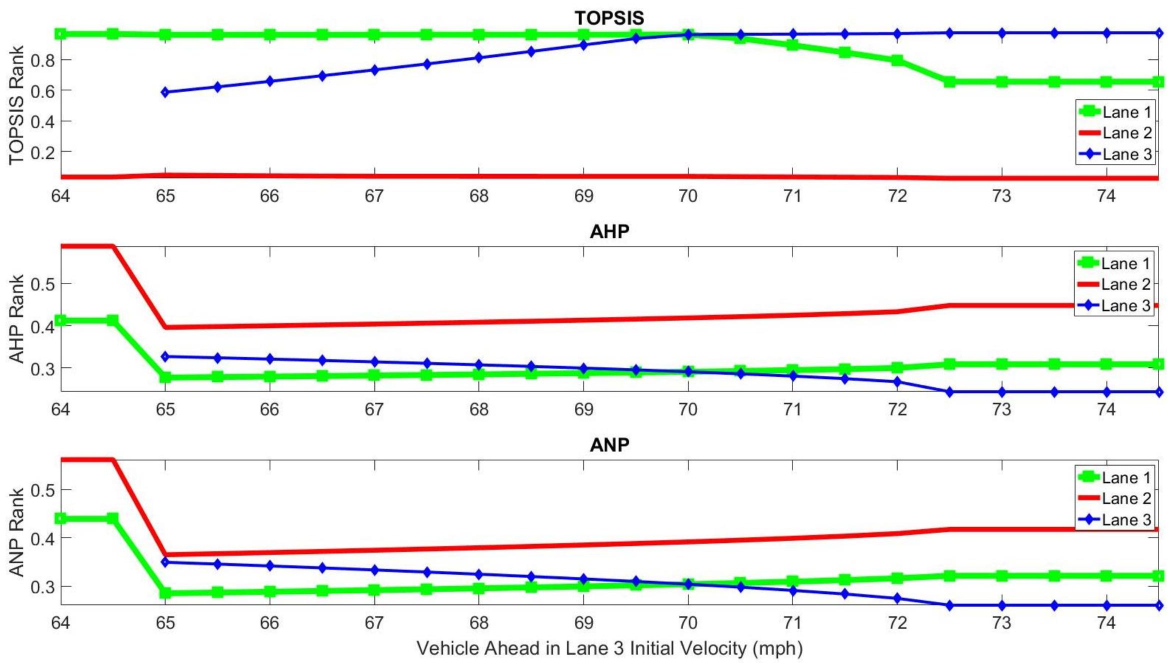

Figure 7. Varying initial velocity of the vehicle ahead in Lane 3

6.7 Vehicle behind initial velocity

The minimum velocity of a vehicle behind in an adjacent lane for a collision to still occur is 65mph. The highest initial velocity of a vehicle behind before an adjacent lane is disqualified due to a collision occurring before the lane-change manoeuvre is complete is 84mph. These limits are of course dependent on the initial headway distances.

Figure 8 shows that the preferred choice of lane-change is not a simple matter of selecting the lane with a lower velocity of the vehicle behind. From 64mph to 67mph, collision avoidance with the vehicle behind is achieved in Lane 3, and all of the MADM methods select this lane as the preferred option. From 68mph to 70mph, the relative impact velocity with the vehicle behind is actually higher in Lane 3 which is due to the autonomous vehicle having a greater available distance to brake in that lane. Therefore, Lane 1 is selected by all of the MADM methods at these given speeds.

From 71mph to 82mph TOPSIS and AHP select Lane 3. TOPSIS however selects Lane 3 at 83mph, which is due to the relative impact velocity being slightly lower in Lane 3. AHP does not select Lane 3 in this case, due to the velocities of the vehicles being much higher in comparison with Lane 1. This is an important consideration because from 77mph onwards, a collision with the vehicle behind occurs before a collision with the vehicle ahead.

Interestingly, the ANP rank disagrees with TOPSIS and AHP at speeds of 78mph to 82mph and instead selects Lane 1, as presented in Figure 8. The feedback which determines the influence of the criteria has guided this decision. Furthermore, the decisions made by TOPSIS and AHP are in-line with the subject expert's decision. However, although the decision made by ANP disagrees with the other decisions made in this scenario, it does still avoid Lane 2 which is the least favourable lane selection for all the MADM methods. The ranking values of Lanes 1 and 3 are very close for all methods from 68mph to 84mph. This demonstrates that the measurable collision severity between these two lanes is very similar.

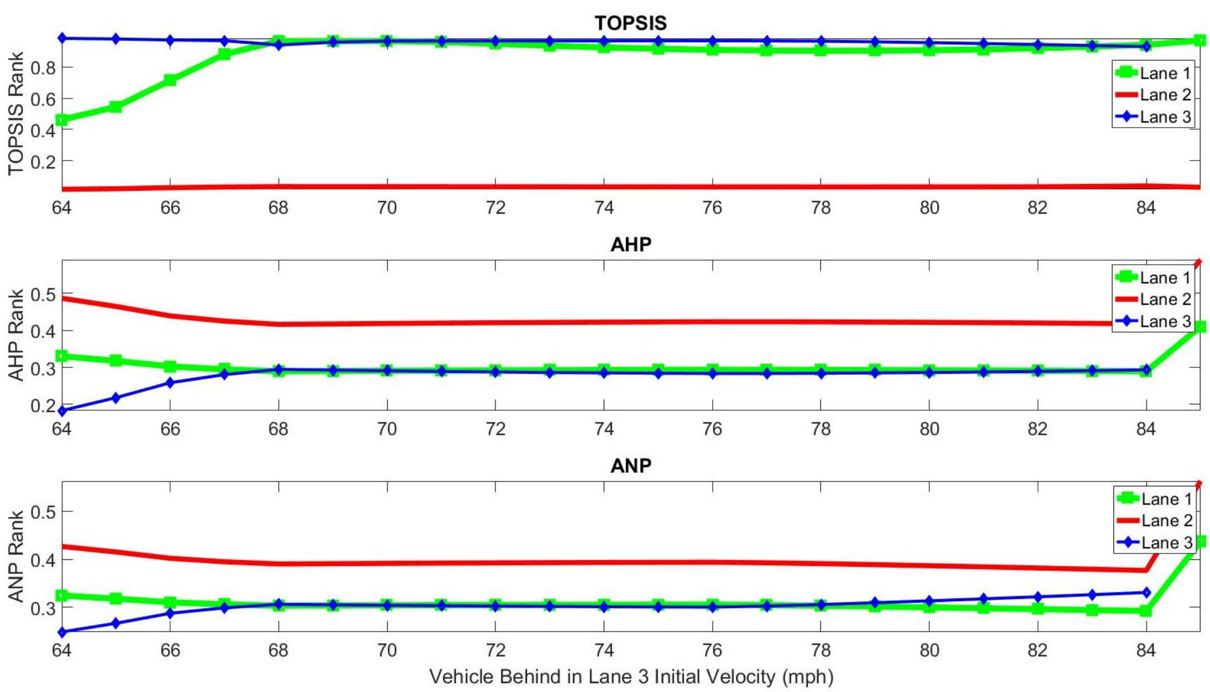

Figure 8. Varying initial velocity of the vehicle behind in Lane 3

6.8. Vehicle ahead headway distance

The minimum initial headway distance for a vehicle ahead in an adjacent lane is 7.5m. At distances less than this, a lane-change manoeuvre cannot be completed before the impact. A greater headway distance is always desirable, giving the autonomous vehicle more distance to decelerate. Collision avoidance will occur when this initial headway distance is set to 17m or greater.

Figure Figure 9 shows that all of the MADM methods have selected the same lane in every simulation. From distances of 7m to the benchmark distance of 12m in Lane 3, the preferred choice is Lane 1. At distances greater than this, the preferred choice is Lane 3.

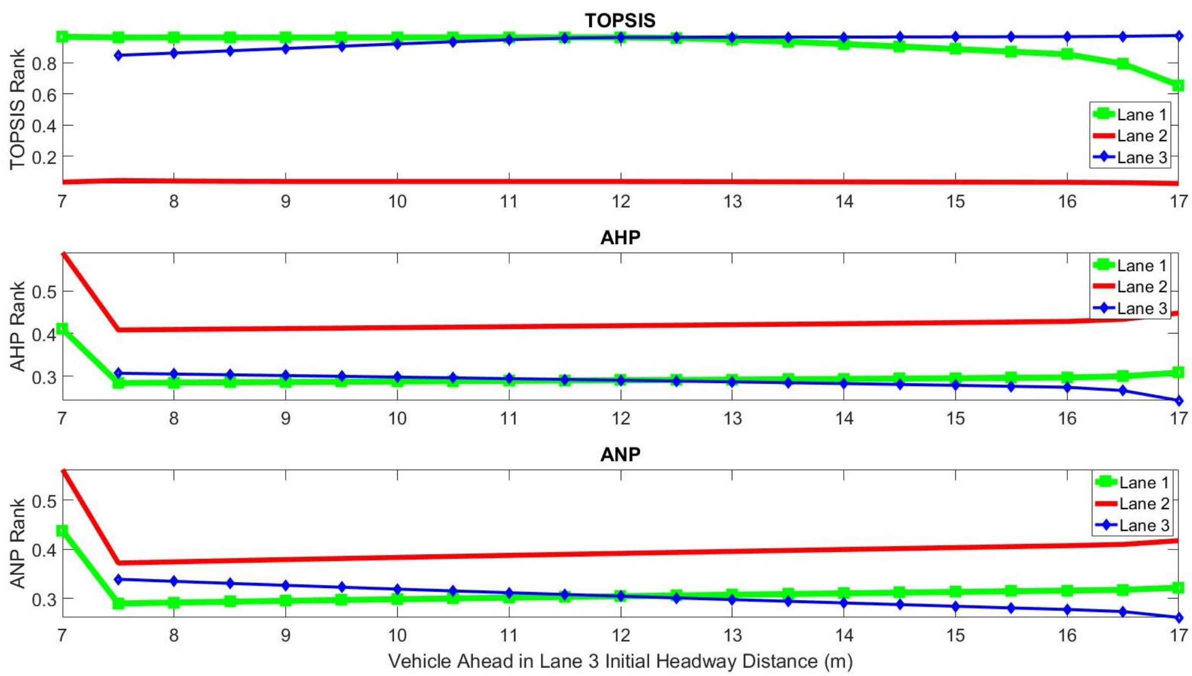

Figure 9. Varying initial headway distance of vehicle ahead in Lane 3

However, although a collision avoidance is calculated for a headway distance of 17m in Lane 3, a closer look at the impact velocities of the vehicle behind and the autonomous vehicle for the rear collision gives an insight into a limitation of the simulator. The impact velocities for the vehicle ahead and autonomous vehicle colliding with it are 0m/s, but the impact velocities for the vehicle behind and autonomous vehicle colliding with it is not 0m/s. The distance between the autonomous vehicle and vehicle ahead in this simulation is only 0.23m. The impact with the vehicle behind may force the autonomous vehicle forward. If this distance is greater than 0.23m, the autonomous vehicle will collide with the vehicle ahead. However, the simulator does not calculate this as it has already determined the collision avoidance with the vehicle ahead. Therefore, the simulator does not consider the vehicle behind. In order to determine the severity of the secondary collision in this situation, further modelling will need to be carried out to consider the velocity of the autonomous vehicle after being impacted by the vehicle behind.

6.9 Vehicles behind headway distance

For the vehicles behind, a greater headway distance is also desirable as this gives those vehicles more distance to decelerate, reducing the impact velocity with the autonomous vehicle. The minimum distance this can be set to before an impact occurs and the lane-change manoeuvre is complete is 8m. A collision avoidance will occur if this distance is set to 35m and greater.

Figure 10 shows the sensitivity analysis results when the preferred lane-changes between Lanes 1 and 3, applying each MADM method, are considered. All of the MADM methods have selected the same lane in every simulation run.

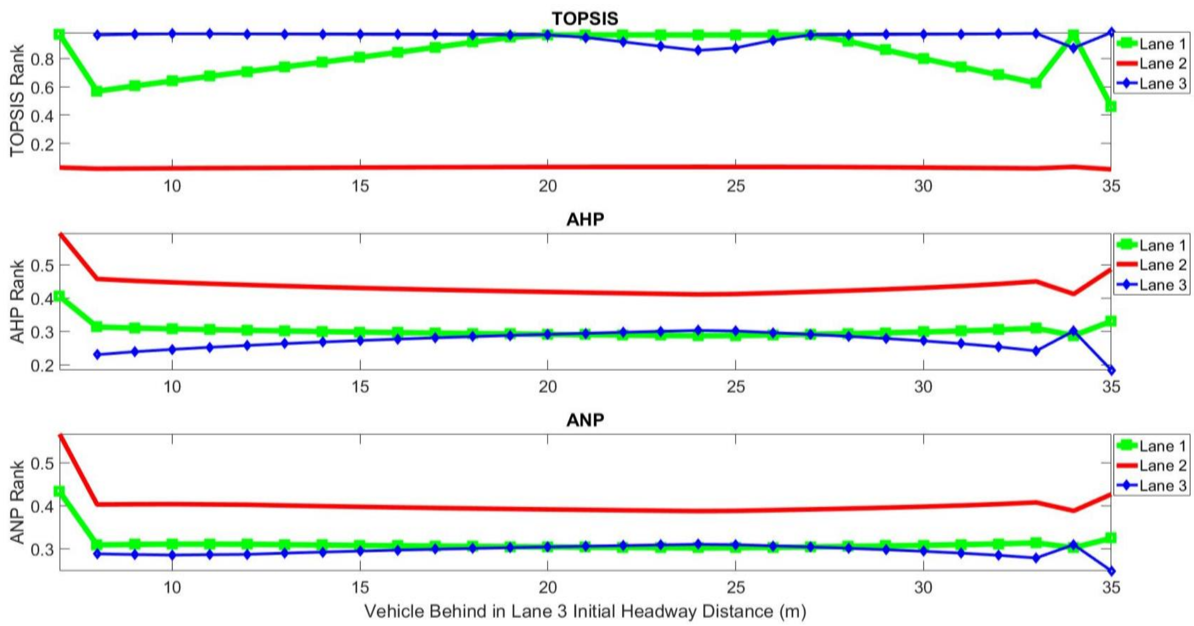

Figure 10. Varying initial headway distance of vehicle behind in Lane 3

When separation distances are from 8m to 19m, the preferred choice by all of the MADM methods is Lane 3, with a shorter headway distance than Lane 1. However, when the distance is shorter, the available braking distance of the vehicle behind is also smaller. Thus, the relative impact velocities are lower in Lane 3 with lower stopping distances for both the vehicle behind and the autonomous vehicle. For distances of 20m to 26m, the preferred choice is Lane 1. At distance 25m, the autonomous vehicle is able to come to a full stop. However, the lane choice decision from the headway distance of 27m to 35m depends entirely on the impact velocity between the vehicle behind and the autonomous vehicle. From 27m to 33m, the preferred choice is Lane 3, but all of the MADM methods select Lane 1 for a distance of 34m, and then again Lane 3 for a distance of 35m. The impact velocity at a distance of 34m is actually lower in Lane 1 than in Lane 3, which is why Lane 1 is selected as the preferred choice.

6.10 Vehicles ahead braking deceleration

Much like the deceleration considered in Section 6.1, the actual rate of deceleration will be greater than the inputted value due to dynamic resistance forces acting on the vehicle. However, the most effective stopping force is from the brakes, and the inputted deceleration determines the braking force. A lane is disqualified due to a collision occurring before the lane-change manoeuvre is complete, if this braking value is set at $-9.4m/s^2$ or greater. Meanwhile collision avoidance will occur if this rate of deceleration is set to $-6.5m/s^2$ or lower, because the autonomous vehicle's deceleration and headway to the vehicle ahead are sufficient to prevent contact between the two vehicles.

The sensitivity analysis results presented in Figure 11 demonstrate a clear conclusion. All of the MADM methods have selected the same lane for every simulation run. Clearly, the end result is better when the deceleration of the vehicle ahead is lower. From $-6.5m/s^2$ to $-6.9m/s^2$ the preferred lane is Lane 3 for all of the MADM methods. From the benchmark of $-7m/s^2$ to higher decelerations of the vehicle ahead in Lane 3, $-9.4m/s^2$, the preferred choice is Lane 1, which will have lower impact velocities.

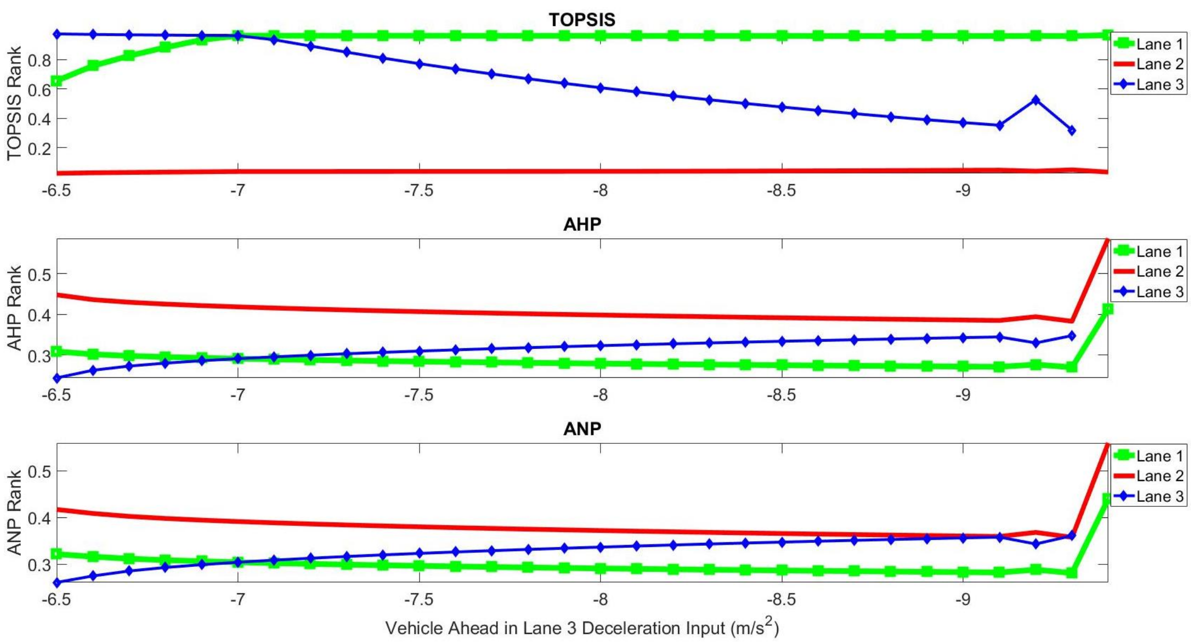

Figure 11. Vehicle ahead in Lane 3 deceleration

6.11 Vehicles behind braking deceleration

For the benchmark parameters of the dynamic braking simulator, there is a sufficient initial headway distance for the vehicles behind not to brake. All of the lanes are available for a lane-change decision as no collision occurs before the autonomous vehicle's lane-change manoeuvre is complete. A collision avoidance will occur if the rate of deceleration is set to $-6.0m/s^2$ or greater.

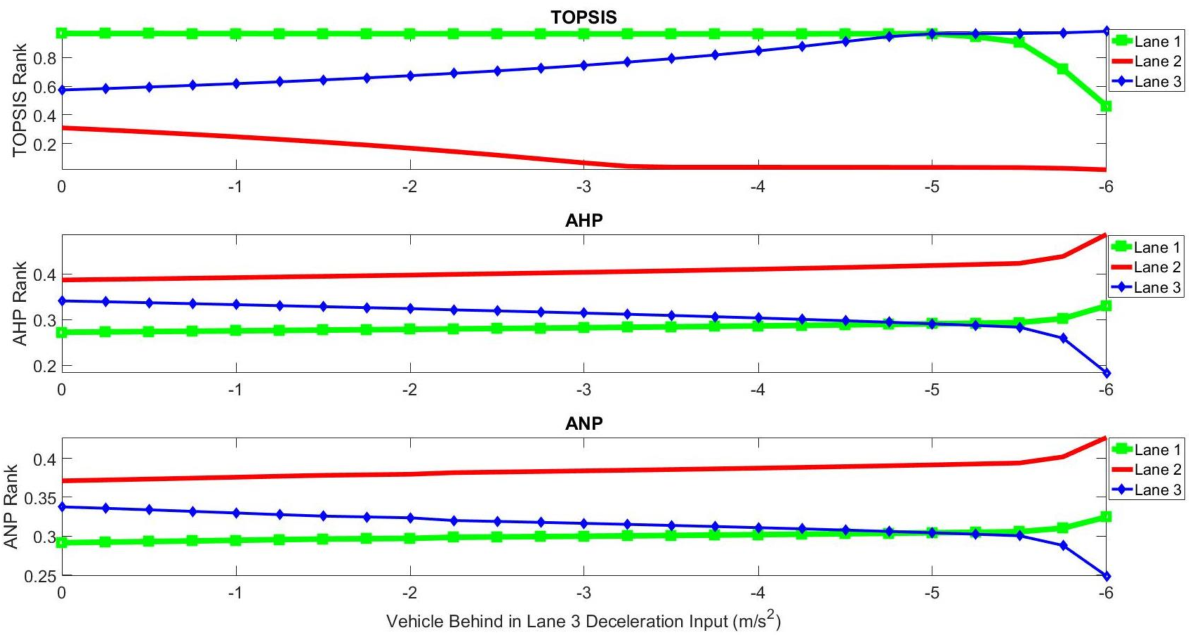

Figure 12. Vehicle behind in lane 3 deceleration

The sensitivity analysis results are presented in Figure 12. All of the MADM methods have selected the same lane in each simulation. For deceleration values of 0m/s^2 to the benchmark deceleration of $-5m/s^2$, the preferred lane is Lane 1. It is the vehicle behind in Lane 3 that has its braking varied, therefore the lower the deceleration the higher the impact velocity. For all deceleration values greater than $-5m/s^2$, it is Lane 3 that is preferred as the resulting impact velocity is lower than that of Lanes 1 and 2. From $0m/s^2$ to $-2m/s^2$, it is the collision behind that occurs before the collision ahead. From $-2.25m/s^2$ to greater deceleration values, it is the collision ahead that occurs first. However, this does not have a great influence on the lane's ranking.

6.12 Summary of V2V dynamic braking MADM results

The simulations show that AHP and TOPSIS give the same preference in each simulation scenario, except for one, when the initial velocity of the vehicle behind in Lane 3 is varied, see Section 6.6. That one scenario is interpreted differently in both MADM methods, but both decisions can be justified as the simulation results are compared for individual vehicle velocity values and impact velocity values. ANP agreed with the ranking of AHP and TOPSIS in most simulation scenarios, but not all. These disagreements are not simple to justify but are the result of the interpretation of the feedback used in ANP.

The ranking of ANP agreed with the other two MADM methods in all cases except in the scenario when the velocity of the vehicle behind is varied. ANP provided a decision that disagreed with the expert's decision, due to the feedback which is calculated from the normalised alternative values. The feedback is beneficial in re-assessing the importance of each criterion, but this is not proved to be always beneficial in this application. The proposed simulator and decision making methods must make an unbiased decision which results in the best outcome for all vehicles involved in the potential collisions. The AHP and TOPSIS methods assess the situation and do not re-evaluate the decision made. The result of the feedback used in ANP is that it may give a greater weight to a criterion which was originally determined to be less important. This is the intended purpose of the feedback, but may not be useful if that criterion should remain less important. In the simulations presented, this does not always occur as a criterion occasionally has its weight reduced when it is not intended or desired to be reduced.

By conducting these scenarios, it is observed that the simulation of varying headway distances for the vehicles ahead does present a limitation of the dynamic braking simulator. This can happen when a collision avoidance is calculated ahead, but the collision behind could still push the autonomous vehicle into the vehicle ahead. This would require further modelling of the autonomous vehicle's longitudinal velocity and displacement from the collision behind.

7. Conclusions

In this paper, the developed simulator of an automated vehicle and surrounding vehicles on a motorway was described. The simulation results were used in MADM methods to select a lane into which the autonomous vehicle should manoeuvre in order to mitigate a collision. Using the benchmark scenario defined as a reference, the decisions made by the MADM methods were analysed.

Sensitivity analyses were performed in different scenarios involving varying of the parameters describing the autonomous vehicle and vehicles ahead and behind. The parameters affected the outputs of the simulation and more critically the decision made. It was observed how a single parameter could change the lane selection.

Generally, it was observed that decisions on changing lanes made by the MADM methods and informed by the simulator were in line with the expected decisions. Therefore, the proposed concept of combining MADM methods and the simulator of autonomous vehicle demonstrates encouraging findings. Out of the three MADM methods, TOPSIS and AHP results were identical in all but one simulation scenario which could be explained and justified. However, the use of ANP was less satisfactory for the lane-change selection. In other examples of ANP applications given in the literature, the feedback was assessed subjectively by human participants. In this research, it was not possible for an expert or a group of experts to assess the influence of the alternatives on criteria; instead the feedback is formed automatically based on the normalised simulation results. It could be concluded that a different method of assessing the influence of the criteria on the alternatives and, consequently the feedback, is needed.

Of course, the proposed simulator and MADM methods require further development before such a system could be employed in a real vehicle. One limitation of the simulator was identified in this research and it refers to the calculation of possible secondary collisions with the vehicle ahead caused by a collision with the vehicle behind, even if an initial collision has been avoided.

Further on, the simulator used to calculate the vehicles' positions and velocities proved essential to inform the decision making. However, more complex simulators do exist, but these are more computationally heavy. New simulators must be developed for real-time use in real world vehicles before a MADM method can decide on the least severe collision for an autonomous vehicle to take. Also, more complexities exist in the real world which must also be simulated, such as slight radius of curvature on the road or weather conditions.

The MADM methods demonstrated an effective way for an autonomous vehicle to select a satisfactory solution between multiple alternatives. It is intended that the proposed MADM method could be utilised in real vehicles. There is of course, a lot of development required before that becomes possible. For example, AHP and ANP can introduce sub-criteria to the criteria hierarchy, which allows for further classification of the criteria. The identified issues with ANP and the feedback could be addressed with developing a new method for determining that feedback.

One of the main concerns is where exactly the information used in MADM methods will come from in a practical scenario. This paper assumes V2V will provide information about the other vehicles on the road. This is an issue which has been addressed by many researchers involved in autonomous vehicles, and in particular vehicle controls, and hardware for the vehicle to steer and brake itself without human input.

The proposed autonomous driving decision making tool is intended to be developed for real-time applications, but it is reliant on other technologies being available and capable. For example, a highly accurate motorway simulation is needed to give the decision-making processes the most accurate data possible. For this, the simulation tools available and computational capabilities of the autonomous vehicle must be very fast to provide this data in real-time.

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