Generalized Love Waves

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Abstract

We present a generalized dispersion equation for shear horizontally polarized acoustic waves in a system of a finite thickness substrate covered by a finite thickness solid layer having a lower shear acoustic speed. The solutions to the equation describe both layer guided Love waves and resonant acoustic plate modes. We identify higher order mode Love waves as continuations from the lowest order acoustic plate mode associated with the previous Love wave mode. Experimental results are presented that confirm the relationship between the multiple Love wave modes and the acoustic plate modes.

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In the more than thirty years since it was suggested that surface acoustic waves (SAWs) could be excited by an interdigital transducer[1] their utility as a tool to investigate fundamental properties of materials has been established. The Rayleigh SAW, with a mechanical component of displacement perpendicular to the surface, has been used to investigate transport properties of one- and two-dimensional electron gas systems, the fractional quantum hall effect and acoustoelectric charge transport in semiconductor quantum wells[2-5]. Shear horizontal SAWs, with their mechanical displacement in the plane of the surface, have also been studied in supported thin films on GaAs wafers and in adsorbed Si-SiO_2 bilayers[6,7]. In recent years, the use of SAW's has been significantly extended into chemical and biochemical applications. A fundamental requirement in these applications is for highly surface mass sensitive techniques capable of operating in an aqueous environment. To avoid the high damping that may be caused by the aqueous environment the acoustic waves must be either shear horizontally polarized or have a phase speed less than the speed of sound in the liquid. A wide range of surface acoustic wave types have been considered, including Love waves and shear horizontally polarized acoustic plate modes (SH-APM) [8]. Love waves were first reported for use as a biosensor in 1992 by Gizeli *et al*[9] and offer one of the highest potential mass sensitivities, but they have generally been regarded as distinct from acoustic plate modes. Therefore, a physical understanding of the spectrum of shear horizontally polarized acoustic modes occurring in a Love wave configuration is of wide interest.

A Love wave is a shear horizontally polarized acoustic wave that is localized to the surface of a semi-infinite half space and guided by a layer which has a shear acoustic speed less than that of the half space material[10,11]. The phase velocity of the Love wave is intermediate between that of the substrate and the layer and is determined by the layer thickness. An acoustic plate mode occurs when a finite thickness substrate is excited at a natural resonant frequency of the substrate[12,13]. The lowest order, n=0, APM mode corresponds to a plane wave whilst higher order APM modes correspond to resonances of the plate. In a Love wave the energy is localized near the surface of the substrate and the phase speed is less than that of the shear acoustic speed of the substrate. In contrast, in an APM the energy is distributed throughout the substrate and the phase speed is higher that of the shear acoustic speed of the substrate.

In this Letter, we generalize the theory of Love waves to a finite thickness substrate and show that Love waves and acoustic plate modes are two branches of solutions of the dispersion equation. It is shown that each Love wave mode is matched by a set of acoustic plate modes. We also demonstrate that as the guiding layer thickness increases, the phase speed of the m=1 acoustic plate mode associated with a Love wave mode reduces until it transforms into the next higher order Love wave mode. We also report new experimental results obtained for 312 MHz acoustic waves on a polymer coated quartz device using an interdigital transducer with a high frequency resolution. The spectrum of shear horizontally polarized modes close to the resonant frequency has been investigated for a wide range of polymer guiding layer thickness. The observed changes in phase speed of the various acoustic modes in the spectrum support the theoretically predicted relationship between Love waves and SH-APM's.

We consider shear horizontal waves with displacements in the x_2 direction travelling along the x_1 direction in a system comprised of an isotropic non-piezoelectric material of thickness, w, covered by an isotropic non-piezoelectric material of thickness, d. A solution for the wave equation is sought by using displacements in the layer, $\underline{u}_l$, and the substrate, $\underline{u}_s$, of

$$\underline{u}_l = (0,1,0)\left[Ae^{-jT_l x_3} + Be^{jT_l x_3}\right]e^{j(\omega t - k_1 x_1)}$$

and

$$\underline{u}_s = (0,1,0)\left[Ce^{T_s x_3} + De^{-T_s x_3}\right]e^{j(\omega t - k_1 x_1)}$$

where ω is the angular frequency. The wave vector $k_1=(\omega/v)^{1/2}$ where v is the phase speed of the Love wave. From the equations of motion, the wave vectors T_l and T_s are given by,

$$T_l^{\,2} = \omega^2\left(\frac{1}{v_l^2} - \frac{1}{v^2}\right) \quad \text{and} \quad T_s^{\,2} = \omega^2\left(\frac{1}{v^2} - \frac{1}{v_s^2}\right)$$

where the substrate and layer shear speeds are related to the rigidities, μ_i, and densities, ρ_i, of the substrate and layer materials by $v_s=(\mu_s/\rho_s)^{1/2}$ and $v_l=(\mu_l/\rho_l)^{1/2}$. Requiring the solution to satisfy boundary conditions of continuity of displacement and stress at the various interfaces gives the dispersion equation,

$$\tan(T_l d) = \xi \tanh(T_s w)$$

where $\xi=\mu_s T_s/\mu_l T_l$.

The dispersion equation has two types of solution. The first has T_s real and non-zero and gives a series of modes with phase speeds lower than the shear acoustic speed of the substrate;

these correspond to Love waves. The second has $T_s=jk_s$ with k_s real and gives a series of modes with phase speeds higher than the shear acoustic speed of the substrate; these correspond to acoustic plate modes. The thickness of the substrate, w, determines the number of Love wave modes and the spacing of the associated acoustic plate modes. At the start of each successive Love wave mode the phase speed of the Love wave is $v=v_s$ and the speeds of the associated plate modes are $v_m=v_s/(1-(m\pi v_s/w\omega)^2)^{1/2}$. The thickness, d_{nm}, at which a new mode appears is,

$$\frac{d_{nm}}{\lambda_l}=\frac{n}{2\sqrt{1-\left(\frac{v_l}{v_s}\right)^2\left[1-\left(\frac{m\lambda_s}{2w}\right)^2\right]}}$$

where n=0,1,2,3, …. labels the successive Love wave modes and m=1,2,3, … labels the acoustic plate modes associated with each Love wave mode. Traditional Love waves correspond to m=0 whilst acoustic plate modes correspond to $m>0$. The n=0 Love wave corresponds to a displacement with a single node located within the substrate and an antinode at the surface of the guiding layer. Each higher order Love wave introduces an additional node within the layered system. Increasing the thickness of the guiding layer by an amount insufficient to change modes causes a reduction in the phase speed of the mode. Although it is not possible to analytically solve the dispersion equation, we have obtained the fractional change in phase speed caused by increasing the thickness of the guiding layer by an amount, Δd, above the guiding layer thickness resulting in the start of a mode. In the case of a Love wave $\Delta v/v_s \propto \omega^2 \Delta d^2$, assuming the substrate is sufficiently thick compared to the perturbation, whereas for the lower order associated acoustic plate modes we find $\Delta v/v_s \propto \Delta d/w$.

In fig. 1 we show the numerical results for the phase speed obtained from the dispersion equation using parameters of w=100 μm f=100 MHz, v_l=1100 ms^{-1}, v_s=5100 ms^{-1}, ρ_l=1000 kgm^{-3} and ρ_s=2655 kgm^{-3}. With the exception of the substrate thickness, these parameters describe high frequency Love waves using a poly(methylmethacrylate) guiding layer on quartz. The substrate thickness in the calculation is thinner than typical substrates used in experiments, but this is solely to enable the acoustic plate modes to be resolved in fig. 1. The points shown in fig. 1 are the start of each mode calculated using the analytical results. Numerically, it is clear that each higher order Love wave mode, labelled by $n=N$, m=0, arises as a continuation of the m=1 acoustic plate mode, labelled by $n=N$-1, m=1, associated with the previous Love wave mode.

The experimental observation of the Love waves and associated acoustic plate modes used propagation orthogonal to the *x*-direction of an ST-cut quartz substrate which supports shear horizontally polarized acoustic waves. The acoustic waves were generated using two sets of interdigital transducers separated by 7 mm to create a delay line device. The transducers were located on the polished side of the 0.5 mm thick quartz substrate. Each transducer had 120 pairs of double fingers with an electrical period of 16 μm and an aperture of 180.5 wavelengths. This arrangement gives a resonant frequency of 312 MHz with a bandwidth, $\Delta f/f_o$, of better than 1% for the transducer. The expected frequency separation between the m^{th} acoustic plate mode and the Love wave of frequency f_o is given by $\Delta f/f_o=(n\lambda/2w)^2/2$ where λ is the electrical period of the transducer. The layered system was created by successively spin coating a polymer (S1813 photoresist from Shipley) at 4000 rpm across the whole device and then softbaking the device at 120 ^{o}C for 90 s on a hotplate. After each step the frequency spectrum of the device was measured (Agilent 8712ET network analyzer) and the resonant frequency of each mode recorded.

The observed change in frequency with polymer guiding layer thickness is shown in fig. 2. Even a thin guiding layer causes a much larger downward shift in the frequency of the Love wave compared to the acoustic plate mode. The guiding layer also initially enhances the transmission of the Love wave, but as the layer thickness increases the mode is increasingly attenuated by the polymer layer until it can no longer be separated from the baseline noise (Fig. 3). The plate mode is also periodically attenuated as the polymer layer thickness increases with the attenuation reaching the baseline just prior to the start of each new plate mode. Further increasing the polymer layer thickness eventually results in the appearance of the next Love wave mode with amplitude similar to that of the uncoated substrate. We have observed five Love wave modes with this type of device.

In conclusion, by developing a generalized dispersion equation for Love waves on a finite thickness substrate we have determined a spectrum that also includes shear horizontally polarized acoustic plate modes. We have demonstrated that a higher order mode of Love wave can be considered to arise from previously existing acoustic plate modes associated with the previous Love wave mode. We have shown experimentally, on a substrate without specific preparation of the back surface, that both multiple Love wave modes and the associated shear horizontally polarized acoustic plate modes can be excited.

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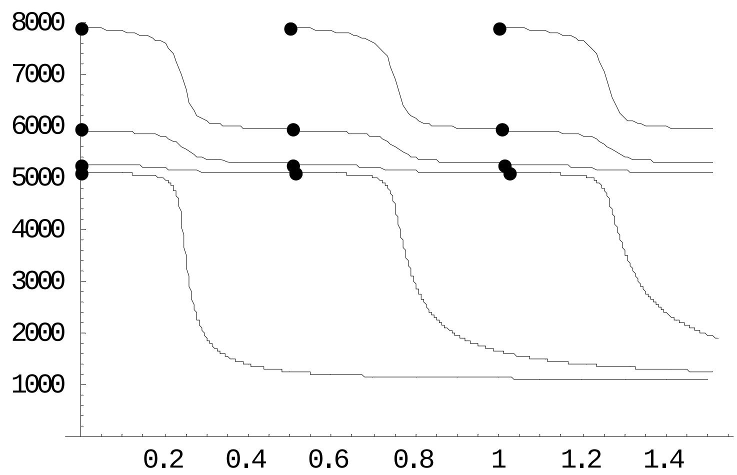
Figure Captions

Figure 1. Theoretical calculated phase speeds showing multiple modes of Love waves and the associated acoustic plate modes.

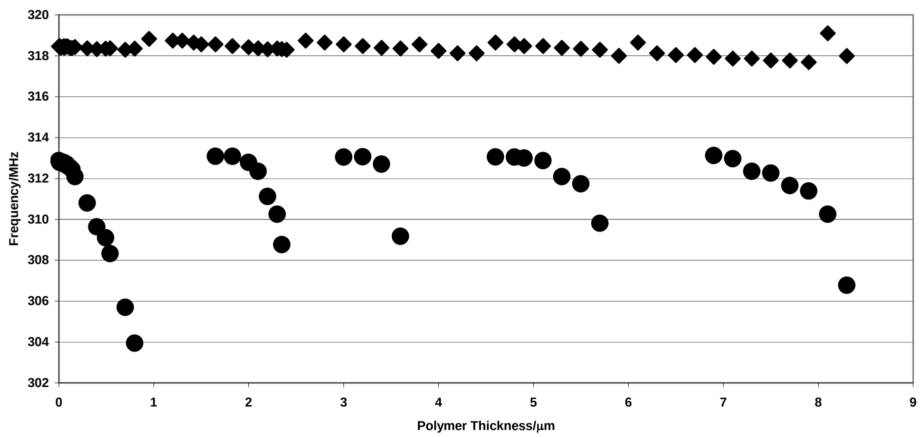
Figure 2. Experimental data for the frequency shift of shear polarized acoustic modes on a device operating at 312 MHz.

Figure 3. Experimental data for the insertion loss of shear polarized acoustic modes on a device operating at 312 MHz.

Figure 1 – Draft version



Draft version of fig. 2. Circles are Love wave and diamonds are the plate mode.



Draft version of fig. 3. Circles are Love wave and diamonds are the plate mode.