From Theory to Automata: A Computational Model of Constructive Alignment

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ABSTRACT

This paper presents a computational model of constructive alignment (Biggs, 1996) that can be conveniently implemented as the main processing framework of a software tool designed to facilitate teaching practitioners to *consistently* and *systematically* produce constructively aligned curricula. The model can provide numerical measures of alignment for both holistic and individual aspects of an educational design. It is realisable on the premise that the desiderata for producing a computational model of constructive alignment are that we: i) adopt a systemic and structural view of teaching and learning; ii) categorise components of a teaching system according to the cognitive ability they elicit from the student; iii) use set theory and linear algebra to express, represent and compute alignment. Using Bloom's taxonomy of cognitive skills (Bloom, 1956) as a basis for measuring the different levels of learning elicited by system components and by borrowing some of the principles of generative linguistics (Chomsky, 1957; Sells, 1985) for generating aligned structures, the model will help teachers to adapt their practices to counter some of the side-effects of governmental agendas such as widening participation and higher student fees.

Keywords: constructive alignment, instructional design, Bloom's taxonomy, automata, alignment systems, learning objectives

INTRODUCTION

Designing learning is considered one of the most fundamental activities of a teaching practitioner and the aim of such a design process is to assist in the development of conscious and purposeful teaching and learning (D'Andrea, 1999). Learning is '*...a cognitive activity that involves the use of intellect for the development and structuring of understanding about oneself and the world in which one lives*' (Wilson, 1980). Teaching is '*...getting most students to use the higher cognitive level processes that the more academic students use spontaneously*' (Biggs, 1999). Although an understanding of such concepts is considered fundamental when constructing teaching and learning strategies, activities and materials (Prosser & Trigwell, 1999), it has been shown that teachers' find it particularly difficult to make the transition from the general learning aims of a programme or module to specific associated learning objectives that encourage students to elicit the appropriate levels of understanding required to meet those aims (Perkins & Blythe, 1993). Such findings implicate teaching practitioners as having misconceptions as to what 'understanding' actually means and naturally this inherently becomes an emergent property of their educational designs. Perkins & Blythe (1993) found that teachers need some framework to help them operationalise what 'understanding' might mean. This finding is disconcerting for students, who naturally rely upon the teacher and the associated educational framework in place to teach them how to effectively achieve the requirements of their chosen programme of study.

Constant shifts within the dynamic of higher education institutions (HEI) to meet government targets and adopt their reforms situates the quality of teaching and learning programmes (and thus educational designs) at the forefront of government and student scrutiny. For example, the government's agenda for widening participation (NCIHE, 1997) encourages universities to admit students with lower academic ability than that usually required for admission onto their degree programmes[1]. Approximately 44% of young people within the United Kingdom (UK) are currently experiencing higher education (HE) (HEFCE, 2003) at a time when only 50% of UK school leavers now have GCSE Mathematics and English to grade C or above (BBC News On-line, 2003). Subsequently, this places significant constraints on the teaching and learning environment. For example, in addition to larger class sizes caused by educational rationalism, teachers must also adapt their practices to accommodate the increased variability in student academic ability. Such pressure is unlikely to subside within the near future as the government aims for 50% participation by 2010. In addition to the forces of widening

[1] Encouraged by way of funding those universities that are seen to be actively encouraging a wider

participation, the government will increase the study costs of UK degree programmes as proposed in 'The Future of Higher Education' document (DfES, 2003). Although controversial, the bill was passed in July 2004 and allows universities to charge up to £3,000 in tuition fees from 2006 thus creating the potential for a UK university market to evolve in an analogous manner to that which currently exists for non-European Union (EU) students. It is envisaged that fee-paying UK students will place considerably more emphasis on educational designs within their selection criteria when pursuing a programme of study i.e. to not only assess the new academic knowledge and understanding potentially offered by a course but to also assess *how* the supplying university can ensure, through its teaching and learning practices and resources, that such intended learning outcomes can be potentially achieved by all students admitted. This is significant, as universities will clearly become more accountable for the correlation between the promotion and delivery of their degree programmes. Failure to ensure a fair and adequate teaching and learning environment can lead to costly litigation processes as evidenced recently (BBC News On-line, 2002).

Constructive alignment (Biggs, 1996) addresses the issues raised by Perkins and Blythe (1993), inter alia, by integrating the main tenets of constructivism (von Glasersfeld, 1996) and instructional design (Cohen, 1987) to form an educational framework that operationalises teaching practice such that the teaching method and assessment are aligned to the learning activities stated in the learning objectives. Given a set of curriculum learning outcomes, it assists the teaching practitioner in making a well-defined transition from generating learning aims to generating compatible learning objectives, teaching and learning activities and assessment tasks that elicit the appropriate cognitive skill from the student for them to attain the curriculum learning outcomes.

Although constructive alignment shows much promise in countering the affects of widening participation by promoting aligned programme and/or module designs, its adoption is optional and in many instances teaching practitioners are unaware of alignment systems per se. The student learning experience is dependent upon individual practitioners and thus subject to variance and misalignment.

This paper presents a computational model and framework for an automaton that assists teaching practitioners to develop and maintain constructively aligned curricula in a *consistent* and *systematic* way. A model that can automate the construction of aligned teaching systems is proposed on the premise that it will facilitate practitioners to adopt such practices and potentially reduce the level of variance in the student learning experience. The remainder of this paper is organised as follows: after briefly reviewing current teaching and learning models, the computational model of constructive

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access to their courses (HEFCE, 2001).

alignment is presented in detail (a worked example of the model is subsequently provided in Appendix B for the interested reader). A discussion then follows outlining the adequacy of the model and suggesting further areas of research. Finally, a conclusion is given summarising the key findings of research and its potential impact on the wider HE community.

Background: Teaching and Learning Models

There are two broad and very different theoretical perspectives of knowledge and effective teaching and learning: *objectivist*, and *constructivist*. The objectivist viewpoint is based on the premise that knowledge exists independently of the 'knower' and teaching is central to any process of knowledge acquisition since teaching is viewed as the medium for knowledge transmission from the knower to the learner (Duffy, 1992). From this perspective, knowledge is decontextualised in order for it to be learned, tested and applied independently of context (Brown et al., 1989). Learning is therefore seen as the practices of receiving, processing, storing and using the transmitted knowledge contextually. In contrast to this, *constructivist* theories (Piaget 1950; Bruner 1960) of teaching and learning reject the *objectivists'* hypothesis that knowledge and knower are independent entities requiring direct instruction (teaching) to transmit knowledge to the student, and argue that the student, rather than the teacher, is responsible for understanding to occur within the student. The fundamental idea behind constructivist theory is based on the notion that initial knowledge structures of the learner are continually being adapted in response to new experiences, actions and knowledge. It is a strong standpoint whereby adaptation of existing knowledge through experience must occur otherwise learning is not considered to have taken place.

Constructivist theories are predominantly providing the framework for contemporary models of student learning, noticeably those by Gibbs (1992) and others (Entwistle, 1995; Savery & Duffy, 1996; Biggs, 1996; Prosser & Trigwell, 1999). The widely accepted theory of experiential learning is also based on constructivist-theoretic motivations (Kolb, 1984; Boud & Walker, 1991; Michelson, 1996) and reflection or reflective practice (Schon, 1983; Boud et al., 1985; Gibbs, 1988). Although it is beyond the scope of this paper to evaluate each learning theory, a review of the literature clearly shows that the most widely-accepted model of experiential learning is the four-stage cyclic model defined by Kolb (1984) motivated by his definition of learning as 'the process whereby knowledge is created through the transformation of experience'.

Biggs (1999) emphasises how effective teaching is partly governed by selecting the appropriate *teaching and learning activities* (TLAs) that maximizes the probability of the student eliciting the appropriate level of cognitive ability required to achieve the desired learning objectives. Educationalists adopting the constructivist paradigm, such as Brown & Atkins (1988), also agree on the notion that effective teaching and learning methods promote *deep learning* whereas poor teaching promotes *surface learning* (Marton & Saljo, 1976). A deep approach to learning is one in which the student intends to gain personal understanding of the learning task (Biggs, 1999). Conversely, a surface approach is adopted when the student's motivation is to avoid failure. Such students tend to memorise information without meaning and organisation (Entwistle & Ramsden, 1983; Ramsden, 1992).

The wide acceptance of the constructivist paradigm of teaching and learning by leading academics is a strong indicator of the efficacy of the approach for designing teaching and learning programmes and subsequently motivates the computational model described in this paper.

Constructive Alignment

Constructive alignment (Biggs, 1996) marries the constructivist viewpoint with instructional design (Cohen, 1987). Instructional design takes an objectivist-theoretic perspective in that the focus remains on what the teacher does to promote student learning. In this case, the teacher defines explicit learning objectives that instruct the students to elicit specific levels of cognitive ability in order to meet associated learning outcomes. Cohen's model aligns the objectives with the assessment tasks to ensure they elicit the same level of cognitive ability from the student. In his model, Biggs (1996) extends instructional design by integrating a constructivist element such that the assessment tasks are also aligned with the teaching and learning activities (i.e., the teaching methods and what the student is actually expected to do to achieve the objectives). Constructive alignment therefore repositions the student to the centre of their learning process. Although this particular form of integration of constructivism and instructional design is not entirely new (for example see Duffy & Johnson, 1992) it is the first educational design that *uses* rather than *identifies* the commonalities between the different strata to improve teaching and learning practice.

According to the main tenet of constructive alignment, efficient student learning is only considered to have been achieved when the learning objectives and assessment tasks are aligned with the TLAs (i.e., what the student actually does). The term '*alignment*' generally refers to a state whereby the affected components of the teaching system reach an equilibrium such that the teaching and learning activities elicit, from the student, the same type of cognitive abilities elicited by the assessment tasks (Biggs, 1996). Constructive alignment offers the teaching practitioner a way to counter the affects of widening participation by enabling the teacher to develop educational designs and environments that purposefully use and make use of the appropriate teaching and learning activities to ensure students of varying academic ability are given the opportunity to elicit the desired level of cognitive skill required to achieve the module's learning objectives and thus outcomes (see Biggs, 2002). As the assessment activities are also aligned with the teaching and learning activities, constructive alignment also promotes fair student assessment.

A COMPUTATIONAL MODEL OF CONSTRUCTIVE ALIGNMENT

When embarking upon this research a fundamental question considered was:

> *Can existing theoretical models of learning and teaching and educational design form the basis of a computational model and automaton engineered to assist the teaching practitioner during the curriculum design, construction and improvement process?*

Subsequently, this provoked further questions as to whether such a model could:

i) provide quantitative measures of module alignment irrespective of subject discipline?;

ii) enable teaching practitioners to design and develop constructively aligned curricula that is fair to all students and enforces inclusivity?;

iii) facilitate teaching practitioners to adapt their practice to improve the alignment of their module designs and variance within their practice?

This section first enumerates the theoretical motivations, which enables the author to address these questions. A computational model of constructive alignment is then presented using set theory to represent component relations and linear algebra to represent and compute alignment.

Theoretical Motivations

It is clear to the author that constructive alignment, through its integration of instructional design and constructivist principles, offers a theoretical and practically proven alignment system (see Biggs, 2002) that can form the basis of a computational system engineered to assist the teacher during curriculum design. It is hypothesised that such a computational system is realisable on the premise that the desiderata for representing and computing alignment are:

- adopt a systemic and structural view of educational design;
- categorise system components according to the level of cognitive ability they elicit from the student;
- apply set theory and linear algebra to express, represent and compute alignment.

The motivations for each of the above important factors will be briefly considered.

A Systemic and Structural Perspective.

Teaching can be thought of as a system and an important characteristic of all systems is the interactions between system components to achieve a common goal or stable state i.e. equilibrium. The author adopts Biggs's (1993) systemic view of teaching and learning within tertiary education. Although, Biggs (1993) identifies several nested micro-systems existing within the tertiary education system, the author focuses on Biggs's *classroom system* which has component parts comprising of students, teachers, and teaching context. Equilibrium occurs within this system when there is a convergence of agreement between the teacher's perceptions of student competences and curriculum needs, setting of tasks, students' perceptions of task demands, teaching and learning processes, and learning outcomes. If a misalignment between these components occurs, e.g. between students' perception of task demands and of teaching processes, then low level outcomes or collaborative student misconceptions could result.

To extend this hypothesis to alignment systems and in particular constructive alignment, the author asserts that Biggs's classroom system inherently embodies constructive alignment in the sense that each align able component of constructive alignment is operated on within and between components of the classroom system. It is an open system that is subject to change in order to adapt its behaviour towards a more stable state and one that represents a more accurate alignment. Such changes would be the result of a modification of learning objective to elicit a different type of cognitive ability or a change in a TLA to better suit one or more learning objectives or assessment tasks. Subsequently, when the classroom system reaches an equilibrium state so too does a system based on constructive alignment. Likewise, a misalignment between components within a constructive alignment system will also lead to disequilibrium.

Alignment systems can also be thought of as being *structural* and *generative*. Before elucidating on this perspective further, it is first important to briefly clarify what the inter-related components of an alignment system are. When designing undergraduate programmes in UK HEIs, the main components of the educational framework are considered to be the *learning aims*, *learning outcomes*, *learning objectives*, *TLAs*, and *assessment tasks (ATs)*. Learning aims, as clarified by Walker (1994), are statements of learning which tend to be generalised. It essentially identifies the learning intentions i.e. what the teacher intends the student to learn[2]. Learning objectives are considered to be teacher-orientated and specify what it is the teacher wants the student to achieve (in terms of levels of understanding of given topics) and underpins the teaching and learning activities they subsequently prescribe (D'Andrea, 1999). TLAs are those teaching methods and techniques that are chosen to get the students to do what the objectives nominate (Biggs, 1999). Although Biggs (2002) does not distinguish between learning outcomes and learning objectives, the author adopts D'Andrea's perspective of learning outcomes as referring to what the students have actually learnt having completed the TLAs. This general view of learning objectives as input specifications and learning outcomes as the outputs or product of the student learning activities are congruent and thus hold amongst other academic viewpoints such as (Otter, 1992; Walker, 1994). Summative ATs refer to those student activities usually prescribed by the teacher to make official judgments vis-à-vis student academic performance on which awards are based (Biggs, 1999).

Considering the above system components further, the author asserts that a computational model of constructive alignment can be realised as a top-down generative system that generates compatible or *aligned* teaching and learning tree structures (both whole and partial structures) in response to each learning outcome. A structural and generative model is motivated in part by the principles of syntactic theory and that of Chomsky's generative linguistic theories (see Chomsky, 1957 and Sells, 1985). As with generative linguistics, which requires a grammar consisting of production rules that when applied describe well-formed syntactic constructions, the generation process requires executable rules based on

[2] Due to the generalised nature of learning aims they are not considered further within the model presented.

the principles of constructive alignment. The model presented in this paper simply requires that the four system components identified can be categorised according to the cognitive ability they elicit and on this basis can make dependency relations across component groups to form structure. Linear algebraic operations can then operate across structures to compute alignment. To understand this structural perspective further, assume that our generative system can only generate three different types of tree structures: a) learning outcome (L^o) trees; b) learning objective (L^b) trees; and c) assessment task (AT) trees. Tree structures have two important properties we must consider, that is *dominance* and *valence*. Dominance refers to the parent/child relationships between nodes (system components) within the tree. For our purposes, a teacher may define a number of learning objectives (or L^bs) for each learning outcome (L^o) thus L^o trees will have learning outcomes (parents) that dominate learning objectives (children). Likewise, since one or more TLAs are employed to stimulate the student to meet a learning objective, L^b trees will subsequently have learning objectives that dominate TLAs. Similarly, an AT may address one or more learning objectives, thus AT trees will have assessment tasks that dominate learning objectives. The three different tree types are shown in Fig 1.

[INSERT FIGURE 1 HERE]

Valence, on the other hand, refers to the number of children each parent can dominate i.e. a parent's power of dominance. For example, a teaching practitioner may define three objectives (L^bs) for outcome 1 (L^o_1) and two for L^o_2. Likewise, to enable a student to achieve L^b_1 to L^b_3, the teacher may ascribe TLA_1 and TLA_2 and for L^b_4 and L^b_5, TLA_3 may be ascribed. This variation effectively causes trees to become imbalanced and this is particularly difficult to model within fixed width vectors and matrices. This problem can be alleviated by balancing the trees via fixing the valence for each tree type. For example, assume a teacher, within their module design, has defined *m* learning outcomes, *n* learning objectives, *p* ATs and *q* TLAs. To balance each of the different tree types we introduce three constants, c_1, c_2 and c_3, whose value determines the valence and thus number of children each parent must dominate. If there are not enough children available then 'filler' elements, referred to as <*empty*> nodes in the context of trees, must be ascribed to make up the required number. The values of these constants need to be empirically established. For our purposes, assume that c_1 corresponds to L^o trees and is fixed at 3, c_2 corresponds to L^b trees and is fixed at 2 and finally, c_3 corresponds to AT trees and is fixed at 4. Fig. 2 shows the three tree types with their fixed valences.

[INSERT FIGURE 2 HERE]

When considered holistically and for an entire module or programme, such a generative system would, given the learning outcomes, generate and coordinate only those L^o trees that dominate aligned L^b nodes that can help, either individually or collectively, the student to meet the associated learning outcome. Subsequently, the learning objectives would generate only the subset of TLAs that collectively elicit the type of student learning required by the learning objective(s). AT structures would then be generated to align and thus dominate one or more L^bs. Clearly there may be more than one path or structure from a given learning outcome to a given set of adequate TLAs. Each different structure can be referred to as a *derivation*. A balanced tree structure generated for a single learning outcome is shown in Fig 3. Note that in Fig. 3, AT trees are pictured as dominating TLA nodes even though they actually dominate L^b nodes. The reason for this is that since ATs dominate L^bs and L^b trees dominate TLAs then AT trees indirectly dominate the TLA nodes associated with the L^b nodes it directly dominates.

[INSERT FIGURE 3 HERE]

Categorising System Components using Bloom's Taxonomy.

Although there is no universally accepted method of aligning elements between the four sets defined above, numerous academics have suggested possible strategies. In particular, Biggs (1999) cites Tang (1991) and Scouller (1998) for utilising verb-matching schemes to determine the level of cognitive ability afforded by an assessment task and provides a comprehensive list of suitable assessment tasks for the different types and levels of learning required by a learning objective (Biggs, 1999). Such verb-matching schemes make allocating each learning outcome and related set of learning objectives an appropriate level of cognitive skill elicited a relatively simple task. The level is obtained by matching the main verb in the outcome or objective with the corresponding entry in Bloom's taxonomy (Bloom, 1956) that contains a matching or synonymous verb. Table I shows each of the six levels representing levels of cognitive ability stimulated by a particular action. Level 6 refers to the *highest* cognitive ability stimulated and level 1 to the *lowest*.

[INSERT TABLE 1 HERE]

It is slightly more difficult, however, to allocate an appropriate level of cognitive skill stimulated by a TLA because there is no such verb defined. Biggs (1999) attempts to bridge this gap by defining and tabling the types of learning elicited by each type of TLA as shown in Appendix A (see table A.2). Biggs's motivation here is to ensure that the selection of a TLA can be governed by a set of learning objectives rather than the TLA governing the objective(s). Consequently, this allows the same verb-matching scheme to be used to associate each TLA with a corresponding entry in Bloom's taxonomy. Biggs also provides similar classification for ATs as shown in Appendix A, table A.1. The level in Bloom's taxonomy assigned for each AT in table A.1 and TLA in table A.2 is based on the author's understanding of Bloom's taxonomy and the information provided in Biggs (1999). The classification schemes for ATs and TLAs presented by Biggs, however, is broad and ambiguous. Biggs acknowledges this by emphasising that such research is unfinished. This is therefore a significant constraint on the computational model presented.

The verb-matching schemes collectively outlined in Biggs (1999) will be used as a basis to cluster the different system components according to the cognitive skill elicited.

Linear Algebra to Represent and Compute Alignment.

The mechanics of linear algebra (Lipschutz, 1997), through its vectors and matrices and associated mathematical operators enables us to numerically represent learning outcomes, objectives, TLAs and ATs and the relationships between them. As discussed in detail in subsequent sections, its operators allow us to perform computations across these structures to yield alignment figures for an entire programme, module or between individual components (e.g., alignment between learning objectives and TLAs). Initially, however, Set theory[3] is used to express the direct and indirect relationships that exist between system components.

Sets for Expressing Component Relations

The four major components of an educational design defined above can be viewed as forming four distinct sets of ordered elements. For example, assume W represents the set of all possible learning outcome declarations for a module, where each declaration, or element w, contains an active verb. The formal declaration for each of the major components is as follows:

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[3] For a concise introduction to Set theory see Lipschutz & Lipson (1997).

W = {w: w is a learning outcome declaration, w contains an active verb}

X = {x: x is a learning objective declaration, x contains an active verb}

Y = {y: y is an assessment task (AT)}

Z = {z: z is a teaching/learning activity (TLA)}

Each of the above sets is considered finite for each programme or module design and thus contains a fixed number of elements. As shown below, the notation *n*(A) (e.g. the number of elements in set A) is used to denote the cardinality of each of the disjoint sets:

n(W) = m i.e. W consists of *m* learning outcomes

n(X) = n i.e. X consists of *n* learning objectives

n(Y) = p i.e. Y consists of *p* ATs

n(Z) = q i.e. Z consists of *q* TLAs

When each element of the four sets, W, X, Y and Z can be associated with a corresponding level in Bloom's taxonomy, a further four sets can be defined to store these levels:

W' = { w': w' is the level in Blooms taxonomy referenced by element w in W 1≤x≤6}

X' = {x': x' is the level in Blooms taxonomy referenced by element x in X 1≤x≤6}

Y' = {y': y' is the level in Blooms taxonomy referenced by element y in Y 1≤x≤6}

Z' = {z': z' is the level in Blooms taxonomy referenced by element z in Z 1≤x≤6}

A number of corresponding relations can then be defined to associate each element of W, X, Y and Z with its corresponding element in W', X', Y' or Z'. For example, suppose R is a relation from W to W' then R is a set of ordered pairs where each first element comes from W and each second element comes from W'. That is, for each pair, w belongs to W (written $w \in W$) and w' belongs to W' (written $w' \in W'$), such that when $(w, w') \in R$ we say that w is R-related to w', written wRw'. Subsequently, the following defines all relationships between the four component sets and their corresponding set representing levels from Bloom's taxonomy:

i) for each pair $w \in W$ and $w^{'} \in W^{'}$, $(w, w^{'}) \in R$ i.e. wRw'

ii) for each pair $x \in X$ and $x^{'} \in X^{'}$, $(x, x^{'}) \in S$ i.e. xSx'

iii) for each pair $y \in Y$ and $y^{'} \in Y^{'}$, $(y, y^{'}) \in T$ i.e. yTy'

iv) for each pair $z \in Z$ and $z^{'} \in Z^{'}$, $(z, z^{'}) \in U$ i.e. zUz'

All relations represent one-to-one mappings between sets and enable us to map a teacher's original textual definitions for each component to a number representing a level in Bloom's taxonomy. Also, the *domain* of a relation is the set of all first elements of the ordered pairs (e.g., w for relation R above) and the *range* of the relation is the set of second elements (e.g., w' for relation R above).

Now that the relationships between the main components of the teaching system and Bloom's taxonomy have been formally established it is now possible to group ordered pairs, across relationship types with respect to the values of w', x', y', and z'. Using the basic principles of set theory this is an easy concept to realise. For example, assume that V refers to a non-empty set containing *all* elements of relations R, S, T and U, that is the 'union' (denoted by the $\cup$ operator) of relations R, S, T and U, written as $V = R \cup S \cup T \cup U$. The number of elements in V is easily determined:

$$n(R \cup S \cup T \cup U) = n(R) + n(S) + n(T) + n(U) = m + n + p + q \,.$$

Partitions of V can be formed based on the type of learning elicited by each element of each relation. Since there are six levels in Bloom's taxonomy, there will be 6 non-overlapping, non-empty subsets. More precisely, a *partition* of V is a collection $\{A_i\}$ of nonempty subsets of V such that:

v) Each *a* in V belongs to one of the A_i.

vi) The sets of $\{A_i\}$ are mutually disjoint; that is, elements in A_i do not occur in A_j (written as $A_i \neq A_j$) thus if we attempted to form a set consisting of only those elements that occur in A_i AND A_j (written as $A_i \cap A_j$) we would have an empty set (written as $A_i \cap A_j = \phi$).

The subsets in a partition are called *cells*. Fig. 4 shows a Venn diagram of a partition of the rectangular set V of points into six cells, A_1, A_2 , A_3 , A_4 , A_5 , and A_6. Clearly, such well defined partitions assume that it is possible to accurately cluster elements of V using some categorisation or matching technique as discussed earlier.

[INSERT FIGURE 4 HERE]

The contents in each cell, A_i , of V would therefore contain *subsets* of relations R, S, T and U where the index *i* refers to the level addressed in the Bloom's taxonomy by the associated learning outcome, learning objective, AT or TLA. Assuming that R_i is a subset of R, such a relationship is formally written as $R_i \subseteq R$. The formal definitions specifying the contents in each cell of A_i are as follows:

vii) $R_i \subseteq R$ and $(w, w') \in R_i$ if and only if (iif) $w' = i$

viii) $S_i \subseteq S$ and $(x, x') \in S_i$ iif $x' = i$

ix) $T_i \subseteq T$ and $(y, y') \in T_i$ iif $y' = i$

x) $U_i \subseteq U$ and $(z, z') \in U_i$ iif $z' = i$

The author asserts that this represents the type of grouping that teaching practitioners should be attempting to perform during the module (or programme) construction process in order to obtain constructively aligned modules.

Vectors and Matrices for Representing and Computing Alignment

The partition of V into six disjoint sets of aligned component elements represents the ideal selections from the teacher's repertoire given a set of learning outcomes. In practice, however, it would be clearly naïve to assume that teachers would naturally select such well-matched learning objectives, ATs and TLAs given the learning outcome(s). A metric of how constructively aligned (or balanced) their selections are would therefore aid them in making alternative, better-suited, selections.

Before defining and computing such a metric using vectors and matrices, a further set of relations needs to be defined to represent the hierarchical relationships that exist between the four major components. Since the learning outcomes are directly related to the learning objectives which are subsequently directly related to both the TLAs and ATs, the alignment between the learning outcomes and the TLAs and also between the learning outcomes and ATs is implicated. The alignment metric proposed here will be based on these relations, which are formally defined as follows:

- The direct relationship between the learning outcomes and learning objectives is defined as:

  for each pair $w' \in W'$ and $x' \in X'$, $(w', x') \in V_1$ i.e. $w'V_1x'$

- The direct relationship between the learning objectives and the TLAs is defined as:

  for each pair $x' \in X'$ and $z' \in Z'$, $(x', z') \in V_2$ i.e. $x'V_2z'$

- The direct relationship between the ATs and learning objectives is defined as:

  for each pair $y' \in Y'$ and $x' \in X'$, $(y', x') \in V_3$ i.e. $y'V_3x'$

- The indirect (transitive) relationship between the learning outcomes and the TLAs is defined as:

  V_1 and V_2 have x' in common which gives rise to the *composition of* V_1 and V_2 written as $V_1 \circ V_2$ and is defined by:

  $$w'(V_1 \circ V_2)z' \text{ if for some } x' \in X' \text{ we have } w'V_1x' \text{ and } x'V_2z'$$

- The transitive relationship between the learning outcomes and the ATs is defined as:

  V_1 and V_3 have x' in common which gives rise to the *composition of* V_1 and V_3 written as $V_1 \circ V_3$ and is defined by:

$$w^{'}(V_1 \circ V_3)y^{'} \text{ if for some } x^{'} \in X^{'} \text{ we have } w^{'}V_1x^{'} \text{ and } x^{'}V_3y^{'}$$

- The transitive relationship between the TLAs and the ATs is defined as:

  V_2 and V_3 have $x^{'}$ in common which gives rise to the *composition of* V_2 and V_3 written as $V_2 \circ V_3$ and is defined by:

$$z^{'}(V_2 \circ V_3)y^{'} \text{ if for some } x^{'} \in X^{'} \text{ we have } x^{'}V_2z^{'} \text{ and } y^{'}V_3x^{'}$$

The vectors and matrices required to compute an alignment metric can now be defined given the above relations. The author asserts that we need only compute an alignment metric for the *direct* relations i.e. individual metrics are computed for V_1, V_2 and V_3. The alignment of the transitive relations is by implication i.e. dependent on the alignment values V_1, V_2 and V_3.

Although Biggs (1993) determines the equilibrium of a classroom system based on one relation, in this paper the author proposes that to determine whether or not a module is constructively aligned we must compute the *degree* to which each of the three direct relations (V_1,V_2 and V_3) have reached their equilibrium. Note that V_1, V_2 and V_3 relations directly corresponds to L^o trees, L^b trees and AT trees respectively.

Assuming that the four major component sets and relations are available for a module, *full* module alignment is calculated as follows:

1. Calculate the equilibrium value for relation V_1.

This is achieved via the 7 following steps:

a. Assume the number of learning outcomes is fixed at *m* and as discussed earlier, we use the constant term c_1 to fix the valence (and thus vector-width) of L^o trees. There must therefore be c_1 learning objectives per learning outcome. As shown in fig. 2 (a), if c_1 objectives are not available for a given outcome then filler elements (i.e. <*empty_nodes*>) must be added to make up the number of dominated elements to c_1. The filler values are set to the level in

Bloom's taxonomy indexed by the associated learning outcome (i.e. a value between 1 and 6 inclusive) to help maintain equilibrium.

b. Let **w** represent the set $W^{'}$ as a row vector such that each element of the vector represents a level in Bloom's taxonomy referenced by a corresponding learning outcome (stored in W).

$$\mathbf{w} = \begin{bmatrix} w_1^{'} & w_2^{'} & \ldots & w_m^{'} \end{bmatrix}^T$$

To recover the actual learning outcome definition we merely retrieve the left-hand side of the corresponding element in the relation R defined in i) above.

c. Let D_1 represent a matrix consisting of c_1 rows and m columns, where each column corresponds to the set of suitable or 'desired' c_1 learning objectives (including filler elements) for a specific learning outcome. The crude assumption made for desired elements is that given a learning outcome i the set of associated c_2 learning objectives should elicit the same cognitive ability from the student as the learning outcome. Since this assumption is made for all learning outcomes, the resulting target value is the summation of all such products. The author accepts that a semi-linear relationship would be more realistic whereby the learning objectives increase in complexity up to the level of the associated outcome and this is discussed later. Each value in D_1 refers to a level in Bloom's taxonomy (i.e., 1 to 6 inclusive) equal to that of the corresponding learning outcome. Matrix D_1 is defined as:

$$D_1 = \begin{bmatrix} d1_{11} & d1_{12} & \ldots & d1_{1i} & \ldots & d1_{1m} \\ d1_{21} & d1_{22} & \ldots & d1_{2i} & \ldots & d1_{2m} \\ & & & & & \\ d1_{j1} & d1_{j2} & \ldots & d1_{ji} & \ldots & d1_{jm} \\ & & & & & \\ d1_{c_1 1} & d1_{c_1 2} & \ldots & d1_{c_1 i} & \ldots & d1_{c_1 m} \end{bmatrix}$$

Let $\mathbf{d1}_i$ represent a column vector from matrix D_1 such that we refer to the set of c_1 'desired' learning objectives associated with learning outcome i as transposed and defined below:

$$\mathbf{d1}_i = \begin{bmatrix} d1_{i1} & d1_{i2} & \dots & d1_{ic_1} \end{bmatrix}^T$$

d. Let X_1 represent a matrix consisting of c_1 rows and m columns, where each column corresponds to the set of c_1 'actual' learning objectives defined (explicitly or implicitly) by the teacher for a specific learning outcome (including filler elements). Since each non-filler value in X_1 refers to some x' in X' it is the actual level in Bloom's taxonomy referenced by the associated learning objective that is stored in the matrix.

$$X_1 = \begin{bmatrix} x'_{11} & x'_{12} & \dots & x'_{1i} & \dots & x'_{1m} \\ x'_{21} & x'_{22} & \dots & x'_{2i} & \dots & x'_{2m} \\ \\ x'_{j1} & x'_{j2} & \dots & x'_{ji} & \dots & x'_{jm} \\ \\ x'_{c_1 1} & x'_{c_1 2} & \dots & x'_{c_1 i} & \dots & x'_{c_1 m} \end{bmatrix}$$

Also, let $\mathbf{x1}_i$ represent a column vector from matrix X_1 such that we refer to the set of c_1 teacher-defined learning objectives associated with learning outcome i as defined below:

$$x1_i = \begin{bmatrix} x'_{i1} & x'_{i2} & \dots & x'_{ic_1} \end{bmatrix}^T$$

e. Calculate the alignment values between the learning outcomes and learning objectives as follows:

- Calculate the *desired alignment* value using the *inner dot product* between each learning outcome, w'_i, and its corresponding set of c_1 desired learning objectives stored in matrix D_1:

$$t1_i = w'_i d1_{1i} + w'_i d1_{2i} + \dots w'_i d1_{ji} \dots + w'_i d1_{c_1 i} = \sum_{j=1}^{c_1} w'_i d1_{ji}$$

where i = 1..m.

Since we are making the naïve assumption that each desired objective elicits the same level as the associated outcome then $t1_i = c_1 * w_i^{'2}$

- Compute the *actual alignment* value, $\mathbf{u1}_i$, using the inner dot product between each learning outcome, $w_i^{'}$, and its corresponding set of c_1 actual learning objectives stored in matrix X_1:

$$u1_i = \sum_{j=1}^{c_1} w_i^{'} x1_{ji}^{'}$$

where i = 1..m.

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual learning outcome. Let **e1** refer to the vector of misalignment between the learning outcomes and learning objectives, where the misalignment value for learning outcome i is defined as:

$$e1_i = u1_i - t1_i$$

where i = 1..m.

The absolute values of **e1**, denoted $\left|\mathbf{e1}_i\right|$, are used to compute alignment and ignores the arithmetic sign of alignment values in favour of magnitude. This allows us to measure according to the magnitude of alignment and is illustrated further using the following piece of Structured English (SE)(Lejk and Deeks, 2002):

For each learning outcome *i* (i=1..m)

Do

If $|\mathbf{e1}_i| <= \tau$

Then If one or more $x'_{ji} = w'_i$ (for each *j*)

Then the learning objectives are *aligned* with learning outcome *i*

Else If $|\mathbf{e1}_i| > \tau$ AND $\mathbf{e1}_i > 0$

Then If one or more $x'_{ji} = w'_i$ (for each *j*)

Then the learning objectives are *positively misaligned* with learning outcome *i*

Else

The learning objectives are *negatively misaligned* with learning outcome *i*

Where τ is a threshold value defined *a priori* and determines the level of acceptable error. The author uses the term '*positively misaligned*' to refer to the situation where a teacher has prescribed learning objectives that elicit cognitive abilities from the student that collectively exceeds that required by the associated module learning outcome. It is considered positive in that the student will still be able to meet the learning outcomes if the learning objectives are achieved. For either actual alignment or *positive misalignment* to occur, at least one learning objective must elicit the same level of cognitive ability from the student as required by the associated learning outcome.

Conversely, the term '*negatively misaligned*' refers to a state where the teacher has defined learning objectives that elicit lower cognitive abilities from the student than that defined in the learning outcome. Clearly, it is referred to as negative as even if the students achieve all of the learning objectives the learning outcome itself is still unobtainable.

g. Finally, calculate the V_1 equilibrium value to measure the *overall* alignment between the learning outcomes and the actual learning objectives assigned to them. This is obtained by calculating the root mean squared error (RMSE) across all elements of **e1**. The alignment errors for each learning outcome are squared to maintain the magnitude of each misalignment value irrespective of sign and obviously to avoid negative error values from cancelling out positive error values. The average error is then computed. Finally, the squared root of the resulting value provides an alignment value representative of the different misalignment errors stored in **e1**. This calculation is expressed as follows:

$$V_1\_equilibrium = \sqrt{\frac{1}{m}\sum_{i=1}^{m} e1_i^{\,2}}$$

2. Calculate the equilibrium value for relation V_2.

The same 7-step process described for V_1 is used and can be summarised for V_2 as follows:

a. As discussed earlier, we use the constant term c_2 to fix the valence of L^b trees. There must therefore be c_2 TLAs for each of the *n* learning objectives defined. As shown in fig. 2 (b), if c_2 TLAs are not available for a given objective then filler elements must again be added to make up the number of dominated elements to c_2. The filler values are set to the level in Bloom's taxonomy indexed by the associated learning outcome (i.e. a value between 1 and 6 inclusive).

b. Let **x2** represent the vector of all *n* teacher-defined learning objectives (no filler elements), where all elements within the vector represent a level in Bloom's taxonomy and thus a value between 1 and 6.

$$x2 = \begin{bmatrix} x2_1 & x2_2 & \dots & x2_n \end{bmatrix}^T$$

To recover the actual learning objective definition, we retrieve the left-hand side of the corresponding element in the relation S, defined in ii) above.

c. Let D_2 represent a matrix consisting of c_2 rows and *n* columns, where each column corresponds to the set of 'desired' c_2 TLAs for each learning objective. Matrix D_2 is defined as:

$$D_2 = \begin{bmatrix} d2_{11} & d2_{12} & \dots & d2_{1j} & \dots & d2_{1n} \\ d2_{21} & d2_{22} & \dots & d2_{2j} & \dots & d2_{2n} \\ \\ d2_{k1} & d2_{k2} & \dots & d2_{kj} & \dots & d2_{kn} \\ \\ d2_{c_2 1} & d2_{c_2 2} & \dots & d2_{c_2 j} & \dots & d2_{c_2 n} \end{bmatrix}$$

Let $\mathbf{d2}_j$ represent a column vector from matrix D_2 such that we refer to the set of c_2 'desired' *TLAs* associated with learning objective *j* as transposed and defined below:

$$\mathbf{d}2_j = \left[d2_{j1} \quad d2_{j2} \quad \dots \quad d2_{jc_2} \right]^T$$

d. Let Z_1 represent a matrix consisting of c_2 rows and *n* columns, where each row corresponds to the actual set of c_2 TLAs (including filler elements) used to help students achieve a specific learning objective. Z_1 is defined as:

$$Z_1 = \begin{bmatrix} z'_{11} & z'_{12} & \dots & z'_{1j} & \dots & z'_{1n} \\ z'_{21} & z'_{22} & \dots & z'_{2j} & \dots & z'_{2n} \\ \\ z'_{k1} & z'_{k2} & \dots & z'_{kj} & \dots & z'_{kn} \\ \\ z'_{c_2 1} & z'_{c_2 2} & \dots & z'_{c_2 j} & \dots & z'_{c_2 n} \end{bmatrix}$$

e. Calculate the V_2 alignment values as follows:

- Calculate the desired alignment value, $\mathbf{t2}_j$, using the inner product between each learning objective, $x2_j$, and its corresponding set of *q* desired TLAs stored in matrix D_2:

$$t2_j = x2_j d2_{1j} + x2_j d2_{2j} + \dots x2_j d2_{kj} \dots + x2_j d2_{c_2 j} = \sum_{k=1}^{c_2} x2_j d2_{kj}$$

where j = 1..n.

Since we are making the naïve assumption that each desired TLA elicits the same level of ability as the associated objective then $t2_j = c_2 * x2_j^2$

- Compute the actual alignment value, **u2**$_j$, using the inner dot product between each learning objective, $x2_j^{'}$, and its corresponding set of c_2 TLAs stored in matrix Z_1:

$$u2_j = \sum_{k=1}^{c_2} x2_j z_{kj}^{'}$$

where j = 1..n.

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual learning objective:

$$e2_j = u2_j - t2_j$$

where j = 1..n.

As before, the absolute values of vector **e2** values indicate the degree to which each *element* of V_2 is aligned as shown in the following logic below:

For each learning objective *j* (j=1..n)

Do

If $|\mathbf{e}2_j| <= \tau$

Then If one or more $z^{'}_{kj} = x2_j$ (for each *k*)

Then the TLAs are *aligned* with learning objective *j*

Else If $|\mathbf{e}2_j| > \tau$ and **e2**$_j$ >0

Then If one or more $z^{'}_{kj} = x2_j$ (for each *k*)

Then the TLAs are *positively misaligned* with learning objective *j*

Else

The TLAs are *negatively misaligned* with learning objective *j*

g. Calculate the V_2 equilibrium value as follows to measure the *overall* alignment between the learning objectives and TLAs:

$$V_2\_equilibrium = \sqrt{\frac{1}{n}\sum_{j=1}^{n} e2_j^{\,2}}$$

3. Calculate the equilibrium value for relation V_3.

The same 7-step process as above is used and can be summarised for V_3 as follows:

a. As discussed earlier, we use the constant term c_3 to fix the valence of AT trees. There must therefore be c_3 learning objectives for each of the *p* ATs used. As shown in fig. 2 (c), if c_3 objectives are not available for a given AT then filler elements must again be added to make up the number of dominated elements to c_3. The filler values are set to the level in Bloom's taxonomy indexed by the associated learning outcome (i.e. a value between 1 and 6 inclusive).

b. Let **y** represent the set Y' as a row vector such that each element of the vector represents a level in Bloom's taxonomy referenced by a corresponding AT (stored in Y).

$$y = \begin{bmatrix} y'_1 & y'_2 & \dots & y'_p \end{bmatrix}^T$$

c. Let D_3 represent a matrix consisting of c_3 rows and *p* columns, where each column corresponds to the set of suitable or 'desired' c_3 learning objectives (including filler elements) assessed by a specific AT. Again, the crude assumption made for desired elements is that the set of associated c_3 learning objectives should elicit the same cognitive ability from the student as the AT. Each value in D_3 refers to a level in Bloom's taxonomy equal to that of the corresponding AT. Matrix D_3 is defined as:

$$D_3 = \begin{bmatrix} d3_{11} & d3_{12} & \dots & d3_{1l} & \dots & d3_{1p} \\ d3_{21} & d3_{22} & \dots & d3_{2l} & \dots & d3_{2p} \\ \\ d3_{j1} & d3_{j2} & \dots & d3_{jl} & \dots & d3_{jp} \\ \\ d3_{c_3 1} & d3_{c_3 2} & \dots & d3_{c_3 l} & \dots & d3_{c_3 p} \end{bmatrix}$$

Let $\mathbf{d3}_l$ represent a column vector from matrix D_3 such that we refer to the set of c_3 'desired' learning objectives associated with learning outcome l as transposed and defined below:

$$\mathbf{d}3_l = \begin{bmatrix} d3_{l1} & d3_{l2} & \dots & d3_{lc_3} \end{bmatrix}^T$$

d. Let X_2 represent a matrix[4] consisting of c_3 rows and p columns, where each column corresponds to the set of c_3 'actual' learning objectives (including filler elements) assessed by the teacher using a specific AT. Since each non-filler element in X_2 refers to some x' in X' it is the actual level in Bloom's taxonomy referenced by the associated learning objective that is stored in the matrix.

$$X_2 = \begin{bmatrix} x'_{11} & x'_{12} & \dots & x'_{1l} & \dots & x'_{1p} \\ x'_{21} & x'_{22} & \dots & x'_{2l} & \dots & x'_{2p} \\ \\ x'_{j1} & x'_{j2} & \dots & x'_{jl} & \dots & x'_{jp} \\ \\ x'_{c_3 1} & x'_{c_3 2} & \dots & x'_{c_3 l} & \dots & x'_{c_3 p} \end{bmatrix}$$

Also, let $\mathbf{x3}_l$ represent a column vector from matrix X_2 such that we refer to the set of c_3 teacher-defined learning objectives assessed by AT l as defined below:

$$x3_l = \begin{bmatrix} x'_{l1} & x'_{l2} & \dots & x'_{lc_3} \end{bmatrix}^T$$

[4] Note that this refers to a matrix and not a vector and is thus very different to **x2** used to reference a

e. Calculate the alignment values between an AT and its associated learning objectives as follows:

- Calculate the desired alignment value using the inner dot product between each AT, $y_l^{'}$, and its corresponding set of c_3 desired learning objectives stored in matrix D_3:

$$t3_l = y_l^{'} d3_{1l} + y_l^{'} d3_{2l} + \ldots y_l^{'} d3_{jl} \ldots + y_i^{'} d3_{c_3 l} = \sum_{j=1}^{c_3} y_l^{'} d3_{jl}$$

where l = 1..p.

Since we are making the naïve assumption that each desired objective elicits the same level as the associated outcome then $t3_l = c_3 * y_l^{'2}$

- Compute the *actual alignment* value, $\mathbf{u3}_l$, using the inner dot product between each AT, $y_l^{'}$, and its corresponding set of c_3 actual learning objectives stored in matrix X_2:

$$u3_l = \sum_{j=1}^{c_3} y_l^{'} x_{jl}^{'}$$

where l = 1..p.

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual AT. Let **e3** refer to the vector of misalignment between ATs and learning objectives, where the misalignment value for AT l is defined as:

$$e3_l = u3_l - t3_l$$

where l = 1..p.

As before, the absolute values of vector **e3** elements indicate the degree to which each *element* of V_3 is aligned as shown in the following logic below:

---

previous vector.

For each AT$_l$ (l =1..p)

Do

If $|\mathbf{e}3_l| <= \tau$

Then If one or more $x'_{jl} = y'_l$ (for each j)

Then the learning objectives are *aligned* with AT_l

Else If $|\mathbf{e}3_l| > \tau$ AND $\mathbf{e3}_l > 0$

Then If $x'_{jl} = y'_l$ (for each j)

Then the learning objectives are *positively misaligned* with AT_l

Else

The learning objectives are *negatively misaligned* with AT_l

g. Calculate the V_3 equilibrium value as follows to measure the *overall* alignment between the learning objectives and ATs:

$$V_3\_equilibrium = \sqrt{\frac{1}{p}\sum_{l=1}^{p} e3_l^{\,2}}$$

4. Calculate the overall equilibrium value.

The equilibrium value consolidating all direct relations and representing constructive alignment for the whole module design is simply:

$$\frac{V_1\_equilibrium + V_2\_equilibrium + V_3\_equilibrium}{3}$$

If each element of $e1_i$, $e2_j$ and $e3_l$ has been classified as either ‘aligned’ or ‘positively misaligned’ then it could be broadly stated that the module as a whole is fully *constructively aligned* otherwise there is some misalignment between the four components of the teaching system. Clearly, in order to determine the cause of any misalignment then the result of the inner dot products for the individual relations (V_1,V_2 and V_3) needs to be examined in order to trace mismatching elements. A worked example is provided in Appendix B for the interested reader.

DISCUSSION

The model presented meets its stated aims in that it:

i. operationalises good teaching practice within a computational framework which can subsequently be implemented as a software tool;
ii. provides a quantitative measure of alignment between individual system components and of full constructive alignment for an entire module;
iii. potentially facilitates teaching practitioners to adapt their practice to better align their modules by making them aware of alignments and misalignments within their educational designs.

Also, it is possible to extend the applicability of the model by enriching information stored within nodes of L^o, L^b, and AT trees to help practitioners develop fair educational designs that enforce inclusivity (e.g. to support students with disabilities such as dyslexia). Since alignment is verb-based we can exploit the powerful features of syntactic theory to generate and enforce well-defined alignment structures. For example, a common phenomenon reported in linguistics is that of *verb subcategorisation* (Sells, 1985) whereby different types of verbs require or 'subcategorise for' different patterns of arguments such as prepositional phrases and object noun phrases. A transitive verb, such as 'slap', requires an object noun phrase to refer to the agent, which is acted upon by the subject e.g. *Jill slapped Jack*. Further, transitive verbs such as 'put' require an additional prepositional phrase to indicate position of the object noun e.g. *Jill put the bucket down*. Intransitive verbs such as 'sleep' and 'run', on the other hand, do not require object noun phrases e.g., *Jill slept.* The computational model proposed can utilise such principles to enforce selectional restrictions based on learning elicited and environmental constraints. For example, a particular learning objective utilising a verb such as 'apply' will restrict the type of TLAs and ATs that can be used to one or more that elicits or assesses (individually or collectively) the cognitive skill of *application*. Environmental restrictions, such as available rooms, resource or student disability, may further reduce the types of allowable TLAs and ATs.

Although the model and its' potential impact on the teaching and learning community is promising, the model presented is not a panacea for implementing good teaching practice via enforcing constructively aligned educational designs. The model could be considered idealistic in its current form. For the model to be realisable within a realistic context, the author identifies four areas requiring further research, these are: a) adequacy of Bloom's taxonomy for categorising system components; b) establishing 'desired' objectives, TLAs and ATs; c) acceptable values for the alignment threshold, τ and finally d) usefulness of alignment metric as tree complexity increases.

This implies that further research is required before the model can be practically implemented. It also assumes that teachers' must know a priori what the main components of an educational design are and how they relate. It is envisaged, however, that a complete software implementation of the model will aid the practitioner in this respect whilst abstracting them away from the actual alignment computations.

CONCLUSIONS

Biggs (1996) states that any discussions about good teaching should include that of alignment models. Biggs integrates instructional design with constructivist principles to produce a framework, referred to as constructive alignment that systematically operationalises the important characteristics of a good teaching practitioner, which are to:

- be able to define what the teacher wants the student to learn and achieve (*learning objectives*);
- be able to define what students have to do to demonstrate they have learned the objectives to the required level (*assessment tasks*);
- be aware of the different cognitive skills each of the teaching and learning activities elicit from the student and be able to instantiate them according to the learning objectives defined (*student-centred teaching and learning activities*)

The computational model of constructive alignment presented in this paper utilises vectorial representations and computations to provide numerical measures of alignment for both holistic and individual aspects of an educational design. A structural and generative perspective of alignment systems is adopted to enable relationships across the above desiderata for good teaching to be represented and manipulated in vectorial form. Crucially, the computation of the alignment metrics is dependent upon three important factors: i) the ability to accurately cluster outcomes, objectives, ATs and TLAs according to the level of cognitive ability they elicit or assess; ii) a priori definitions of acceptable prototypes of perfect or 'desired' alignment values from which to 'benchmark' against and iii) defining realistic alignment threshold values. Although, further research is required on all three counts, the model is a significant step towards the realisation of a software tool to facilitate teachers to systematically and consistently produce *and* manage constructively aligned programmes of teaching and learning.

It is envisaged that the model will have a positive impact on HE across the UK sector as it allows the quality of educational designs to be measured and it is applicable to all subject disciplines since it works on the principle of 'practice techniques' and 'learning elicited' as opposed to content.

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FIG. 1. a) L^o tree showing relationships between outcomes and objectives; b) L^b tree showing relationships between objectives and TLAs and c) AT tree showing relationships between ATs and objectives.

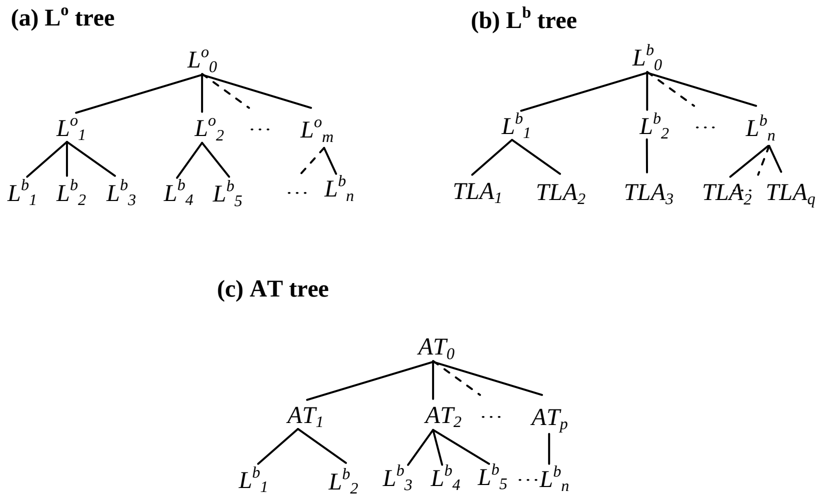

(a) L^o tree
L^o_0
L^o_1 L^o_2 $\cdots$ L^o_m
L^b_1 L^b_2 L^b_3 L^b_4 L^b_5 $\cdots$ L^b_n
(b) L^b tree
L^b_0
L^b_1 L^b_2 $\cdots$ L^b_n
TLA_1 TLA_2 TLA_3 TLA_2 TLA_q
(c) AT tree
AT_0
AT_1 AT_2 $\cdots$ AT_p
L^b_1 L^b_2 L^b_3 L^b_4 L^b_5 $\cdots$ L^b_n

FIG. 2. a) Balanced L^o tree with a fixed valence of 3; b) Balanced L^b tree with a fixed valence of 2 and c) Balanced AT tree with a fixed valence of 4.

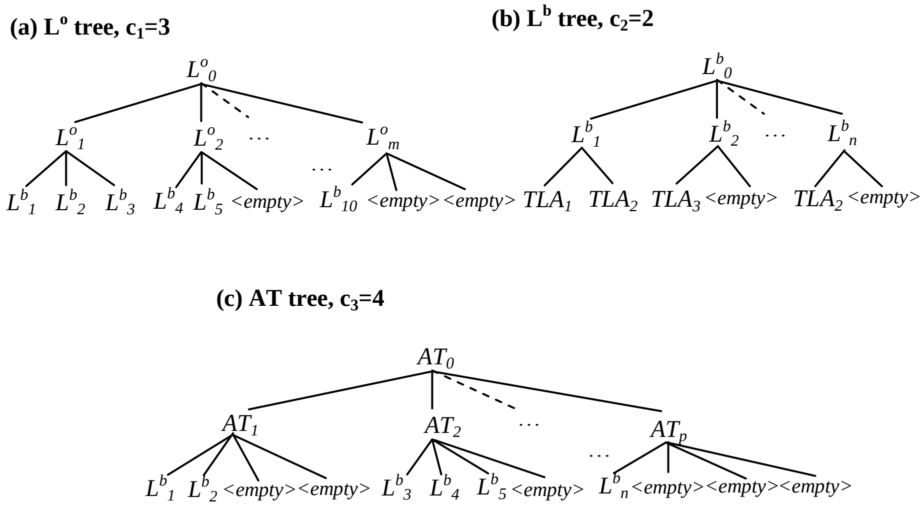

(a) L° tree, c1=3
L°0
L°1 L°2 … L°m
Lb1 Lb2 Lb3 Lb4 Lb5 <empty> … Lb10 <empty> <empty>
(b) Lb tree, c2=2
Lb0
Lb1 Lb2 … Lbn
TLA1 TLA2 TLA3 <empty> TLA2 <empty>
(c) AT tree, c3=4
AT0
AT1 AT2 … ATp
Lb1 Lb2 <empty><empty> Lb3 Lb4 Lb5 <empty> … Lbn <empty><empty><empty>

FIG. 3. A balanced tree structure showing relationships between system components for a single learning outcome.

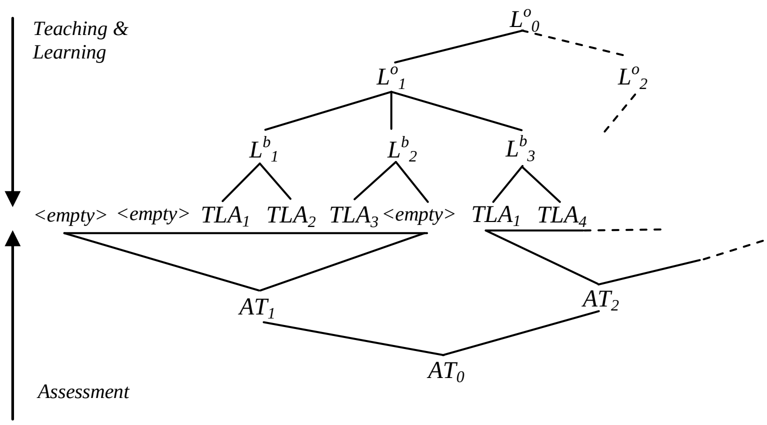
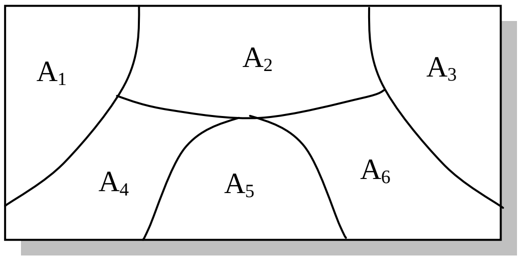

Teaching & Learning
L^o_0
L^o_1
L^o_2
L^b_1
L^b_2
L^b_3
<empty> <empty> TLA_1 TLA_2 TLA_3 <empty> TLA_1 TLA_4
AT_1
AT_2
AT_0
Assessment

TABLE 1

Bloom's taxonomy of learning objectives containing six levels of learning stimulated as described by Bloom (1956). Adapted from D'Andrea (1999) and Brown et al., (1997).

Level	Cognitive Ability Stimulated	Action Elicited	Suitable Action Verbs
6	Evaluation	Ability to make a judgment of the worth of something.	judge, appraise, evaluate, compare, assess
5	Synthesis	Ability to combine separate parts into a whole.	design, organise, formulate, propose
4	Analysis	Ability to divide a problem into its constituent parts and establish the relationship between each one.	distinguish, analyse, calculate, test, inspect
3	Application	Ability to apply rephrased knowledge to novel situations.	apply, use, demonstrate, illustrate, practice
2	Comprehension	Ability to rephrase knowledge.	describe, explain, discuss, recognise
1	Knowledge	That which can be recalled.	define, list, name, recall, record

FIG. 4 Venn diagram showing partition of V into six disjoint sets containing subsets of relations corresponding to different levels of learning elicited according to Bloom's taxonomy (Bloom, 1956).



$$A_i = R_i \cap S_i \cap T_i \cap U_i$$

APPENDIX A

The level in Bloom's taxonomy assigned for each AT in table A.1 and TLA in table A.2 is based on the information provided in Biggs (1999) and the author's understanding of Bloom's taxonomy. It is not a precise grouping and as Biggs (1999) noted, research into such groupings is so far incomplete and much work still needs to be done.

TABLE A.1
Assessment tasks and the types of learning assessed by those tasks (adapted from Biggs, 1999). Assessment task coding scheme (AT^{type}_n) and index to Bloom's taxonomy added.

Assessment type and task	**Type of learning assessed**	**Bloom's tax. (1-6)**
1. Extended prose, essay-type (AT^e)		
essay exam (AT^e_1)	rote, question spotting, speed structuring	5
open book (At^e_2)	as above but less memory and greater coverage	2
assignment, take-home (AT^e_3)	read widely, interrelate, organise, apply, copy	5
2. Objective test (AT^o)		
multiple-choice (AT^o_1)	recognition, strategy, comprehension, coverage	2
ordered outcome (AT^o_2)	hierarchies of understanding	3
3. Performance assessment (AT^p)		
practicum (AT^p_1)	skills needed in real life, procedural knowledge	4
seminar, presentation (AT^p_2)	communication skills	3
posters (AT^p_3)	concentrating on relevance, application	3
interviewing (AT^p_4)	responding interactively, recall, application	3
critical incidents (AT^p_5)	reflection, application, sense of relevance	6
project (AT^p_6)	application, research, problem solving	4
reflective journal (AT^p_7)	reflection, application, sense of relevance	6
case study, problems (AT^p_8)	application, professional skills	3
portfolio (AT^p_9)	reflection, creativity, unintended outcomes	6
4. Rapid ATs (large class) (AT^r)		
concept maps (AT^r_1)	coverage, relationships, some holistic understanding	5
Venn diagrams (AT^r_2)	Relationships	2
three-minute essay (AT^r_3)	level of understanding, sense of relevance	3
gobbets (AT^r_4)	realising importance of significant detail, some multistructural understanding across topics	2
short answer (AT^r_5)	recall units of information, coverage	2
letter to a friend (AT^r_6)	holistic understanding, application, reflection	3
cloze (AT^r_7)	Comprehension of main ideas	2

TABLE A.2

Teaching and learning activities and the types of learning they elicit. Adapted from Biggs (1999). TLA coding scheme (TLA^{type}_{n}) and index to Bloom's taxonomy added.

T LA	**A form of learning**	**Bloom's tax. (1-6)**
1. Teacher-controlled (TLA^{t})		
lecture, set texts (TLA^{t}_{1})	reception of selected content	2
think aloud (TLA^{t}_{2})	demonstrate conceptual skills	3
questioning (TLA^{t}_{3})	clarifying, seeking error	4
advance organizer (TLA^{t}_{4})	structuring, preview	5
concept mapping (TLA^{t}_{5})	structuring, overview	5
tutorial (TLA^{t}_{6})	elaboration, clarification	2
laboratory (TLA^{t}_{7})	procedures, application	4
excursion (TLA^{t}_{8})	experiential knowledge, interest	2
seminar (TLA^{t}_{9})	clarify, presentation skill	3
2. Peer-controlled (TLA^{p})		
various groups (TLA^{p}_{1})	elaboration, problem-solving, metacognition	4
learning partners (TLA^{p}_{2})	resolve differences, application	3
peer teaching (TLA^{p}_{3})	depends whether teacher or taught	3?
spontaneous collaboration (TLA^{p}_{4})	breadth, self-insight	2?
3. Self-controlled (TLA^{s})		
generic study skills (TLA^{s}_{1})	basic self-management	6?
content study skills (TLA^{s}_{2})	information handling	6?
metacognitive learning skills (TLA^{s}_{3})	independence and self-monitoring	6?

APPENDIX B

The worked example is for one of the author's undergraduate computing modules, Introduction to Information Systems (IIS). IIS accounts for 10 credit points of a degree programme and is run in the first Semester. It is a compulsory level 1 module of all Computing degree programmes administered by the author's School. The assessed learning outcomes for IIS are shown in table B.1 and have been formally prescribed by the school's management team. Each outcome is linked to an associated level in Bloom's taxonomy based on the main active verb and is shown in parentheses.

TABLE B.1
Module learning outcomes.

Introduction to Information Systems Module : Assessed Learning Outcomes		
L^o_1	Explain the role and skills of the Systems Analyst.	(2)
L^o_2	Explain Systems Development Lifecycles and Methodologies.	(2)
L^o_3	Formulate a set of balanced data flow diagrams (DFDs) for a simple information system.	(5)
L^o_4	Specify the processing logic for simple DFD processes using a logic modelling technique.	(5)
L^o_5	Formulate a normalised data model (to third normal form) showing entities, entity attributes, entity relationships and data dictionary entries for a simple information system.	(5)

The IIS module framework is summarised as follows:

Class: 250 first-year undergraduate computing students.

Teaching structure (per week): one plenary lecture, one tutorial of 10 groups of 25 students - all classes are thus evidently large. There are eight major topics introduced and variously elaborated in the lectures and tutorials over the 12-week semester. A reading schedule is given and students are expected to produce questions to be answered during lectures and tutorials. Each lecture and corresponding tutorial explicitly has a set of teacher-defined learning objectives which the students must achieve to meet the associated learning outcome.

Staff: One senior lecturer, who is the module leader and responsible for: creating all teaching and learning materials, delivering lectures, taking some tutorials, managing assessment marking, moderation and reporting of results to administration; three teaching assistants who between them take the remaining tutorials and help with assessment.

Summative Assessment: coursework worth 30% of the module and consists of an individual MC test and 3 group take-home assignments to be worked on between tutorials (students are informed a priori to encourage preparation); 2 hour individual examination worth 70% of the module which addresses the learning objectives associated with the group coursework.

For reasons of brevity, the worked example will compute alignment for a single learning outcome, *learning outcome 3* or L^o_3. This will be sufficient to show how the computational model works with a real module specification. It is envisaged that the reader will then find it intuitive to extend the example to an entire module given the repetitious nature of the computations.

Let us define the appropriate sets for L^o_3 as follows:

W = {"L^o_3 – *Formulate a set of balanced Data Flow Diagrams (DFDs) for a simple information system*"}

n(W) = m = 1

$W' = \{5\}$

Let us define the associated set of learning objectives as:

X = {"L^b_1 – *Define systems modelling and differentiate between logical and physical system models*",
"L^b_2 – *Define process models and describe its benefits*",
"L^b_3 – *Demonstrate an understanding of the basic concepts and constructs of a DFD*",

"L^b_4 – *Explain the differences among four types of DFDs: current physical, current logical, new physical and new logical*",
"L^b_5 – *Formulating level-0 (context) and level 1 DFDs*",
"L^b_6 – *Decompose DFDs into lower-level diagrams (DFD levelling)*",
"L^b_7 – *Demonstrate an understanding of DFD balancing*"
}

$n(X) = n = 7$

$X' = \{1, 1, 3, 2, 5, 4, 3\}$

Since we are computing alignment for one outcome, L^o_3, then L^o_3 simply "dominates" all of the objectives in X' (i.e. $w'V_1x'$) and represents the L^o_3 tree.

Assume that Z is the set of all TLAs used to encourage students to achieve the objectives in X:

$Z = \{TLA^t_1, TLA^t_2, TLA^t_3, TLA^t_4, TLA^t_6, TLA^t_9, TLA^p_1, TLA^p_2\}$

$n(Z) = q = 8$

$Z' = \{2, 3, 4, 5, 2, 3, 4, 3\}$

(Note that the TLA coding scheme from table A.2 is used for clarity).

Since an objective may be associated with multiple TLAs, the direct relationship between X' and Z' (i.e. $x'V_2z'$) requires us to define V_2 as being a *class* of sets whereby each element of V_2 refers to a subclass of Z'. Each element in X represents a parent node and the corresponding element in V_2 represents the children nodes therefore forming L^b trees. For clarity, the actual TLA code is used to express association (z) rather than the level of learning elicited (z') as different TLAs can elicit the same level of learning. Each element of V_2 is therefore:

$$V_2 = \begin{bmatrix} \{TLA^t_1, TLA^t_2, TLA^t_3, TLA^t_6\}, \\ \{TLA^t_1, TLA^t_2, TLA^t_3, TLA^t_6\}, \\ \{TLA^t_6, TLA^t_9\}, \\ \{TLA^t_1, TLA^t_6, TLA^p_2\}, \\ \{TLA^t_1, TLA^t_2, TLA^t_3, TLA^t_4, TLA^p_1, TLA^p_2\}, \\ \{TLA^t_1, TLA^t_2, TLA^t_3, TLA^t_4, TLA^p_1, TLA^p_2\}, \\ \{TLA^t_6, TLA^t_9\} \end{bmatrix}$$

Assume that Y is the ordered set of all ATs used to assess students' ability to achieve the learning objectives stated in X:

$Y = \{AT^r_5, AT^p_2, AT^e_1, AT^e_3\}$

$n(Y) = p = 4$

$Y' = \{2, 3, 5, 5\}$

(Note that the AT coding scheme from table A.1 is used for clarity).

Multiple objectives may be associated with each assessment task, the direct relationship between Y' and X' (i.e. V_3) therefore requires us to define V_3 as being a class of ordered sets whereby each element of V_3 refers to a subclass of X'. V_3 is an ordered set in that the first element in V_3 corresponds to the first element in Y and so on. Moreover, each element in Y represents a parent node and the corresponding element in V_3 represents the children nodes therefore forming AT trees. For clarity, the actual learning objective code (L^b_j) is used to express association (x) rather than the level of learning elicited (x') as different active verbs can elicit the same level of learning. Each element of V_3 is therefore:

$$V_3 = \begin{bmatrix} \{L_1^b, L_2^b, L_4^b\}, \\ \{L_3^b, L_7^b\}, \\ \{L_5^b, L_6^b\}, \\ \{L_5^b, L_6^b\} \end{bmatrix}$$

Now that the four major component sets and relations have now been defined we need only set the alignment threshold, τ. For this example, assume that τ is set to 30. Alignment for L^o_3 can now be calculated in four main steps as follows:

1. **Calculate the equilibrium value for relation V_1.**
   a. Recall that we use the constant term c_1 to fix the valence of L^o trees. We set c_1 to 7 as we are only computing alignment for one L^o that dominates seven L^bs.

   b. Let **w** represent the set W' as a row vector such that:

   $$\mathbf{w} = [5]$$

   In this case, **w** is actually a scalar.

   c. Let **d1** represent the vector of c_1 'desired' learning objectives for L^o_3 and **x1** represent the vector of c_1 'actual' learning objectives:

   $$\mathbf{d1} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} \quad x\mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \\ 5 \\ 4 \\ 3 \end{bmatrix}$$

   d. Calculate the alignment values between the learning outcomes and learning objectives as follows:

   - Calculate the *desired alignment* value:

   $$t1 = \sum_{j=1}^{c_1} w' d1_j = 175$$

   - Compute the *actual alignment* value:

   $$u1 = \sum_{j=1}^{c_1} w' x1_j = 95$$

   e. Calculate the difference or *misalignment* between the desired and actual alignment value for L^o_3 represented by **e1**:

   $$e1 = u1 - t1 = -80$$

   Since $|\mathbf{e1}|$ is greater than τ and e1 is negative V_1 is *negatively misaligned*.

   f. Calculate the V_1_equilibrium value:

$$V_1\_equilibrium = \sqrt{\frac{1}{m}\sum_{i=1}^{m} e1_i^{\;2}} = 80$$

The V_1_equilibrium is considerably higher than τ and would typically indicate a significant imbalance between the outcome and its objectives. After assessing the outcome/objective associations, however, this level of disequilibrium can be apportioned to the D_1 matrix in that unrealistic ‘desired’ values are given. As mentioned previously, a semi-linear relationship between components would be more realistic.

2. Calculate the equilibrium value for relation V_2.

a. Recall that we use the constant term c_2 to fix the valence of L^b trees. We set c_2 to 6, as it is the maximum number of TLAs dominated by a single L^b. Filler elements are equal to the parent L^b_j value.

b. Let **x2** represent the vector of all *n* learning objectives from X’:

$$\mathbf{x2} = \begin{bmatrix}1 & 1 & 3 & 2 & 5 & 4 & 3\end{bmatrix}^T$$

To retrieve actual objective we obtain the corresponding element from X.

c. Let D_2 represent the matrix consisting of c_2 rows and *n* columns, where each column corresponds to the set of ‘desired’ c_2 TLAs for each learning objective. Matrix D_2 is defined as:

$$D_2 = \begin{bmatrix} 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \end{bmatrix}$$

Let $\mathbf{d2}_j$ represent a column vector from matrix D_2 such that we refer to the set of c_2 ‘desired’ *TLAs* associated with learning objective *j* as transposed and defined below:

$$\mathbf{d}2_j = \begin{bmatrix} d2_{j1} & d2_{j2} & \ldots & d2_{jc_2} \end{bmatrix}^T$$

d. Let Z_1 represent a matrix consisting of c_2 rows and *n* columns, where each row corresponds to the actual set of c_2 TLAs (including filler elements) used to help students achieve a specific learning objective. Z_1 is defined as:

$$Z_1 = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 2 & 3 & 3 & 3 \\ 4 & 4 & 3 & 3 & 4 & 4 & 3 \\ 2 & 2 & 3 & 2 & 5 & 5 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 & 3 \\ 1 & 1 & 3 & 2 & 3 & 3 & 3 \end{bmatrix}$$

e. Calculate the V_2 alignment values as follows:

- Calculate the *desired alignment* values:

$$\mathbf{t2}_j = \sum_{k=1}^{c_2} x2_j d2_{kj}$$
$$\mathbf{t2} = [6 \quad 6 \quad 54 \quad 24 \quad 150 \quad 96 \quad 54]$$

- Compute the *actual alignment* values:

$$u2_j = \sum_{k=1}^{c_2} x2_j z'_{kj}$$

$$\mathbf{u2} = [13 \quad 13 \quad 51 \quad 26 \quad 105 \quad 84 \quad 51]$$

f. Calculate the misalignment between the desired and actual alignment values:

$$e2_j = u2_j - t2_j$$
$$e\mathbf{2} = [7 \quad 7 \quad -3 \quad 2 \quad -45 \quad -12 \quad -3]$$

Since $|\mathbf{e}2_5|$ is greater than τ and $\mathbf{e2}_5$ is negative, the "L^b_5 tree" is negatively misaligned. All other L^b trees are aligned since the absolute error values are less than τ and at least one TLA from each of the other trees elicit the required cognitive skill.

g. Calculate the V_1_equilibrium value:

$$V_2\_equilibrium = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e2_j^{\,2}} = 18.1$$

V_2_equilibrium is less than τ and could generally be considered aligned. The high misalignment value, however, indicates that some learning objective/TLA associations need to be assessed and modified to further reduce this disequilibrium.

3. Calculate the equilibrium value for relation V_2.

a. Recall that we use the constant term c_3 to fix the valence of AT trees. We set c_3 to 3, as it is the maximum number of L^bs dominated by a single AT. Filler elements are equal to the parent AT value.

b. Let **y** represent the set Y' as a row vector:

$$y = [2 \quad 3 \quad 5 \quad 5]^T$$

c. Let D_3 represent a matrix consisting of c_3 rows and p columns, where each column corresponds to the set of suitable or 'desired' c_3 learning objectives (including filler elements) assessed by a specific AT:

$$D_3 = \begin{bmatrix} 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 \end{bmatrix}$$

Let $\mathbf{d3}_l$ represent a column vector from matrix D_3 such that we refer to the set of c_3 'desired' learning objectives associated with learning outcome l as transposed and defined below:

$$\mathbf{d}3_l = \begin{bmatrix} d3_{l1} & d3_{l2} & \dots & d3_{lc_3} \end{bmatrix}^T$$

d. Let X_2 represent a matrix consisting of c_3 rows and p columns, where each column corresponds to the set of c_3 'actual' learning objectives (including filler elements) assessed by the teacher using a specific AT. X_2 is defined as follows:

$$X_2 = \begin{bmatrix} 1 & 3 & 5 & 5 \\ 1 & 3 & 4 & 4 \\ 2 & 3 & 5 & 5 \end{bmatrix}$$

Also, let $\mathbf{x3}_l$ represent a column vector from matrix X_2 such that we refer to the set of c_3 teacher-defined learning objectives assessed by AT l as defined below:

$$x3_l = \begin{bmatrix} x'_{l1} & x'_{l2} & \dots & x'_{lc_3} \end{bmatrix}^T$$

e. Calculate the alignment values between an AT and its associated learning objectives as follows:

- Calculate the *desired* alignment:

$$t3_l = \sum_{j=1}^{c_3} y'_l d3_{jl}$$

$$\mathbf{t}3 = \begin{bmatrix} 12 & 27 & 75 & 75 \end{bmatrix}$$

- Compute the *actual* alignment value:

$$u3_l = \sum_{j=1}^{c_3} y'_l x'_{jl}$$
$$u3 = \begin{bmatrix} 8 & 27 & 70 & 70 \end{bmatrix}$$

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual AT:

$$e3_l = u3_l - t3_l$$
$$e3 = \begin{bmatrix} -4 & 0 & -5 & -5 \end{bmatrix}$$

All AT trees are aligned since all absolute error values are less than τ and at least one L^b within each AT tree elicits the required level of cognitive ability to be assessed.

g. Calculate the V_3 equilibrium value:

$$V_3\_equilibrium = \sqrt{\frac{1}{p}\sum_{l=1}^{p} e3_l^{\,2}} = 4.1$$

V_3_equilibrium is less than τ and is considered aligned. Clearly, the desired and actual objective vectors are much more similar than those for the V_1 and V_2 alignment computations. This similarity is reflected numerically in **e3** and V_3_equilibrium.

4. Calculate the overall equilibrium value.

The equilibrium value consolidating all direct relations and representing constructive alignment, in this case for a single outcome of a module, is simply:

$$\frac{V_1\_equilibrium + V_2\_equilibrium + V_3\_equilibrium}{3}$$

$$\frac{80 + 18.1 + 4.1}{3} = 34.1$$

If each element of $e1_i$, $e2_j$ and $e3_l$ has been classified as either being 'aligned' or 'positively misaligned' then it could be broadly stated that the module is *constructively aligned* for the learning outcome addressed. Clearly, V_1 is responsible for the majority of misalignment and thus disequilibrium and it is envisaged that the practitioner would first inspect the relationships between the learning outcome and the objectives they had subsequently prescribed. The significance of the magnitude for the individual error and equilibrium values requires more research for it to be considered an accurate and truly representative measure of alignment.