Corrected formula for uncertainty in estimations of gestational age from fetal head circumference measurements

Over recent years, numerous publications have proposed methods for estimating gestational age (GA) using fetal measurements, including biparietal diameter, head circumference (HC) and crown–rump length, among others. The paper by Altman and Chitty\(^1\) presented statistical modelling of data from 663 fetuses to produce charts and tables for pregnancy dating based upon such measures. The resulting outputs were tables of estimates of mean GA based upon each measurement, each with a corresponding standard deviation that encompasses the uncertainty in the prediction. Here, we address an erroneous result in the appendix of this work, associated with the uncertainty in GA prediction based upon derived HC measurements.

Altman and Chitty report the following formulae for calculating the mean loge GA (in weeks) and the associated standard deviation, as a function of (derived) HC:

\[
\text{mean} = 1.848 + 0.010611HC - 0.000030321HC^2 + 0.43498 \times 10^{-7}HC^3, \tag{1}
\]

\[
\text{SD} = 0.08024 - 0.00052635HC + 0.000014204HC^2. \tag{2}
\]

Predictions for mean GA are calculated by taking the exponential of the mean (1), and the uncertainty in these predictions is described by the 5th and 95th centiles, calculated according to:

\[
e^{\text{mean} \pm 1.64\text{SD}}. \tag{3}
\]

The resulting values are presented in a reference table for estimated GA based upon these (derived) HC measurements (Table 5 in Altman and Chitty\(^1\)). While the estimated GA aligns with values produced when using the formula for mean (1), the formula for SD (2) gives rise to exponentially divergent 5th and 95th centiles, with a significantly greater degree of uncertainty than is quoted in Altman and Chitty’s reference table (Figure 1). We stress here that the values quoted in their paper do seem reflective of the data in their original study, and we do not call these into question. Indeed, these tables have been recommended for routine clinical use by the British Medical Ultrasound Society\(^2\). The formula in (2), however, predicts unfeasible values of GA, for large HC measurements in particular, suggesting that this formula is subject to typographical error in the original publication. We address this here due to the need for a continuous analog to Altman and Chitty’s reference table, to enable these predictions and uncertainty measures to be incorporated easily into commercial healthcare software currently under development.

We correct for the above anomaly as follows: using Altman and Chitty’s tabulated values at each measured HC, which we index by \(i\), we denote the estimated GA by \(GA_i\) and the 5th and 95th centiles by \(c^-\) and \(c^+\) respectively. (Note that, from (1), \(GA_i = \exp(\text{mean}(HC_i))\).) Using the absolute log-difference between the estimate and centiles, we compute the SD for each HC measurement independently, according to:

\[
SD_i = \frac{|\log_e(GA_i) - \log_e(c^\pm_i)|}{1.64}, \quad i = 1, \ldots, 49. \tag{4}
\]

We then use a least-squares approach to fit a second-degree polynomial to the resulting data in Matlab, to obtain the following corrected formula for the SD of GA, as a function of (derived) HC:

\[
\hat{SD} = 9.5934 \times 10^{-2} - 6.3256 \times 10^{-4}HC + 1.7103 \times 10^{-6}HC^2. \tag{5}
\]

The GA predictions resulting from this adjusted formula are shown in Figure 2, in which the mean GA was calculated using the original formula (1) and the centiles were calculated by replacing SD with \(\hat{SD}\) in (3). As Figure 2 demonstrates, the corrected formula in (5) reproduces accurately the tabulated results produced by Altman and Chitty\(^1\), for the full range of HC measurements. This formula is readily amenable to implementation within...
Figure 2 Prediction of gestational age using head circumference measurements, with corrected 5th and 95th centiles (outer solid lines) using the amended SD formula (equation 5), compared with the estimates of mean gestational age (•) and 5th and 95th centiles (○) in Table 5 of Altman and Chitty¹.

clinical software alongside the remainder of Altman and Chitty’s results.

References