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Optimum Design of Composite Prestressed Concrete Girder Railway Bridges

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1. Abstract
This paper deals with the formulation of design optimisation of pretsressed concrete bridges. The bridge is of a slab-on-girder type, hence modeled as an equivalent orthotropic plate. The whole bridge system is considered as a simply supported right angle plate. Following linear elastic behaviour, the governing fourth order differential equation of the plate for patch load is solved in order to find out load distribution on the girders forming the bridge as well as the deflections and internal forces at critical sections of the whole bridge. The optimisation problem is formulated for various cross sectional geometries including rectangular, symmetrical I, unsymmetrical I, box, T and inverted T sections. The design variables are the main cross sectional dimensions, prestressing force and tendon eccentricity. The objective function comprises the cost of concrete material, formwork and prestressing steel tendons. The constraint functions are set to satisfy design requirements as per British Standards for bridges (BS 5400). Nonlinear optimisation method based on sequential unconstrained minimisation technique (SUMT) is employed to achieve optimum bridge configuration for specific design parameters of span length, concrete compressive strength and railway loading patterns. A purpose built computer program is set up to carry out the solution of the design optimisation problem efficiently in terms of time and effort. A typical example of unsymmetrical I-section having a small bottom flange as compared to the top flange width with composite deck effect is presented. The results show that the total cost increases as the span increases due to the increase of the initial prestressing force. Furthermore, the total cost decreases as the concrete compressive strength increases in spite of the increasing of the prestressing force. This is due to decrease of the overall depth, top and bottom flange widths, hence leading to a smaller girder size. Such finding will encourage engineers to adopt high strength concrete for bridges as it will help reducing not only the initial cost but also the life cycle cost of the bridge over its entire life.
2. Keywords: Prestressed, Concrete, Railway, Bridge, Optimisation

3. Introduction
A bridge is a structure providing passage over an obstacle without closing the way beneath. The required passage may be for a road, a railway, pedestrian, a canal or a pipeline. The obstacle to be crossed may be a river, a road, railway or a valley. Bridge building has occupied much of the working lives of the great civil engineering in the last century and a half. Early bridge builders had little choice of materials for their structures. Timber and masonry reigned supreme until the nineteenth century and over the years engineers developed a good general knowledge of the working limits of these materials. In the nineteenth century the further development of iron and beginnings of the industrial revolution in Britain created the new railways with their accompanying need for bridges. The gradual transition from timber to steel during the nineteenth century was to be followed by a more rapid change from masonry to concrete. Another major contribution was the development of prestressed concrete by French *Freyssinet*. The improvement of methods of concrete production permitting higher working stresses with the availability of high tensile steels has eventually paved the way to the widespread adoption of prestressed concrete bridges for spans of less than 180 m (600 ft). The great advantage of prestressed concrete lies in the reduction of dead weight in comparison with conventional reinforced concrete. For medium spans, advances in pre-cast construction and post-tensioning techniques have made possible the imaginative use of segmental construction in concrete. In the range of shorter spans up to 40 m (130 ft) the trend is towards an increasing use of prefabricated beams of steel or prestressed concrete with addition of a reinforced concrete slab to make up some form of composite deck [1].

4. Bridge Deck Analysis
The main categories of bridge deck of concrete construction are illustrated in Figure 1. The simplest form of concrete deck is the solid slab which is usually of conventional reinforced concrete. Since much of the design load is accountable to self-weight, various forms of voided slabs have been adopted. In many countries standard pre-cast prestressed beams are available for short and medium spans. After the beams have been positioned across the span which is to be bridged, it is usual to employ in situ concrete to give transverse connection in order to create a slab-type deck. This form of deck and the voided slab may be considered as pseudo-slabs. Concrete bridge decks which are transversely connected only by in situ slabs are usually referred to as beam and slab decks. The transition from pseudo-slab to beam and slab deck is difficult to define with complete precision. For the purpose of analysis a beam and slab deck with many longitudinal beams may be idealized as a slab or plate and can therefore be described as a pseudo-slab [1]. Over the last five decades, the science of bridge analysis has undergone major changes. Following the advent of the digital computer and the consequent development of analytical techniques based upon its use, a number of so-called refined methods for the analysis of load distribution of bridge of various types have been well established. They are:

a. Orthotropic plate method
b. Grillage analogy method
c. Articulated plate method

d. Space frame method
e. Finite element method
f. Finite strip method
g. Finite difference method

The main parameters which govern the choice of analytical technique to be identified for the bridge deck are form of construction, plan geometry and supporting condition. For more details about the key features of these methods and their advantages and applicability, see for example references [1, 2].

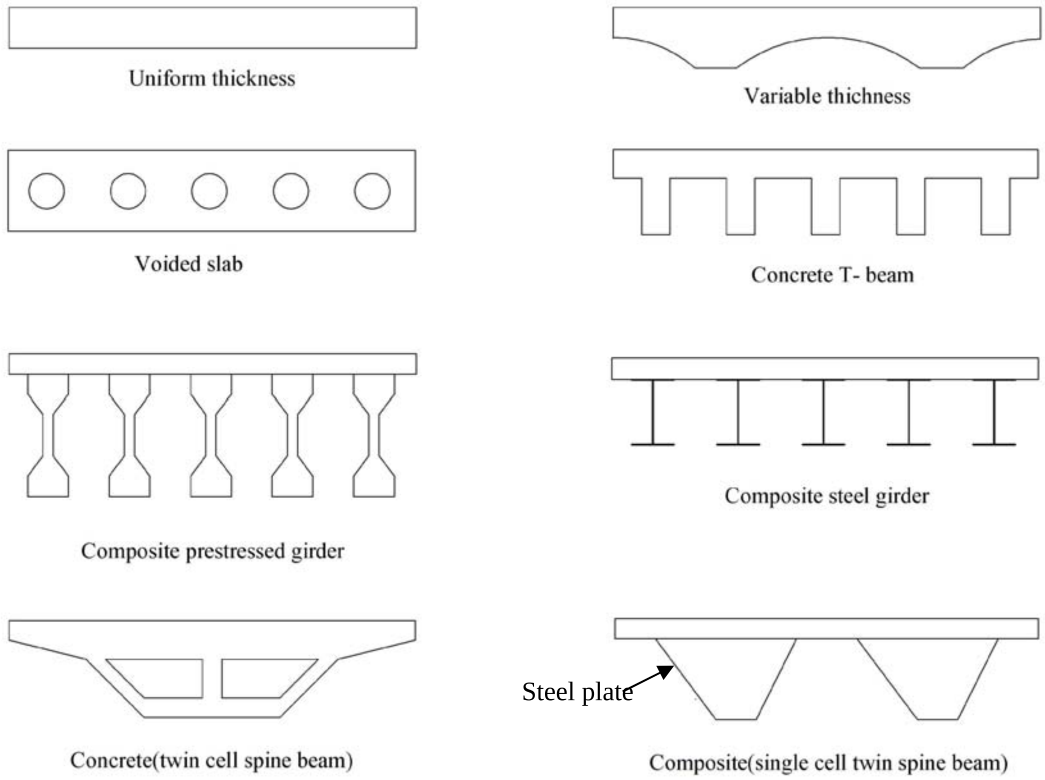

Figure 1. Types of concrete bridge deck [1, 2].

The present study deals with a bridge of a slab-on-girder type treated as an orthotropic plate that has different specified elastic properties in two perpendicular directions. The deck is right and simply supported. The governing fourth order differential equation of the equivalent orthotropic plate is [1]:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P(x,y) \quad (1)$$

Where:
w = Deflection
H = Total torsional rigidity
= $(D_{xy} + D_{yx} + D_1 + D_2)/2$
D_x = Flexural rigidity in x direction
D_y = Flexural rigidity in y direction
D_{xy} = Torsional rigidity in x direction
D_{yx} = Torsional rigidity in y direction
D_1 = Coupling rigidity in x direction
D_2 = Coupling rigidity in y direction
$P(x,y)$ = Lateral load on a plate

The solution of Eq. (1) for various loading conditions is elaborately explained in [1, 2].

5. Railway Bridge Live Load
BS 5400 [3] gives two types of standard railway loadings known as *RU* and *RL* loading.

5.1. Type (*RU*) loading:
Normal type (*RU*) loading consists of four 250 kN concentrated loads preceded, and followed, by uniformly distributed load of 80 kN/m, see Figure 2.

5.2. Type (*RL*) loading:
Normal type (*RL*) loading consists of single 200 kN concentrated load coupled with a uniformly distributed load of 50 kN/m for loaded length up to 100 m. For loaded lengths in access of 100 m, the distribution nominal load shall be 50 kN/m for the first 100 m and shall be reduced to 25 kN/m for lengths in access of 100 m, see Figure 3.

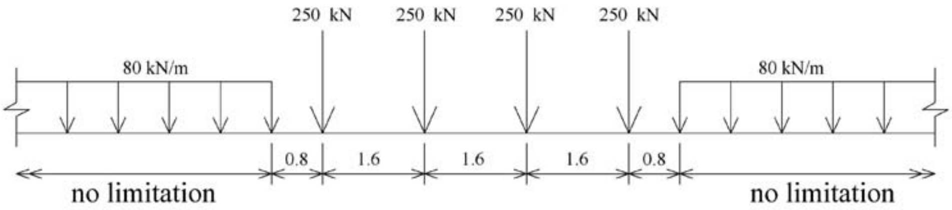

Figure 2. Type (*RU*) loading [3].

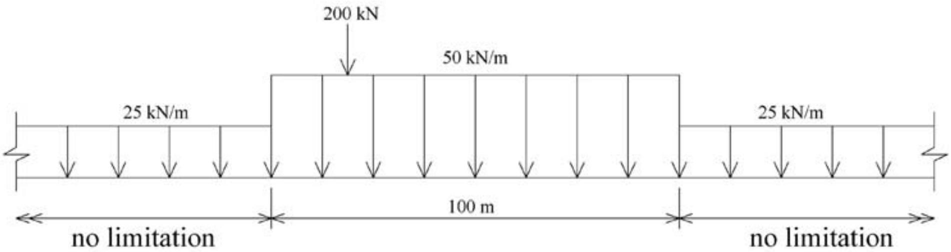

Figure 3. Type (*RL*) loading [3].

Alternatively, two concentrated nominal loads, one of 300 kN and the other of 150 kN, spaced at 2.4 m intervals along the track, shall be used on deck elements where this gives a more severe condition. These two concentrated loads shall be deemed to include dynamic effects [3]. Concentrated loads applied to the rail will be distributed both longitudinally by the continuous rail to more than one sleeper, and transversely over a certain area of deck by the sleeper and ballast. It may be assumed that only two-third of a concentrated load applied to one sleeper will be transmitted to the bridge deck by that sleeper, and that the remaining one-third will be transmitted by the two sleepers either side. The load acting on the sleeper under each rail may be assumed to be distributed uniformly over the ballast at the level on the underside of the sleeper for a distance of 800 mm symmetrically about the centre line of the rail or twice the distance from the centre line of the rail to the nearer end of the sleeper, whichever is the less. Dispersal of this load through the ballast onto the supporting structure shall be taken at 5^o to the vertical [3].

6. Review of Bridge Deck Optimisation
The advent of mathematical programming techniques and the availability of powerful computers at affordable prices have permitted comprehensive structural optimisation studies to be carried out. In 1966, Guzman-Barron Torres et. al. [4] developed a programming design the superstructure of simple-span prestressed concrete highway bridges using linear programming. The independent design parameters were the number of girders, the depth of the girders, and the initial prestressing force per girder and tendon eccentricity. In 1971, Goble and Lapay [5] presented a method for obtaining the optimum design of prestressed concrete beam considering the total cost unit surface area of the member as the objective function. The objective function was including the cost per unit volume of concrete, prestressing steel, and the cost of forms. The optimum design selected to satisfy the limitations of ACI 318-63. The design variables comprise the cross-section variables, initial prestressing force, area of mild steel and the prestressing steel area. The eccentricity of the tendon was considered as design parameter. The constraints represent inequality limitations on stress, deflection and ultimate section strength. The shear design problem and the spacing of the stirrups were ignored in the optimization technique. This problem however, was non-linear and the technique used was a sequential method.
In 1972, Kirsch [6] developed a method for the optimum design of prestressed indeterminate beam with uniform cross-section. Optimum values of prestressing force, tendon configuration and cross-sectional dimensions were determined subjected to constraints on design variables and stresses. The objective function has been selected as a linear combination of prestressing force and the cross-

sectional area to minimise the cost of the problem.

In 1982, Kirsch [7] studied the optimum design of girders considering the total cost of concrete and reinforcing steel as the objective function. He assumed dimensions for the length and width for the member. The design variables were to find the design moments obtained from the redistribution moments, depth of the cross-section and steel reinforcement. The constraints on the depth and deflection were included. The constraints on the longitudinal reinforcement was to resist the bending moments and to increase the amount of steel reinforcement on the lower limit, while the constraint on the transverse reinforcement was to resist the shearing force. The constraints on the crack width and the maximum limit of reinforcement were not considered.

In 1996, Fereing [8] presented minimum cost preliminary design of single-span bridge structures consisting of cast-in-place RC deck and girders based on the AASHTO specifications 1992. The author linearised the problem by approximating the nonlinear constraints with straight lines and solves the resulting linear problem by the simplex method. The author concluded that it is always more economical to space the girder at the maximum practical spacing.

In 2004, Al-Lihibi [9] constructed a computer program involving analysis and optimisation of steel bridges. One and two dimensional bridges were investigated and designed according to AISC and AASHTO specifications, respectively. He used the sequential unconstraint minimisation technique (SUMT) to minimise the objective function which is the initial cost of bridge. Super structure and substructure were considered in the optimisation problem. Design charts had been presented by the author to be available for using by bridge designers.

Other similar works on the optimum design of bridge decks can be found, for instance, in references [10, 11, 12].

7. Formulation of the Optimisation Problem

7.1. Design Variables

The selection of perstressed concrete section is a common problem confronting the designer. This is because the depth of the section will not be the only variable changed, but also the web thickness, flanges width and thickness varied independently for a given case. The types of cross sections generally used for precast prestressed concrete beams are rectangular, symmetrical I, unsymmetrical I, box, T and inverted T sections as shown in Figure 4. Therefore, there are, in general, five independent design variables for perstressed concrete girders which are:

a. Top flange width (b_t)
b. Bottom flange width (b_b)
c. Depth of the section (h)
d. Initial prestressing force (F_i)
e. Eccentricity of the prestressing force (e)

On the other hand, the span length, imposed load, ultimate strength of prestressing steel and concrete compressive strength at transfer and service are treated as design parameters. The prestressing steel area will be dependent variable as it is function of the initial prestressing force and ultimate strength of the prestressing steel.

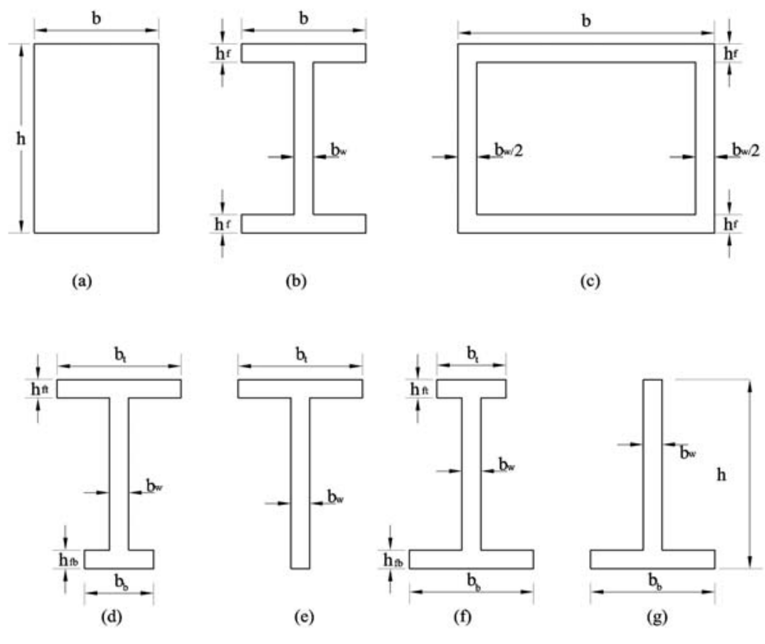

Figure 4. Types of beam cross-section; (a) Rectangular, (b) Symmetrical I-section, (c) T-section, (d) unsymmetrical I-section, (e) T-section, (f) Inverted unsymmetrical I-section, (g) Inverted T-section.

7.2. Constraint Functions

The design optimisation is made to satisfy the allowable working stresses and deflection both at transfer and service according to British Standards specifications for bridges (BS 5400), [3].

The loading stages, for a typical composite prestressed concrete girder are illustrated in Figure 5 with the initial pre-stress force *(F_i)* in stage (1), stage (2) comprise the initial combination with the self weight of the member. In stage (3) the all time-depend losses would be assumed to occur at this stage. The above three-stages occurred without the associated non-composite dead load. At stage (4) the load resulting from the weight of the wet concrete slab is added to stage (3). After this stage, the effect centroid shift upward of that of the composite section, when the freshly poured concrete slab has hardened and acquired its strength. Stage (5) represents the super-imposed dead and live load acts on the composite section [13].

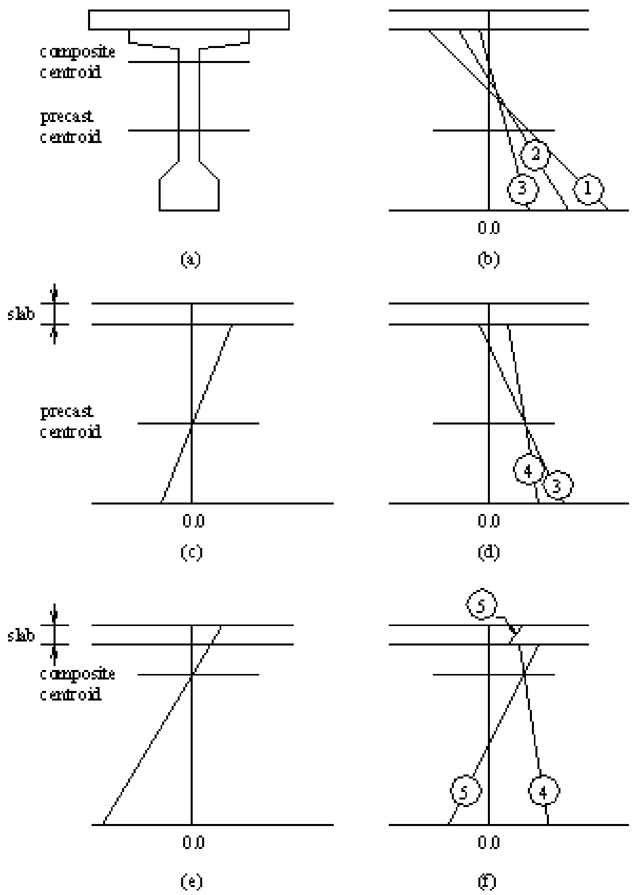

Figure 5. Elastic stresses in an uncracked composite beam; (a) Cross-section, (b) Prestress plus self weight, (c) Increment due to non-composite loads, (d) Prestress plus non- composite loads, (e) Increment due to composite loads, (f) Prestress plus non-composite loads and composite loads [13].

At transfer stage, the governing equations of the applied stresses at the bottom and top fibres are:

$$f_{bir} = -\frac{F_i}{A} - \frac{F_i e}{Z_b} + \frac{\alpha M_d}{Z_b} \geq f_{bi} \tag{2}$$

$$f_{tir} = -\frac{F_i}{A} + \frac{F_i e}{Z_t} - \frac{\alpha M_d}{Z_t} \leq f_{ti} \tag{3}$$

At service stage, the applied stresses at the top and bottom fibres are:

$$f_{tr} = -\eta \frac{F_i}{A} + \eta \frac{F_i e}{Z_t} - \frac{M_d}{Z_t} - \frac{M_c}{Z_t} - \frac{mt M_s}{Z_t} \geq f_t \tag{4}$$

$$f_{br} = -\eta \frac{F_i}{A} - \eta \frac{F_i e}{Z_b} + \frac{M_d}{Z_b} + \frac{M_c}{Z_b} + \frac{mb M_s}{Z_b} \leq f_b \quad (5)$$

Where:

A = Area of gross concrete sections
F_i = Initial prestressing force
F_e = Effective prestressing force
e = Eccentricity of the prestressing force
Z_b = Elastic section modulus of bottom fibre
Z_t = Elastic section modulus of top fibre
f_{bi} = Maximum allowable stresses at bottom fibre at transfer;
= $-0.5F_{ci} \geq -0.4F_{cu}$
f_{ti} = Maximum allowable stresses at top fibre at transfer;
= $0.36(F_{ci})^{1/2}$ for post-tension (mid span section)
= $0.45(F_{ci})^{1/2}$ for pre-tension (end section)
f_b = Maximum allowable stresses at bottom fibre at service (after all prestress losses);
several values for f_b are recommended by BS 5400 [6] depending on the grade of the concrete.
f_t = Maximum allowable stresses at top fibre at service (after all prestress losses);
= $-0.4F_{cu}$
f_{bir} = Real stresses at bottom fibre at transfer
f_{tir} = Real stresses at bottom and top at transfer
f_{br} = Real stresses at bottom fibre at service (after all prestress losses)
f_{tr} = Real stresses at top fibre at service (after all prestress losses)
M_a = Moment due to superimposed dead and live loads at service
M_c = Moment due to dead load added after erection but before the composite
M_d = Moment due to the girder self-weight
M_s = Moment due to the additional load added after the composite section has been formed
mb = Ratio of the section modulus of the precast portion alone to section modulus of the composite section at bottom fibre;
= Z_b / Z_{bco}
= 1 for non-composite members
mt = Ratio of the section modulus of the precast portion alone to section modulus of the composite section at top fibre;
= Z_t / Z_{tco}
= 1 for non-composite members
α = 0 for pre-tensioned beam (end section)
α = 1 for post-tensioned beams (mid span section)
η = F_e / F_i

It must be noted that the sign convention is the tensile stress is positive and the compressive stress is negative, the eccentricity is considered positive when it lies below the centroid.

The deflection at initial stage and long-term are considered as an important phase for the design of prestressed concrete member. Accordingly, the following deflection constraints should be satisfied at various stage of member load.

At transfer stage, due to initial prestressing and self-weight only:

$$\Delta = -\Delta_{pi} + \Delta_o \geq \Delta_p \quad (6)$$

At service loading stage, due to superimposed live load only:

$$\delta_{ll} \leq \Delta_{ll} \quad (7)$$

At long term stage, due to effective prestressing, self weight and imposed load after all losses and creep effect:

$$\Delta = -\Delta_{pe} - \frac{\Delta_{pi} + \Delta_{pe}}{2} C_u + (\Delta_o + \Delta_d)(1 + C_u) + \delta_{ll} \leq \Delta_u \quad (8)$$

Where:

Δ_{pi} = Instantaneous camber due to initial prestressing;
= $5 F_i e L^2/(48EI)$ for a parabolic tendon profile
= $F_i e L^2/(8EI)$ for a straight tendon profile
Δ_o = Deflection due to self weight of the member
Δ_p = Allowable camber
Δ_{pe} = Deflection due to effective prestressing after all losses taken place
Δ_{pe} = $\Delta_{pe}\eta$
C_u = Creep coefficient.
δ_{ll} = Deflection due to superimposed live load
Δ_{ll} = Allowable live load deflection
Δ_d = Additional dead load deflection
Δ_u = Maximum allowable deflection

7.3. Objective Function

The objective function for the design of prestressed concrete girder is the minimum cost which comprises the cost of the concrete, strands and formwork:

$$Z = C_1 F_i + C_2 A_c L + C_3 A_f \quad (9)$$

Where:

Z = Objective function
C_1 = Unit price of prestressing force, which implies cost of strand, (units/MN)
C_2 = Unit price of concrete, (units/m^3)
C_3 = Unit price of form work, (units/m^2)
A_c = Area of girder cross section, (m^2)
A_f = Area of formwork, (m^2)
L = Span length, (m)

7.4. Optimisation Technique

The optimum design problem of the prestressed girder entails highly nonlinear objective and constraint functions. Therefore, nonlinear programming method is required to solve the problem efficiently. In this work, Sequential Unconstrained Minimisation Technique *(SUMT)* is adopted to find the minimum cost of prestressed concrete girder subject to stress and deflection constraints at various stages of the girder span life as shown above according to the British Standards specifications for bridges (BS 5400). In this method, the constrained design problems are solved by transforming the constraints into a sequence of unconstrained minimisation problem using penalty functions. This method has already been extensively used for tackling design optimisation problem for various civil and structural engineering systems due to its several advantages and merits [14, 15].

8. Application and Results

A purpose–built computer program suitable for the optimum design of prestressed concrete girder bridges under railway loading is developed. The program can be used for various cross-sectional geometries as shown in Figure 4 above, taking into account the deck effect, composite or non-composite. Post-tensioning and pre-tensioning are considered in the program. The program is applied to one detailed problem of a bridge deck subjected to railway loading type (*RU*).

Optimum design of an unsymmetrical I-section having a small bottom flange width as compared with the top flange width with composite deck effect is carried to find out the minimum cost using the developed program.

The layout and geometry of the bridge deck system is shown in Figure 6 with girder cross-sectional dimensions shown in Figure 7. The following data have been assumed for the selected problem:

Number of girders (N_g) = *6*
Total deck width (*2b*) = *6 m*
Number of diaphragms (N_d) = 6
Span length (L) = *15, 20, 25 and 30 m*
Characteristic compressive strength of prestressed concrete at transfer (F_{ci}) = *28 MPa*
Characteristic compressive strength of prestressed concrete at service (F_{cu}) = *40 MPa*
Unit price of prestressing force (C_1) = *1 units/MN*
Unit price of concrete of girder (C_2) = *1 units/m^3*
Unit price of forming (C_3) = *1 units/m^2*

The optimum prestressing force with its eccentricity, section depth, top flange width, bottom flange width, and the total cost are presented in Figures 8 to 13. It is very clear as shown in Figure 8 to be concluded that the cost increases as the span increases due to the increase of the initial prestressing force as shown in Figure 9.

Figure 10 shows that the total cost decreases as compressive strength of the prestressing concrete (F_{cu}) increases in spite of increasing of the prestressing force, as Figure 11 shows. This is due to the decreasing of the section depth, top and bottom flange widths as shown in Figures 12, 13 and 14 respectively. This leads to a smaller cross section for the girders. Such finding will encourage engineers to adopt high strength concrete for bridges as it will help reducing not only the initial cost but also the life cycle cost of the bridge over its entire life.

The convergence of the total cost and the design variables are studied. Two convergence limits are taken into account for this application, the first one on the objective function (e_1 = 0.001) and the other on the penalty function (e_2 = 0.00001). Figures 15 to 18 introduce the convergence behaviour for different span length values, from which one can easily observe that the convergence achieved and stable results obtained after relatively few number of cycles. Such finding most likely demonstrates the efficiency of the SUMT technique. Hence, SUMT would be highly recommended as a powerful optimisation tool for tackling a wide range of nonlinear engineering systems.

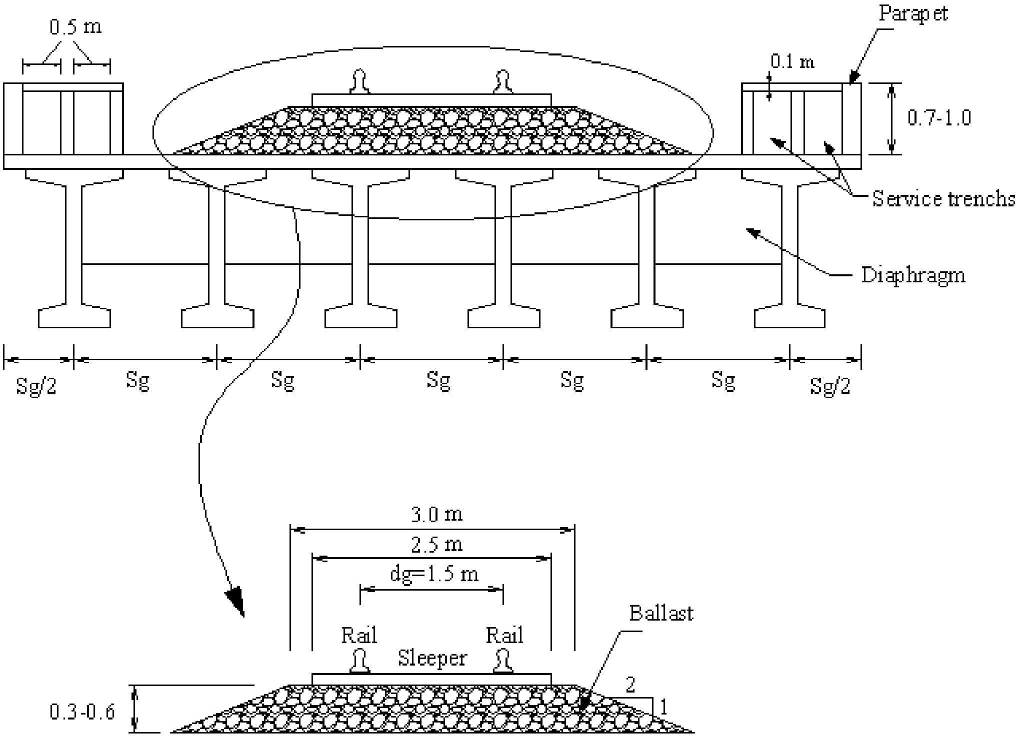

Figure 6. Transverse section of the bridge deck.

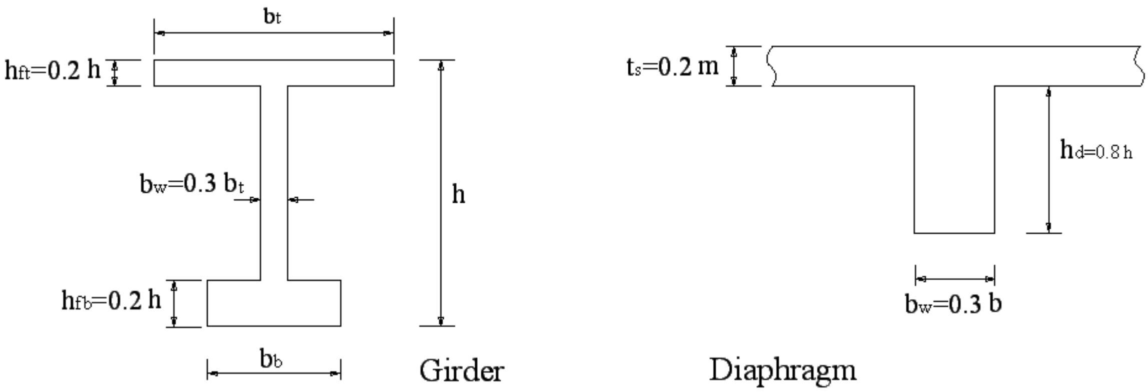

Figure 7. Sections of girder and diaphragms

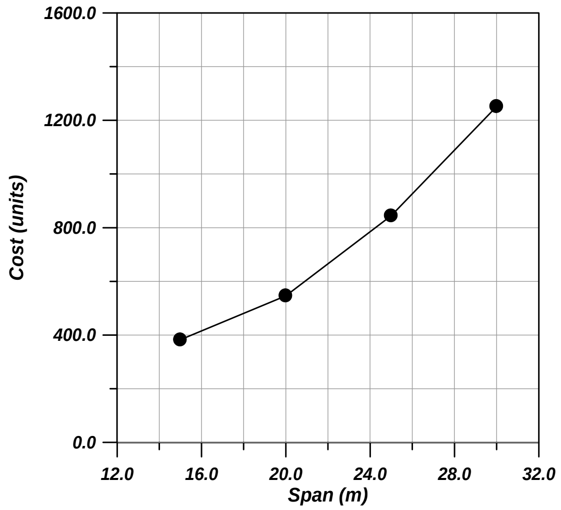

Figure 8. Span effect on the objective function.

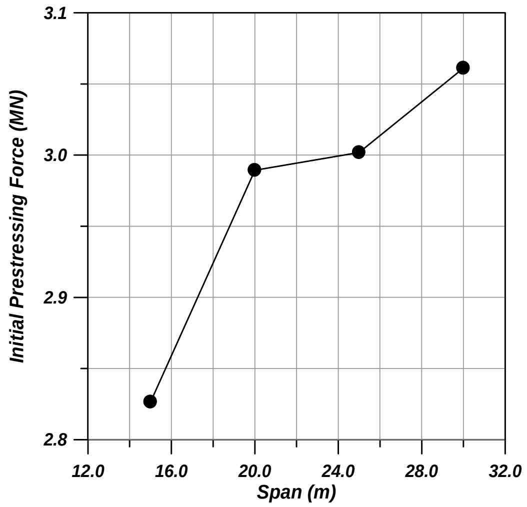

Figure 9. Span effect on the initial prestressing force.

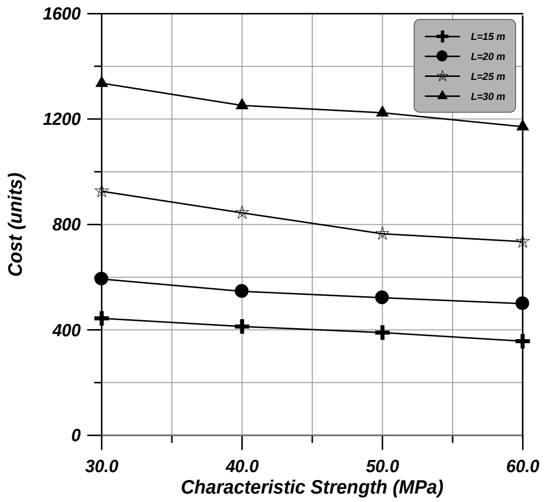

Figure 10. Effect of (F_{cu}) on the objective function.

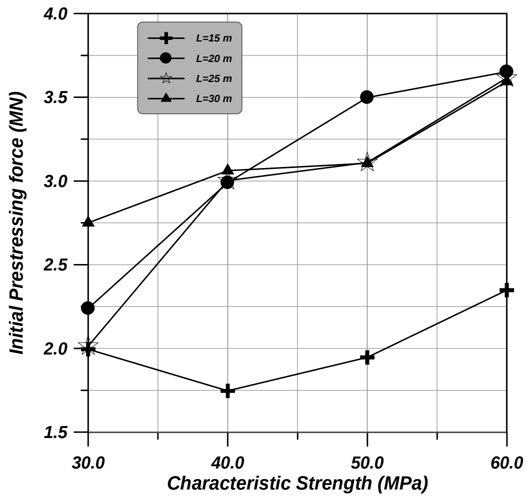

Figure 11. Effect of (F_{cu}) on the initial prestressing force.

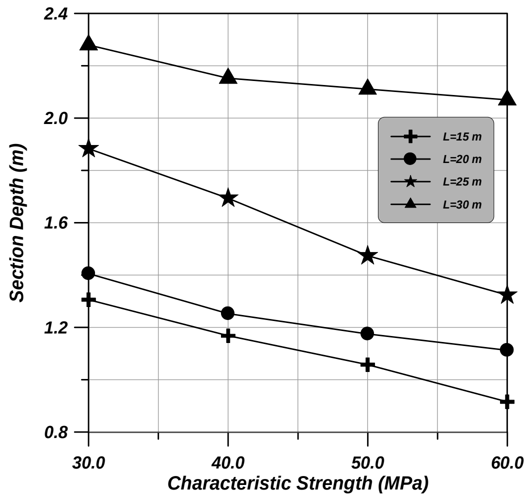

Figure 12. Effect of (F_{cu}) on the section depth.

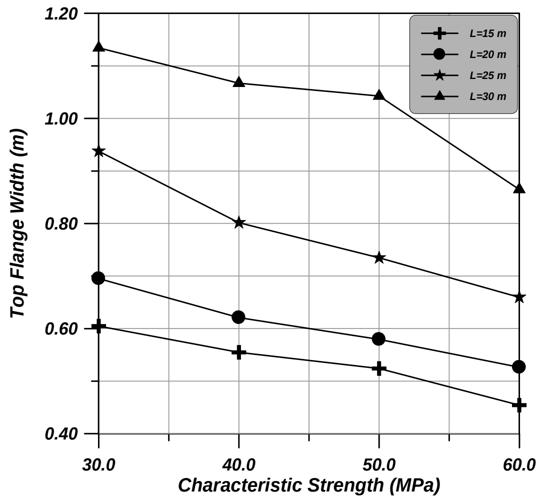

Figure 13. Effect of (F_{cu}) on the top flange width.

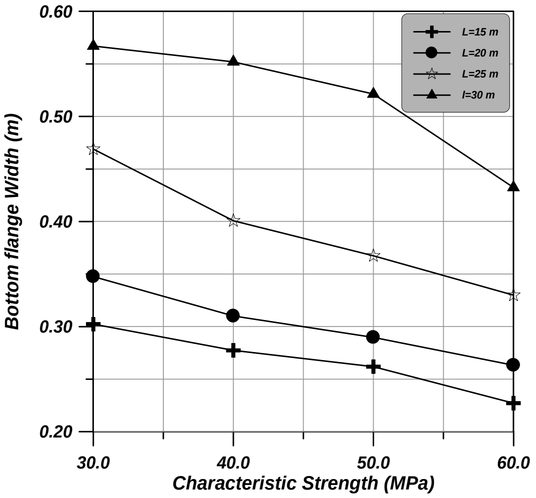

Figure 14. Effect of (F_{cu}) on the bottom flange width.

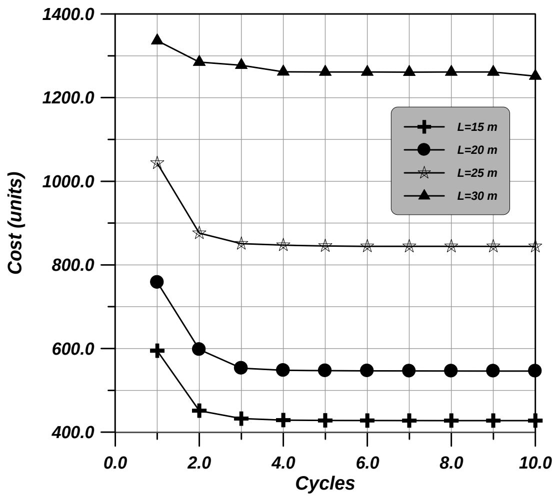

Figure 15. Convergence of total cost.

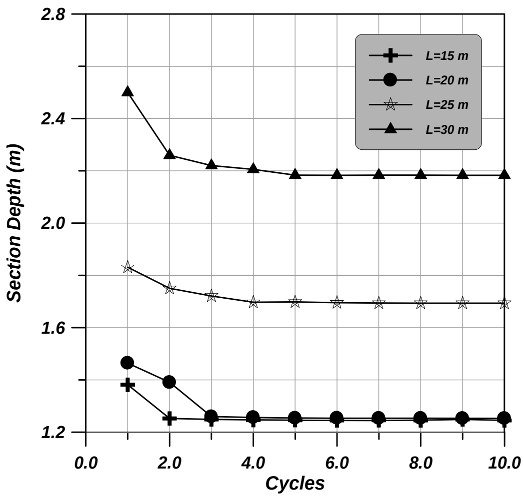

Figure 16. Convergence of section depth.

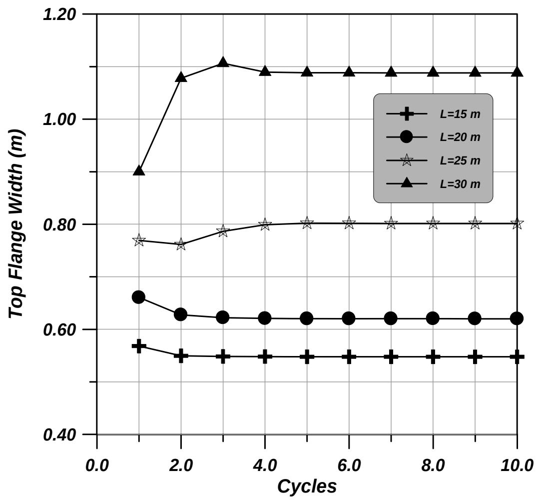

Figure 17. Convergence of top flange width.

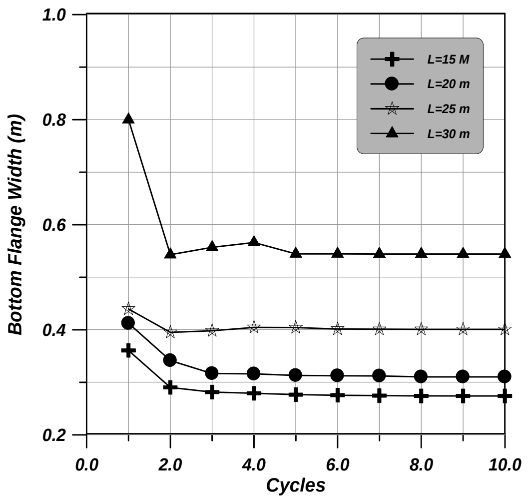

Figure 18. Convergence of bottom flange width.

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