Can the Fiscal Theory of the Price Level explain UK inflation in the 1970s? *

Jingwen Fan†
(Cardiff University)

Patrick Minford
(Cardiff University and CEPR)

August 2013

Abstract

We investigate whether the Fiscal Theory of the Price Level can explain UK inflation in the 1970s. We find that fiscal policy was non-Ricardian and money growth entirely endogenous in this period. The implied model of inflation is tested in two ways: for its trend using cointegration analysis and for its dynamics using the method of indirect inference. We find that it is not rejected. We also find that the model's errors indicate omitted dynamics which merit further research.

Keywords: UK Inflation; Fiscal Theory of the Price Level; Cointegration test; Bootstrap simulation; Indirect inference; Wald statistic

JEL Classification: E31, E37, E62, E65

---

*We are grateful for helpful comments to Michael Hatcher, Vo Phuong Mai Le, David Meenagh, Zhirong Ou and Andrew Scott; and to two anonymous referees, and participants in the Cardiff University economics workshop, the Irish Economic Association 2010 Annual Conference and the 42$^{\text{th}}$ Money, Marco and Finance Research Group Annual Conference.

†Corresponding Author: Jingwen Fan, Cardiff Business School, Cardiff University, Colum Drive, Cardiff, CF10 3EU, UK. Tel: +44 (0)29 20876545. Email: jingwenfan6@gmail.com

1 Introduction

In 1972 the UK government floated the pound while pursuing highly expansionary fiscal policies whose aim was to reduce rising unemployment. To control inflation the government introduced statutory wage and price controls. Monetary policy was given no targets for either the money supply or inflation; interest rates were held at rates that would accommodate growth and falling unemployment. Since wage and price controls would inevitably break down faced with the inflationary effects of such policies, this period appears to fit rather well with the policy requirements of the Fiscal Theory of the Price Level: fiscal policy appears to have been non-Ricardian (not limited by concerns with solvency) and monetary policy accommodative to inflation - in the language of Leeper (1991) fiscal policy was 'active' and monetary policy was 'passive'. Furthermore, there was no reason to believe that this policy regime would come to an end: both Conservative and Labour parties won elections in the 1970s and both pursued essentially the same policies. While Margaret Thatcher won the Conservative leadership in 1975 and also the election in 1979, during the period we study here it was not assumed that the monetarist policies she advocated would ever occur, since they were opposed by the two other parties, by a powerful group in her own party, as well as by the senior civil service. Only after her election and her actual implementation of them was this a reasonable assumption. So it appears that in the period 1972-79 there was a prevailing policy regime which was expected to continue. These are key assumptions about the policy environment; besides this narrative background we also check them empirically below.

Under FTPL the price level or inflation is determined by the need to impose fiscal solvency; thus it is set at the value necessary for the government's intertemporal budget constraint to hold at the market value of outstanding debt. Given this determinate price level, money supply growth, interest rates and output are determined recursively as the values required by the rest of the model to permit this price level.

The theory implies a relationship between the trend in the level of inflation and trends in fiscal variables. This relationship can be tested by cointegration analysis in a familiar way. Indirect Inference, less familiarly, can be used to evaluate the model's dynamics by checking whether its simulated dynamic behaviour is consistent with the data. The data under Indirect Inference is described by some time-series equation. The model's simulated behaviour implies a range of time-series behaviour depending on the shocks hitting it; this range can be described by the parameters of the same time-series equation that fits the data. We can derive the implied statistical joint distribution for the parameters of this equation and test whether the parameters of the time-series equation from the data lie jointly within this distribution at some confidence level.

The FTPL has been set out and developed in Leeper (1991), Sims (1994, 1997), Woodford (1996, 1998, 2001), and Cochrane (2000, 2001) - see also comments by McCallum (2001,2003), Buiter (1999, 2002), and for surveys Kocherlakota and Phelan (1999), Carlstrom and Fuerst (2000) and Christiano and Fitzgerald (2000). Empirical tests have been proposed by Canzoneri, Cumby and Diba (2001), Bohn (1998), Cochrane (1999) and Woodford (1999), Davig et al (2007).

In particular Loyo (2000) argues that Brazilian policy in the late 1970s and early 1980s was non-Ricardian and that the FTPL provides a persuasive explanation for Brazil's high inflation during that time. The work of Tanner and Ramos (2003) also finds evidence of fiscal dominance for the case of Brazil for some important periods. Cochrane (1999, 2000) argues that the FTPL with a statistically exogenous surplus process explains the dynamics of U.S. inflation in the 1970s. This appears to be similar to what we see in the UK during the 1970s.

With fiscal policy of this type, the financial markets - forced to price the resulting supplies of government bonds - will take a view about future inflation and set interest rates and bond prices accordingly. It will set bond prices so that the government's solvency is assured ex post (i.e. in equilibrium); thus it will be ensuring that buyers of the bonds are paying a fair price. Future inflation is expected because if the bonds were priced at excessive value then consumers would have wealth to spend, in that their bonds would be worth more than their future tax liabilities; this would generate excess demand which would drive up inflation. However this mechanism would only come into play out of equilibrium. We would not observe it because markets anticipate it and so drive interest rates and expected inflation up in advance; inflation follows because of the standard Phillips Curve mechanism by which workers and firms raise inflation in line with expected inflation. Thus the FTPL can be regarded as a particular policy regime within a sequence of different policy regimes.

Our aim in this paper is to test the Fiscal Theory of the Price Level (FTPL) as applied to the UK in the 1970s episode we described above. Cochrane (1998, 2000, and 2001) has noted that there is a basic identification problem affecting the FTPL: in the FTPL fiscal policy is exogenous and forces inflation to close the government constraint while monetary policy is endogenous and responds to that given inflation; but the same economic behaviour can be consistent with an exogenous monetary policy determining inflation in the normal way, with Ricardian fiscal policy endogenously responding to the government budget constraint to ensure solvency given that inflation path. Hence testing either or both models is not straightforward. Our procedure is in two stages. In the first stage we identify which model set-up is operating with tests of exogeneity for the two policy regimes: we test for and do not reject, both the endogeneity of monetary policy and the exogeneity of fiscal policy. In the second stage we go on to test whether the FTPL can account for the behaviour of inflation in terms of the behaviour of fiscal variables alone; we do so first by testing for the trend cointegrating relationship and secondly carrying out a test for the dynamic relationship.

Our paper is organised as follows. We review the history of UK policy during the 1970s in section 2; in this section we establish a narrative that in our view plainly supports the exogeneity features required for the FTPL to be identified. In section 3 we set up the model of FTPL for this UK episode. In section 4 we discuss the data and test the two key policy exogeneity/endogeneity assumptions of the theory econometrically. In section 5 we carry out the cointegration test for the inflation trend. In section 6 we explain and carry out the indirect inference tests for the dynamics of inflation. In section 7 we discuss what evidence our model throws on other dynamic factors affecting inflation that are included via the model's error term. Section 8 concludes.

2 The nature of UK policy during the 1970s

From WWII until its breakdown in 1970 the Bretton Woods system governed the UK exchange rate and hence its monetary policy. While exchange controls gave some moderate freedom to manage interest rates away from foreign rates without the policy being overwhelmed by capital movements, such freedom was mainly only for the short term; the setting of interest rates was dominated in the longer term by the need to control the balance of payments sufficiently to hold the sterling exchange rate. Pegging the exchange rate implied that the price level was also pegged to the foreign price level. Through this mechanism monetary policy ensured price level determinacy. Fiscal policy was therefore disciplined by the inability to shift the price level from this trajectory and also by the consequent fixing of the home interest rate to the foreign level. While this discipline could in principle be overthrown by fiscal policy forcing a series of devaluations, the evidence suggests that this did not happen; there were just two devaluations during the whole post-war period up to 1970, in 1949 and 1967. On both occasions a Labour government viewed the devaluation as a one-off change permitting a brief period of monetary and fiscal ease, to be followed by a return to the previous regime.

However, after the collapse of Bretton Woods, the UK moved in a series of steps to a floating exchange rate. Initially sterling was fixed to continental currencies through a European exchange rate system known as 'the snake in the tunnel', designed to hold rates within a general range (the tunnel) and if possible even closer (the snake). Sterling proved difficult to keep within these ranges, and was in practice kept within a range against the dollar and an 'effective' (currency basket) rate. Finally it was formally floated in June 1972.

UK monetary policy was not given a new nominal target to replace the exchange rate. Instead the Conservative government of Edward Heath assigned the determination of inflation to wage and price controls. A statutory 'incomes policy' was introduced in late 1972. After the 1974 election the incoming Labour government set up a 'voluntary incomes policy', buttressed by food subsidies and cuts in indirect tax rates. Fiscal policy was expansionary until 1975 and monetary policy was accommodative, with interest rates kept low to encourage falling unemployment. In 1976 the Labour government invited the IMF to stabilise the falling sterling exchange rate; the IMF terms included the setting of targets for Domestic Credit Expansion. These were largely met by a form of control on deposits (the 'corset') which forced banks to reduce deposits in favour of other forms of liability. But by 1978 these restraints had effectively been abandoned and prices and incomes controls reinstated in the context of a pre-election fiscal and monetary expansion - see Minford (1993), Nelson (2003) and Meenagh et al (2009b) for further discussions of the UK policy environment for this and other post-war UK periods.

Our description of policy suggests that the role of the nominal anchor for inflation will have been played during the 1970s by fiscal policy, if only because monetary policy was not given this task and was purely accommodative.

3 The FTPL Model for the UK in the 1970s

We assume that the UK finances its deficit by issuing nominal perpetuities, each paying one pound per period and whose present value is therefore $\frac{1}{R_t}$where R_t is the long-term rate of interest. We use perpetuities here rather than the usual one-period bond because of the preponderance of long-term bonds in the UK debt issue: the average maturity of UK debt at this time was approximately ten years. All bonds at this time were nominal (indexed bonds were not issued until 1981).

The government budget constraint can then be written as

(1) $\frac{B_{t+1}}{R_t} = G_t - T_t + B_t + \frac{B_t}{R_t}$

where G_t is government spending in money terms, T_t is government taxation in money terms, B_t is the number of perpetuities issued. Note that when perpetuities are assumed the debt interest in period t is B_t while the stock of debt at the start of period t has the value during the period of $\frac{B_t}{R_t}$; end-period debt therefore has the value $\frac{B_{t+1}}{R_t}$. Note too the perpetuity interest rate is by construction expected to remain constant into the future.

We can derive the implied value of current bonds outstanding by substituting forwards for future bonds outstanding:

(2) $\frac{B_t}{R_t} = E_t \sum_{i=0}^{\infty} (T_{t+i} - G_{t+i}) \frac{1}{(1+R_t)^{i+1}}$

We represent this equation in terms of each period's expected 'permanent' tax and spending share, t_t and g_t, and assume that $E_t T_{t+i} = t_t E_t P_{t+i} y_{t+i}$ and $E_t G_{t+i} = g_t E_t P_{t+i} y_{t+i}$.

We can then simplify (2) (see Appendix B) to:

(3) $\frac{B_t}{R_t P_t y_t} = \frac{t_t - g_t}{(1+\pi_t+\gamma)(r_t^* - \gamma)}$

where $R_t = r_t^* + \pi_t$ (respectively the perpetuity real interest rate and perpetuity inflation rate, both 'permanent' variables), γ is the 'permanent' growth rate of real GDP. All these expected permanent variables are by construction expected to be constant in the future at today's level. Permanent growth in this period we assume to be constant so that output (which is an I(1) variable during this period) is assumed to be a random walk with constant drift equal to γ.

In the case of inflation we impose on the model the simplifying assumption that it is a random walk, so that future expected inflation is equal to current inflation and is also therefore permanent inflation. Notice that in the rest of the model we have equations for output and real interest rates, in the IS and Phillips Curves; but these cannot determine inflation as well. Hence if inflation had some dynamic time-path other than the random walk we would have to determine it exogenously; we choose the random walk for simplicity, on the basis that the off-equilibrium wealth effect would operate so powerfully on excess demand that it would drive inflation at once to its permanent value.

The pricing condition on bonds in equation (3) thus sets their value consistently with expected future primary surpluses. Suppose now the government reduces the present value of future primary surpluses. At an unchanged real value of the debt this would be a 'non-Ricardian' fiscal policy move. According to the FTPL prices will adjust to reduce the real value of the debt to ensure the government budget constraint holds and thus the solvency condition is met. This is to be compared with the normal Ricardian situation, in which fiscal surpluses are endogenous so that fiscal shocks today lead to adjustments in future surpluses, the price level remaining unaffected.

Since the pricing equation sets the ratio of debt value to GDP equal to a function of permanent variables, it follows that this ratio b_t follows a random walk [1] such that:

(4) $b_t = \frac{B_t}{R_t P_t y_t} = E_t b_{t+1}$ and (5) $\Delta b_t = \eta_t$, an $i.i.d.$process.

This in turn allows us to solve for the inflation shock as a function of other shocks (especially shocks to government tax and spending). With the number of government bonds issued, B_t, being pre-determined (issued last period) and therefore known at t−1, equation (3) could be written as follows (taking logs and letting $\log x_t^{ue} = \log x_t - E_{t-1} \log x_t$, the unexpected change in $\log x_t$)

(6) $\log b_t^{ue} = -\log R_t^{ue} - \log P_t^{ue} - \log y_t^{ue}$ [LHS of equation (3)]

$= \log (t_t - g_t)^{ue} - \log (1 + \pi_t + \gamma)^{ue} - \log(r_t^* - \gamma)^{ue}$ [RHS of equation (3)]

With all the variables in the equation defined to follow a random walk, we can rewrite the above expression as (note that for small γ $\log (1 + \pi_t + \gamma)^{ue} \approx \pi_t^{ue} = \log P_t^{ue}$)

(7) $-\Delta \log (\pi_t + r_t^*) = \Delta \log(t_t - g_t) - \Delta \log(r_t^* - \gamma)$

Using a first-order Taylor Series expansion around the sample means we can obtain a solution for $\Delta\pi_t$

[1] A 'permanent' variable $\overline{x}_t$ is by definition a variable expected not to change in the future so that
$E_t \; \overline{x}_{t+1} = \overline{x}_t$

Thus $\overline{x}_{t+1} = \overline{x}_t + \epsilon_{t+1}$
where ϵ_{t+1} is an iid error making the process a random walk.

as a function of change in government expenditure and tax rates

(8) $\Delta\pi_t = \kappa(\Delta g_t - \Delta t_t) + \eta_t$

where $\eta_t = \lambda\Delta r_t^*; \kappa = \frac{\overline{\pi}+\overline{r^*}}{\overline{t}-\overline{g}}, \lambda = \frac{\overline{\pi}+\gamma}{r^*-\gamma}$; $\overline{\pi}, \overline{r^*}, \overline{t}$ and $\overline{g}$ are mean values of the corresponding variables. The term $\eta_t = \lambda\Delta r_t^*$ is treated as an error term because we cannot observe r_t^* though we know it is a random walk and hence its first difference is random.

We can now complete the DSGE model as in Meenagh et al (2009a), by adding a forward-looking IS curve, derived in the usual way from the household Euler equations and the goods market-clearing condition, and a New Classical Phillips Curve (which we have found in Meenagh et al, op. cit., to come closest to accounting for UK inflation over the post-war period other than this episode):

(9) $(y_t - y_t^*) = -\alpha(r_t - r_t^*) + \zeta E_t(y_{t+1} - y_{t+1}^*) + v_t$

(10) $\pi_t = E_{t-1}\pi_t + \delta(y_t - y_t^*) + u_t$

Since inflation follows a random walk by assumption (so that expected inflation is simply lagged inflation), we can now establish from these two equations that:

(11) $y_t = y_t^* + \frac{1}{\delta}(\Delta\pi_t - u_t)$

(12) $r_t = r_t^* - \frac{1}{\alpha\delta}(\Delta\pi_t - u_t) + \frac{1}{\alpha}v_t - \frac{\zeta}{\alpha\delta}E_t u_{t+1}$

Thus both output and real interest rates are stationary processes around their natural rates. Both u_t and v_t may be serially correlated. Our full model consists of (8), (9) and (10).

4 Data, estimation and exogeneity testing

4.1 Time series properties of the data

We begin with some notes on the time-series behaviour of inflation and the other macro variables we are dealing with for this period (1970q4-1978q4). Table 1 shows that inflation, output, interest rates and money supply (M4) growth are all I(1).

Unit Root Tests	π_t		y_t		R_t		M_t	
	Levels	1^{st} Diff.	Levels	1^{st} Diff.	Levels	1^{st} Diff.	Levels	1^{st} Diff.
ADF Test Statistic	-2.107	-5.218	-0.906	-2.165	-1.952	-4.782	-3.177	-6.058
	(0.243)	(0.000)**	(0.772)	(0.031)*	(0.605)	(0.000)**	(0.108)	(0.000)**
PP Test Statistic	-2.127	-7.561	-1.104	-2.322	-2.129	-4.666	-3.010	-8.591
	(0.236)	(0.000)**	(0.702)	(0.022)*	(0.511)	(0.000)**	(0.146)	(0.000)**

Table 1: Test for Non-Stationarity

*Notes on Table: MacKinnon's critical values for rejection of hypothesis of a unit root: values in parentheses are p-values, while *, ** indicate significance at the 5% and 1% level, respectively. Number of lags in the ADF test is set upon AIC criterion and PP test upon Newey-West bandwidth.*

We now go on to estimate the best fitting $ARMA$ for the inflation first difference. Starting with $ARMA$ (0,0), we raise the order of the AR and MA each by one, and apply an F-test to test the validity of the lower order restriction. We find that any $ARMA$ coefficients added to a random walk are insignificant, suggesting that UK inflation first difference, $\Delta\pi_t$, may well simply be $ARMA$ (0,0), a pure random walk. However, of course it is also possible the dynamics are more complex, even if we cannot reject the simple random walk at the 5% level. We show below how the AIC varies as one raises the order- Table 2. The approach we take to the dynamics of $\Delta\pi_t$ is to examine all these $ARMA$ equations (except order 3,3 whose MA roots lie outside the unit circle), in order to achieve robustness in the face of possibly more complex dynamics.

	$AR(1)$	$AR(2)$	$AR(3)$	$MA(1)$	$MA(2)$	$MA(3)$	AIC	$\overline{R}^2$
$ARMA$(0,0)	-	-	-	-	-	-	-6.261	0.000
$ARMA$(1,1)	-0.371	-	-	0.512	-	-	-6.138	-0.009
$ARMA$(1,2)	0.270	-	-	-0.296	-0.310	-	-6.185	0.065
$ARMA$(1,3)	-0.403	-	-	0.664	-0.351	-0.791	-6.280	0.174
$ARMA$(2,0)	0.056	-0.360	-	-	-	-	-6.213	0.097
$ARMA$(2,1)	0.382	-0.367	-	-0.393	-	-	-6.173	0.088
$ARMA$(2,2)	0.421	-0.463	-	-0.441	0.115	-	-6.110	0.097
$ARMA$(2,3)	-0.666	-0.312	-	0.859	-0.020	-0.597	-6.251	0.203
$ARMA$(3,0)	-0.003	-0.374	-0.160	-	-	-	-6.179	0.103
$ARMA$(3,1)	-0.835	-0.333	-0.458	0.952	-	-	-6.186	0.135
$ARMA$(3,2)	-0.955	-0.432	-0.449	1.094	0.131	-	-6.119	0.101
$ARMA$(3,3)*	-0.599	-0.482	0.377	1.565	0.738	-1.814	-7.356	0.746

Table 2: ARMA Regressions:*=AR or MA roots outside unit circle

The fiscal variables, G/GDP and T/GDP, are shown in Figure 1. G/GDP is non-stationary: both the ADF and PP test suggest that it follows a pure random walk (Table 3), which implies that its current value is also its trend or permanent value, g_t. T/GDP is stationary around its mean with no significant deterministic trend; hence its trend or permanent value, t_t, is simply a constant. We conclude that government expenditure is the only driving force for inflation that we can observe in the data.[2]

[2]For model convergence, the amount of government expenditure is required be less than taxation for government bonds to have a positive value. We note that since government expenditure of a capital variety is expected to produce future returns in line with real interest rates, we should deduct the trend in such spending from the trend in g (derived from the data shown in the Figure 1). To implement this we assume that the average share of expenditure in the period devoted to fixed capital, health and education can be regarded as the (constant) trend in such capital spending; of course the 'capital' element in total government spending is essentially unobservable and hence our assumption is intended merely to adjust the level of the g trend in an approximate way but not its movement over time which we regard as accurately capturing changes in current spending. The adjustment for these is of the order of 10% of GDP.

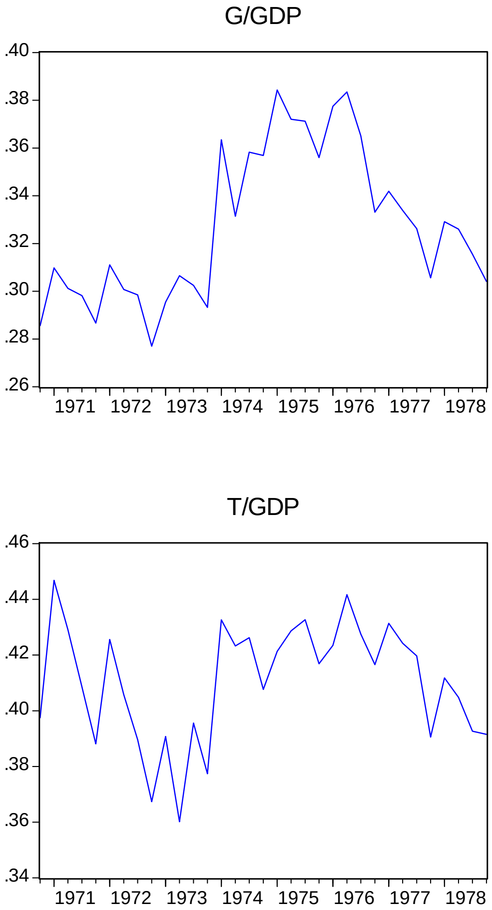

Figure 1: The patterns of government expenditure rate G/GDP and tax rate T/GDP.

Unit Root Tests	*G/GDP*		*T/GDP*	
	Levels	1^{st} Diff.	Levels	1^{st} Diff.
ADF Test Statistic	-0.814	-8.393	-2.180	-
	(0.953)	(0.000)**	(0.030)*	-
PP Test Statistic	-1.576	-8.410	-3.379	-
	(0.780)	(0.000)**	(0.001)**	-

Table 3: Test for Non-Stationarity of deseasonalised government expenditure and tax rates

*Notes on Table: MacKinnon's critical values for rejection of hypothesis of a unit root: values in parentheses are p-values, while *, ** indicate significance at the 5% and 1% level, respectively. Number of lags in the ADF test is set upon AIC criterion and PP test upon Newey-West bandwidth.*

4.1.1 Endogeneity of Money Supply

Our focus is on M4, as M0 is generally agreed to have been supplied on demand during this period and indeed generally since WWII. The question therefore we ask here is whether M4 responds to the lagged behaviour of inflation, output and interest rates, all of which should enter the demand for money; we did not attempt to estimate a stable demand for money function as this has proved elusive (see for instance Fisher and Vega (1993), Astley and Haldane (1995), Fiess and MacDonald (2001)). However our aim is narrower: to check on whether M4 responds to these minimum determinants. We found that the growth of M4 was I(1); other I(1) variables were inflation, the log of output, and the level of interest rates. Thus we checked an equation in the first differences of these variables, relating the change in M4 growth to the lagged changes in inflation, in output and in interest rates - Table 4. One can see that this equation finds highly significant feedback of money growth to these determinants.[3]

We also find below- section 7- that there is no effect of lagged money growth on the error in our model. Thus there is evidence here that money growth is endogenous.

Dependent Variable	$\Delta_2 M_t$
Variable	coefficient(std. error)
constant	-0.000(0.001)
$\Delta\pi_{t-1}$	-0.117(0.157)
Δy_{t-1}	0.133(0.048)**
ΔR_{t-1}	-0.484(0.109)**
F-test on joint significance (p-value)	0.000**
S.E.of regression	0.01
Misspecification tests (p-values)	
F-AR	0.13
F-$ARCH$	0.12
Normality	0.40
F-Het	0.41

Table 4: Money endogeneity test -standard errors in parenthesis: **=significant at 1

Notes to table: Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-ARCH is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is

[3]It might be suggested that a test should be made for whether there is an interest rate setting, Taylor rule in this period. However, as noted by Minford et al (2002), a Taylor rule equation is not identified on its own, since a variety of full models imply an equation indistinguishable from it. For example with the FTPL model here we can substitute (12) into the Fisher identity for the short-term interest rate $R_{st} = r_t + E_t\pi_{t+1}$ using that inflation is a random walk to obtain $E_t\pi_{t+1} = \pi_t$ and we obtain $R_{st} = r_t + E_t\pi_{t+1} = \pi_t + r_t^* - \frac{1}{\alpha\delta}(\Delta\pi_t - u_t) + \frac{1}{\alpha}v_t - \frac{\zeta}{\alpha\delta}E_t u_{t+1}$. The long-run cointegrating relationship here gives a unit coefficient on inflation plus a relationship with the natural real rate; dynamics add further relationships with inflation both directly and via the two errors' correlation with inflation. Hence after detrending one will obtain some relationship between interest rates and inflation; this cannot be distinguished from the Taylor rule family of relationships. One can achieve identification by specifying a full alternative DSGE model with a Taylor rule. This could be tested by indirect inference in the manner of this paper (Minford and Ou, 2009, do this for the US data post-1984); however, there are difficulties in specifying a Taylor rule for this period as it would need to permit very high annual rates of inflation (up to 50%) in the mid-1970s. Some, eg Nelson (2003), have argued that the correct rule would imply indeterminacy, hence a sunspot solution. Since the sunspot can be any number and has infinite variance, a model that includes it is simply untestable. One could take the approach of Minford and Srinivasan (2009) and impose a terminal condition to create determinacy in which case the Taylor rule could have a time-varying inflation target to accommodate the large swings in inflation. However, resolving these issues and specifying an alternative Taylor rule model for testing against the FTPL model here lie well outside the scope of this paper.

F-test for residuals heteroskedasticity.

4.1.2 Is Fiscal Policy exogenous?

We test the fiscal policy exogeneity assumption with the following equation

$$\Delta s_t = \alpha + \sum_{i=0}^{T} \gamma_i \Delta d_{t-i} + u_t \tag{1}$$

where s_t is government primary surplus as a percentage to GDP and d_t is the debt to GDP ratio. Both variables are I(1)-confirmed by both ADF and PP tests - Table 5. Thus we test whether the budget surplus responds to the public debt, with both variables in first differences –Table 6 . There is evidently no feedback from changes in the debt/GDP ratio onto the primary surplus: this is clear evidence therefore of a fiscal regime that is exogenous with respect to the state of the public finances, as assumed in the FTPL.

Unit root tests	s_t		d_t	
	Levels	1^{st} Diff.	Levels	1^{st} Diff.
ADF test statistic	-2.679	-11.251	-2.920	-1.810
	(0.251)	(0.000)**	(0.172)	(0.067)
PP test statistic	-0.963	-12.422	-3.051	-7.580
	(0.293)	(0.0000)**	(0.135)	(0.000)**

Table 5: Tests for non-stationarity of deseasonalised government surplus and debt - both in GDP ratio

*Notes on Table: MacKinnon's critical values for rejection of hypothesis of a unit root: values in parentheses are p-values, while ** indicate significance at the 1% level. Number of lags in the ADF test is set upon AIC criterion and PP test upon Newey-West bandwidth.*

	$T=1$	$T=2$	$T=3$	$T=4$
$\widehat{\alpha}$	-0.001(0.003)	0.001(0.003)	0.000(0.003)	0.000(0.003)
$\widehat{\gamma}_0$	0.407(0.215)	0.625(0.255)*	0.544(0.278)	0.497(0.316)
$\widehat{\gamma}_1$	-0.020(0.217)	-0.046(0.215)	-0.221(0.268)	-0.368(0.306)
$\widehat{\gamma}_2$	-	0.401(0.251)	0.396(0.267)	0.448(0.319)
$\widehat{\gamma}_3$	-	-	-0.165(0.262)	-0.258(0.292)
$\widehat{\gamma}_4$	-	-		0.167(0.297)
F-test on joint significance (p-value)	0.19	0.13	0.43	0.54
S.E.of regression	0.02	0.02	0.02	0.02
Misspecification tests (p-values)				
F-AR	0.25	0.30	0.44	0.01
F-ARCH	0.66	0.16	0.23	0.56
Normality	0.93	0.03	0.08	0.24
F-Het	0.64	0.83	0.13	0.13

Table 6: Fiscal policy exogeneity test- standard errors in parenthesis *=significant at 5

Notes to table: Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-ARCH is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.

5 Can FTPL account for the trend in inflation?

Our theory above implies that there is a cointegrating relation between inflation and the other arguments of (8): if one integrates (8) one obtains:

(8a)$\pi_t = \kappa g_t + \lambda r_t^* - \kappa t_t + c$

While both t_t and r_t^* are in principle random walks, we found empirically that t_t was constant during this period. As for r_t^* again it is entirely possible that permanent real interest rates moved little. Thus it is likely that the only non-stationary variable on the right hand side that moves substantially is g_t. If so one would expect government spending and inflation to be cointegrated. Figure 2 compares the pattern of inflation (π_t) and public spending (g_t): both are I(1) variables and plainly share some similarities in behaviour. We examine the relationship with an Engle and Granger (1987) cointegration test – Table 7. The stationarity of the estimated cointegrating vector $\widehat{cv} = \pi_t - \widehat{\alpha} - \widehat{\beta} g_t$ is established on ADF and PP tests, both of which reject the null hypothesis of non-stationarity. The result suggests there is a strong positive association between these two as suggested by the theory. Thus the FTPL's implication that fiscal trends drive inflation is quite consistent with the data.

	Engle-Granger(1987) Approach
Estimated equation	1970Q4-1978Q4
$\pi_t = \alpha + \beta g_t + \varepsilon_t$	
$\widehat{\alpha}$	-0.065(0.021)**
$\widehat{\beta}$	0.295(0.064)**
t-ADF test on $\widehat{\varepsilon}_t$	-4.018
p-value	0.000**
t-PP test on $\widehat{\varepsilon}_t$	-3.937
p-value	0.000**
S.E.of regression	0.01
Misspecification tests (p-values)	
F-AR	0.12
F-ARCH	0.37
Normality	0.49
F-Het	0.04

Table 7: Cointegration Analysis of inflation and public expenditure- standard errors in parenthesis **=significant at 1

Notes to table: Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for

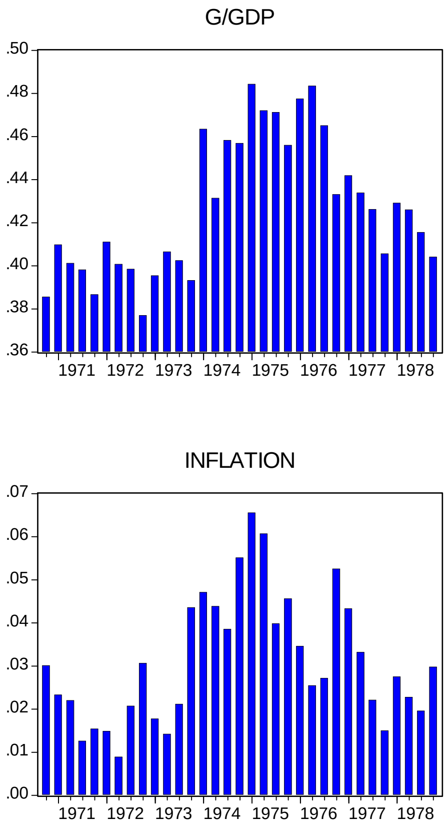

Figure 2: Patterns of inflation (π_t) and public spending (g_t)

residual serial correlation up to forth order. F-ARCH is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.

6 Inflation dynamics: bootstrapping and the method of indirect inference

We now turn to the dynamics of inflation and replicate the stochastic environment for the FTPL model to see whether our estimated dynamic equations for $\Delta\pi_t$ could have been generated by this model. This we do by bootstrapping the model above with their error processes. Meenagh et al. (2009a, 2009c) explain how this procedure is derived from the method of indirect inference. This method uses an 'auxiliary model' - such as our time-series representation here - to describe the data - see Smith (1993), Gregory and Smith (1991, 1993), Gourieroux et al. (1993), Gourieroux and Monfort (1995) and Canova (2005). The method is used here to evaluate the fit of a given structural model (rather than for estimation). This is relevant as here, when we are interested in the behaviour of a structural model whose structure is rather precisely specified by the theory.

The idea of this evaluation is to create pseudo data samples - here 1000 - for inflation. We randomly draw *i.i.d.* shocks in our error processes with replacement; we then input them into their error processes and these in turn into the model to solve for the implied path of inflation over the sample period. We then run $ARMA$ regressions of the inflation first difference on all the pseudo-samples to derive the implied 95% confidence intervals for all the coefficient values found. Finally we compare the $ARMA$ coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals: under the null hypothesis these values represent the sampling variation for the $ARMA$ coefficients which are generated by the model. The portmanteau Wald statistic - the 95% confidence limit for the joint distribution of the $ARMA$ parameters- is also computed. The Wald statistic is derived from the bootstrap joint distribution of the $ARMA$ parameters under the null hypothesis that the structural model holds. For the particular case of an $ARMA(0,0)$ we use the joint distribution of the coefficients of an $ARMA(1,1)$ to check whether it encompasses the two zero coefficients found in the data.

Figure 3 below illustrates the method for two parameters in the auxiliary equation such as in an $ARMA$(1,1). The bootstrap distribution of these two parameters under the null are shown for two cases: one where the two parameter estimates are uncorrelated, the other where they are highly correlated (with a coefficient of 0.9). The typical situation is one where they are correlated. One can think of estimation via indirect inference as changing the parameters of the structural model, thus changing the implied distribution, so as to push the observed data point as far into the centre of the distribution as possible. The test however takes the structural parameters (and hence the bivariate distribution) as given and merely notes the position of the observed data point (here given as 0.1 and 0.9) in the distribution. The Wald statistic is computed as this position expressed as a percentile; thus for example 96 indicates that the observed parameter estimates lie on the 96% 'contour', ie in the 95% rejection region.

6.1 The Indirect Inference Results

We now use the FTPL equation above for $\Delta\pi_t$ and bootstrap the random components of these Δg and η processes (since t_t is a constant, it drops out on first-differencing). We obtain 1000 pseudo-samples of $\Delta\pi_t$ then run an $ARMA$ on each of these samples to generate the distribution of the $ARMA$ parameters. The Wald statistic then tests the model at the 95% level of confidence on the basis of the complete set of $ARMA$ parameters.

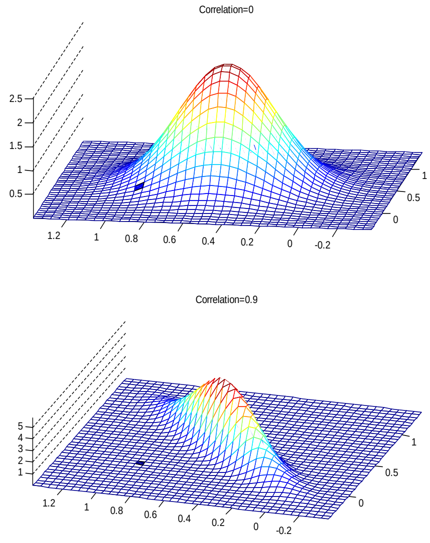

Figure 3: Bivariate Normal Distributions (0.1, 0.9 shaded) with correlation of 0 and 0.9.

6.1.1 Test results

Next we use the bootstrapped samples to compare the model with the data on its dynamic aspects - here the coefficients of the $ARMA$ for $\Delta\pi_t$. Of course we have already established that g is a pure random walk and that inflation is close to that too, which suggests that the model will generate similar dynamics. We run 1000 $ARMA$ regressions on the pseudo-samples to derive the implied 95% confidence intervals for both AR and MA coefficients. Then we compare the $ARMA$ coefficients estimated from the observed data to see whether they lie within these 95% confidence intervals. The Wald-statistic is derived from the bootstrap distribution of the $ARMA$ parameters under the null hypothesis of the model. The Wald-statistic (Meenagh 2009a, 2009c) is calculated using the following formula

$(\widehat{\gamma}-\overline{\gamma})' \sum_{\gamma}^{-1} (\widehat{\gamma}-\overline{\gamma})'$

where, $\sum_{\gamma}^{-1}$ is the inverse of the variance-covariance matrix of $\widehat{\gamma}$,the $ARMA$ parameter vector here generated by the bootstrap ($\overline{\gamma}$ is the mean of the bootstrap distribution). We arrange the values in ascending order and get the 5% critical percentile value for the model to be accepted as a whole. Table 9 lists the results of this exercise. We first show the $ARMA(1,1)$ case for illustration in Table 8.

	Model	Estimated	95% Confidence Interval		IN/OUT
			Lower	Upper	
	$AR(1)$	-0.371	-0.936	0.913	IN
	$MA(1)$	0.512	-1.052	1.379	IN
Wald statistic	16.9				

Table 8: Confidence Limits of first-differenced inflation process for Theoretical ARMA(1,1)

If we disregard the $ARMA$(3,3) as unstable, Table 9 reports that the Wald statistics for all the $ARMA$s lie inside the 95% bounds. Detailed test results can be found in Appendix C. The more elaborate the dynamics that are estimated, the closer the model gets to being rejected; but this is a normal occurrence with indirect inference. The DSGE models impose stringent theoretical assumptions on behaviour, so that the more complex the representation of the data's behaviour the less well does the model replicate that behaviour. Here we find the model very easily encompasses the random walk but encompasses less well the more complicated dynamic schemes that can be found in the data.

	Wald Statistic
$ARMA$(0,0)	0.01
$ARMA$(1,1)	16.9
$ARMA$(1,2)	21.9
$ARMA$(1,3)	76.8
$ARMA$(2,0)	85.7
$ARMA$(2,1)	67.5
$ARMA$(2,2)	45.2
$ARMA$(2,3)	58.7
$ARMA$(3,0)	80.2
$ARMA$(3,1)	94.3
$ARMA$(3,2)	84.3

Table 9: Wald Statistics for variety of ARMA representations

7 What other dynamic factors could be affecting inflation?

We have seen from the above that our theoretical model does replicate the dynamic behaviour of inflation. It is worth noting however, that it does so when the model's structural error, η_t, in equation (8) is included as implied by the model and the data. This error, which has the interpretation of omitted variables, we find to be serially correlated which implies that the theory above only works approximately because if it were exact then this error would serially uncorrelated. In the rest of this section we explore what these omitted influences on inflation may have been.

We begin by carefully identifying the time-series properties of this error. Note that since ε_t, the error in the cointegrating equation (8a), is stationary, the error in the dynamic equation (8), $\eta_t = \Delta\varepsilon_t$, is also stationary but will in general not be *i.i.d.*, rather an $ARMA$ process. Estimating it we find indeed that this is the case - Table 10. If we ignore $ARMA$s with roots outside the unit circle (asterisked) we find that the best relationship is $ARMA$(1,3).

	$AR(1)$	$AR(2)$	$AR(3)$	$MA(1)$	$MA(2)$	$MA(3)$	AIC	$\overline{R}^2$
$ARMA$(1,1)	0.115	-	-	0.262	-	-	-6.166	0.101
$ARMA$(1,2)	-0.695	-	-	1.422	0.774	-	-6.352	0.274
$ARMA$(1,3)	-0.479	-	-	1.139	0.313	-0.394	-6.363	0.302
$ARMA$(2,0)	0.408	-0.181	-	-	-	-	-6.152	0.118
$ARMA$(2,1)	1.264	-0.417	-	-0.955	-	-	-6.133	0.126
$ARMA$(2,2)	1.107	-0.272	-	-0.779	-0.190	-	-6.082	0.107
$ARMA$(2,3)	0.144	0.507	-	0.439	-0.611	-0.819	-6.295	0.297
$ARMA$(3,0)	0.392	-0.141	-0.114	-	-	-	-6.062	0.096
$ARMA$(3,1)	-0.364	0.190	-0.411	0.979	-	-	-6.279	0.293
$ARMA$(3,2)*	0.098	0.665	-0.460	0.858	-0.990	-	-6.665	0.533
$ARMA$(3,3)*	0.315	0.794	-0.440	1.527	-1.215	-2.120	-7.555	0.813

Table 10: ARMA Regressions for change in error of cointegrating vector. *= AR or MA roots outside unit circle

What now interests us is what lagged factors are influencing this error; current factors we know include all the innovations in the shocks to the economy. Our method of investigation is to regress this error as an Error Correction Mechanism on a variety of candidate variables that could influence the dynamics of inflation via the usual channels of aggregate demand and supply that are omitted from the model's inflation determination. Significant factors could suggest ways the model could be enriched dynamically in future versions. The significance of both the ϕ_i and δ –Table 11– suggest the presence of dynamic effects on inflation that the model does not capture; there is rather rapid error correction and positive reaction to lagged interest rate rises. Thus the model's dynamics could be enriched in ways that further work could investigate. Notice that money growth is insignificant, consistently with our earlier finding that it is entirely endogenous.

Estimated equation	1970Q4-1978Q4
$\eta_t = \Delta\varepsilon_t = \alpha + \sum_{i=1}^{T} \beta_i \Delta_2 M_{t-i} + \sum_{i=1}^{T} \gamma_i \Delta y_{t-i} + \sum_{i=1}^{T} \phi_i \Delta R_{t-i} - \delta\varepsilon_{t-1} + u_t$	
$\widehat{\alpha}$	-0.000(0.002)
$\widehat{\beta}_1$	0.256(0.188)
$\widehat{\gamma}_1$	-0.022(0.050)
$\widehat{\phi}_1$	0.306(0.147)*
$\widehat{\phi}_2$	0.271(0.131)*
$\widehat{\delta}$	-0.587(0.186)**
S.E.of regression	0.01
Misspecification tests (p-values)	
F-AR	0.17
F-$ARCH$	0.35
Normality	0.57
F-Het	0.83

Table 11: Omitted variable test- standard errors in parenthesis *,**= significant at 5 and 1 respectively

Notes to table: Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-$ARCH$ is an F-test for autoregressive conditional het-

eroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.

8 Conclusions

We investigate whether the Fiscal Theory of the Price Level can explain UK inflation in the 1970s, a period in which the government greatly increased public spending without raising taxes and monetary policy was accommodative; we find evidence that fiscal policy behaved exogenously with respect to the state of the public finances and that money growth behaved entirely endogenously, thus identifying the policy assumptions of the Fiscal Theory. Its implied model of inflation is tested in two ways: for its trend using cointegration analysis and for its dynamics using the method of indirect inference. We find that it is not rejected. We also find that the model's errors indicate omitted dynamics which merit further research.

References

[1] Astley, M. and Haldane, A.G. (1995) 'Money as an indicator' Bank of England Working Paper, No. 35.

[2] Bohn. H. (1998) 'The Behaviour of U.S. Public Debt and Deficits' *Quarterly Journal of Economics*, 113(3): 949-63.

[3] Buiter, W. H. (1999) 'The fallacy of the fiscal theory of the price level' CEPR discussion paper no. 2205, Centre for Economic Policy Research, London, August.

[4] Buiter, W. H. (2002) 'The Fiscal Theory of the Price Level: A Critique' *The Economic Journal*, 112: 459-480.

[5] Canova, F. (2005) '*Methods for Applied Macroeconomic Research*' Princeton University Press, Princeton.

[6] Canzoneri, M, B., Cumby, R. E. and Diba, B. T. (2001) 'Is the Price Level Determined by the Needs of Fiscal Solvency?' *American Economic Review*, 91(5): 1221-38.

[7] Carlstrom, C. T. and Fuerst, T. S. (2000) 'The fiscal theory of the price level' *Economic Review*, Federal Reserve Bank of Cleveland, issue QI: 22-32.

[8] Christiano, L. J. and Fitzgerald, T. J. (2000) 'Understanding the Fiscal Theory of the Price Level' *Economics Review*, Federal Reserve Bank of Cleveland, 36(2):1-38.

[9] Cochrane, J. H. (1999) 'A Frictionless View of U.S. Inflation' NBER Chapters, *NBER Macroeconomics Annual 1998*, 13: 323-421, National Bureau of Economic Research.

[10] Cochrane, J. H. (2000) 'Money as Stock: Price Level Determination with no Money Demand' NBER Working Papers 7498, National Bureau of Economic Research.

[11] Cochrane, J. H. (2001) 'Long Term Debt and Optimal Policy in the Fiscal Theory of the Price Level' *Econometrica*, 69(1): 69-116.

[12] Dickey, D. A. and Fuller, W. A. (1979) 'Distribution of the estimators for autoregressive time series with a unit root' *Journal of the American Statisticl Association*, 74: 427-431.

[13] Dickey, D. A. and Fuller, W. A. (1981) 'Likelihood ratio statistics for autoregressive time series with a unit root' *Econometrica*, 49: 1057-1072.

[14] Davig, T., Leeper, E. M. and Chung, H. (2007) 'Monetary and Fiscal Policy Switching' *Journal of Money, Credit and Banking*, 39 (4): 809-842.

[15] Engle, R.F. and Granger, C.W.J., (1987) 'Cointegration and Error-Correction: Representation, Estimation, and Testing' *Econometrica* 55: 251-276.

[16] Fiess, N. and MacDonald, R. (2001) 'The instability of the money demand function: an I(2) interpretation' *Oxford Bulletin of Economics and Statistics*, 63(4): 475-495.

[17] Fisher, P. F. and Vega, J. L. (1993) 'An empirical analysis of M4 in the United Kingdom' Bank of England Working Paper No. 21.

[18] Gourieroux, C. and Monfort, A. (1995) 'Simulation Based Econometric Methods' *CORE Lectures Series*, Louvain-la-Neuve.

[19] Gourieroux, C., Monfort, A. and Renault, E. (1993) 'Indirect inference' *Journal of Applied Econometrics* 8: 85-118.

[20] Gregory, A. and Smith, G. (1991) 'Calibration as testing: Inference in simulated macro models' *Journal of Business and Economic Statistics*, 9: 293-303.

[21] Gregory, A. and Smith, G. (1993) 'Calibration in macroeconomics' in G. Maddala, ed. *Handbook of Statistics*, 11: 703-719.

[22] Kocherlakota, N and Phelan, C. (1999) 'Explaining the Fiscal Theory of the Price Level' *Federal Reserve Bank of Minneapolis Quarterly Review*, 23(4): 14-23.

[23] Leeper, E. (1991) 'Equilibria Under Active and Passive Monetary and Fiscal Policies' *Journal of Monetary Economics*, 27: 129-147.

[24] Loyo, E. (2000) 'Tight money paradox on the loose: a fiscalist hyperinflation' *mimeo.*

[25] McCallum, B.T. (2001) 'Indeterminacy, Bubbles, and the Fiscal Theory of Price Level Determination' *Journal of Monetary Economics*, 47(1): 19-30.

[26] McCallum, B.T. (2003) 'Is The Fiscal Theory of the Price Level Learnable?' *Scottish Journal of Political Economy*, 50: 634-649.

[27] Meenagh, D., Minford, P. and Theodoridis, K. (2009a) 'Testing a model of the UK by the method of indirect inference' *Open Economies Review*, Springer, 20(2): 265-291.

[28] Meenagh, D., Minford, P., Nowell, E., Sofat, P. and Srinivasan, N.K. (2009b) 'Can the facts of UK inflation persistence be explained by nominal rigidity?' *Economic Modelling*, 26 (5): 978-992.

[29] Meenagh, D., Minford, P. and Wickens, M. (2009c) 'Testing a DSGE model of the EU by the method of indirect inference' *Open Economies Review*, 20(4): 435-471.

[30] Minford, P (1993) 'Monetary policy in the other G7 countries: the United Kingdom' Monetary policy in Developed Economies, *Handbook of comparative economic policies*, 3: 405–32.

[31] Minford, P., Perugini F. and N. Srinivasan (2002) 'Are interest rate regressions evidence for a Taylor Rule?' *Economics Letters*, 76(1): 145-150.

[32] Minford, P., and Ou, Z. (2009) 'Taylor Rule or Optimal Timeless Policy? Reconsidering the Fed's behaviour since 1982' working paper 2009/19, Cardiff University, www.cf.ac.uk/carbs/econ/workingpapers/papers/E2009_19.pdf

[33] Minford, P., and Srinivasan, N. (2009) 'Determinacy in New Keynesian models: a role for money after all?' working paper 2009/21, Cardiff University, www.cf.ac.uk/carbs/econ/workingpapers/papers/E2009_21.pdf, updated version Sept 2010.

[34] Nelson, E (2003) 'UK Monetary Policy 1972-1997: A Guide Using Taylor Rules' in P. Mizen (ed.), *Central Banking, Monetary Theory and Practice*: Essays in Honour of Charles Goodhart, 1: 195-216.

[35] Phillips, P. and Perron, P. (1988) 'Testing for a unit root in time series regression' *Biometrica*, 75: 333-346.

[36] Sims, C. A. (1994) 'A simple model for the study of the price level and the interaction of monetary and fiscal policy' *Economic Theory*, 4: 381-399.

[37] Sims, C. A. (1997) 'Fiscal Foundations of Price Stability in Open Economies' *mimeo*.

[38] Smith, A. (1993) 'Estimating nonlinear time-series models using simulated vector autoregressions' *Journal of Applied Econometrics*, 8: S63-S84.

[39] Tammer, E. and Ramos, A.M. (2003) 'Fiscal sustainability and monetary versus fiscal dominance: Evidence from Brazil, 1991-2000' *Applied Economics*, 35: 859-873.

[40] Woodford, M. (1996) 'Control of the Public Debt: A Requirement for Price Stability?' NBER Working Papers 5684, National Bureau of Economic Research.

[41] Woodford, M. (1998) 'Doing without money: controlling inflation in a post-monetary world' *Review of Economic Dynamics*, 1(1): 173-219.

[42] Woodford, M. (1999) 'Comment on John Cochrane, 'A Frictionless View of U.S. Inflation' in Bernanke, B. and Rotemberg, J. (eds.) *NBER Macroeconomics Annual 1998*, 390-418.

[43] Woodford, M. (2001) 'Fiscal Requirements for Price Stability', *Journal of Money, Credit and Banking*, 33(3): 669-728.

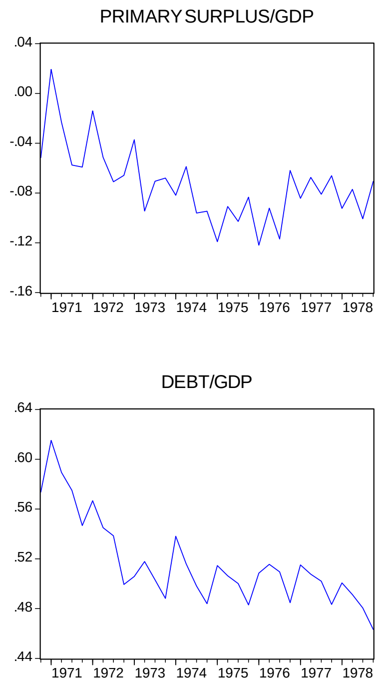

Figure 4: Patterns of Government Primary Suplus-to-GDP ratio and Debt-to-GDP ratio.

AppendixA Data and Sources

- Inflation: defined as the Consumer Price Level (CPI) deflator, $\frac{\text{Nominal Total Consumption (NTC)}}{\text{Real Total Consumption (RTC)}}$

- Government Expenditure: Total Managed Expenditure excludes debt interest payment. (TME= Total current expenditure + Net Investment +Depreciation)

- Government Revenue: Total Current Receipts

- GDP: Gross Domestic Product: chained volume measures: Seasonally adjusted

- Nominal Interest Rate: Sterling certificates of deposit: 3 months: bid rate: end period observation

- Money Supply M4: Money Stock M4 (end period), Level, Seasonally adjusted

- Primary Surplus: Difference between Government Expenditure and Revenue - Figure 4

- Debt: Public Sector Finances Net Debt (data is unavailable in quarterly frequency, authors convert) - Figure 4

Source: *UK Office for National Statistics (ONS) databank*

AppendixB Derivation of Government budget constraint

The government budget constraint gives us

$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t + \frac{B_t}{R_t}$

Where,

G_t is the government spending in money terms,

T_t is the government taxation in money terms,

R_t is the amount of nominal interest the government must pay. The value of the bonds outstanding is $B \times \frac{1}{R}$.

We can derive an expression for government budget constraint in the forward direction by substituting forwards for future bonds outstanding, yields

$\frac{B_t}{R_t} = \sum_{i=0}^{\infty} (T_{t+i} - G_{t+i}) \frac{1}{(1+R_t)^{i+1}}$

If T_{t+i} and G_{t+i} are growing with money GDP,

i.e. $T_{t+i} = t_t P_{t+i} y_{t+i}$

$G_{t+i} = g_t P_{t+i} y_{t+i}$

$\frac{B_t}{R_t} = \sum_{i=0}^{\infty} \frac{(t_t-g_t)P_{t+i}y_{t+i}}{(1+R_t)^{1+i}}$

$= \sum_{i=0}^{\infty} \frac{(t_t-g_t)P_t y_t (1+\gamma+\pi_t)^i}{(1+R_t)^{1+i}}$ (note that real output grows at rate γ)

$= (t_t - g_t) P_t y_t \sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^i}{(1+R_t)^{1+i}}$

$= (t_t - g_t) P_t y_t \sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^{1+i}}{(1+R_t)^{1+i}(1+\gamma+\pi_t)}$

If γ and π_t are both small enough,

$\sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^{1+i}}{(1+R_t)^{1+i}} = \sum_{i=0}^{\infty} \left(\frac{1}{1+R_t-\gamma-\pi_t}\right)^{1+i} = \left(\frac{1}{1-\frac{1}{1+r^*_{t}-\gamma}} - 1\right) = \left(\frac{1}{r^*_t-\gamma}\right)$

Hence, $\frac{B_t}{R_t P_t y_t} = \frac{(t_t-g_t)}{(1+\gamma+\pi_t)(r^*_t-\gamma)}$.

AppendixC Details of Indirect Inference tests

AppendixC.1 $ARMA$(1,2)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.270	-0.929	0.902	IN
$MA(1)$	-0.296	-1.215	1.233	IN
$MA(2)$	-0.310	-0.586	0.917	IN
Wald Statistic	21.9			

Table 12: Confidence Limits of change in inflation process for Theoretical ARMA(1,2)

AppendixC.2 $ARMA$(1,3)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.403	-0.947	0.948	IN
$MA(1)$	0.664	-1.343	1.329	IN
$MA(2)$	-0.351	-0.718	0.739	IN
$MA(3)$	-0.791	-0.872	0.865	IN
Wald statistic	76.8			

Table 13: Confidence Limits of change in inflation process for Theoretical ARMA(1,3)

AppendixC.3 $ARMA$(2,0)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.056	-0.361	0.360	IN
$AR(2)$	-0.360	-0.371	0.299	IN
Wald statistic	85.7			

Table 14: Confidence Limits of change in inflation process for Theoretical ARMA(2,0)

AppendixC.4 $ARMA$(2,1)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.382	-1.033	1.040	IN
$AR(2)$	-0.367	-0.423	0.344	IN
$MA(1)$	-0.393	-1.060	1.407	IN
Wald statistic	67.5			

Table 15: Confidence Limits of change in inflation process for Theoretical ARMA(2,1)

AppendixC.5 $ARMA$(2,2)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.421	-1.296	1.219	IN
$AR(2)$	-0.463	-0.953	0.713	IN
$MA(1)$	-0.441	-1.529	1.502	IN
$MA(2)$	0.115	-0.983	2.101	IN
Wald statistic	45.2			

Table 16: Confidence Limits of change in inflation process for Theoretical ARMA(2,2)

AppendixC.6 $ARMA$(2,3)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.666	-1.274	1.323	IN
$AR(2)$	-0.312	-0.911	0.745	IN
$MA(1)$	0.859	-1.584	1.511	IN
$MA(2)$	-0.020	-1.084	1.831	IN
$MA(3)$	-0.597	-0.903	0.938	IN
Wald statistic	58.7			

Table 17: Confidence Limits of change in inflation process for Theoretical ARMA(2,3)

AppendixC.7 $ARMA$(3,0)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.003	-0.383	0.364	IN
$AR(2)$	-0.374	-0.382	0.319	IN
$AR(3)$	-0.160	-0.358	0.353	IN
Wald statistic	80.2			

Table 18: Confidence Limits of change in inflation process for Theoretical ARMA(3,0)

AppendixC.8 $ARMA$(3,1)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.835	-0.977	0.995	IN
$AR(2)$	-0.333	-0.466	0.383	IN
$AR(3)$	-0.458	-0.398	0.394	OUT
$MA(1)$	0.952	-1.468	1.426	IN
Wald statistic	94.3			

Table 19: Confidence Limits of change in inflation process for Theoretical ARMA(3,1)

AppendixC.9 $ARMA$(3,2)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.955	-1.193	1.239	IN
$AR(2)$	-0.432	-0.984	0.743	IN
$AR(3)$	-0.449	-0.449	0.446	OUT
$MA(1)$	1.094	-1.624	1.568	IN
$MA(2)$	0.131	-1.099	2.129	IN
Wald Statistic	84.3			

Table 20: Confidence Limits of change in inflation process for Theoretical ARMA(3,2)