Real-World Utility of Non-Singleton Fuzzy Logic Systems: A Case of Environmental Management

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***Abstract*—The potentials of non-singleton fuzzy logic systems (NSFLSs) in dealing with uncertainties are widely known. However, their utilities and possible challenges in real-world applications, particularly beyond fuzzy controls, are still not widely examined. This paper presents some user-centric design approaches in making NSFLSs usable in a real-world problem of environmental management. In previous work, a singleton FLS was developed based on an established environmental management framework. After further investigation of the users' requirements, it was realized that the effective capture, representation and visualization of the system's inputs and outputs are critical, particularly when there are uncertainties involved in data collection and decision-making processes. For addressing the new requirements, the system has been extended to a NSFLS, so it can make use of non-singleton fuzzification in handling uncertain (e.g., noisy) environmental data. Inspired by the user-centric design of this particular system extension, the contribution of this paper is the development of some practical methods to capture/represent input/output uncertainties in NSFLSs. Subject to further users evaluation, the explained methods have potential to be employed in many similar real-world applications, thus extending the NSFLSs applicability to a wider context than the present.**

I. Introduction

FUZZY logic systems (FLSs) have been widely used in dealing with uncertainty and imprecision in practical applications ranging from human resource allocation to stock market prediction and industrial control [1]–[3]. Handling uncertain and vague information has been at the forefront of FLSs since the introduction of fuzzy sets by Zadeh [4], [5].

While Singleton FLSs (SFLSs) are the most common type of FLS, Non-Singleton FLSs (NSFLSs) [6], which are specifically designed for handling the uncertainties associated with the inputs to a FLS, also exist. A NSFLS is a type of FLS where the input uncertainty is modelled by fuzzy sets (FSs) [7], rather than singleton FSs as is the case for SFLSs. NSFLSs have shown their effectiveness in a wide range of applications including engineering, natural sciences and time-series prediction [1]–[3].

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Although the theory of NSFLSs has been established for many years (e.g., [6]), real-world application of NSFLSs are still reasonably rare in comparison to SFLSs, largely due to the associated additional computational and design complexity [8]. There are a limited number of research works dedicated to comparing the performances of singleton and non-singleton FLSs (e.g., [6], [9], [10]), generally highlighting the superior performance of NSFLSs. However, a series of practical challenges exist in real-world multi-disciplinary NSFLSs' applications (part of which apply to FLSs generally) that have limited their usage, such as the choice of input FSs (e.g. Gaussian or Triangular) and fuzzification type (type-1 or type-2) etc.

Furthermore, another challenge (not specific to, but practically important for NSFLSs) is how to quantify and/or represent the output uncertainties. In singleton systems, although the whole system captures some level of uncertainty in the form of antecedent and consequent FSs, the whole system mostly maps crisp inputs into crisp outputs. In NSFLSs, as a result of allowing users to quantify their input uncertainties, the users are more likely to expect a more effective delivery of output uncertainties.

In FLSs, particularly in NSFLSs, users rightfully expect the system outputs to indicate the level of (un)certainty encountered. While the ability to describe output uncertainty is more frequently discussed in the context of type-2 FLSs, this paper uses a case study of transitioning from a type-1 SFLS to a type-1 NSFLS, in order to highlight the challenges in such a transition for this particular environmental management application.

In a previous paper [11], we described a fuzzy logic based approach for operationalizing an established environmental conservation framework (called *value-driven framework*), which is currently being employed by the Western Australian Department of Parks and Wildlife. The presented system highlighted the challenging domain of environmental policy design, particularly in relation to the incorporation of a large number of heterogeneous and uncertain information sources. In this system, the complex and uncertain nature of relevant variables in the challenging area of environmental conservation makes fuzzy logic a highly suitable modeling approach. The early results and feedback from stakeholders and experts highlighted the capability of FSs to capture this uncertainty as well as the high interpretability of the results as key strong points of the fuzzy logic based approach.

The described system however, was designed as a singleton system. Effectively, it disregarded the uncertainties attached to the environmental data. In other words, even though it was well known that inputs to the system, such as the "size" of a population of birds, was uncertain, it was modelled as a crisp value. To address this, the system has been upgraded to a NSFLS which enables the more effective capture of actual uncertainties in the environmental data, that are frequently uncertain and/or noisy. In this paper our study on real-world suitability and implications of utilizing a NSFLS in operationalizing the environmental conservation framework is presented, with a focus on representing input/output uncertainties.

In the rest of this paper, we first provide the background on the developed environmental management FLS and the benefits of developing NSFLSs for such a purpose (Section II). Using the developed NSFLS as a case study, we will then provide the practical approaches in capturing/representing input/output uncertainties in the developed NSFLS (Sections III and IV). We then conclude and finally draw our plan for future work in Section V.

II. Background

In this section we briefly introduce some background materials for the reader, including a short brief about using FLSs for environmental management, particularly for the framework used to develop our case study FLS. Later in this section, we briefly introduce NSFLSs.

A. Environmental Management Using FLSs

1) Overview: Environmental management is highly challenging in a real world setting. Particularly challenging is the complexity of natural environments, and the related uncertainties in capturing environmental data and evaluating the resulting heterogeneous information. Often, as here, data including both biological and stakeholder views must be integrated in the decision process. It is widely shown that FLSs systematically deal with decision-making problems in uncertain and complex systems [12], [13], so the challenges of environmental management make FLSs suitable solutions in this regard [14].

Using FLSs in environmental management is not new, however not many working implementations exist. The group of decision-making support systems (DSS) that are used in environmental management is usually called EDSS [15] in the literature. In [16] for instance, a FS is used as an EDSS in the context of air pollution management. The work in [17] reviews rule-based fuzzy logic modeling of environmental information. In [18], environmental information was represented as FSs in order to classify and quantify environmental facts, as well as dealing with uncertain or missing data. Further, a fuzzy logic based approach for wetlands' classification is provided in [14]. Finally, there are studies considering the incorporation of fuzziness into geographical contexts of environmental management, such as the fuzzy logic based model of geographical extents of vegetation using remotely-sensed imagery [19].

One key difference between the works described above and our approach [11] lies in the underlying environmental management framework, which will be described briefly below.

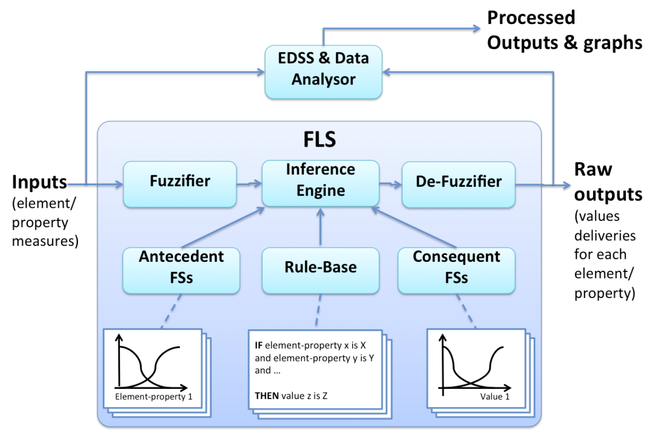


Fig. 1. The standard FLS architecture, used for environmental management

2) The Value-Driven Framework: A substantial question in any environmental management is defining the goal. Modern environmental planning frameworks such as the Value-Driven Framework [20] follow focus on the preservation and enhancement of *human values*, i.e. those required for human survival and well-being, as the main management *drivers*. The framework, further describes how to identify the human values (e.g., *Adequate Resources*), and thus focus on the *delivery* of human values, which support wellbeing, as management goals. The framework also describes how certain environmental *elements* (e.g., *Mammals*) are determined that can deliver the human values and how a set of quantifiable *properties* (e.g., *Richness*) can be assigned to each element.

A group of inputs of the framework which is focused in this paper, is the collected data from the environment, i.e. the quantification of the properties of the identified elements. Key decision outputs from this framework include the prioritization of environmental elements in order of their utility in delivering identified, priority values; information on the relative merits of various management options; the capture of key risk factors, or to assess the sensitivity of the elements expected value to the changes in the environment. These have been the main requirements for developing the FLS in [11] to operationalize the framework, which will be further described in the next sub-section.

B. The Developed FLS

In order to operationalize the described value-driven framework as a policy-making tool, we have used a FLS to structure a computational model for estimating the human value delivery of a given element (or a group of them). An overview of the system structure is shown in Fig. 1. Briefly, the FLS processes the inputs (element property measurements/assessments) using a rule-based inference engine and produces raw and processes outputs (e.g., value deliveries from elements). More details of the developed web-based FLS is explained in [11]. A practical instance with real-world data based on a current conservation exercise that is being used in Western Australia's Department of Parks and Wildlife, has also been described there.

Quantifying the elements' properties in such a FLS is where it is vital to appropriately capture the uncertainty, at least as

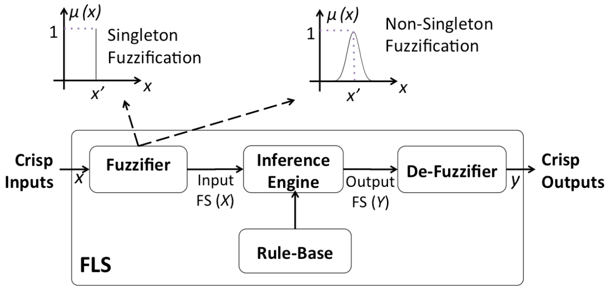


Fig. 2. FLS components and the illustration of different fuzzification methods

far or to a level as it is known within the context/discipline. In this regard, although a SFLS handles such uncertainties, more specialised types of FLSs may be used that can handle them more effectively, which makes non-singleton and type-2 FLS an intuitive choice. The developed system, in its previous state, was a type-1 FLS. This paper examines its real-world potential and challenges in this design when further developed to a NSFLS. Its further extensions towards type-2 and non-singleton type-2 FLSs are the directions of future work.

C. Non-Singleton FLSs

NSFLSs [6] are introduced in order to more explicitly address input uncertainties, compared to SFLSs. The fuzzifier block shown in classical FLSs structures (such as Fig. 3) converts a given crisp input to a *fuzzy input set*, rather than to a fuzzy singleton - as is the case in SFLSs. It is noticeable that in the both FLSs, the input (x in Fig. 3) is commonly crisp. The difference is that the crisp input is treated differently throughout the system.

In NSFLSs, the actual type of fuzzification is application-dependent, with the most common being a type of fuzzy number, i.e. a convex, normal FS. Mostly, the membership function (MF) of the input FS is evenly distributed around the crisp input [7]. In Fig. 3, a Gaussian distribution is shown as an example, such that x is located at the centre of a Gaussian distribution. In real-world applications, the input (x) may in fact not be crisp. It could be an interval or even a distribution, making fuzzification in he traditional sense redundant.

The defuzzifier component also converts a generated *fuzzy output set* (Y in Fig. 3) to a crisp number (y). However, unlike the input set, the MF of Y may not be evenly distributed around y. Different methods of defuzzification (such as centroid) are aimed to make y the best *representative* of Y. This is where a question of how best to capture the *output uncertainty* arises.

III. Real-World Representation of Input Uncertainties

The extension of the developed system from a SFLS to a NSFLS has been a result of an iterative users' requirement analysis. In our case, the users were government planning officers in Western Australia. In this section, first we explore our given set of user requirements, then the proposed NSFLS design addressing these requirements will be presented.

A. Users' Requirements

After the first development stage, the users expressed their requirements for capturing input uncertainties. The uncertainties that they wanted to be accounted for either related to errors in quantifying the environmental, such as sensor noises, or due to the natural uncertainties embedded in the captured quantities such as the continuous spatio-temporal changes in the environment.

The users expressed their requirements in having flexibility in defining the attributes of a suitable fuzzifier to any individual input. For example, the property *size* (number) of a specific tree is required to be uniformly fuzzified (i.e. with a min/max bound), but *rarity* may be best captured with a triangular or Gaussian fuzzification. The choice of input FSs by the users of the environmental management system was determined to be based on three different approaches:

1) Data-driven Approach: This approach is mainly taken by the users when historical data or data mining techniques are used for determining or predicting the investigated quantity. Since this is a statistical approach based on relatively large amount of data, the natural choice is Gaussian (Normal) Distribution for statistical modelling (if the data fits this model), which consequently leads to Gaussian fuzzification for the FLS's input. Since a Gaussian fuzzification is attributed by its mean and SD, it may be difficult to directly realize the range of the quantity's change. However, since SD is very well known in a statistical context, the users could easily map between data density and SD in Normal Distribution using the widely available Normal Distribution Tables (also called Z-table).

2) Technique-driven approach: This approach is not based on the existence of any historical data, rather it is based on the current measurements such as counts, satellite images, local sensors, etc. The uncertainties involved in such measurements are mostly leading to some bounded fuzzification, mainly in uniform, triangular or trapezoidal shapes. The advantage of such an approach is the sensible description of the uncertainty for the bounded quantities based on the limited information known, i.e. avoiding a weighting of the model which may not be warranted - as for example if a Gaussian model was applied.

3) Tailored approaches: Specifically designed applications, such as when data is collected from experts, input models here can be any type of FS. For example, see Fig ... for an example of an input model based on expert input using the Interval Agreement Approach [21].

B. Case Study: Input Uncertainties in the Environmental Management System

According to the user requirements, all the above approaches are applicable for different inputs. At this stage, the approaches (1) and (2) have been included in the system. In order to enable the system's users to provide input uncertainties in any of the two approaches, new columns were added to the user interface for each element-property tuple: *fuzzification* and *parameters* (Fig. 4). The fuzzification column determines the distribution type of the input FS, to

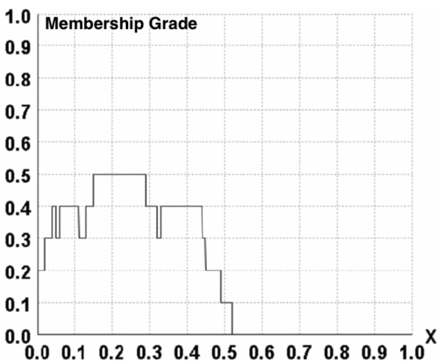


Fig. 3. Interval Agreement Approach [21] is used to make the illustrated FS for modelling expert inputs.

#	element	property	fuzzification	parameters
	Microbialites	Richness	Triangular	70,75,80
	Microbialites	Size	Singleton	5000
	Reptiles	Loss	Gaussian	15,3
	Reptiles	Rarity	Singleton	50

Fig. 4. A sample list of inputs and their uncertainty specifications in the developed NSFLS

be either a singleton (by default) or a choice among other basic distributions (Gaussian, triangular or trapezoidal). The type and parameters of each input's fuzzification can be set in the user interface shown in Fig. 5.

Having singleton fuzzification as an available choice, provides a backward-compatibility for the system to its previous singleton version. Fig. 6 shows a sample plot of the MF of the fuzzy input (of Fig. 5), which is a Gaussian fuzzification of the property *loss* of element *Reptiles*, based on mean and standard deviation calculated by the environmental experts.

IV. Real-World Representation of Output Uncertainties

When inputs are uncertain in NSFLSs, representing output uncertainties to the users of such a systems becomes even more important, as it is vital to show the variations in output uncertainty in response to variations in input uncertainties. In this section a number of approaches in representing FLS output's uncertainty, particularly in NSFLSs, are provided.

A. Overall Output Uncertainty

Measuring the output uncertainty is intuitively linked to measuring how widely the output FS is distributed around

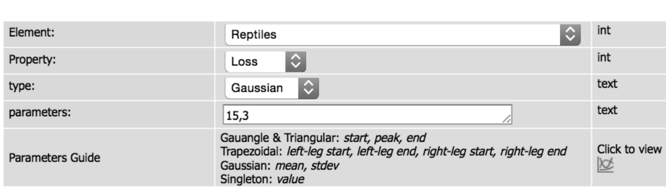


Fig. 5. The user interface used for editing input parameters for property *loss* of element *Reptile* in the NSFLS

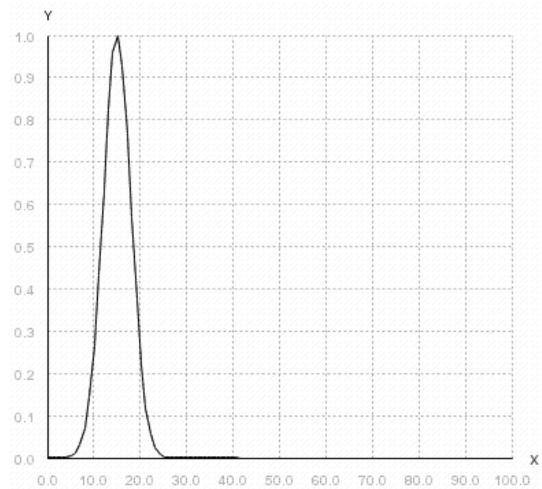


Fig. 6. MF plot related to the input illustrated in Fig. 5.

the calculated centroid. For this purpose, a similar method to calculating the standard deviation around a mean can be utilized. For this purpose, we define a random variables whose probability density function (PDF) graphically matches the normalized version of the FS's membership function. If Y (in Fig. 3) is the discrete output FS defined as $Y = \{\mu_Y(y_i)/y_i\}_{i=1\ldots n}$, and if the set of the random variable is defined as R=$\{r_i\}_{i=1\ldots n}$ with a PDF of $p_R(r)$, in such a way that:

$$p_R(r_i) = \frac{\mu_Y(y_i)}{\sum_{i=1}^{n} \mu_Y(y_i)}, \tag{1}$$

then we define *overall output uncertainty* of Y (U_Y), and define it as the standard deviation of R, i.e.

$$U_Y = \sigma_R \tag{2}$$

By the above definition, the mean of such a random variable also computationally matches the FS's centroid.

The definition is only based on a graphical match, and it clearly does not express any other (e.g., conceptual) match beyond this. While a number of alternative quantifications are also possible, some of which we review in a future publication, in this paper we focus on (2) as one example of uncertainty quantification in FLS output sets.

The overall output uncertainty is applicable to both singleton and non-singleton FLSs. For example in Fig. 7, the developed environmental management FLS has produced an output FS representing the expected value of an element, in which the overall uncertainty of the value delivery is represented as 0.2321 around the centroid (0.5876).

The overall output uncertainty is determined by the characteristics of the output FS, and calculated for any given FLSs' output (singleton or non-singleton). The next sub-section focuses on an NSFLS-specific type of output uncertainty.

B. Input-Dependent Uncertainty

Various factors (e.g. uncertainties embedded in antecedents' and consequents' FSs) are involved in calculating the overall

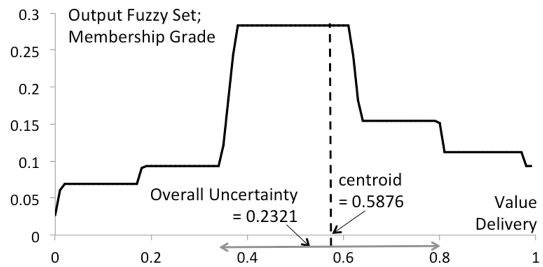


Fig. 7. A sample output FS, in which its overall uncertainty around its centroid is shown.

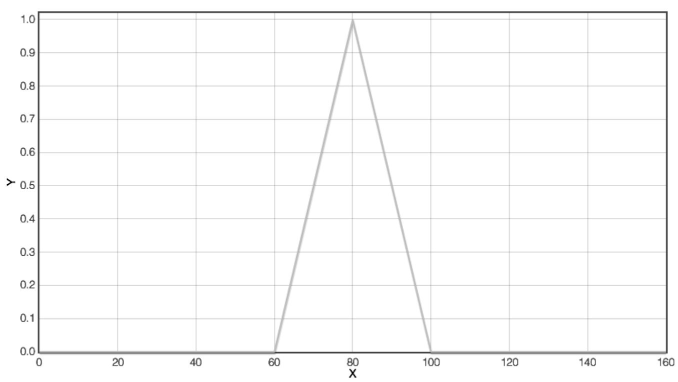


Fig. 8. A sample MF for triangular fuzzification used for the property *richness* of element *mammals*.

output uncertainty whereas the input uncertainty is one particular aspect. Plainly speaking, we need to be able to answer questions such as: "How does a variation (in uncertainty) in an input FS affect the overall output uncertainty?". It is thus an important point from the user perspective, to specify the contribution of the input fuzzification to the output uncertainty, i.e. the *Input-Dependent Uncertainty* (IDU) between each input/output pair.

To have an estimation of the IDU, we first notice that a FLS output is a highly nonlinear function of its inputs [7], so developing a closed mathematical relation for the IDU can be very complex. Instead of developing a formal relationship, our approach is to suggest a set of estimates in such a way that they are sufficiently expressive to users. Three approaches will be provided here in this regard. For each approach, an example will be provided from the developed environmental management NSFLS. To simplify the examples, the NSFLS is configured using a single human value (*knowledge and heritage*) and its relationship to a single element (*mammals*) as determined by one property of the element (*richness*), for which there is uncertainty concerning its quantification. In the provided examples, the non-singleton input is captured by a triangular fuzzification, as shown in Fig 8.

The three approaches are described here, in order of increasing complexity:

1) Min-Max Approach: In this method, the fuzzifier is temporarily set as a singleton. Two crisp extreme inputs (minimum and maximum possible values in the input's uncertainty range) are used, and the difference between the two crisp extreme outputs are used to estimate the IDU. If x is a crisp input and

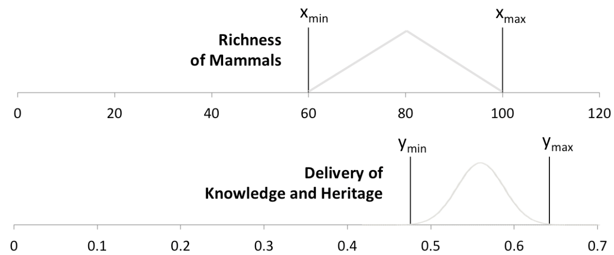


Fig. 9. The sample illustration of inputs/outputs in the min/max approach. Two extreme inputs (richness of mammals) have generated two extreme outputs (delivery of knowledge and heritage).

y is a crisp output, we measure the extreme range of output change, as $\Delta y = y_{max} - y_{min}$ where y_{min} and y_{max} are the extreme outputs. However, Δy is a range and is not readily comparable to the overall output uncertainty.

In order to convert Δy to IDU, we first notice that y_{max} and y_{min} are not the bounds of an actually generated output FS, since the only available outputs are two generated crisp numbers. Secondly, we notice that in a Gaussian distribution, the samples that are less than 3 SDs from the mean, account for 99.73% of the whole samples. By analogy, an output FS whose bounds are known can be approximately matched, for example, to a Gaussian distribution with a spread of IDU. In this case, the IDU can be estimated as:

$$IDU \approx \frac{1}{6}\Delta y \tag{3}$$

For example in the developed FLS, the SFLS (that is the NSFLS temporarily having a singleton fuzzification) is run twice with crisp inputs $x_{min} = 60$ and $x_{max} = 100$ as the two extremes of the input fuzzification range that would have been used in the real NSFLS (see Fig. 8). The generated outputs are $y_{min} = 0.4755$ and $y_{max} = 0.6422$. In this case $IDU = \frac{1}{6}\Delta y = 0.0277$. This example is illustrated in Fig. 9.

Clearly, this simplistic approach is only possible for bounded fuzzifications. Moreover, there are two aspects of disregarding the uncertainty information in this method. Firstly, the input uncertainty characteristics are mostly ignored, since an arbitrary input FS is replaced by two extreme values, such that a triangular fuzzification can generate the same result as a uniform fuzzification over the same range. Secondly, the generated output uncertainties are not explicitly considered.

2) Monte Carlo Method: While the Min-Max method disregards some valuable fuzzification information, in this method we take the input fuzzification into account, but the method still does not fully account for the output uncertainties. While the system is still a singleton FLS, a statistical model simulates the real-world input, i.e. generates random inputs in line with the fuzzification model, then the statistical model of the generated crisp outputs is examined (as known as Monte Carlo simulation). The IDU in this case is defined as the statistical SD of the output.

For example in the developed FLS, first a PDF with a symmetric triangular distribution between 60 and 100 (a normalized version of Fig. 8) is defined as:

Fig. 10. The sample (triangular) distribution of inputs (*Richness* of *Mammals*) and outputs (Delivery of *Knowledge and Heritage*) in the Monte Carlo method.

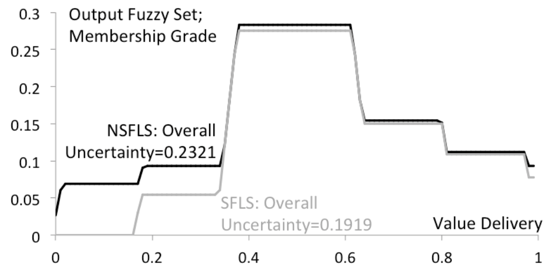


Fig. 11. The MF of the output FS (delivery of knowledge and heritage) in the designed SFLS and NSFLS.

$$p_X(x) = \begin{cases} \frac{x-60}{400} & 60 \leq x < 80 \\ \frac{100-x}{400} & 80 \leq x < 100 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Secondly, using the Inverse Transform Sampling method [22], another function $f(r)$ is defined that transforms a uniformly-distributed random number r (between 0 and 1) to the random variable x which its probability density is defined in (4). Using the method, function $f(r)$ is defined as:

$$f(r) = \begin{cases} 60 + 20\sqrt{2r} & 0 \leq r \leq 0.5 \\ 100 - 20\sqrt{2-2r} & 0.5 \leq r \leq 1 \end{cases} \tag{5}$$

Thirdly, using a uniform random number generator, the system is run in singleton mode using 100 crisp inputs generated by (5), and finally, the crisp outputs (after defuzzification) are statistically analyzed in order to calculate the IDU according. In this example, The logged outputs show a mean of 0.5874 and a SD of 0.0393. The distribution of inputs and outputs in this example are illustrated in Fig. 10. Note how the distribution reflects the shape of the input MF and made another shape for the output MF.

3) Improved Statistical Approach: Although the Monte Carlo method takes the fuzzification type into account, it still disregards the spread of the output FS by simply defuzzifying it. In the third method, both the input fuzzification and output FS characteristics are involved in the IDU estimation. We notice that in finding IDU, the part of the output's overall uncertainty that is exclusively caused by the input uncertainty is required. The IDU in this method is thus estimated as the difference between the overall output uncertainty delivered by singleton and non-singleton fuzzifications. If the overall output uncertainty calculated in singleton mode is U_Y^S and in non-singleton mode is U_Y^N, the IDU is estimated as:

$$IDU = U_Y^N - U_Y^S \tag{6}$$

IDU in this case represents how much uncertainty is "injected" into the output because of the non-singleton fuzzification. For example in our simplified FLS, if the fuzzification is a singleton at x=80 (centroid of the non-singleton input illustrated in Fig. 8), then the output FS will have an overall uncertainty of 0.1919. On the other hand, if the non-singleton fuzzification is used, the overall output uncertainty is increased to 0.2321. According to (6), the IDU estimate is 0.0402. This example is illustrated in Fig. 11.

C. Case Study: Output Uncertainties in the Environmental Management System

Although the concepts of overall uncertainty and IDU have been explored in the previous sub-sections, our example has been a simplified (single-value and single-property) system. In this sub-section, the real-world scenario of representing the overall uncertainty and IDU with real settings will be examined. Thus, a NSFLS with the full range of values, elements and their properties together with a number of rules are considered, similar to what presented in [11]. In this regard, we provide the NSFLS-specific features designed for representing the overall uncertainty and IDU.

1) Representing Overall Uncertainties: The previous (SFLS) version of the system did not provide any quantification of the outputs' uncertainties. In the current NSFLS version of the system, the users are provided with a new column (spread) along with each crisp output in the "value delivery results" page (Fig. 12). In this regard, the new column represents the overall uncertainty (called spread) based on equation (2).

As also shown in Fig. 12, a new column for spread is provided for the delivery of both "human values" and "elements' relative human values". Since the expected values are derived by averaging a number of fuzzy outputs (as described in [11]), each spread in this case is also an average of the spreads of the corresponding individual output FSs. For example, while the delivery of value *Recreation* is the average of the value deliveries among elements *Amphibians*, *Melaleuca Shrubland*, *Fungi*, etc., the spread of the value delivery is the average of the spreads of the value deliveries among the same elements. To illustrate the overall output uncertainties, the spread can also be shown as standard error bars. Fig. 13 shows such an illustration produced for the list of environmental elements.

2) Representing IDU: The user interface described in Fig. 12 does not show the IDU along with the overall uncertainty. This is because in calculating the IDU a specific input/output pair must be specified, so it cannot be a single column next to the spread in the same user interface. Another user interface of the system has already been dedicated to the input/output pairs, i.e. the sensitivity analysis interface. The sensitivity analysis interface repeatedly runs the FLS with different quantities of a single property of a single element, and shows the resulting changes in the delivery of human values. For example in Fig. 14, the sensitivity of "total human values' delivery" to the changes in the property *richness* of element *Mammals* is

Value delivery across all elements - sorted

Name	Weight	Delivery	Spread
Adequate Resources	0	0.5476	0.2268
Recreation	0.57	0.5441	0.246
Knowledge and heritage	1	0.5314	0.219

Elements' relative human value across all [weighted] values

Name	Relative human value	Spread
Terrestrial birds	0.6288	0.233
Mallee shrubland	0.6113	0.2351
Other woodlands	0.6011	0.2421
Fungi	0.5556	0.2301
Terrestrial inverts	0.5459	0.2369
Mammals	0.5454	0.2296
Muehlenbeckia	0.5449	0.2288
Reptiles	0.5385	0.2494
Salmon gum woodland	0.5346	0.2233
Samphire communities	0.5314	0.2213
Melaleuca shrubland	0.516	0.2359
Water birds	0.514	0.2286
Yate swamp vegetation community	0.4961	0.2255
Aquatic inverts	0.4431	0.2335
Amphibians	0.4326	0.2321

Fig. 12. The *spread* of each output is provided along with each numerical (crisp) output value.

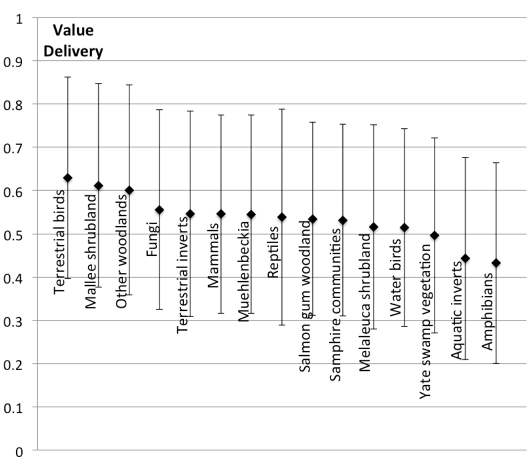


Fig. 13. The list of elements, their value deliveries (in the range between 0 and 1) and their individual uncertainty illustrated as standard error bars.

graphically shown. Each row has an extra column for showing the spread of the output uncertainty for each variation in the property *richness*. In the NSFLS implementation of the system, this interface can also be used to graphically represent the IDU, as well as to compare it with the overall uncertainty. Among the three provided methods of IDU estimation in the previous sub-section, we use the "improved statistical approach" since it takes into account both the input and output uncertainties. For each FLS run during the sensitivity analysis, the overall output uncertainty is the result of averaging the individual spreads. The IDU for the same FLS run is the average of the IDUs calculated by (6). For calculating the IDU in this method, it is also required that the system is repeatedly run as a SFLS.

Fig. 15(a) shows the sample sensitivity graph when a singleton fuzzifier at *richness*=80 is used. The spreads are represented in this figure as standard error bars. In Fig.15(b) a non-singleton fuzzifier (see Fig. 6) is used for the property

Sensitivity Analysis of Elements' Relative Human Values

Element: Mammals Property: Richness

% of actual input from : 25.0 to: 150.0 step (%): 5

Go

Actual property input for this element is set to: 80.0

% of actual input	input	Total relative human value	Spread
25.0	20.0	0.29201667380906654	0.20014
30.0	24.0	0.291832804237584	0.2
35.0	28.0	0.2948785976429088	0.20414
40.0	32.0	0.30379201177817283	0.21493
45.0	36.0	0.31865849112344247	0.22861
50.0	40.0	0.3226810173862715	0.23267
55.0	44.0	0.3286138426395056	0.2318
60.0	48.0	0.3320587623124233	0.2321
65.0	52.0	0.3332423517177142	0.23226
70.0	56.0	0.32975548737220955	0.23193
75.0	60.0	0.32652749690681393	0.23162
80.0	64.0	0.37047343588595794	0.23755
85.0	68.0	0.4284508450963188	0.23267
90.0	72.0	0.438470114776832	0.2283
95.0	76.0	0.43852400692934457	0.22807
100.0	80.0	0.43842472257121506	0.2284

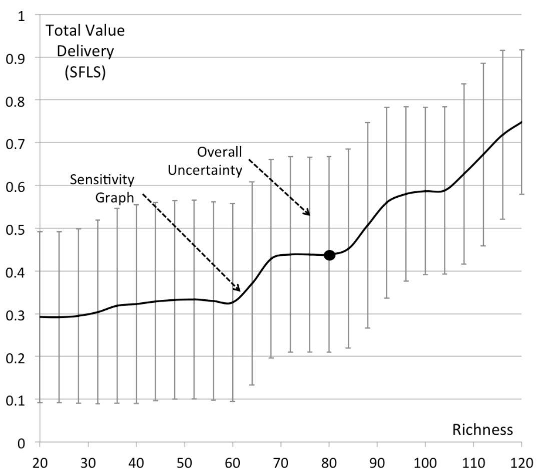
Fig. 14. The user interface designed for sensitivity analysis of the total value delivery to the variation of a single property. The column "spread" represents the overall output uncertainty.

richness. In this case, the overall uncertainty and IDU are shown as two series of standard error bars. The user can clearly see the contribution of the input uncertainty to each output, and where this contribution plays any major or minor role, which is important in the process of his/her decision making. For example in Fig. 15(b), the decision maker firstly realizes that NSFLS is more sensitive to the input change from its current state (black circle) than SFLS, so if NSFLS better captures the real-world information, the NSFLS user is more aware of the possible output sensitivity, and thus more likely to make a better decision than the SFLS user. Secondly, from the IDU bars, he/she realizes that in the middle ranges of the *richness*, the uncertainty of data collection has less importance towards the overall output uncertainty than the other parts, so the level of his/her trust on the output increases in the middle ranges.

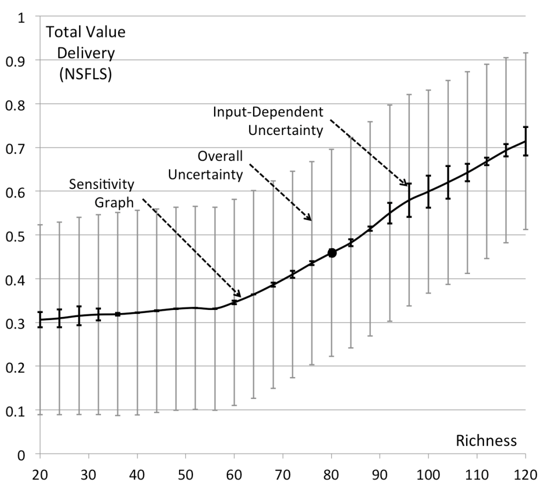
V. Conclusions and Future Work

In this paper the applied methods of representing input/output uncertainties in a practical real-world NSFLS, lead by the a user-centric design, are presented. By focusing on a FLS previously designed for environmental management [11], our aim has been to bridge between the NSFLSs functionalities and the real user's requirements in handling uncertainties, thus making the FLS more usable in uncertain conditions. This is reached not only by upgrading the system to a NSFLS, but also by representing the input/output uncertainties in some effective and usable methods.

Regarding the input uncertainty capture, the NSFLS is equipped with tools that allow the user to tune the fuzzification method according to the environmental facts (e.g. the noise generating model of a sensor). As far as the output uncertainties are concerned, firstly two types of uncertainties are defined (overall and input-dependent) that each can reveal



(a)



(b)

Fig. 15. The sensitivity graph of the total value delivery to the variation of the property *richness*; (a) in the SFLS together standard error bars representing overall uncertainty; and (b) in the NSFLS together with standard error bars representing overall uncertainty and IDUs. The black circles specify the current state of the system (*richness*=80).

a different aspect of output uncertainties, and thus help the decision makers comprehending different characteristics of output uncertainties. Secondly, a number of approaches in calculating the two defined uncertainties are presented. Finally, the described methods have been applied and tested on the designed NSFLS to make some usable output tables and graphs. While the proposed approach in uncertainty measure in NSFLSs is an early prototype, it highlights the potential of NSFLSs in this area.

Following this research, there are many possibilities to go forward. This firstly includes a more accurate estimation of the output uncertainties, by exploring alternative measures. Secondly, more advanced visual representation of the output uncertainties are possible, such as three-dimensional views where the MF of the output FSs can be the third dimension of the produced input/output graphs (such as in the sensitivity analysis). It is noticeable that if the output is an aggregation of some different output FSs (as it is the case in the studied FLS of environmental management), it will be necessary to involve fuzzy arithmetic methods [23]. Finally, a major upgrade of the designed NSFLS will be to extend it to a type-2 NSFLS, in which more uncertainty aspects can be captured from the environment and/or represented to the users.

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