Government interventions in banking crises: 
Assessing alternative schemes in a banking model 
of debt overhang

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Abstract

We evaluate policy measures to stop the fall in loan supply following a banking crisis. We apply a dynamic framework in which a debt overhang induces banks to curtail lending or to choose a fragile capital structure. Government assistance conditional on new banking activities, like on new lending or on debt and equity issues, allows banks to influence the scale of the assistance and to externalize risks, implying overinvestment or excessive risk taking or both. Assistance granted without reference to new activities, like establishing a bad bank, does not generate adverse incentives but may have higher fiscal costs.

Keywords Banking crisis, debt overhang, bank lending, capital structure.

JEL Classification G01, G21, G28
1 Introduction

The recent financial crisis has led to large devaluations of assets which have eroded the capital basis of banks. As a consequence, many banks have suffered from a debt overhang as it forms an obstacle to raising fresh funds for new businesses. In this paper, we evaluate different policy measures to stop the fall in bank loan supply following a banking crisis.

Starting with Myers (1977), it has been argued in the corporate finance literature that a debt-laden firm may be unable to raise further funds, even for projects with positive NPV. As regards banks, a debt overhang is special because, in contrast to nonbanks, a default on pre-existing short-term debt may trigger a bank run.\(^1\) Such a bank run, however, creates externalities not only because new lending of a single bank is impeded. It may also result in a systemic failure.\(^2\) Both effects have determined much of the real costs of past banking crises (Dell’Ariccia et al, 2008). Policy makers, therefore, face a set of interrelated problems. One is to deal with the inability of banks to finance new valuable investment projects. Another is to prevent systemic spillover effects that impend when an important financial institution fails. Finally, for a measure being sustainable, banks should be assisted in a way that does not encourage further excessive risk taking.

In order to evaluate the effectiveness of different policy measures, we apply a dynamic model in which a sudden illiquidity of (some) assets causes a debt overhang for banks. A bank responds either by curtailing new loans or by choosing a fragile capital structure, a feature that many banking crises have shared (Laeven and Valencia, 2008). We argue that government assistance conditional on new banking activities allows banks to influence the scale of the assistance and to externalize risks, which results in excessive risk taking or overinvestment or both. By contrast, granting assistance without reference to new activities does not generate these adverse incentives.

\(^1\)This aspect has largely been neglected so far. For example, although dedicated to banks, the analysis by Philippon and Schnabl (2009) is applicable to non-financial and financial firms alike.

\(^2\)Systemic risks may stem from contractual linkages (Allen and Gale, 2000), from identification problems in funding markets (Freixas et al, 2000), from a liquidity-solvency spiral (Diamond and Rajan, 2005), or because shocks are common to all banks (Acharya, 2009). The specific reason for systemic risks is not explicitly considered here. Instead, we take the existence of systemically important bank liabilities and the need to protect them as granted.
These results are derived from a banking model based on work by Diamond and Rajan (2000, 2001). They argue that a banker has an informational advantage in collecting loans and that this advantage allows her to extract an information rent. Deposits are useful to prevent such rent extraction. They serve as a commitment device because depositors will run on the bank whenever a banker is suspected to misbehave. When asset values are risky, however, deposits imply that a bank would sometimes be run even without any misbehavior of the banker. Therefore, a banker will also issue non-deposit debt or equity shares, which serve as a buffer in bad times but allow her to withhold some rents in good times.

In this setting, we consider a banker who has to repay outstanding deposits at some date. In the past, these deposits have been used to refinance bank assets. These assets, however, have turned out to be non-performing. Their market liquidity has dried up so that selling them does not generate any immediate return. Funding liquidity is also weak. The reason here is that a banker cannot borrow against the full future value of non-performing assets, because their returns are uncertain, so that the banker, to some extent, relies on non-deposit debt or equity shares. As a result, the bank may be insolvent if outstanding deposits are too large compared to what the banker can raise by borrowing against non-performing assets.

These liquidity and solvency problems for a bank form an obstacle to new business loans and to bank stability. Stability of the bank not only requires to serve outstanding deposits but also to issue only little new deposits as the prospective asset returns are uncertain. Though non-deposit debt and equity shares buffer against this risk, they do not allow the banker to raise much because of their lacking disciplinary effect. Hence, the higher the volume of outstanding deposits, the more limited is new lending when the banker wants to keep the bank stable. When the banker chooses to issue more new deposits, she opts for a fragile capital structure and puts the existence of the bank at risk, but at least can commit to repay more if everything goes well. This commitment eases the funding constraint for new loans, although it is associated with financial instability.

How do different policy measures influence a bank’s lending behavior, capital structure and risk taking in this situation? We consider deposit guarantees, capital injections,
and bad banks. We argue that although government interventions are needed to mitigate the externalities of a debt overhang, success crucially depends on their terms. When the goal is to re-establish the intermediation function of banks in order to reduce shortages in the supply of new loans and to restore financial stability, government should not support new banking activities, neither directly nor indirectly. Instead, government should assume the overhanging debt of banks. The reason is that different measures of subsidizing banks in their efforts to raise new funds will in many cases result in overinvestment and in financial fragility. Government intervention may result in future instability of the bank, not because of incentives for banks to take excessive risk in anticipation of a future bailout (as in Nier and Baumann, 2006; Panageas, 2010). We exclude repeated bailouts. Instead, excessive risk taking may occurs because overinvestment and financial fragility are interlinked. When banks are incited to overinvest, they have to raise more funds. As the marginal source of funding is deposits, banks issue more deposits than compatible with safety.

Based upon these insights, we uncover the underlying, more general problem with government interventions in banking crises. The debt overhang, which hampers new business lending, is the result of past lending behavior and capital structure decisions. In order to release new lending from the debt burden, government could either cope with the existing debt or support new lending. These two options are not equivalent, however. Only with the first option, government does not make its financial assistance depending on new contracts, and thus does not incite banks to externalize risks that lead to excessive risk taking and overinvestment.

By exploring the effects of a bank capital crunch on loan supply, this paper is related to Peek and Rosengren (1995) and Holmström and Tirole (1997). In contrast to these studies, however, we also focus on the question of how policy measures should be shaped in order to mitigate the real costs of banking crises. Moreover, while Holmström and Tirole (1997) conclude that a recapitalization of banks with taxpayer money can improve banks’ incentive to monitor loans, we argue that this may also imply exces-

\(^3\)Monetary policy measures are excluded from the analysis. Although these measure are helpful in overcoming pure liquidity problems, they can at best collaterally support fiscal measures in the presence of insolvency problems, see Diamond and Rajan (2005).
sive risk taking and overinvestment. Diamond (2001), Demirgüç-Kunt and Detragiache (2002), and Flannery (2009) study the effects of single policy measures but do not compare them as we do in this paper. Aghion et al. (1999) and Mitchell (2001), who compare different policy measures, do not consider the dynamic linkage between bank lending and capital structure.

The paper is organized as follows. In section 2, we present the benchmark model. In section 3, we fit some recently applied policy instruments in the model in order to analyze their likely effects on bank lending and risk taking. In section 4 we further discuss the results and in section 5 we briefly summarize our findings.

2 A banking model of debt overhang

2.1 Agents and technologies

We consider a bank that is run by a banker. While the banker has no funds on her own, there are many investors endowed with plenty of funds. All agents are risk neutral and have access to a safe investment opportunity with a zero rate of return.

At date $t$, the banker has to repay outstanding deposits $D_0$. In the past, these deposits were used to originate loans supposed to be collected at $t$. However, in the meantime the economic environment has worsened unexpectedly and these legacy assets have turned out to generate nothing at $t$. Some assets are completely written off, others only delay. The latter will earn an uncertain return at date $t + 1$ if the bank survives until then. With probability $p$, the economy has recovered at $t + 1$ and legacy assets pay $x_g > 0$ at this date. If the economy fails to turn upward again (with probability $1 - p$), the return will be only $x_b \in [0, x_g)$.

Also at $t$, the banker faces new lending opportunities. Let $L$ denote the volume of new business loans. They will earn $RL$ at $t + 1$ if the economy recovers and nothing otherwise. With $pR \in (1, 2)$, expected returns per unit of new loans cover the initial outlay but are not too large. The banker can also invest $C$ in a safe asset, which will pay $C$ at $t + 1$. 
For all bank assets to generate a return, specific skills are required. Investors do not have these skills so that only the banker can collect loans and other bank assets (Diamond and Rajan, 2001). Maintaining these collection skills causes private costs to the banker. Managing the safe asset is not very demanding, and the costs of managing old loans are predetermined at date \( t \). Hence, the variable costs for these two asset classes are negligible. By contrast, the more new loans the banker grants the more complex and demanding the task of managing them will be. Total costs \( c(L) \) are thus an increasing and convex function of \( L \) with \( c(0) = c'(0) = 0 \) and \( c''(L) > 0 \). These costs are non-verifiable. The first-best lending volume \( L^{fb} : pR - 1 = c'(L^{fb}) \) is thus unknown to an outside third party, so that although a supervisor can observe the actual lending volume, it cannot tell whether it is the efficient one.\(^4\)

### 2.2 Bank capital structure

The banker has an informational advantage in collecting the returns on bank assets. This advantage gives her some leeway to renegotiate contractual payments to investors by threatening to withdraw her skills. The extent to which the banker can renegotiate ex post, and thus the banker’s ability to raise funds ex ante, depends on the bank’s capital structure.

Deposits serve as a commitment device for a banker (Diamond and Rajan, 2001). They are payable on demand based on a first-come-first-served principle. If the banker tried to renegotiate deposits, every single depositor would run on the bank and demand the liquidation of assets until being fully paid. Since assets are assumed to have (almost) no liquidation value, a run destroys the value of the bank. It is the threat of being run that effectively prevents the banker from renegotiating with depositors. The disadvantage of deposits is that they cannot be renegotiated even when the value of the bank’s assets slumps down for reasons outside the banker’s control. A bank run, although inefficient, cannot be avoided then.

\(^4\)Assuming that not only new loans and legacy assets but also safe assets require specific skills is not necessary for our results to hold. However, treating all assets symmetrically reduces complexity.
The banker can also issue non-deposit debt and equity shares. Both claims are junior to deposits and not subject to a first-come-first-served constraint. Hence, debt and equity can be renegotiated by the banker in bad times and thus act as a buffer against low loan earnings. However, the banker can threaten to withhold her skills when the total value of loans, legacy assets, and the safe asset exceeds deposits. This threat allows her to extract a personal rent. Adapting the approach of Diamond and Rajan (2000), shareholders and debtholders could force the banker to pay them half of the returns on all bank assets net of deposits. Hence, the banker can pledge to them only a share $\lambda \leq 0.5$ of her earnings net of deposits and diverts the remaining share $1 - \lambda$. The ability to extract a rent at $t + 1$ eventually restricts the amount of funds she can raise at $t$.\footnote{If $\lambda = 0.5$, the banker issues equity shares with shareholders being always repaid half of the asset returns net of deposits. If $\lambda < 0.5$, the banker issues non-deposit debt with debtholders being repaid either some fixed amount or at most half of the asset returns net of deposits, depending on which amount is smaller.} Abusing terminology slightly, we henceforth refer to both, the holders of equity shares and the holders of non-deposit debt as shareholders.

### 2.3 Restrictions on bank loan supply

The banker can avoid an immediate run at $t$ only by repaying the existing deposits $D_0$. Accordingly, if she plans to extend new loans $L$ and to hold some amount $C$ in the safe asset, the banker needs to raise a total of $D_0 + L + C$ at $t$.

The liquidity that a banker can raise at $t$ is limited to the amount she can borrow against prospective asset values at this date. Let $D_1$ denote the face value of new deposits issued at $t$. If $D_1 \leq x_b + C$, the banker is able to repay $D_1$ to new depositors at $t + 1$ irrespective whether the economy recovers or not. There is thus no risk of a run at $t + 1$ and the banker operates in a safe mode. Accordingly, the new shareholders, whose share in the bank’s net earnings is $\hat{\lambda}$, expect a payment of $\hat{\lambda} \left( E[x] + pRL + C - D_1 \right)$. The budget constraint of the banker reads

$$D_0 + L + C = D_1 + \hat{\lambda} \left( E[x] + pRL + C - D_1 \right)$$ (1)
in the safe mode. The banker can increase her safe assets $C$ simply by issuing more deposits $D_1$ one-to-one. There is thus no limit to safe assets, and they do not influence the banker’s ability to raise funds for new business loans.

However, since operating in a safe manner requires $D_1 \leq x_b + C$, it also follows from (1) that the new lending volume is constrained. This constraint is least tight when $\lambda = \frac{1}{2}$, which yields

\[
L \leq \frac{E[x] + x_b - 2D_0}{2 - pR} =: L_{\text{max}}(D_0). \tag{2}
\]

Intuitively, in order to keep her bank run-proof, the banker can issue only equity shares (or non-deposit debt) against future returns on new business loans. They have the highest value if $\lambda = \frac{1}{2}$. However, as $\frac{1}{2}pR < 1$, equity shares are worth less than one dollar per dollar of new loans. Consequently, the banker will obtain sufficient funds for extending new loans only if she borrows against legacy assets as well. The funding liquidity of these assets is limited. Therefore, the volume of new loans is limited, too. The banker can grant more new loans when the expected future value $E[x]$ of legacy assets is high. A larger loan portfolio is also possible, the higher is $x_b$, since then she is able to issue more new deposits (as a commitment device) without risking a run. Finally, more loans are feasible when the existing debt burden $D_0$ is small. It follows from (2) that, in the safe mode, $L \geq 0$ if

\[
D_0 \leq \frac{E[x] + x_b}{2}. \tag{3}
\]

That is, the banker can operate in the safe mode only if outstanding deposits are not too large.

The banker can also choose to operate in a risky mode. There, she issues deposits with face value $D_1 \in (x_b + C, x_g + RL + C]$ so that deposits will be repaid only if the economy recovers. If the economy does not recover, deposits exceed the value of the bank’s assets so that depositors will run and demand immediate repayment of their deposits.\(^6\) Hence, depositors expect to obtain $pD_1$ at $t + 1$ while shareholders expect

\(^6\)This run is socially inefficient. Although some loans turn out to be worthless in the bad state, other assets will melt down only because of the run. The loss in bank asset values owing to a run in the bad state determines the real social cost of operating in the risky mode.
\[ p \lambda (x_g + RL + C - D_1). \] The budget constraint reads

\[ D_0 + L + C = pD_1 + p \lambda (x_g + RL + C - D_1). \] (4)

In contrast to the safe mode, the banker cannot increase her safe assets \( C \) by increasing deposits \( D_1 \) one-to-one. Raising one additional dollar in deposits at \( t \) requires to repay more than one dollar in the good state at \( t + 1 \) because the bank will be run in the bad state. Hence, putting an additionally raised dollar in the safe asset alone does not suffice to meet the depositors claim in the good state at \( t + 1 \). Instead, the banker must cross-pledge earnings from new business loans, too. The safe asset is thus detrimental to the banker’s ability to raise funds for new loans in the risky mode.

The risky mode differs from the safe mode in another important aspect. When operating in a risky manner, a banker could issue deposits with face value \( R \) per unit of new loans. Depositors would then expect to receive \( pR > 1 \). Accordingly, they would be willing to provide more than one dollar per unit of new loans, implying that there is no upper bound on new business loans \( L \) in the risky mode.

### 2.4 Implications

New lending is linked to the future stability of the bank. Only when new loans fulfill restriction (2), the banker can avoid a bank run. Once the banker has decided to make new loans in a safe manner, loans \( L \), safe assets \( C \), deposits \( D_1 \) and the share \( \lambda \) are chosen to solve

\[
\max_{L,C,D_1,\lambda} \left( (1 - \lambda) (E[x] + pRL + C - D_1) - c(L) \right)
\]

\[
s.t. (1), \quad D_1 \in [0, x_b + C], \quad \lambda \in \left[0, \frac{1}{2}\right], \quad and L, C \geq 0,
\] (5)

with \((1 - \lambda) (E[x] + pRL + C - D_1)\) being the amount the banker can withhold from shareholders. Similarly, if the banker has decided to lend in a risky manner, the opti-
max_{L,C,D_1,\lambda} \left[ p (1 - \lambda) (x_g + RL + C - D_1) - c(L) \right]
\text{s.t. (4), } D_1 \in (x_b + C, x_g + RL + C], \; \lambda \in [0, \frac{1}{2}], \text{ and } L, C \geq 0.

We obtain

**Proposition 1** For a given outstanding amount of deposits $D_0$, the banker

1. operates in a safe manner and invests according to

   \[ L^* = \begin{cases} 
   L_f^b & \text{if } D_0 \leq \frac{E[x] + x_b - (2 - pR)L_f^b}{2} \\
   L_{\max}(D_0) & \text{if } D_0 \in \left( \frac{E[x] + x_b - (2 - pR)L_f^b}{2}, \min\left\{ D_0, \frac{E[x] + x_b}{2} \right\} \right) 
   \end{cases} \]

2. operates in a risky manner and invests according to

   \[ L^* = L_f^b \text{ if } D_0 \in \left( \min \left\{ D_0, \frac{E[x] + x_b}{2} \right\}, px_g + (pR - 1)L_f^b - c(L_f^b) \right) \]

3. is run already at $t$ if $D_0 > px_g + (pR - 1)L_f^b - c(L_f^b)$,

where $\overline{D}_0$ is the outstanding amount of deposits for which

\[ \left[ (pR - 1)L_f^b - c(L_f^b) \right] - \left[ (pR - 1)L_{\max}(\overline{D}_0) - c(L_{\max}(\overline{D}_0)) \right] = (1 - p)x_b. \]

**Proof.** See appendix. \qed

With a small outstanding amount of deposits, the banker grants loans according to the first best and operates in a safe manner. The reason is twofold. First, the restriction on loan supply (2) is not binding. Second, expected repayments to investors do not depend on the bank’s mode of operation while earnings from legacy assets contribute to her rents in both states only when she opts for the safe mode of refinancing.

With a moderate outstanding amount of deposits, restriction (2) is binding, and the banker faces the following trade-off when deciding on the operation mode. On the one hand, operating in the safe mode requires to cut loan supply to $L_{\max} < L_f^b$. This reduces the banker’s expected future loan earnings by an amount given by the LHS of (9), with
the fall in loan earnings being larger the higher is the outstanding amount of deposits $D_0$. On the other hand, operating in the risky mode does not require to reduce loan supply. However, the run in the bad state, occurring with probability $(1 - p)$, implies that the return $x_b$ of the legacy assets is lost (see RHS of (9)). The banker thus decides in favor of the safe mode and of reducing her loan supply when the safe mode is feasible (see (3)) and when underinvestment in the safe mode reduces her rents by less than the foreseeable run in the risky mode (see (9)). When the debt overhang problem is more severe, the banker pursues the risky strategy as long as this does not imply a negative expected profit for her. If the outstanding amount of deposits cannot be covered by the expected asset values (net of financing and portfolio management cost) the bank will be run already at $t$.

A further implication is that access to safe assets does not ease the banker’s financial constraint with respect to new loans. In the safe mode, marginal financing costs for safe assets are equal to their marginal returns. Hence, financing safe assets can be separated from financing risky loans so that safe assets do not influence the lending decision of a debt-laden banking firm operating in a safe manner. In the risky mode, safe assets are no longer safe from the perspective of depositors as the bank has chosen a fragile capital structure. Since a run occurs with probability $(1 - p)$, marginal returns on safe assets are only $p$. With new business loans being more profitable on expectation and safe assets crowding out finance for new loans, the banker does not invest in safe assets but focuses on new risky loans.

3 Effects of government interventions

As argued in the preceding section, unanticipated changes in the risk-return structure of bank assets may result in a debt overhang that forces banks either to reduce new lending or to jeopardize their financial stability. The externality created by the debt overhang, therefore, is two-dimensional and so is the objective of government interventions. First, government should enable banks to provide the efficient level of intermediation services. Second, in so doing, banks should not be incited to take on excessive risks. In
this section, we analyze how banks change their lending behavior and capital structure in response to different policy measures. We consider capital injections, deposit guarantees, and bad banks. In the 2007-09 crisis, these interventions were among the most frequently taken measures around the globe (Praet and Nguyen, 2008; Panetta et al, 2009).

3.1 Capital injections

Capital injections can take two forms. Either the government makes the assistance conditional on new activities of the bank, e.g., on new lending or on new equity or deposit issues. Or the government provides taxpayer money independently of new banking activities through, e.g., lump sum transfers or transfers conditional on the existing debt burden of a bank. Both forms of capital injections have in common that they foster new lending only if the government does not require the banker to fully pay them back. Otherwise they merely crowd out private funds. However, the two forms of capital injections differ with respect to their contributions to the government’s policy objective.

3.1.1 Unconditional on new activities

A lump sum subsidy $S$ granted by the government to the banker is a good illustrative example for financial assistance that is independent from new banking activities. In principle, this transfer could be used to repay (some of) the old depositors—a straightforward way to free a bank from its debt overhang as the government directly assumes responsibility for the outstanding deposits $D_0$ (or parts of it). A standard result in public finance is that a lump sum transfer does not alter first-order conditions and is thus efficient. Hence, we expect a lump sum transfer not to change a banker’s investment incentives. However, as we will show, it does affect a banker’s decision on capital structure and thus the stability of the bank.

Consider a bank which operates in a safe manner at first. If the government pays a lump sum $S$, the banker’s budget constraint (1) modifies to

$$D_0 + L + C = D_1 + \lambda (E[x] + pRL + C - D_1) + S.$$  \hspace{1cm} (10)
This budget constraint together with $D_1 \leq x_b + C$ and $\lambda = \frac{1}{2}$ implies

$$L \leq \frac{E[x] + x_b - 2(D_0 - S)}{2 - pR} =: L_{\text{max}}(D_0 - S).$$  \tag{11}$$

Hence, the higher is the subsidy, the higher is the volume of new loans that is feasible if the banker wishes to operate safely. The banker is able to make new loans in a safe manner only if

$$D_0 - S < \frac{E[x] + x_b}{2}. \tag{12}$$

Comparing with the benchmark model in the previous section, we see that in the safe mode an outstanding amount of deposits $D_0$ and a subsidy $S$ is equivalent to an outstanding amount $D_0 - S$ and no subsidy.

For the risky mode, in which the budget constraint of the banker changes to

$$D_0 + L + C = pD_1 + p\lambda (x_g + RL + C - D_1) + S, \tag{13}$$

similar conclusions can be drawn. Hence, we obtain

**Proposition 2** When government offers a lump sum subsidy $S$, a banker with an outstanding amount of deposits $D_0$

1. operates in a safe manner and invests according to

$$L^* = \begin{cases} 
L^{fb} & \text{if } D_0 - S \leq \frac{E[x] + x_b - (2 - pR)L^{fb}}{2 - pR} \\
L_{\text{max}}(D_0 - S) & \text{if } D_0 - S \in \left(\frac{E[x] + x_b - (2 - pR)L^{fb}}{2 - pR}, \min\left\{D_0, \frac{E[x] + x_b}{2}\right\}\right)
\end{cases} \tag{14}$$

2. operates in a risky manner and invests according to

$$L^* = L^{fb} \text{ if } D_0 - S \in \left(\min\left\{D_0, \frac{E[x] + x_b}{2}\right\} \cdot p x_g + (pR - 1)L^{fb} - c(L^{fb})\right) \tag{15}$$

3. is run already at $t$ if $D_0 - S > p x_g + (pR - 1)L^{fb} - c(L^{fb})$. 

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where $D_0$ is the outstanding amount of deposits for which

$$\left[ (pR - 1)L^{fb} - c(L^{fb}) \right] - \left[ (pR - 1)L_{\text{max}}(D_0 - S) - c(L_{\text{max}}(D_0 - S)) \right] = (1 - p)x_b.$$  

(16)

**Proof.** See appendix. ■

A lump sum subsidy $S$ mitigates the debt overhang problem without distorting the lending incentives of the banker. It just implies that a banker with an outstanding amount of deposits $D_0$ behaves as if she had a debt burden $D_0 - S$ without being subsidized. Accordingly, only the incentives to take the risky mode change whereas the marginal conditions for lending do not, leading to the following conclusions. First, banks that would grant loans according to the first-best loan volume in a safe manner even without the subsidy continue to do so when they are subsidized. Second, banks that would operate in a safe manner but underinvest without a subsidy are enabled to lend more. Third, no bank operating in the risky mode has an incentive to overinvest and thus to additionally jeopardize the future stability of the banking system. Fourth, some banks are even incited to abandon the risky mode, so that they operate in the safe mode and underinvest instead. Finally, some banks, which were unable to operate at all without assistance because of severe debt overhang problems, are enabled to grant new loans although only in a risky manner.

The safe asset only serves to absorb public money in excess of what is needed for the first-best loan volume. For example, if the subsidy net of outstanding deposits $(S - D_0)$ exceeds the efficient loan volume $(L^{fb})$, the banker does not have to issue any new deposits, non-deposit debt or equity shares $(D_1 = \lambda = 0)$. However, waiving (parts of) the subsidy is not rational from the banker’s perspective. She will take the total subsidy and, if necessary, invest it in the safe asset.

### 3.1.2 Conditional on new activities

In the world financial crisis, governments in many countries made capital injections conditional on new banking activities. In Japan, for example, the Financial Services Agency required banks to lend out some minimum amounts to small and medium-sized
enterprises in exchange for public assistance. In the UK, public funds were injected into Lloyds Banking Group with the proviso that the bank will expand its loan portfolio by some GBP 28bn within two years. In Germany, the state-owned KfW–Kreditanstalt für Wiederaufbau co-finances bank loans to small and medium sized enterprises, assuming some 90 percent of the default risk. In the US, banks that have participated in public rescue programs have been required to prefer Americans in any future personnel decisions. In these countries, governments also issued guarantees for new non-deposit debt. Although all these forms of conditional interventions differ, they have similar effects. Hence, we can focus on conditional capital injections that directly or indirectly rely on the volume of new loans, which are among the most commonly applied measures.

Let $\gamma$ be the parameter that matches taxpayer money with new loans with $\gamma < 1 - \frac{1}{2} pR$.\footnote{Restricting to $\gamma < 1 - \frac{1}{2} pR$ simplifies the analysis without changing results qualitatively.} When the banker pursues a strategy of safeness ($D_1 \leq x_b + C$), she raises $D_1$ from new depositors and $\lambda (E[x] + pRL + C - D_1)$ from shareholders, and obtains $\gamma L$ as a subsidy from the government. The budget constraint thus reads

$$D_0 + L + C = D_1 + \lambda (E[x] + pRL + C - D_1) + \gamma L. \quad (17)$$

New lending $L$ is again least constrained when $\lambda = \frac{1}{2}$, for which (17) along with $D_1 \leq x_b + C$ leads to

$$L \leq \frac{E[x] + x_b - 2D_0}{2(1 - \gamma) - pR} =: L_{\text{max}}^{\text{ccs}}(D_0) > L_{\text{max}}(D_0). \quad (18)$$

For a given $D_0$, the subsidy allows to refinance more loans, and the maximum volume of loans $L_{\text{max}}^{\text{ccs}}$ is higher, the higher is the level of the subsidy $\gamma$. It follows from (18), however, that the banker can grant new loans in a safe manner only if

$$D_0 < \frac{E[x] + x_b}{2}, \quad (19)$$

which is the same condition as in the case without subsidy (see (3)): The subsidy does not alter the availability of the safe mode because the banker must be able to extend at least some new loans without assistance in order to benefit from the subsidy.
In the risky mode of operation, a capital subsidy \( \gamma \) per unit of new business loans implies that the banker’s budget constraint (4) modifies to

\[
D_0 + L + C = pD_1 + p\lambda (x_g + RL + C - D_1) + \gamma L. \tag{20}
\]

We conclude

**Proposition 3** When government offers a capital subsidy \( \gamma \) per unit of new business loans, a banker with an outstanding amount of deposits \( D_0 \)

1. operates in a safe manner and invests according to

\[
L^* = \begin{cases} 
L^{ccs} & \text{if } D_0 \leq \frac{E[x] + x_b - (2(1 - \gamma) - pR)L^{ccs}}{2} \\
L^{ccs}_{\max}(D_0) & \text{if } D_0 \in \left( \frac{E[x] + x_b - (2(1 - \gamma) - pR)L^{ccs}}{2}, \min\left\{ D^{ccs}_0, \frac{E[x] + x_b}{2} \right\} \right) 
\end{cases}
\] \tag{21}

2. operates in a risky manner and invests according to

\[
L^* = L^{ccs} \text{ if } D_0 \in \left( \min\left\{ D^{ccs}_0, \frac{E[x] + x_b}{2} \right\}, px_g + (pR - 1)L^{ccs} - c(L^{ccs}) + \gamma L^{ccs} \right)
\] \tag{22}

3. is run already at \( t \) if \( D_0 > px_g + (pR - 1)L^{ccs} - c(L^{ccs}) + \gamma L^{ccs} \),

where \( L^{ccs} : pR - (1 - \gamma) = c'(L^{ccs}) \), and where \( D^{ccs}_0 \) is the outstanding amount of deposits for which

\[
\gamma(L^{ccs} - L^{ccs}_{\max}(D^{ccs}_0)) = (1 - p)x_b + \left[ (pR - 1)L^{ccs}_{\max}(D^{ccs}_0) - c(L^{ccs}_{\max}(D^{ccs}_0)) \right] - \left[ (pR - 1)L^{ccs} - c(L^{ccs}) \right]. \tag{23}
\]

**Proof.** See appendix. □

The capital subsidy not only eases the budget constraint but also incites the banker to overinvest irrespective of the operation mode. This is because it reduces the marginal refinancing costs of new business loans to \( 1 - \gamma \) in either mode of operation.

As long as the outstanding amount of deposits \( D_0 \) is sufficiently small so that the banker can balance her (reduced) marginal costs and expected marginal returns on new
loans in the safe mode, she will do so and remain safe. This way, she benefits from the return on legacy assets even if the economy does not recover. If, however, the debt overhang problem is too severe to make new loans with a volume $L^{ccs}$ in the safe mode, the banker faces a trade off. Either she remains safe and cuts lending or she switches to the risky mode, for which balancing marginal costs and marginal returns for new business loans is still possible. Compared to the risky mode, the safe mode may be associated with higher expected earnings for two reasons (see RHS of (23)). First, the volume of new loans is smaller and thus possibly more efficient. Second, legacy assets yield a return $x_b$ in the bad state. The opportunity cost of the safe mode, however, is the reduction in the subsidy because of the lower lending volume (the LHS of (23)). As long as the outstanding amount of deposits $D_0$ is not too large, the benefits of the safe mode outweigh its costs so that the banker remains safe and cuts lending to $L^{ccs}_{max}$. Otherwise, she opts for the risky mode and overinvests by choosing a lending volume $L^{ccs}$.

As for investment in the safe asset, the capital subsidy has no effect. In the safe mode, a bank is still indifferent w.r.t. the safe asset. In the risky mode, the safe asset’s returns become risky because a bank run occurs in the bad state, implying that the banker does not invest in the safe asset.

### 3.2 Deposit guarantees

In fall 2008, governments of many countries responded to the financial crisis also by declaring blanket guarantees for new debt, in particular deposits. Although coverage differed across countries, the general aim was to reduce systemic risks due to spillover effects and to prevent a freezing of the loan markets.

Guarantee schemes tend to reduce the proper incentives for bankers. They lower market discipline, and thus create moral hazard, because depositors no longer have an incentive to control the risk of a bank (Nier and Baumann, 2006). However, even when deposit guarantees are supplemented with a proper regulatory supervision which substitutes for market discipline, they still influence the banker’s attitude toward risk and her ability to grant new loans.
To see this, assume that government ensures that only depositors, who are not repaid by the bank when a run occurs, but not the banker benefit from the guarantee. Then, guaranteeing new deposits has no effect on the banker’s behavior when she operates in the safe mode, in which there is no run. By contrast, when operating in the risky mode, a guarantee serves to fully repay depositors in the bad state, in which the bank is run. Accordingly, the banker’s budget constraint (4) modifies to

\[ D_0 + L + C = D_1 + p\lambda (x_g + RL + C - D_1). \]  

(24)

The guarantee effectively subsidizes deposits in the risky mode. The banker has to pay only an expected return \( p \) per unit of deposits (\( 1 - p \) is covered by the guarantee) while she would have to compensate shareholders for their opportunity costs completely. Therefore, the banker capitalizes the most on the subsidy if she does not issue any equity or non-deposit debt at all, \( \lambda = 0 \). But bank capital is not only more expensive. It is also useless since opting for the risky mode implies that capital is too small to prevent a run in the bad state. Accordingly, bank capital is strictly dominated by deposits. The banker’s budget constraint in the risky mode (24) changes to

\[ D_1 = D_0 + L + C. \]  

(25)

This leads us to

**Proposition 4** When government offers a deposit guarantee, a banker with an outstanding amount of deposits \( D_0 \)

1. operates in a safe manner and invests according to

\[ L^* = L^{fb} \text{ if } D_0 \leq D_{0g} < E[x] + x_b - \frac{(2 - pR)L^{fb}}{2} \]  

(26)

2. operates in a risky manner and invests according to

\[ L^* = L^{dg} \text{ if } D_0 \in \left( D_{0g}, \frac{px_g + (pR - p)L^{dg} - c(L^{dg})}{p} \right) \]  

(27)
3. is run already at $t$ if $D_0 > \frac{px_b + (pR - p)L^{dg} - c(L^{fb})}{p}$, where $L^{dg} = p(R - 1) = c'(L^{dg})$, and where $D_0^{dg}$ is the outstanding amount of deposits for which

$$(1 - p)D_0^{dg} + (1 - p)L^{dg} = (1 - p)x_b + \left[ (pR - 1)L^{fb} - c(L^{fb}) \right] - \left[ (pR - 1)L^{dg} - c(L^{dg}) \right].$$

**Proof.** See appendix. ■

In the presence of a deposit guarantee, the risky mode is thus associated with overinvestment ($L^{dg} > L^{fb}$). This is because the guarantee effectively depresses marginal funding costs for the banker to $p$. Accordingly, with respect to new business loans, the banker is willing to take on additional monitoring costs $c(L)$ in order to further capitalize on the subsidy. In the case of a moderate outstanding amount of deposits, in which operating in the safe mode with first-best investment is still feasible, the guarantee leads to the following trade-off. On the one hand, when the banker switches to the risky mode, she obtains a subsidy $(1 - p)D_0 + (1 - p)L^{dg}$ from the deposit guarantee, which is increasing in $D_0$. On the other hand, she sacrifices the return $x_b$ from the legacy assets in the bad state and, since the guarantee induces overinvestment, the expected return of the new business loans decreases by $\left[ (pR - 1)L^{fb} - c(L^{fb}) \right] - \left[ (pR - 1)L^{dg} - c(L^{dg}) \right]$. As long as $D_0 \leq D_0^{dg}$, the subsidy associated with the guarantee does not exceed the opportunity costs of the risky mode so that the banker opts for the first best loan supply and the safe mode. Otherwise, the banker switches to the risky mode and overinvests, as long as this yields a non-negative expected profit. Guarantees thus can incite the banker to opt against the safe mode even when safety would be compatible with first-best lending. Only if the debt overhang is small enough so that the value of the guarantee is rather small, she refrains from opting for the risky mode and from overinvestment.

The deposit guarantee makes a banker always indifferent with respect to the amount invested in the safe asset. In the safe mode, in which guarantees do not apply, the marginal return of the safe asset and the marginal financing costs are both equal to one. In the risky mode, the bank is run in the bad state. Then the guarantee applies but the
safe asset yields no return. The safe asset’s marginal cost and marginal returns are, therefore, both equal to \( p \).

Neither a guarantee for outstanding deposits nor a guarantee with a fair insurance fee would free up additional funds to mitigate the shortage in the loan market. A guarantee makes payments only if the bank fails to repay depositors. If the bank never fails, a banker behaves just like without a guarantee. This holds not only for any new deposits but also for the outstanding deposits. Hence, if the bank fails already at \( t \), old depositors would get their money back from the government but the bank then ceases to exist so that new loans will not be granted.

With a fair insurance of new deposits, the banker pays \((1 - p)D_1\) upfront to the government. As this is exactly the amount that the banker can raise additionally from depositors, there is no effect on bank lending. One may still be tempted to argue that granting a guarantee and imposing a fair fee would avoid negative allocative effects but may still contribute to calm down stressed financial markets. However, since the model here takes the effects of looming bank runs already into account, this argument loses much of its bite.

3.3 Bad bank

In response to banking crises, several countries encouraged banks to establish so called bad banks. This has been also the case in the recent crisis. The general aim of bad banks is to rescue distressed financial institutions by relieving them of legacy assets in order to foster new lending. While the design of bad banks varies across countries, all concepts have one major principle in common. When a financial institution opts to establish a bad bank, it substitutes its legacy assets, which are characterized by a high degree of risk and a low expected value, for a payment stream, which is less risky.

To translate the concept of a bad bank into our model, suppose that the bank can replace its risky legacy assets by a certain quantity \( y \) of the safe asset. That is, we assume that when participating in a bad bank, the banker can get rid of her existing risks completely and faces only the risks associated with new loans. In this case, the volume of new deposits is restricted by \( D_1 \leq y + C \) in the safe mode, with \( C \) being the quantity of
the safe asset acquired in addition to what the banker gets in exchange for legacy assets. Accordingly, if $y > x_b$ the bad bank allows the banker to issue more deposits without having to switch to the risky mode. After replacing legacy assets, the budget constraint of the banker in the safe mode is

$$D_0 + L + C = D_1 + \lambda (y + pRL + C - D_1), \quad (28)$$

and the maximum volume of new loans in the safe mode is

$$L \leq \frac{2(y - D_0)}{2 - pR} =: L_{\text{max}}^{bb}(D_0), \quad (29)$$

implying that the banker can operate safely if

$$D_0 \leq y. \quad (30)$$

Compared to the case without government interventions (see (2) and (3)), the existence of a bad bank thus improves the banker’s ability to extend loans safely if the funding liquidity of her assets improves, i.e. if $y > E[x] + x_b$. Hence, although the quantity $y$ of the safe asset obtained by the swap may be smaller than the expected value of the legacy assets, the bank can extend more new loans in the safe mode as long as $y$ is not too small. The reason is that due to the swap of assets, the banker can issue deposits against the complete expected value $y$ of the safe asset. Therefore, in contrast to keeping the legacy assets on the balance sheet, the banker relies less on equity issues, which is an inefficient financing instrument as the banker can extract rents from the shareholders.

If $y > \frac{E[x] + x_b}{2}$ is not met, the banker has no incentive to take part in a bad bank program. This result indicates that a bad bank can be a useful instrument. By lowering the risk of the bank’s future earnings, it allows for a stronger commitment of the banker vis-a-vis her financiers through the use of deposits and thus eventually improves her ability to obtain funding for new businesses.

With a bad bank, the banker can also operate in the risky mode by issuing deposits with face value $D_1 \in (y + C, y + RL + C]$, so that depositors are repaid only if the econ-
omy recovers. Accordingly, the banker’s budget constraint in the risky mode reads

\[ D_0 + L + C = pD_1 + p\lambda (y + RL + C - D_1). \] (31)

We conclude

**Proposition 5** When a banker with an outstanding amount of deposits \( D_0 \) has participated in a bad bank, she

1. operates in a safe manner and invests according to

\[
L^* = \begin{cases} 
L^{fb} & \text{if } D_0 \leq \frac{2y - (2 - pR)L^{fb}}{2} \\
L^{bb}_{\text{max}}(D_0) & \text{if } D_0 \in \left( \frac{2y - (2 - pR)L^{fb}}{2}, \min\{D_0, y\} \right] 
\end{cases} 
\] (32)

2. operates in a risky manner and invests according to

\[ L^* = L^{fb} \text{ if } D_0 \in \left( \min\{\overline{D}_0^{bb}, y\}, py + (pR - 1)L^{fb} - c(L^{fb}) \right] \] (33)

3. is run already at \( t \) if \( D_0 > py + (pR - 1)L^{fb} - c(L^{fb}) \),

where \( \overline{D}_0^{bb} \) is the outstanding amount of deposits for which

\[
\left[(pR - 1)L^{fb} - c(L^{fb})\right] - \left[(pR - 1)L^{bb}_{\text{max}}(\overline{D}_0^{bb}) - c(L^{bb}_{\text{max}}(\overline{D}_0^{bb}))\right] = (1 - p)y. \] (34)

**Proof.** See appendix. ■

The proposition shows that a bad bank does not alter the banker’s incentives to provide new loans. This is not surprising. A bad bank concept is backward looking in the sense that it only affects the existing balance sheet. It thus does not influence new businesses of the bank so that it does not affect the marginal costs and benefits of new loans. Therefore, a bad bank has similar implications as a lump sum transfer.
4 Discussion

4.1 An assessment of policy measures

Our assessment of policy measures follows a two-stage selection process. In a first stage, a policy measure qualifies if it 1) avoids a default on systemically important bank liabilities, 2) fosters new business lending without inciting banks to overinvest, and 3) does not encourage further excessive risk taking. In the second stage, when two or more measures sort out these problems equally well, the one should be preferred which is associated with the smallest expected fiscal burden because a measure that involves less taxpayer money implies less distortions due to taxation.

Starting with deposit guarantees, we have seen that they primarily incite banks to choose a fragile capital structure and to overinvest in risky assets, a potentially dangerous combination which may prepare the ground for the next bubble and subsequent crisis. Therefore, guarantees fulfill neither function 2) nor 3).

As for conditional capital injections, we have seen that subsidizing new lending is associated with unintended effects. Like deposit guarantees, the subsidy induces banks operating in the risky mode to overinvest. However, even those banks that would remain safe and provide new business loans according to the first best without a subsidy, are incited to overinvest. Hence, subsidizing new business loans is efficiency-enhancing only for banks with an intermediate debt overhang, which still prefer to operate safely but are unable to grant more new loans than associated with the first best. For them, the subsidy may be helpful because it eases the financial constraint without giving scope for overinvestment. Altogether, conditional capital injections do not fulfill function 2) properly.

Like subsidies conditional on new activities, a government assistance that does not depend on new businesses of the banker also releases banks from their debt burden, thereby expanding the capacity of banks to provide new loans in a safe manner. Unlike subsidies conditional on new activities, however, they do not incite a bank to overinvest. A bad bank, which allows banks to swap risky legacy assets for a certain quantity of the safe asset, is also suitable to implement the same allocation as unconditional capital
injections. We conclude that only unconditional capital injections and the bad bank qualify as effective policy measures.

Notwithstanding their merits, unconditional capital injections and the bad bank are by no means perfect when taking their fiscal effects for taxpayer into account. As long as the government is unable to discriminate between those banks who are really in need of support and those who are not, government assistance will imply a waste of taxpayers money. Consider the lump sum transfer. There, banks with a small debt overhang do not need support but will also draw on this kind of assistance and invest into the safe asset. Banks with a severe debt overhang will not change their behavior at all. They would grant loans according to the first-best anyway but still in a risky manner when the lump sum is too small to provide them with the proper incentives to switch into the safe mode. For these banks, taxpayer money is again wasted. Finally, for banks with a moderate debt overhang public funds would not be wasted, but banks may be granted too little support to remove their refinancing restrictions so that they still underinvest.

To address this selection problem, government could make its support conditional on the existing debt burden $D_0$ because banks with a relatively small level of outstanding deposits shall be likely to invest efficiently and safely without support from the government. This, however, is not necessarily true. The extent of investment inefficiency does not only depend on the size of the pre-existing debt but also on the first-best loan supply $L^{fb}$ (which is unknown to the government). That is, it may be that banks with a relatively small debt burden will invest rather inefficiently due to a large $L^{fb}$ while other banks with a large $D_0$ are able and willing to provide first best lending in a safe manner. It is also not advisable to prevent banks from investing in the safe asset. In the safe mode, banks are always indifferent with respect to the safe asset, irrespective whether they underinvest in new loans or not. Hence, investment in safe assets is not a good indicator for the strength of the debt overhang problem ex ante. Moreover, conditioning on investment in the safe asset makes the financial assistance again dependent on new banking activities ex post and thus incites banks to overinvest in new loans.

Compared to a lump sum subsidy, funding a bad bank is even less cost efficient. In order to have a bank granting new loans of some $\bar{L}$ in a safe manner, the government
has to inject \((1 - \frac{1}{2}pR)\bar{L} + D_0 - \frac{1}{2}(E[x] + x_b)\) as lump sum, whereas taxpayers have to pay \((1 - \frac{1}{2}pR)\bar{L} + D_0 - X\) in the case of a bad bank, with \(X\) being the expected value of legacy assets to taxpayers. As the specific skills of the banker are needed to extract the full value of these assets, we have \(X < \frac{1}{2}(E[x] + x_b)\). Hence, although a bad bank does not induce any disincentives with respect to new loans, a lump sum subsidy has the same effect at a lower fiscal cost. The comparative disadvantage of bad banks thus mainly stems from the loss in the bankers’ specific expertise in collecting loans.\(^8\)

### 4.2 Government subsidy versus financial investment

A further feature of our analysis is that banks should not be obliged to pay the government a market return on its financial stake. Either this kind of aid will not be accepted, or government funds just crowd out private funds. Accordingly, government is not advised to require, e.g., fair insurance premiums for granting deposit guarantees amid a banking crisis. This claim has to be qualified, though. It does not mean that government help should always be free of charge. A proper pricing of the safety net is indeed an important lesson learned from the recent crisis. This pricing should, first and foremost, mitigate adverse incentives that arise from the provision of the safety net, but not primarily reduce taxpayers money at risk. It is, however, unlikely that properly priced prevention measures will completely rule out the emergence of a crisis. Once the banking system suffers from a debt overhang, it is too late for charging a price as bankers have only little scope to influence the value of legacy assets and of debt that had been issued in the past.\(^9\)

---

8. A bad bank is also less cost efficient than a conditional capital injection. For the latter to induce a banker to grant \(L\) in a safe manner, \(\gamma\) should equal \(\frac{(1 - \frac{1}{2}pR)\bar{L} + D_0 - \frac{1}{2}(E[x] + x_b)}{L}\) implying a capital injection of the same magnitude as in the case of a lump sum transfer.

9. This may explain why government interventions have sometimes been disappointing. For example, Bank of America announced in December 2009 that it would be about to repay 45bn dollars in TARP funds. In the 12 month since the government first made its investment in the bank, the US government earned about 3.6bn dollars for its one-year stake in Bank of America. Within this time period, the bank originated a total of only 760m dollars in new loans.
4.3 Further proposals

It has also been proposed that the government should subsidize equity or deposit issues. This might be rational, so the argument goes, because banks which are able to raise more funds on the market are often supposed to be more valuable. If there is still a shortage in loan supply, these banks are considered worthy of being supported.\textsuperscript{10} Such a subsidy, however, is just another type of capital injections conditional on new banking businesses. Although it does not refer to new lending activities, a banker can influence the amount of the subsidy by adapting equity and deposit issues. By reducing effective marginal refinancing costs, the subsidy distorts the banker’s investment incentives and induces her to overinvest irrespective of the operation mode. While this disadvantage applies to both forms of subsidies, subsidizing new deposits has a further disadvantage. Like deposit guarantees, it makes deposit financing more attractive relative to equity financing so that the banker has a stronger tendency to opt for the risky mode of operation, in which she issues a higher volume of deposits.

Hart and Zingales (2009) proposed that large financial intermediaries should be subject to regulatory margin calls. Whenever an intermediary becomes risky (in the sense that the probability of default on its debt exceeds some pre-specified threshold), shareholders shall inject fresh funds in order to make debt safe. Imposing such a margin call prevents a bank from operating in a risky mode, but is associated with underinvestment and some banks dropping out of the market. Although this measure cannot deal with the problem of shortages in the loan market, it may reasonably supplement other interventions, in particular conditional capital injections, in order to reduce the associated incentives to pursue a risky business model. The drawback then is that CDS spreads may decrease just because of expected government interventions.

\textsuperscript{10}For example, Calomiris (2008) proposed that government should inject preferred stocks into banks in order to recapitalize them. In particular, being matched by common stock issues underwritten by private investors, it would recapitalize banks in “an incentive compatible manner to facilitate banks’ abilities to maintain and grow assets.”
5 Summary

In this paper, we have evaluated policy measures that aim at stopping the fall in bank loan supply following a banking crisis. We have presented a dynamic banking model in which a debt overhang induces banks either to curtail lending or to choose a fragile capital structure. Different government interventions influence banks’ lending behavior, capital structure and risk taking in different ways. Measures can be grouped into two categories. One group comprises measures that cope with the existing debt. Measures that deal with the adverse effects of the debt overhang from the second group. When government grants assistance in order to mitigate the adverse effects of the debt overhang by supporting new lending or the issue of new debt and equity, banks can influence the scale of the assistance by changing new contracts. Since banks can externalize risks then, there will be excessive risk taking and overinvestment which put the future stability of the banking system at risk. Providing unconditional financial assistance to banks is a better solution as it does not incite banks to have a fragile capital structure risk or to overinvest in risky loans. In the end, this solution recognizes that the debt overhang is the result of past lending behavior and capital structure decisions of banks.

Appendix

Proof of Proposition 1

1. Suppose the banker has opted for the safe mode. Solving (1) for $D_1$ and substitution in (5) yields

$$\max_{L,C,\lambda} \left[ E[x] + (pR - 1)L - D_0 - c(L)\right] =: \Gamma(L)$$

s.t. $L \in \left[ \frac{\lambda E[x] - C - D_0}{1 - \lambda pR}, \frac{\lambda E[x] + (1 - \lambda)x_0 - D_0}{1 - \lambda pR} \right], \lambda \in \left[0, \frac{1}{2}\right], \text{ and } L, C \geq 0.$

(35)

Note that $\Gamma$ is independent of $\lambda$ and $C$. Moreover, the upper bound of $L$ is increasing in $\lambda$ while the lower bound of $L$ is decreasing in $C$ for all $C \leq \frac{\lambda E[x] - D_0}{1 - \lambda}$. Therefore, we can conclude that $\lambda = \frac{1}{2}$ and $C = \max \{E[x] - 2D_0, 0\}$ is among the
max \_L \_Γ(L) = [E \_x + (pR - 1) \_L - D_0 - c(L)] \_s.t. \_L \in \left[0, \frac{E\_x + x_b - 2D_0}{2 - pR}\right].

As \frac{\partial Γ}{\partial L} = (pR - 1) - c'(L) is decreasing in L and equal to zero for \_L = L^{fb}, the optimal loan volume is \_L^*_S = \min \{L^{fb}, L_{\text{max}}(D_0)\}. The safe mode is feasible if \_L^*_S \geq 0, implying \_D_0 \leq \frac{E\_x + x_b}{2}. Then, the banker’s expected payoff is

\_Γ(\_L^*_S) = E \_x + (pR - 1) \min \{L^{fb}, L_{\text{max}}(D_0)\} - D_0 - c \left(\min \{L^{fb}, L_{\text{max}}(D_0)\}\right).

(36)

2. Suppose the banker has opted for the risky mode. Solving (4) for \_D_1 and substitution in (6) yields

\max \_L, \_C, \_λ \_\_px + (pR - 1) \_L - (1 - p)\_C - D_0 - c(L) =: \_Ω(L)

s.t. \_λ \in [0, 1], and \_L, \_C \geq 0.

(37)

We neglect \_D_1 \in (x_b + C, x_g + RL + C] but show that it will be met in equilibrium. As \_Ω is independent from \_λ and decreasing in \_C, \_λ = 0 and \_C = 0 is among the set of optimal decisions of the banker, so that (37) simplifies to

\max \_L \_Ω(L) = \_px + (pR - 1) \_L - D_0 - c(L) \_s.t. \_L \geq 0.

As \frac{\partial Ω}{\partial L} = (pR - 1) - c'(L) is decreasing in L and equal to zero for \_L = L^{fb}, the optimal loan volume is \_L^*_R = L^{fb}. Then, the banker’s expected payoff is

\_Ω(\_L^*_R) = \_px + (pR - 1) \_L^{fb} - D_0 - c \left(L^{fb}\right).

(38)

3. Comparing \_Γ(\_L^*_S) and \_Ω(\_L^*_R) yields

(a) If \_D_0 \leq \frac{E\_x + x_b - (2 - pR)L^{fb}}{2 - pR} and thus \_L^*_S = L^{fb}, it follows from (36) and (38) that \_Γ(\_L^*_S) > \_Ω(\_L^*_R): the banker operates in a safe manner and invests according to \_L^{fb}.
(b) If \( D_0 \in \left( \frac{E[x]+x_b-(2-pR)L^{fb}}{2}, \frac{E[x]+x_b}{2} \right) \) and thus \( L^*_S = L_{\max}(D_0) \), it follows from (36) and (38) that \( \Gamma(L^*_S) \geq \Omega(L^{fb}) \) only if

\[
(1-p)x_b \geq \left[ (pR-1)L^{fb} - c(L^{fb}) \right] - \left[ (pR-1)L_{\max}(D_0) - c(L_{\max}(D_0)) \right].
\]

The RHS of (39) is increasing in \( D_0 \). Therefore, the banker

- operates in a safe manner and invests according to \( L_{\max}(D_0) \) if \( D_0 \in \left( \frac{E[x]+x_b-(2-pR)L^{fb}}{2}, \min \left\{ D_0, \frac{E[x]+x_b}{2} \right\} \right) \),
- operates in a risky manner and invests according to \( L^{fb} \) if \( D_0 \in \left( \frac{E[x]+x_b}{2}, D_0 \right) \).

(c) If \( D_0 > \frac{E[x]+x_b}{2} \), only the risky mode is available. The banker invests according to \( L^{fb} \) only if \( \Omega(L^{fb}) \geq 0 \) implying \( D_0 \leq px_g + (pR-1)L^{fb} - c(L^{fb}) \).

Otherwise, the bank is run already at \( t \).

It remains to show that \( D_1 \in (x_b+C, x_g+RL+C] \) is met whenever the banker operates in the risky mode in equilibrium. Substitution of (4), \( \lambda = 0 \), \( C = 0 \) and \( L = L^{fb} \) yields \( D_0 \in (px_b-L^{fb}, px_g+(pR-1)L^{fb}] \), which is indeed met whenever the banker opts for the risky mode in equilibrium.

**Proof of Proposition 2**

The proof is essentially equivalent to the proof of Proposition 1 with an outstanding amount of deposits of \( D_0 - S \) instead of \( D_0 \).

**Proof of Proposition 3**

We follow the same three steps as in the proof of Proposition 1.

1. Suppose the banker has opted for the safe mode. Her optimization problem is

\[
\max_{L,C,D_1,\lambda} \left[ (1-\lambda) (E[x] + pRL + C - D_1) - c(L) \right] =: \Gamma^{css}(L)
\]

s.t. (17), \( D_1 \in [0, x_b + C] \), \( \lambda \in [0, \frac{1}{2}] \), and \( L, C \geq 0 \).
Solving (17) for \( D_1 \) and substitution in (40) yields

\[
\max_{L,C,\lambda} \Gamma^{ccs}(L) = [E[x] + (pR - (1 - \gamma))L - D_0 - c(L)]
\]

\[
s.t. \ L \in \left[\frac{\lambda E[x] - (1 - \lambda)C - D_0}{1 - \lambda pR - \gamma}, \frac{\lambda E[x] + (1 - \lambda)x_b - D_0}{1 - \lambda pR - \gamma}\right], \ \lambda \in \left[0, \frac{1}{2}\right], \ and \ L, C \geq 0.
\]

(41)

Note that \( \Gamma^{ccs} \) is independent of \( \lambda \) and \( C \). Moreover, the upper bound of \( L \) is increasing in \( \lambda \) while the lower bound of \( L \) is decreasing in \( C \) for all \( C \leq \frac{\lambda E[x] - D_0}{1 - \lambda} \).

Therefore, we can conclude that \( \lambda = \frac{1}{2} \) and \( C = \max \{E[x] - 2D_0, 0\} \) is among the set of optimal decisions, so that (41) simplifies to

\[
\max_{L} \Gamma^{ccs}(L) = [E[x] + (pR - (1 - \gamma))L - D_0 - c(L)] \ s.t. \ L \in \left[0, \frac{E[x] + x_b - 2D_0}{2(1 - \gamma - pR)}\right].
\]

(42)

As \( \frac{\partial \Gamma^{ccs}}{\partial L} = pR - (1 - \gamma) - c'(L) \) is decreasing in \( L \) and equal to zero for \( L = L^{ccs} \), the optimal loan volume is \( L^*_S = \min \{L^{ccs}, L^{ccs}_{max}(D_0)\} \). The safe mode is feasible if \( L^*_S \geq 0 \), implying \( D_0 \leq \frac{E[x] + x_b}{2} \). Then, the banker’s expected payoff is

\[
\Gamma^{ccs}(L^*_S) = (pR - (1 - \gamma)) \min \{L^{ccs}, L^{ccs}_{max}(D_0)\} + E[x] - D_0 - c(\min \{L^{ccs}, L^{ccs}_{max}(D_0)\}).
\]

(43)

2. Suppose the banker has opted for the risky mode. Her optimization problem is

\[
\max_{L,C,\lambda} \Omega^{ccs}(L) = \max_{L,C,\lambda} [p \ (1 - \lambda) \ (x_g + RL + C - D_1) - c(L)] =: \Omega^{ccs}(L)
\]

s.t. (20), \( D_1 \in (x_b + C, RL + x_g + C) \), \( \lambda \in \left[0, \frac{1}{2}\right] \), and \( L, C \geq 0 \).

(44)

Solving (20) for \( D_1 \) and substitution in (49) yields

\[
\max_{L,C,\lambda} \Omega^{ccs}(L) = [(pR - (1 - \gamma))L + px_g - (1 - p)C - D_0 - c(L)]
\]

\[
s.t. \ \lambda \in \left[0, \frac{1}{2}\right], \ and \ L, C \geq 0,
\]

(45)

We neglect again that \( D_1 \in (x_b + C, RL + x_g + C) \) but show that it will be met in equilibrium. As \( \Omega^{ccs} \) is independent from \( \lambda \) and decreasing in \( C \), \( \lambda = 0 \) and \( C = 0 \) is among the set of optimal decisions of the banker, so that (45) simplifies to

\[
\max_{L} \Omega^{ccs}(L) = [(pR - (1 - \gamma))L + px_g - D_0 - c(L)] \ s.t. \ L \geq 0.
\]
As \( \frac{\partial \Omega^{ccs}}{\partial L} = pR - (1 - \gamma) - c'(L) \) is decreasing in \( L \) and equal to zero for \( L = L^{ccs} \), the optimal loan volume is \( L^* = L^{ccs} \) and the banker’s expected payoff is

\[
\Omega^{ccs}(L^*_R) = (pR - (1 - \gamma))L^{ccs} + px_g - D_0 - c(L^{ccs}).
\] (46)

3. Comparing \( \Gamma^{ccs}(L^*_S) \) and \( \Omega^{ccs}(L^*_R) \) yields

(a) If \( D_0 \leq \frac{E[x] + x_b - (2(1 - \gamma) - pR)L^{ccs}}{2} \) and thus \( L^*_S = L^{ccs} \), it follows from (43) and (46) that \( \Gamma^{ccs}(L^*_S) > \Omega^{ccs}(L^*_R) \) so that the banker operates in a safe manner and invests according to \( L^{ccs} \).

(b) If \( D_0 \in \left( \frac{E[x] + x_b - (2(1 - \gamma) - pR)L^{ccs}}{2}, \frac{E[x] + x_b}{2} \right) \) and thus \( L^*_S = L^{ccs}(D_0) \), it follows from (43) and (46) that \( \Gamma^{ccs}(L^*_S) \geq \Omega^{ccs}(L^*_R) \) only if

\[
(1 - p)x_b - [(pR - (1 - \gamma))L^{ccs} - c(L^{ccs})] \geq -[(pR - (1 - \gamma))L^{ccs}(D_0^{ccs}) - c(L^{ccs}(D_0^{ccs}))]
\] (47)

The RHS of (47) is decreasing in \( L \) for all \( L \leq L^{ccs} \). Therefore, the banker

- operates in a safe manner and invests according to \( L^{ccs}(D_0) \) if \( D_0 \in \left( \frac{E[x] + x_b - (2(1 - \gamma) - pR)L^{ccs}}{2}, \min \left\{ \frac{D_0^{ccs}}{E[x] + x_b}, \frac{E[x] + x_b}{2} \right\} \right) \).
- operates in a risky manner and invests according to \( L^{ccs} \) if \( D_0 \in \left( \frac{D_0^{ccs}}{E[x] + x_b}, \frac{E[x] + x_b}{2} \right) \).

(c) If \( D_0 > \frac{E[x] + x_b}{2} \), only the risky mode is available. The banker invests according to \( L^{ccs} \) only if \( \Omega^{ccs}(L^{ccs}) \geq 0 \) implying \( D_0 \leq px_g + (pR - 1)L^{ccs} - c(L^{ccs}) + \gamma L^{ccs} \). Otherwise, the bank is run already at \( t \).

It remains to show that \( D_1 \in (x_b + C, x_g + RL + C] \) is met whenever the banker operates in the risky mode in equilibrium. Substitution of (20), \( \lambda = 0 \), \( C = 0 \) and \( L = L^{ccs} \) yields \( D_0 \in (px_b - (1 - \gamma)L^{ccs}, px_g + (pR - (1 - \gamma))L^{ccs}] \), which is indeed met whenever the banker opts for the risky mode in equilibrium.
Proof of Proposition 4

We follow the same three steps as in the proof of Proposition 1.

1. Suppose the banker has opted for the safe mode. Then, the banker will not draw on the guarantee. Therefore, the safe mode is feasible if $D_0 \leq \frac{E[x] + x_0}{2}$. Then, the optimal loan volume is $L^*_S = \min \{ L^{fb}, L_{\max}(D_0) \}$, and the banker’s expected payoff is

$$\Gamma^{dg}(L^*_S) = E[x] + (pR - 1) \min \{ L^{fb}, L_{\max}(D_0) \} - D_0 - c \left( \min \{ L^{fb}, L_{\max}(D_0) \} \right).$$

(48)

2. Suppose the banker has opted for the risky mode. Then, her optimization problem is

$$\max_{L,C,D_1,\lambda} \left[ p(1 - \lambda) (x_g + RL + C - D_1) - c(L) \right] =: \Omega^{dg}(L)$$

s.t. \(D_1 \in (x_b + C, x_g + RL + C], \lambda \in [0, \frac{1}{2}], \text{ and } L, C \geq 0.\)

(49)

Solving (24) for $D_1$ and substitution in (49) yields

$$\max_{L,C,\lambda} \Omega^{dg}(L) = \left[ p(1 - \lambda) \left( x_g + (R - 1)L - D_0 \right) - c(L) \right]$$

s.t. $\lambda \in [0, \frac{1}{2}], \text{ and } L, C \geq 0,$

(50)

We neglect that $D_1 \in (x_b + C, x_g + RL + C]$ but show that it will be met in equilibrium. As $\Omega^{dg}$ is decreasing in $\lambda$ and independent of $C$, $\lambda = 0$ and $C = 0$ is among the set of optimal decisions of the banker, so that (50) simplifies to

$$\max_{L} \Omega^{dg}(L) = \left[ p \left( x_g + (R - 1)L - D_0 \right) - c(L) \right] \text{ s.t. } L \geq 0.$$

(51)

As $\frac{\partial \Omega^{dg}}{\partial L} = p(R - 1) - c'(L)$ is decreasing in $L$ and equal to zero for $L = L^{dg}$, the optimal loan volume satisfies $L^*_R = L^{dg}$ and the banker’s expected payoff is

$$\Omega^{dg}(L^*_R) = p \left( x_g + (R - 1)L^{dg} - D_0 \right) - c(L^{dg}).$$

(52)

3. Comparing $\Gamma^{dg}(L^*_S)$ and $\Omega^{dg}(L^*_R)$ yields
(a) If \(D_0 \leq \frac{E[x] + x_b - (2-p)R L^{fb}}{2}\) and thus \(L^*_S = L^{fb}\), it follows from (48) and (52) that \(\Gamma^{dg}(L^*_S) \geq \Omega^{dg}(L^*_R)\) only if

\[
(1-p)x_b + \left[ (pR-1)L^{fb} - c(L^{fb}) \right] - \left[ (pR-1)L^{dg} - c(L^{dg}) \right] \geq (1-p)D_0^{dg} + (1-p)L^{dg}.
\]

The RHS of (53) is increasing in \(D_0\). Therefore, the banker

- operates in a safe manner and invests according to \(L^{fb}\) if \(D_0 \leq D_0^{dg}\),
- operates in a risky manner and invests according to \(L^{dg}\) if \(D_0 \in \left( D_0^{dg}, \frac{E[x] + x_b - (2-p)R L^{fb}}{2} \right)\).

(b) If \(D_0 \in \left( \frac{E[x] + x_b - (2-p)R L^{fb}}{2}, \frac{E[x] + x_b}{2} \right)\) and thus \(L^*_S = L_{max}(D_0)\), the banker will operate in a risky manner and invest according to \(L^{dg}\). To see this, note first that \(\Gamma^{dg}(L_{max}(D_0)) < \Gamma^{dg}(L^{fb})\) and \(\Omega^{dg}(L^{fb}) < \Omega^{dg}(L^{dg})\), since otherwise, the banker would choose \(L^{fb}\) instead of \(L^{dg}\) in the risky mode. Accordingly, it remains to show that \(\Gamma^{dg}(L^{fb}) < \Omega^{dg}(L^{fb})\), which is true if \(D_0 > x_b - L^{fb}\), which is true in the case considered here.

(c) If \(D_0 > \frac{E[x] + x_b}{2}\), only the risky mode is available. The banker operates in this risky mode and invests according to \(L^{dg}\) only if \(\Omega^{dg}(L^{dg}) \geq 0\) implying \(D_0 \leq \frac{px + (pR-p) L^{dg} - c(L^{dg})}{p}\). Otherwise, the bank is run already at \(t\).

It remains to show that \(D_1 \in (x_b + C, x_g + RL + C]\) is met whenever the banker operates in the risky mode in equilibrium. Substitution of (24), \(\lambda = 0\), \(C = 0\) and \(L = L^{dg}\) yields \(D_0 \in (x_b - L^{dg}, x_g + (R-1)L^{dg})\), which is indeed met whenever the banker opts for the risky mode in equilibrium.

**Proof of Proposition 5**

The proof is essentially equivalent to the proof of Proposition 1 with \(x_b = E[x] = y\).
References


Flannery, M (2009), Stabilizing Large Financial Institutions With Contingent Capital Certificates. mimeo, Available at http://ssrn.com/abstract=1485689


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