

1                   **The role of micro cavitation on EHL: a study using a**  
2                   **multiscale mass conserving approach**

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13                   **KEYWORDS**

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15                   Cavitation; Multiscale; Textured Surface; Elastohydrodynamic Lubrication (EHL).

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18                   **ABSTRACT**

19                   The role of micro cavitation in Elastohydrodynamic Lubrication is numerically investigated  
20                   using a multiscale approach whereby both the small scale topographical features and the  
21                   micro-cavitation of the lubricant due to the features are resolved. Micro-cavitation and the  
22                   fluid's shear-thinning property are modelled at the small scale of topological feature. The  
23                   effects of topographical features on the film thickness of the line contact bearings and friction  
24                   coefficient are presented with a focus on the role of micro-cavitation. This highlights how a  
25                   mass conserving small scale model can be used to model both micro-cavitation and cavitation  
26                   occurring at the bearing scale, and how topological features can be designed to reduce friction  
27                   while maintaining bearing load.  
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## 1 1 INTRODUCTION

2  
3 In this paper the role of cavitation in an Elastohydrodynamic Lubrication (EHL) converging-  
4 diverging line contact is investigated. The bearing surfaces are a smooth moving roller surface  
5 relative to a stationary, textured flat surface. Topographical changes to a lubricated surface of  
6 industrial components have been experimentally and numerically shown to improve their  
7 tribological performance in three main aspects, the load carrying capacity, the friction  
8 coefficient and the lubricant fluid film [1, 2]. Such applications include piston rings [3, 4],  
9 mechanical seals [5, 6], journal bearings [7, 8], pad bearings [9-12] and roller bearings in line  
10 contacts [13-17] and point contacts [18, 19].

11 A number of numerical approaches have been proposed to represent lubrication of surfaces  
12 with topographical features [7, 17, 18, 20, 21]. One of the challenges of numerically describing  
13 these problems is the order of magnitudes difference in the size of bearing surface topography  
14 and the bearing itself. This has led to a number of multiscale methodologies to analyse the  
15 problem and overcome the limitation in terms of computing costs [22-26]. Among the  
16 multiscale models, many of them employ an adapted Reynolds equation based on Patir and  
17 Cheng's average flow model [27] to solve the large scale fluid pressure, and the Stokes or  
18 Navier-Stokes equations to solve the small scale fluid flow [22, 24, 26]. Recently, the  
19 homogeneous multiscale approach has been developed, in which the large scale fluid flow was  
20 governed by a homogeneous pressure-gradient function whose coefficient was obtained from  
21 the small scale simulations. These include the work of Nyemeck et al. [25] on the  
22 hydrodynamic lubrication with rigid bearing surfaces of seals, and the authors' work [11, 12] on  
23 the EHL simulation of micro-textured pad bearings.

24 The role of micro-cavitation on lubrication has been studied by a number of investigators  
25 arising from experimental observation of cavitation occurring in the vicinity of surface  
26 roughness [20, 28]. The role of cavitation raises further questions regarding the validity of  
27 using a form of the lubrication equation, where cavitation effects may not be uniform across  
28 the film thickness due to the underlying topography; this cannot be captured by the lubrication  
29 approximation where a constant pressure is assumed across the film thickness. Olver et al. [29]  
30 proposed an 'inlet suction' effect due to fluid flow driven by cavitation pressures located in the

1 inlet region of the pad bearing surface. Ausas et al. [30] and Qiu and Khonsari et al. [31]  
2 studied micro-cavitation in textured bearing lubrication using a mass conserving model and  
3 compared different boundary conditions of cavitation; the half-Sommerfeld condition, Swift-  
4 Steiber (Reynolds) condition and the Floberg–Jakobsson–Olsson (JFO) condition. It was found  
5 that the Reynolds condition largely underestimated the cavitation area and predicted a higher  
6 load-carrying capacity than the JFO results. Other studies of micro-cavitation have used Navier-  
7 Stokes based Computational Fluid Dynamics (CFD) simulations to solve the fluid flow, for  
8 example, Shi and Ni [32], Wahl et al. [33] and Meng and Yang [34]. However, these studies of  
9 micro-cavitation were all modelled at a single scale, where the topographical features were  
10 described over the entire lubrication domain. The number of simulated micro dimples or  
11 grooves in these studies was limited to up to 10 due to the very fine mesh required to resolve  
12 the small scale features and cavitation. In real engineering applications the number of micro  
13 dimples (and roughness) could be much larger on a real textured bearing’s surface, and a  
14 multiscale method is especially relevant to solve such problem.

15 In this paper the heterogenous multiscale method (HMM) [35] is applied to EHL as derived  
16 by the authors [11, 12, 36] and extended to include cavitation effects, via the application of a  
17 mass-conserving approach at both small and large scales. This enables the model to capture  
18 cavitation at both scales. The pressure gradient-mass flow rate relationship is obtained from a  
19 homogenised local scale solution. This relationship is subsequently used at the global scale as a  
20 governing equation of fluid flow, and solved along with the conservation of mass. In this work  
21 cavitation is considered at the local scale via a predefined threshold cavitation pressure. The  
22 effects of the micro-texture’s geometrical parameters on the bearings’ lubrication film  
23 thickness and friction coefficient are presented. The piezo-viscous and shear-thinning effects  
24 are discussed and the importance of the role of micro cavitation at the small scale is  
25 highlighted.

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## 27 **2 NUMERICAL METHODOLOGY**

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### 29 **2.1 Geometry and Materials**

1 In this study, the global geometry of the lubrication model is a two-dimensional cylindrical  
 2 line contact. The smooth cylinder rotates relative to a textured stationary surface, as shown in  
 3 Fig. 1. The material of the plane is PTFE with an elastic modulus ( $E$ ) of 0.5 GPa and Poisson's  
 4 ratio ( $\nu$ ) of 0.4. The cylinder is assumed to be rigid compared to the soft PTFE bearing surface.  
 5 The radius of the cylinder ( $r$ ) is 25 mm and the rotation speed ( $\omega$ ) is 80 rad/s, and equivalent to  
 6 a sliding speed ( $U_0$ ) of 2 m/s. The micro-pocket length ( $L$ ) ranges from 20  $\mu\text{m}$  and 100  $\mu\text{m}$  and  
 7 the depth ( $d$ ) from 0  $\mu\text{m}$  and 30  $\mu\text{m}$ . The geometrical and material parameters are listed in  
 8 Table 1.

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## 10 2.2 Large Scale Simulation

11 The large scale simulation describes the fluid-structure interaction in the global lubrication  
 12 domain, where the fluid pressure is solved simultaneously with the elastic deformation of the  
 13 bearing surfaces. The difference between the current study and classical EHL analysis is that  
 14 the governing equation for the hydrodynamic pressure is a homogenised equation from the  
 15 small scale simulations, rather than the Reynolds equation, expressed as

$$\frac{d\hat{p}}{dx} = f(g, \hat{p}, \dot{m}) \quad (1)$$

16 together with the mass conservation equation

$$\frac{d\dot{m}}{dx} = 0 \quad (2)$$

17 The pressure gradient ( $\frac{d\hat{p}}{dx}$ ) is a homogenised function of the pressure ( $\hat{p}$ ), mass flow rate ( $\dot{m}$ )  
 18 and film gap ( $g$ ), interpolated from a series of small scale solutions. The large scale boundary  
 19 conditions used to solve Eqs. (1) and (2) are that the pressure at the bearing inlet and outlet  
 20 boundaries is equal to zero:

$$\hat{p}_{in} = \hat{p}_{out} = 0 \quad (3)$$

21 The line contact bearing load is balanced by an integral of the average small scale pressure  
 22 (i.e., load per unit length  $p^*$ ), along the line contact domain. The average small scale pressure  
 23 ( $p^*$ ) was defined in Eq. (17) in the Section 2.3.1 'Small Scale Simulations'.

$$w = \int_{x_{in}}^{x_{out}} p^* dx \quad (4)$$

1 The geometry equation is expressed as a sum of the rigid displacement ( $e$ , an unknown  
2 constant determined by load  $w$ ), rigid gap geometry and the surface deformation ( $\delta$ ):

$$h = e + \frac{x^2}{2r} + \delta \quad (5)$$

$$\delta = \mathbf{K} \times p^* \quad (6)$$

3 where the displacement influence coefficient matrix  $\mathbf{K}$  was obtained using the Green's function  
4 [37] for linear elastic contact model.

5

## 6 **2.3 Small Scale Simulations**

7 The small scale problem is described by the flow equations and those governing the elastic  
8 deformation of the small scale features. The coupling is facilitated through the application of  
9 the Arbitrary Lagrangian Eulerian (ALE) method to describe the fluid domain as the solid  
10 domain deforms and the inclusion of non-Newtonian, piezo-viscous and cavitation effects are  
11 included.

12

### 13 **2.3.1 Solid Structure Model**

14 The small scale solid domain is described by a plain strain model with the separation of the  
15 displacement influence coefficient matrix  $\mathbf{K}$  into local (diagonal,  $\mathbf{K}_1$ ) and global (off-diagonal,  
16  $\mathbf{K}_2$ ) terms [12]. The film gap ( $g$ ) at the small scale is described by the sum of undeformed gap  
17 and the non-local deformation ( $\mathbf{K}_2 \times p$ ), not including the local deformation ( $\mathbf{K}_1 \times p$ ).

$$g = e + \frac{x^2}{2r} + \mathbf{K}_2 \times p \quad (7)$$

18 The local terms are represented as part of the local fluid-structure interaction simulation, and  
19 the global terms are applied as they would be in a conventional EHL simulation. An equivalent  
20 height of the solid domain is introduced in order to ensure that the local deformation in the  
21 small scale simulations is the same as the column deformation obtained from the diagonal  
22 matrix. For further details of the structure model the reader is referred to references [12].

23

### 24 **2.3.2 Laminar Flow Model**

1 The steady-state, isothermal and laminar flow is governed by the compressible Navier-  
 2 Stokes equations:

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathbf{I} + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - 2\eta/3(\nabla \cdot \mathbf{u})\mathbf{I}] \quad (8)$$

$$\nabla \cdot (\rho\mathbf{u}) = 0 \quad (9)$$

3 where,  $\mathbf{u}$  is the fluid velocity vector and  $\mathbf{I}$  the unit tensor.  $\rho$  is the generalized fluid density  
 4 which is a function of pressure based on Dowson-Higginson formula [38],

$$\rho = \theta \cdot \rho_0 \frac{c_1 + c_2 \cdot p}{c_1 + p} \quad (10)$$

5 where,  $c_1$  and  $c_2$  are the density-pressure coefficients,  $\rho_0$  is the density at ambient pressure,  $\theta$   
 6 is the density fraction of liquid in the liquid and gas mixture and is defined in the cavitation  
 7 model in Section 2.3.2. Piezo-viscous effects are described by an exponential relationship [39],  
 8 the viscosity of liquid and gas mixture ( $\eta$ ) is expressed as:

$$\eta = \theta \cdot \eta^* \exp(\alpha \cdot p) \quad (11)$$

9 with the pressure-viscosity coefficient  $\alpha$ . Considering the fluid shear-thinning property, the  
 10 generalized viscosity  $\eta^*$  is defined using a Carreau viscosity model [40] below.

$$\eta^* = \eta_\infty + (\eta_0 - \eta_\infty)[1 + (\eta_0 \dot{\gamma}/G_{cr})^2]^{\frac{m-1}{2}} \quad (12)$$

11 where,  $\dot{\gamma}$  is shear rate.  $\eta_0$  and  $\eta_\infty$  represent the dynamic viscosity at zero shear rate and  
 12 infinite shear rate, respectively.  $G_{cr}$  is critical stress at ambient pressure. The piezo-viscous  
 13 effect described in Eq. (11) and the shear-thinning property defined in Eq. (12) are illustrated in  
 14 Fig. 2.

15 The boundary conditions are shown in Fig. 1b. The lower boundary CD (Fig. 1b) is a sliding  
 16 wall. The upper fluid-structure interface is a no slip boundary. In heterogenous multiscale  
 17 method, periodic boundary conditions are required on the AD and BC boundary in terms of  
 18 fluid velocity and pressure. Due to small scale deformation the two boundary geometries are  
 19 not exactly the same, therefore, the boundary BC was scaled to the same length of boundary  
 20 AD in the reference coordinates (undeformed gap) [12]. Near-periodic velocity boundary  
 21 conditions are derived from the mass conservation at the two boundaries, scaled by the local  
 22 strain  $\varepsilon$ :

$$\rho_1 \mathbf{u}_1(1 + \varepsilon_1) = \rho_2 \mathbf{u}_2(1 + \varepsilon_2) \quad (13)$$

1 and a pressure jump ( $\Delta p$ ) is applied onto the scaled boundaries:

$$p_2 = p_1 + \Delta p \quad (14)$$

2 where, subscript 1 and 2 represent the scaled boundary AD and BC in Fig. 1b respectively. Since  
3 the moving wall (lower surface) was fully constrained, i.e. there was no deformation allowed  
4 and the strain was zero, the velocities at both sides of the fluid domain on the moving wall  
5 surface were the same. Thus the nearly periodic conditions described in Eq. (**Error! Reference**  
6 **source not found.**) are the same as periodic conditions at the moving surface and it satisfies  
7 the no-slip boundary conditions.

8 The homogenised pressure gradient ( $\frac{d\hat{p}}{dx}$ ) across a unit cell and a pressure jump across the small  
9 scale cell is described by:

$$\frac{d\hat{p}}{dx} = \frac{\Delta p}{L} \quad (15)$$

10 The mass flow rate ( $\dot{m}$ ) is calculated as:

$$\dot{m} = \int_0^{g+K_1 \times p} \rho u \, dy \quad (16)$$

11 An average pressure ( $p^*$ ) is defined to represent the cell pressure in large scale solutions:

$$p^* = \int_0^L p \, dx / L \quad (17)$$

12 The shear stress  $\tau$  (shear force per unit length) is calculated as:

$$\tau = \int_0^L \eta^* \frac{du}{dy} \, dx / L \quad (18)$$

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### 14 **2.3.3 Cavitation Model**

15 The lubricant is assumed to be a homogeneous mixture of liquid and gas. When the fluid  
16 pressure drops below the saturation pressure cavitation occurs and some gas dissolved in fluid  
17 will come out of solution. The density fraction of liquid ( $\theta$ ) is defined as a continuous function  
18 of pressure using the hyperbolic tangent function:

$$\theta = 0.5 \times \left( 1 + \tanh \frac{p - p_c}{k} \right) \quad (19)$$

1 The constants  $k$  are used to determine the steep gradient of the density fraction with respect  
 2 to a threshold cavitation pressure  $p_c = -30$  KPa. The variation of  $\theta$  against pressure is shown  
 3 in Fig. 3. The relationship described here is similar to the polynomial based approach used by  
 4 Almqvist and Larsson [41], to describe the density of lubricant with the fluid pressure. The  
 5 parameters of fluid properties are given in Table 2, and are based on mineral oil of the type  
 6 typically used as bearing lubricant [42, 43].

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#### 9 **2.4 Homogenisation of the pressure gradient equation**

10 The homogenised relationship between the pressure gradient and mass flow rate links the  
 11 small scale and large scale simulations. This relationship is obtained via interpolation. In order  
 12 to obtain an accurate representation of the small scale model, a range of small scale  
 13 simulations were undertaken for a range of gaps ( $g$ ), homogenised pressure gradients ( $\frac{\Delta p}{L}$ ) and  
 14 cell inlet pressures ( $p_1$ ). A linear interpolation function was adopted, based on a Delaunay  
 15 triangulation of the data using Quickhull algorithm as implemented in Matlab [44]. To obtain  
 16 effective data samples for the interpolation, the range of input parameters are selected as  
 17 shown in Table 3 with total number of 3000 sample points, based on the corresponding results  
 18 of the smooth surface case of Reynolds equation.

19

20 Small scale solutions were obtained using the finite element method as implemented in  
 21 COMSOL Multiphysics. The variables are transformed to the non-dimensional forms for  
 22 convenience of numerical computing, for the global scale,

$$X, Y = x, y/R_{Hz}, G = \frac{g}{L}, \hat{P} = \frac{\hat{p}L}{\eta_0 U_0}, \frac{d\hat{P}}{dX} = \frac{d\hat{p}}{dx} \cdot \frac{L^2}{\eta_0 U_0}, \dot{M} = \frac{\dot{m}}{\rho_0 U_0 L} \quad (20)$$

23 and for the small scale,

$$P_1 = \frac{p_1 L}{\eta_0 U_0} \quad (21)$$

24

25 where  $R_{Hz}$  is the Hertzian contact radius,



$$R_{Hz} = \sqrt{\frac{8wr}{\pi E'}} \quad (22)$$

1 Subsequently, the pressure gradient equation is obtained via linear interpolation,

$$\frac{d\hat{P}}{dX}(i) = f \left[ P_1, G, \dot{M}, \frac{d\hat{P}}{dX}, \hat{P}(i), G(i), \dot{M} \right], i = 1, \dots, n \quad (23)$$

2 where the first four parameters on the right-hand side ( $P_1, G, \dot{M}, \frac{d\hat{P}}{dX}$ ) were known and obtained  
 3 from small scale analysis.  $n$  denotes the mesh points at the large scale domain. The non-  
 4 dimensional mass conservation equation is expressed as,

$$\frac{d\dot{M}}{dX}(i) = 0, i = 1, \dots, n \quad (24)$$

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### 7 **3 Results**

8 A non-dimensional large scale domain of  $X = [-4, 2]$  and a fixed load of 2500 N was  
 9 considered in this study. Large scale mesh independence tests were undertaken from 60 to 960  
 10 points in the large scale domain. For the Newtonian case with a smooth surface, the relative  
 11 errors in the large-scale pressure and mass flow rate using different mesh are presented in  
 12 Fig.4 (a), and the large-scale pressure distributions are compared in Fig.4 (b) and (c). The  
 13 presence of smooth surface solutions allowed comparison with the solution obtained using  
 14 Reynolds equation. In current study the number of mesh nodes  $n$  was set at 120, at which level  
 15 the relative errors were approximately 7% and 5% in pressure and mass flow rate respectively.  
 16 Four fluid viscosity models were investigated, i.e. (i) Newtonian, (ii) Newtonian and piezo-  
 17 viscous, (iii) shear-thinning, and (iv) both piezo-viscous and shear-thinning. In each model, a  
 18 range of cell lengths ( $L = 20, 50$  and  $100 \mu\text{m}$ ) and depths ( $d = 0 \sim 30 \mu\text{m}$ , increased by  $5 \mu\text{m}$ )  
 19 were considered. The friction coefficient and minimum film thickness are presented in Fig. 5.  
 20 Typical results showing how cavitation is captured at the large scale is shown in Fig. 6 (a) and  
 21 (b) for the case of  $L = 100 \mu\text{m}$  and  $d = 30 \mu\text{m}$ , where the large scale homogenised pressure and  
 22 viscosity are presented from the small scale simulations, and the elastic deformation of the  
 23 bearing surface also presented. The development of cavitation at small scale is demonstrated

1 in Fig. 6 (c) and (d), in the large scale outlet zone in the region of  $X = [0.8, 1.8]$ . The role of  
2 shear thinning fluid properties is demonstrated in Fig. 7, where the homogenised viscosity is  
3 clearly observed to decrease in the main loading domain at the large scale. The combination of  
4 both piezo-viscosity and shear-thinning effect on the pressure, viscosity and film thickness is  
5 shown in Fig. 8.

## 6 7 **4 Discussion**

### 8 9 **4.1. Fluid Rheology**

10 It can clearly be seen from these results that through the careful selection of small scale  
11 depth and length, that the friction coefficient can be reduced. The friction coefficient is  
12 presented as a function of the depth to length ratio ( $d/L$ ) in Fig. 5, these results imply that  
13 there is an optimal cell depth and length to achieve minimum friction. For example, in the  
14 Newtonian cases, for the cell length  $L = 50 \mu\text{m}$  the minimum friction coefficient was observed  
15 with the cell depth  $d = 10 \mu\text{m}$  and the reduction in the friction coefficient is 42% compared to  
16 the smooth surface; for the cell length  $L = 100 \mu\text{m}$  the friction coefficient is reduced by 52%  
17 when the cell depth  $d$  is  $15 \mu\text{m}$ . This is similar to the results previously obtained by Gao and  
18 Hewson [36] who obtained a similar trend for a slider bearing with the same small scale  
19 surface features. In these cases there was a monotonic decrease in the friction with increasing  
20 small scale length to depth ratio ( $d/L$ ). What is interesting to note is that contrary to the  
21 previous case there is a clear minimum friction coefficient predicted for a cell depth to length  
22 ratio of around 0.15 for the cases of  $L \geq 50 \mu\text{m}$ . In the previous case it was observed that the  
23 friction coefficient decreased then plateaued out to a near constant friction coefficient with  
24 increasing cell depth.

25 While the different fluid rheologies considered all exhibit similar characteristics it should be  
26 noted that the reduction in friction is most pronounced for the shear thinning fluid  
27 characteristics, where there is a 3 fold decrease in the friction coefficient for the largest cell  
28 geometry  $L = 100 \mu\text{m}$ . The effect of piezo-viscous is not significant for a low pressure values  
29 encountered in the current study. The variations in the pressure, viscosity and film thickness

1 are less than 10% by comparing the results with or without piezo-viscous effect considered, as  
2 shown in Figs. 7 and 8.

3 The minimum film thickness decreases with increasing cell depth as shown in Fig. 5, and this  
4 has been reported in other EHL studies of textured surface [18, 20, 45]. Examining the  
5 minimum film thickness shows how there is a clear compromise to be made between reducing  
6 friction and maintaining a reasonable fluid film, as the effect of topography is to reduce the  
7 minimum film thickness, with the greatest effect observed for the shear thinning fluid model.  
8 The reasonable fluid film should have a minimum value which is double or triple the surface  
9 roughness, i.e. the lambda ratio is 2 or 3, which means the bearing could operate in the mixed  
10 or full film lubrication regime.

#### 11 **4.2. Micro-Cavitation**

12 Cavitation was included in the small scale geometry, permitting the modelling of the  
13 converging-diverging geometry to be modelled without a specific large scale treatment of  
14 cavitation. The pressure distributed over the whole lubrication domain was governed by the  
15 homogenised pressure equation. This is different from classical EHL models, where the  
16 Reynolds boundary condition (pressure is positive everywhere) is commonly applied in the  
17 diverging geometry. Since the small scale pocket itself is a divergent-convergent geometry, the  
18 local pressure usually decreases at the inlet divergent edge, and then increase at the outlet  
19 convergent edge. When there is limited cavitation in the small scale the local pressure  
20 distributed is nearly anti-symmetrically (as shown by the top curve in Fig. 6 (c), at location  $X =$   
21  $0.8$ ). When cavitation extended towards outlet zone, i.e.  $X$  increases, the pressure field  
22 diverges from this. The cavitation region can be observed in Fig. 6 (d) where the region of low  
23 density fraction indicates a larger cavitated zone as  $X$  increases. As the region of cavitation  
24 increases further the local pressure became nearly constant of  $-30$  KPa (as shown by the  
25 bottom curve in Fig. 6 (c), at location  $X = 1.8$ ).

26 What is interesting is that there is a rise in viscosity in the diverging region before cavitation  
27 occurs as shown in Fig. 7. This can also be observed when piezo-viscosity is also added to the  
28 model as is shown in Fig. 8.  
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## 5 CONCLUSION

A heterogeneous multiscale model has been developed for the fluid-structure interaction in cylindrical line contact EHL with the bearing surface topography addressed. Fluid cavitation is explicitly modelled at the small scale via a continuous function of the fluid density and viscosity with pressure. The small scale cavitation effects are passed to the large scale model via the homogenised small scale relationship without further large scale treatment of cavitation. Such an approach also allows a range of rheological models to also be considered. The shear-thinning effects have been found to have significant effect on the bearing performance as well as the optimum small scale features required for optimum performance.

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## NOMENCLATURE

$d$	Cell depth [m]
$E$	Young's modulus [Pa]
$E'$	Equivalent Young's modulus [Pa]
$e$	Rigid displacement [m]
$h$	Large scale film thickness [m]
$g$	locally undeformed gap [m]
$\mathbf{K}$	Displacement influence coefficient matrix [ $\text{m}^3/\text{N}$ ]
$L$	Cell length [m]
$n$	Number of large-scale mesh grid
$p$	Pressure [Pa]
$p_c$	Threshold cavitation pressure [Pa]
$\dot{m}$	Mass flow rate per unit length [kg/m/s]
$r$	Radius of cylinder [m]
$t'$	Equivalent small-scale cell height [m]
$U_0$	Sliding speed of the roller [m/s]

$\mathbf{u}$	Fluid velocity vector [m/s]
$w$	One-dimensional load per unit length [N/m]
$x$	Coordinate in direction of fluid flow [m]
$X$	Dimensionless coordinate of $x$
$\alpha$	Pressure-viscosity coefficient
$\dot{\gamma}$	Shear rate [1/s]
$\delta$	Elastic deformation [m]
$\varepsilon$	Strain
$\eta_0$	Viscosity at zero shear rate [Pa s]
$\eta_\infty$	Viscosity at infinite shear rate [Pa s]
$\eta$	Generalized viscosity in Carreau model [Pa s]
$\theta$	Density fraction in cavitation model
$\mu$	Friction coefficient
$\sigma$	Normal stress [Pa]
$\tau$	Shear stress [Pa]
$\nu$	Poisson's ratio
$\rho_0$	Ambient fluid density [kg/m <sup>3</sup> ]
$\rho$	Generalized fluid density [kg/m <sup>3</sup> ]
$\omega$	Rotation velocity of cylinder [rad/s]

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## 1 **Tables and Figures**

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27 with optimal parameters compared to smooth surface; the arrow shows location of the  
28 minimum film thickness)

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31 with optimal parameters compared to smooth surface; the arrow shows location of the  
32 minimum film thickness)

33  
34 Fig. 9 Shear-thinning and piezo-viscous solutions: (a) pressure and viscosity, (b) film thickness  
35 (textured surface with optimal parameters compared to smooth surface; the arrow shows  
36 location of the minimum film thickness)



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Table 1 Geometrical and material parameters

Cylinder radius	$r = 25 \text{ mm}$
Micro pocket length	$L = 20 \text{ }\mu\text{m} - 200 \text{ }\mu\text{m}$
Micro pocket depth	$d = 5 \text{ }\mu\text{m} - 30 \text{ }\mu\text{m}$
Young's modulus of PTFE	$E = 0.5 \text{ GPa}$
Poisson's ratio of PTFE	$\nu = 0.4$
Sliding speed of smooth surface	$u = 2 \text{ m/s}$

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Table 2 Fluid Properties

Ambient pressure	$p_a = 10^5 \text{ Pa}$
Ambient fluid density	$\rho_0 = 870 \text{ kg/m}^3$
Viscosity at zero shear rate	$\eta_0 = 0.01 \text{ Pas}$
Viscosity at infinite shear rate	$\eta_\infty = 0.006 \text{ Pas}$
Density-pressure coefficient	$c_1 = 5.9 \times 10^8 \text{ Pa}$ $c_2 = 1.34$
Pressure-viscosity coefficient	$\alpha$ $= 2.2 \times 10^{-8} \text{ Pa}^{-1}$
Critical stress at ambient pressure	$G_{cr} = 2 \times 10^4 \text{ Pa}$
Power in Carreau viscosity model	$m = 0.6$
Constants in cavitation model	$k_1 = 5$ $k_2 = 1.5 \times 10^5 \text{ Pa}$

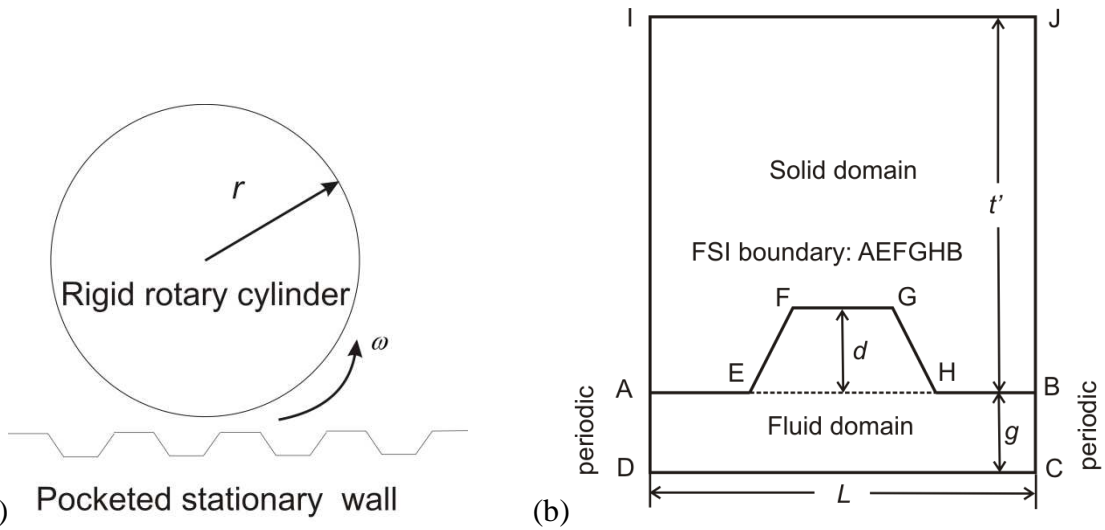
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Table 3 Date selection of small scale simulations

	Range	Number of mesh point
Gap	1- 55 $\mu\text{m}$	30
Pressure gradient	-2e10 -2e10 (N/m <sup>3</sup> )	10
Cell inlet pressure	-0.2 – 4 (MPa)	10

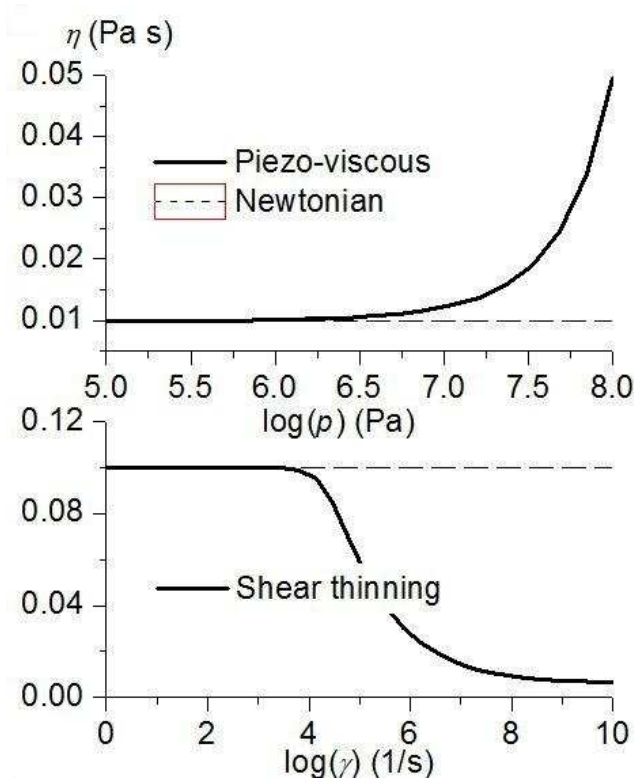
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4 (a) Pocketed stationary wall (b)  
5 Fig. 1 (a) Global geometry of a cylinder bearing in line contact, (b) micro pocket geometry of a  
6 unit cell on the stationary wall surface (side length  $|AE| = |FG| = |HB| = L/5$ ,  $|EF| = |GH| =$   
7  $\sqrt{(L/5)^2 + d^2}$ .)

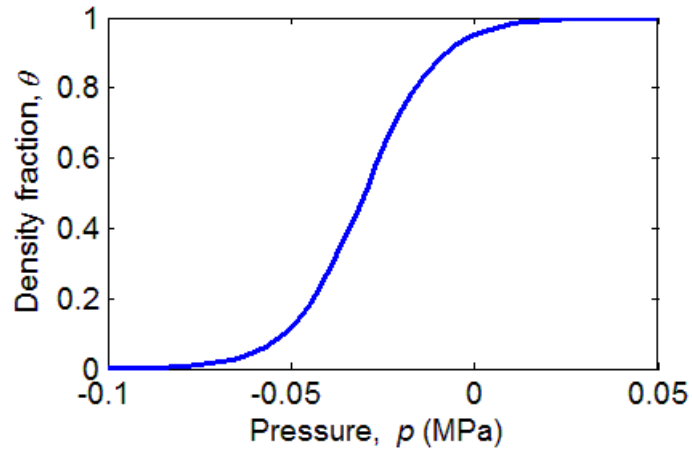
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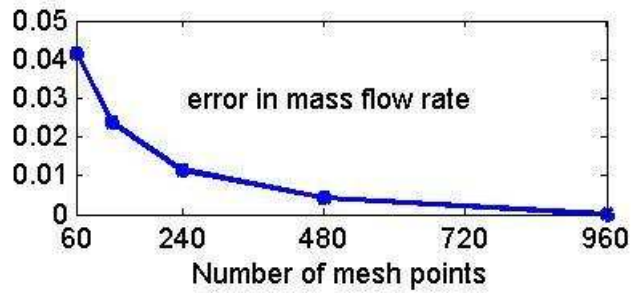
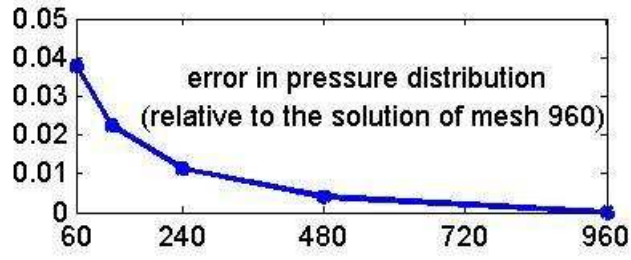
Fig. 2 Shear-thinning and piezo-viscous fluid property



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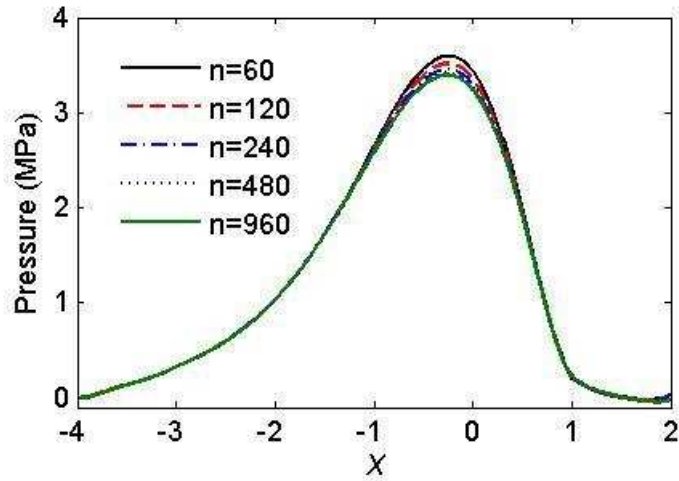
Fig. 3 Variations of the density fraction against fluid pressure described by hyperbolic tangent function

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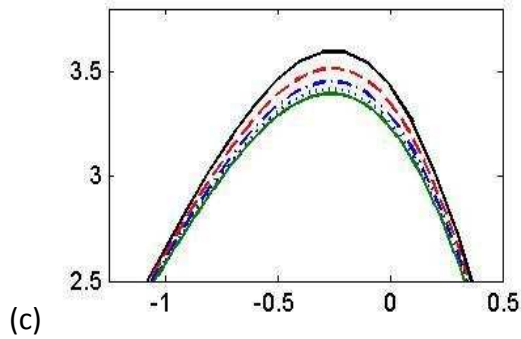
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(a)



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(b)

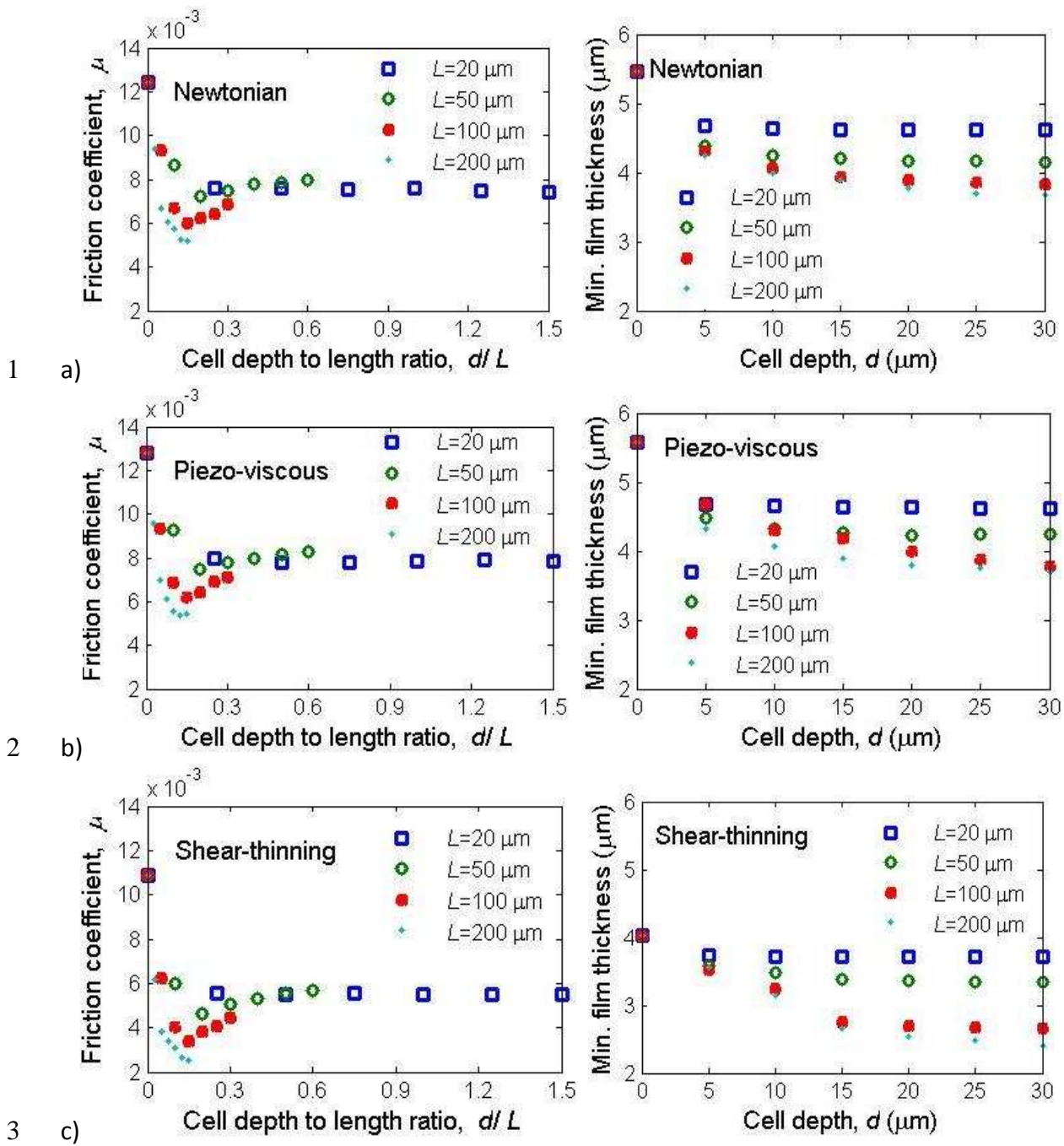


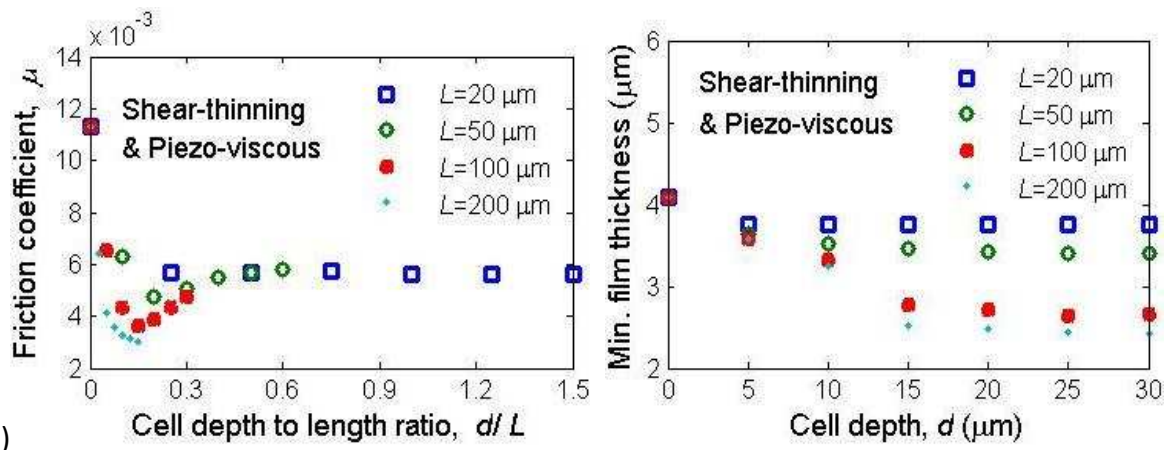
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(c)

1 Fig. 4 Mesh sensitivity analysis on smooth surface: (a) relative errors in the large-scale pressure  
2 and mass flow rate using the solution of the finest mesh (960) as reference, (b) the large-scale  
3 pressure distributions, and (c) enlarged local pressure details of figure (b)  
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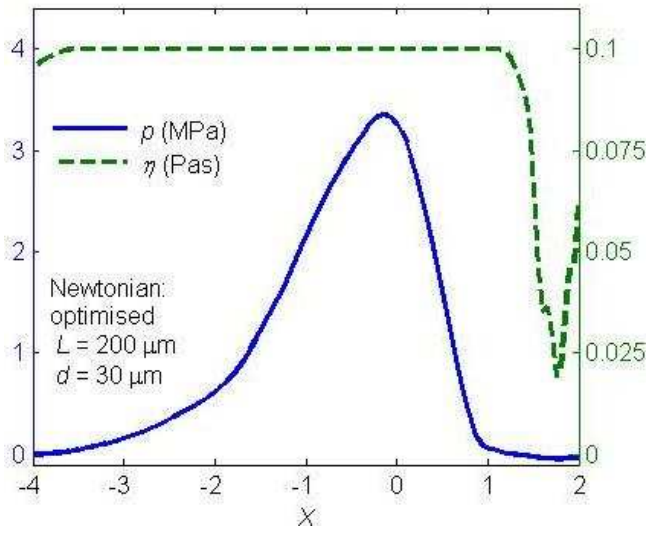




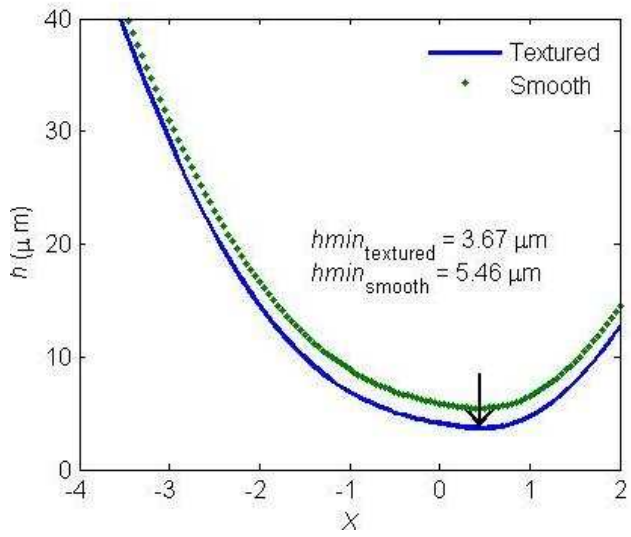
1 d)

2 Fig. 5 Friction coefficient against ratio ( $d/L$ ) (left) and the minimum film thickness against cell  
 3 depth ( $d$ ) (right); a) Newtonian; b) Peizo-viscous; c) Shear-thinning; d) Shear-thinning & Piezo-  
 4 viscous

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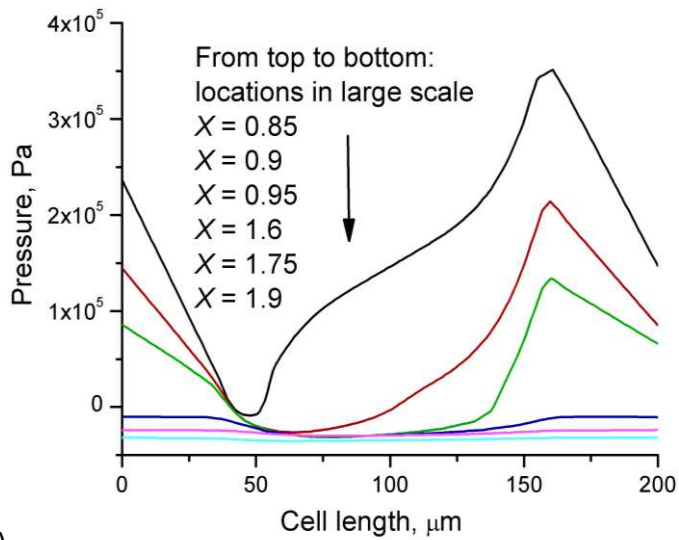


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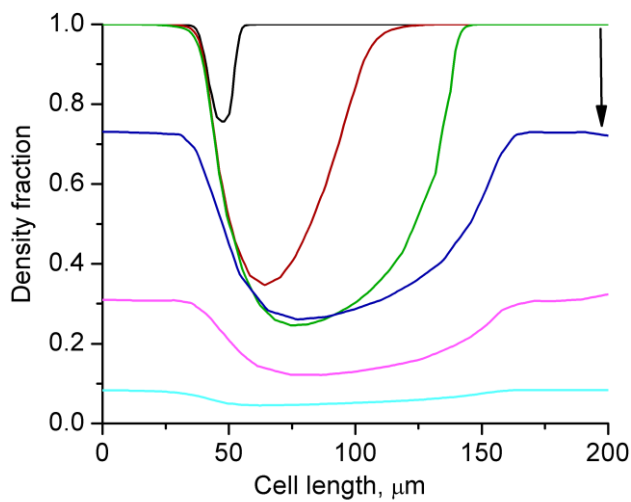
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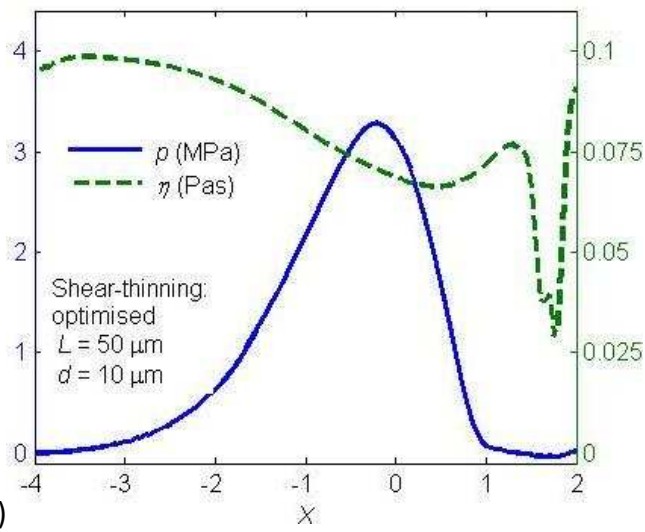


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4 Fig. 6 Newtonian solutions: (a) pressure and viscosity fraction, (b) film thickness (textured  
 5 surface with optimal parameters compared to smooth surface), (c) small scale pressure and (d)  
 6 density fraction variations at different locations convergent zone  $X = [0.85, 1.9]$  in the large  
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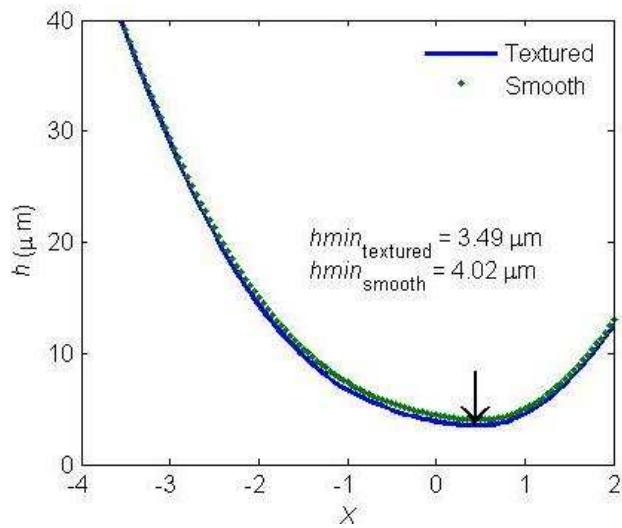
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2 a)

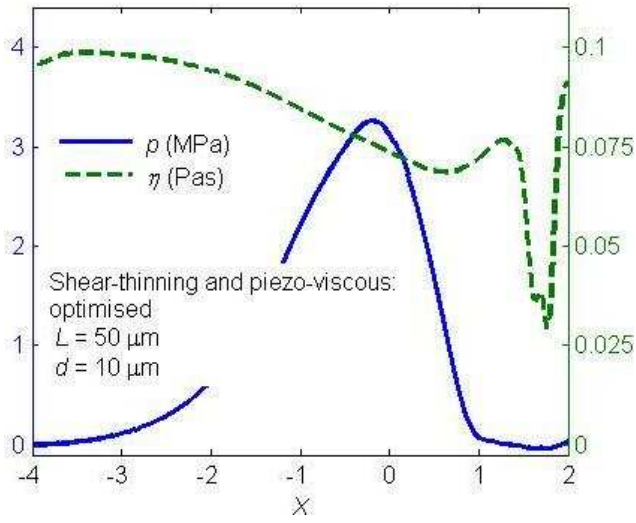
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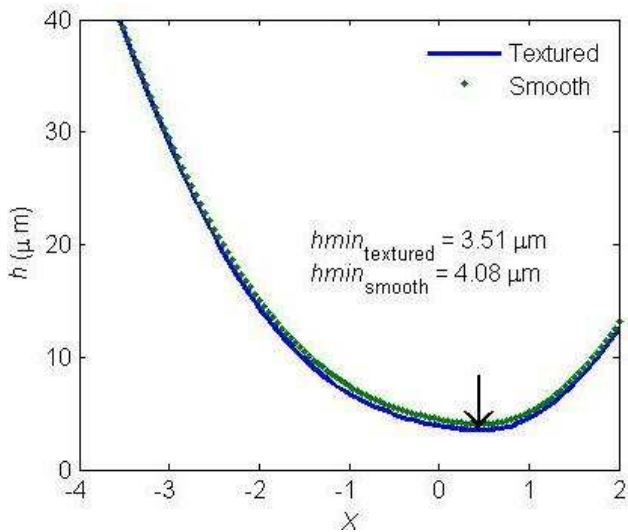
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4 Fig. 7 Shear-thinning solutions: (a) pressure and viscosity, (b) film thickness (textured surface  
5 with optimal parameters compared to smooth surface; the arrow shows location of the  
6 minimum film thickness)

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1 a) b)



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3 Fig. 8 Shear-thinning and piezo-viscous solutions: (a) pressure and viscosity, (b) film thickness  
4 (textured surface with optimal parameters compared to smooth surface; the arrow shows  
5 location of the minimum film thickness)  
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