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# Non-Cooperative Power Control Game in D2D Underlying Networks with Variant System Conditions

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**Abstract:** In this paper, the problem of power control using a game theoretic approach based on sigmoid cost function is studied for device-to-device (D2D) communications underlying cellular networks. A non-cooperative game, where each D2D transmitter and a cellular user select their own transmit power level independently, is analyzed to minimize their user-serving cost function and achieve a target signal to interference-plus-noise-ratio (SINR) requirement. It is proved analytically that the Nash equilibrium point of the game exists and it is unique under certain constraints. Numerical results verify the analysis and demonstrate the effectiveness of the proposed game with variant system conditions, such as path loss exponents, target SINR, interference caused by the cellular user, pricing coefficients, and sigmoid control parameter.

**Keywords:** D2D Networks; non-cooperative game; sigmoid function; cost function; power control; Nash equilibrium

## 1. Introduction

Device-to-Device (D2D) communication has been increasing in recent years, which represents an add-on communication paradigm to the modern 5G wireless cellular networks [1]. Thus, D2D communications reduce the traffic seen by the Base Station (BS), and thus increase the spectral efficiency, energy efficiency, and system capacity [2–5]. In addition, the quality of service (QoS) in D2D communication is necessary to guarantee high reliability in data transmission. To solve the problem of spectrum scarcity and increase the system capacity, a D2D user can share the licensed spectrum with cellular users. Moreover, the information data increases proportionally with the number of D2D and cellular users, and therefore the interference caused by these devices will deteriorate the received signal of these data [6]. Therefore, the reduction of aggressive interference and guarantee of reliable transmission have emerged as the key issues in D2D networks. To address such issues, appropriate power control algorithms are needed in both D2D and cellular user devices. In addition, a power control algorithm is necessary for energy efficiency and to increase the battery life of devices [7]. This can be done by ensuring that the most efficient strategy is implemented by selecting the minimum power level that achieves the desired QoS requirement.

Power control has been studied in various wireless networks and scenarios, in which the objective was to achieve reliable communication for wireless devices and maintain the QoS requirement. Recently, game theory is exploited to address the problem of power control in modern wireless networks such as cognitive radio (CR) and Femtocell networks [8]. Among all these algorithms, the signal-to-interference-and-noise-ratio (SINR) based on power control is the most well-known.

The QoS objective based only on SINR is not appropriate in wireless data networks because the error-free communication had high priority [9]. Typically, a user would prefer receiving a higher signal quality (high SINR) while using less amount of power to extend its battery life. The relationship between user's transmits power and quality of received signal conflict objectives is an important issue in network resources [9]. To find the right balance between the users' transmit power and the received SINR, game theory based on the utility (or cost) function presents a promising solution.

The players, strategy, and the utility (cost) function are the most important elements of the game model. In the game model, each player selects its strategy from the available strategy set to maximize its own utility or minimize its cost. Similarly, each user in wireless networks selects its transmitting power from the power strategy set, to improve its received signal with less power consume. Therefore, the conflict between users transmit power and received signal SINR can be expressed in the mathematical formula of the utility or cost function.

In this paper, we use a sigmoid cost function introduced in previous work [10] to adapt the game in our framework. Here, we examine the cost function in a different topology of D2D networks with multiple D2D transmitter–receiver pairs that communicate together and share the spectrum with cellular users. The following contributions are thus made in this work.

- We formulate a non-cooperative game for multiple (D2D) users, where each pair of users can communicate with each other without exchanging information from a base station, and determine the Nash equilibrium of the game based on extreme value theorem.
- We describe the existence of Nash solution of the proposed game and prove the uniqueness of this Nash equilibrium. We show that the analysis in [10] is also valid in the D2D system. In addition, we derive the dependence of the Nash equilibrium on the distance between the transmitter device and receiver.
- We prove that the Nash equilibrium of the game may not exist in some variant system conditions. Unlike previous studies, we assume that the cellular user does not change its transmit power, and we proved through extensive simulations that the existence of Nash equilibrium of the power control game is also depend on other system parameters, such as the target SINR, distance, sigmoid, and pricing coefficient parameters.

The remainder of the paper is structured as follows. In Section 2, we present an overview of the related work. The system model of underlying D2D network is introduced in Section 3. In Section 4, the non-cooperative power control game, including the Nash equilibrium of the game, bounds on transmitter power at equilibrium, properties of equilibrium, and dependence of the equilibrium on distance, is described. The simulations results and performance of the proposed game are given in Section 5. Finally, the concluding remarks of this paper are made in Section 6.

## 2. Related Work

Many of the previous researches focused on the mathematical expression of their proposed cost function to design and develop the power allocation algorithm. The famous research in this context was proposed by Koskie and Zoran [11], which aimed to slightly decrease the users' SINR to obtain a significant reduction in their power consumption. They proposed a Nash game model, in which the cost function is defined as a weighted sum of power and square of signal-to-interference-and-noise-ratio (SINR) error. On the other hand, the authors of [12] proposed a new power control game based on the cost function and the target of transmit power, which has been included as well as the target SINR. The cost function in [12] is defined as a weighted sum of the Logarithm of SINR error and the Logarithm

function of power error. The algorithm has many advantages, such as fast convergence and better anti-noise performance and capacity compared with the previous algorithm. In addition, the authors of [13] proposed a non-cooperative power control algorithm also based on the cost function, similar to the authors of [12], by using the square root function instead of the logarithm function to improve the acceleration of the algorithm. In [14], the authors proposed a power control game in CDMA cognitive radio networks based on the cost function, and they used two SINR thresholds to adjust the interference factor of power and improve the fairness among users. Moreover, the author of [15] investigated how to decide the transmission power levels of cognitive radio users using non-cooperative game theory. They proposed a novel cost function, in which the thresholds of the SINR and transmission power level were considered. The numerical results obtained by executing the algorithm indicate better performance in terms of anti-noise.

In [10], the SIR-based sigmoid power control game in cognitive radio networks (CRNs) is proposed to reduce the power consumption of cognitive radios. The authors proposed a new cost function to formulate the power control algorithm, which consists of a weighted sum of power and square function of SINR error, based on sigmoid function. The numerical results indicate that the sigmoid power control can achieve a better reduction in power with the same SINR comparing to other previous algorithms. The acceleration of power control algorithm based on the sigmoid cost function that proposed in [10], has been improved in [16] by increasing the speed of convergence. The sigmoid power control algorithm can be accelerated using numerical methods such as the Newton Iterative Method. In a further improvement, the authors of [17] proposed a new chaos-based cost function to design the power control algorithm and analyzed the dynamic spectrum sharing issue in the uplink of cellular CRNs. The chaos cost function is defined by taking into account the interference and interference tolerance of the primary users. The algorithm led to significantly lower power consumption and fast convergence. On the other hand, a game theory approach based on cost function has been applied in the Femtocell networks with local gain, the results of which were efficient in terms of power-saving and SINR achievement [18]. In [19], the authors introduced a cost function for MTC NOMA networks that is supposed to be convex, non-negative, and has a minimum value. The objective was to minimize the power consumption as much as possible and keep SINR beyond an acceptable level. They found that their algorithm satisfied the SINR requirement with less power consumption and higher power efficiency with the same constrains.

In the context of D2D underlying cellular networks, the interference between D2D and cellular user links that sharing the same spectrum is one of the critical problems [20]. Therefore, efficient power control algorithms are necessary to avoid failure of both the D2D and cellular user links. Many researches addressed the problem of the interference between D2D and cellular user links based on power allocation scheme in D2D underlying networks [21,22]. The power control based on game theory has been used in some literature for interference management and to solve the resource allocation problem. In [23], the problem of uplink power control has been addressed in D2D underlying cellular networks, in which the interaction between D2D users modeled through a non-cooperative power control game. The authors of [24] developed a Nash bargaining solution and Nash competitive game frameworks to model the power allocation problem. In other work [25], the problem of spectrum sharing and power allocation has been addressed for D2D underlying networks. The problem was modeled based on the Stackelberg game model, in which the base station represents the leader that coordinates the interference from the D2D transmission to the cellular users. The D2D users act as followers in the game and compete the spectrum in a distributive fashion. On the other hand, the authors of [26] proposed a two-step approach in a multicell D2D underlay cellular network, where the first step concerns the efficient resource block (RB) allocation to the users, and the second is the transmission power allocation. In the first step, the RB allocation problem is formulated as a bilateral symmetric interaction game; whereas, in the second step, the power allocation problem is formulated as a linear programming problem per RB. Numerical results showed improvement in performance in terms of interference reduction, power minimization, and increase number of supported users.

In addition, the power control problem has been considered based on Nash equilibrium to eliminate the interference in [27]. The problem modeled as a non-cooperative game model; whereas, by adjusting the residual energy factor, the algorithm obtained a better equilibrium income, and the power control problem has been adopted for successive interference cancellation. Moreover, the authors of [28] investigated the power allocation problem for a distributed multiple-radar. The new cost function proposed in [28], considered the target detection performance and the received interference power was used to formulate the non-cooperative power allocation strategy in distributed multiple-radar spectrum sharing system. The simulation results indicate that the distributed algorithm is effective for power allocation and could protect the communication system with limited implementation overhead.

In [29], the authors proposed a new system model for D2D users based game theory to study the behavior of users under different channel conditions. They found that a user with poor channel conditions could significantly improve its rate especially when the channel conditions of the paired users are quite different. The resource allocation and power control in D2D has been also studied based on non-cooperative game in [30] to prolong the overall system survival time of mobile cells. In [31], the Stackelberg game based interference control scheme with full frequency reuse in D2D underlaying mmWave small cell network has been proposed. The scheme aims to optimize the transmit power of D2D links, reduce the interference caused by D2D communication to the mmWave small cell and take full advantage of the bandwidth of the millimeter band. Numerical results show rapid convergence, keeps signal to interference-plus-noise-ratio (SINR) in a high range and achieves excellent throughput performance. In further study, two utility functions of cellular and D2D users has been proposed in [32] to enhance the system spectral efficiency and to increase the SINR. The authors proved the existence and uniqueness of the Nash equilibrium (NE), and the simulation results indicate that the performance of both cellular and D2D users are improved by using the mixed game, in terms of average power.

### 3. System Model

Let us consider a D2D system consisting of one base station and multiple different users. As shown in Figure 1, there are two kinds of users: cellular users that communicate directly with the base station via uplink mode and D2D users that each pair have a direct data transmission. The pair of D2D users satisfies the constraint of the distance of D2D communication and other D2D pairs can have communicating demands at the same time. We assume that the cellular users communicate with the base station in uplink direction and share their spectrum resources with D2D pairs. In this paper, we assumed that the interference from other cells is controlled by intercell interference coordination [33].

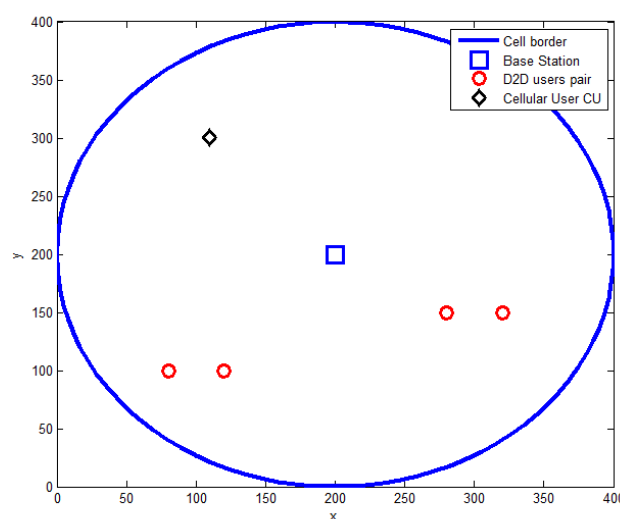


Figure 1. System model of Device-to-Device (D2D) system with uplink resource sharing.

We denote  $UE_{d1}$  and  $UE_{d2}$  to the D2D transmitter and receiver pair and  $UE_c$  to the cellular user. Therefore, we suppose that any resource blocks occupied by cellular users can be reassigned to multiple D2D pairs. Thus, if  $UE_{d1}$  and  $UE_c$  are sharing the same resources they may interfere with each other, in which the interference produced by  $UE_c$  will be received by the receiver  $UE_{d2}$ . We model the system based on distance dependent path loss condition, and we used the free space propagation path-loss model  $h_i = A(r_0/r_{i,k}^j)^{\alpha/2}$ , where  $A$  is a constant,  $\alpha$  is the path loss exponent,  $r_{i,k}^j$  is the distance of the  $i - k$  link, and  $r_0$  is the reference distance.

Denote the D2D user set as  $\mathcal{N} = \{1, 2, \dots, N\}$ , the cellular user set as  $\mathcal{M} = \{1, 2, \dots, M\}$ , and the set of all user as  $\mathcal{Q} = \mathcal{N} \cup \mathcal{M}$ . We denote  $p_i$  to the power of D2D user, where  $i \in \mathcal{N}$  and  $p_k$  to the power of cellular user where  $k \in \mathcal{M}$  and we have used the subscript “ $-i$ ” in  $p_{-i}$  to indicate the power of all users in  $\mathcal{Q}$  except the D2D user  $i$ . let  $g_{ii}$  and  $g_{ij}$  be the channel gain between the D2D user  $i$  and the receiver (pair) and between the D2D user  $i$  and the receiver of other  $j$ th D2D users respectively. In addition, we denote  $g_{ik}$  to the channel coefficient between D2D user  $i$  and the cellular users  $k$ . The channel gain between the D2D user  $i$  and base station is denoted by  $h_{i,b}$ , the channel gain between cellular user by  $k$ , the base station is denoted by  $h_{k,b}$ , and  $\sigma_i^2$  and  $\sigma_k^2$  are the additive white zero-mean Gaussian noise at D2D pair  $i$  and cellular user  $k$  receiver, respectively.

Due to the coexistence of D2D pairs and cellular users, the QoS of both users will be degraded due to the interference caused during their communications. Therefore, the QoS of all users can be guaranteed if all users achieve a higher SINR than a threshold and the total interference accumulated at BS must be less than interference temperature limit ( $I_{TL}$ ) as

$$\sum_{i=1}^N p_i h_{i,b} + \sigma_i^2 + \sum_{k=1}^M p_k h_{k,b} + \sigma_k^2 \leq I_{TL} \tag{1}$$

At the D2D receiver, the desired signal is the signal from the D2D transmitter  $i$ . Therefore, the signal from other D2D transmitters,  $j \neq i$ , plus the signal of cellular users will be treated as noise at the D2D receiver. Thus, the SINR at the D2D receiver of user  $i$  is

$$\gamma_i = \frac{p_i g_{ii}}{\sum_{j=1, j \neq i}^N p_j g_{ij} + \sigma_i^2 + \sum_{k=1}^M p_k g_{ik} + \sigma_k^2} \tag{2}$$

The denominator of the Equation (2) represent the sum of the interference plus noise and can be denoted by  $I(p_{-i})$ . Therefore, the Equation (2) can be rewritten as

$$\gamma_i = \frac{p_i g_{ii}}{I(p_{-i})} \tag{3}$$

It is found from Equation (3) that the SINR at any D2D receiver can be maximized by increasing the transmission power of its paired transmitter. However, increasing power will cause more interference to the neighboring D2D pairs and cellular users, as well as increasing transmission power, will accelerate the drain of battery. Therefore, to meet this trade-off, an efficient power control algorithm has to be assigned to all D2D and cellular transmitters. The objective of power control algorithm is used to guide all users to achieve the target value of SINR, ( $\Gamma_i$ ), that guarantees successful transmission with a minimum expense of power  $p$ .

#### 4. Non-Cooperative Power Control Game

In this paper, we propose a non-cooperative power control game in which the D2D transmitters and the cellular users are the players of the game and each user seeks to minimize its own cost by choosing its transmit power  $p_i$  from the power strategy set. Practically, the base station and D2D receivers are able to compute the amount of interference based on the received signal strength. Both cellular and D2D transmitter are acting individually (no user has information about other power strategy) and they update their transmit power depend only on the value of interference received from

their receivers via the feedback channel. The proposed non-cooperative power control game can be expressed in a basic form as

$$G = [\mathcal{Q}, P_i, J_i] \tag{4}$$

where  $\mathcal{Q} = 1, 2, \dots, N + M$  are the set of players,  $P_i = [0, P_i^{max}]$  is the strategy set for the  $i$ th user, where  $P_i^{max}$  is the user maximum transmit power and  $J_i$  is the cost function of  $i$ th user. Given a reference target SINR  $\Gamma_i$ , we propose the sigmoid cost function in [10] as

$$J_i(p_i, \gamma_i) = \frac{2g_{ii}}{I(p_{-i})} \sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) p_i + c_i(\Gamma_i - \gamma_i)^2 \tag{5}$$

where  $b_i$  and  $c_i$  are the weighting factors and  $a$  is the sigmoid factor. The function  $\sigma(\cdot)$  represents the sigmoid function, which is defined as

$$\sigma(x, a) = \frac{2}{1 + e^{-ax}} - 1 \tag{6}$$

Therefore, according to the cost function defined in Equation (5), cellular and D2D users have conflicting objectives. They are seeking to higher SINR by increasing their transmission power but the cost increase due to the increase of energy consumption in their battery-based devices and the limits of interference constraint. Thus, non-cooperative power control game problem can be expressed as the following optimization problem,

$$\min_{p_i \in P_i} J_i(\gamma_i, p_i) = \min_{p_i \in P_i} \frac{2g_{ii}}{I(p_{-i})} \sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) p_i + c_i(\Gamma_i - \gamma_i)^2 \tag{7}$$

With some necessary conditions, the optimization problem in Equation (7) can be solved by obtaining the Nash equilibrium.

#### 4.1. Nash Equilibrium

The Nash equilibrium point(s) means that no user can improve its individual cost function unilaterally. Mathematically, for  $i \in N$

$$J_i(p_i^*, \gamma_i(\mathbf{p}_{-i}^*)) < J_i(p_i, \gamma_i(\mathbf{p}_{-1}^*, \mathbf{p}_{-2}^*, \dots, \mathbf{p}_{-i-1}^*, \mathbf{p}_i, \mathbf{p}_{i+1}^*, \dots, \mathbf{p}_N^*)) \forall i = 1, 2, \dots, M + N \tag{8}$$

where  $\mathbf{p}_{-i}^*$  is the Nash Equilibrium power vector for cellular and D2D users except user  $i$ , and  $J_i(p_i^*, \gamma_i(\mathbf{p}_{-i}^*))$  is the cost function of D2D user  $i$  with transmit power  $p_i$  which the other users transmit with power  $p_i^* \in \mathbf{p}_{-i}^*$ . The Nash equilibrium can be obtained by taking the first derivative of cost function in (5) with respect to  $p_i$  and equating it with zero as

$$\frac{\partial J_i(\gamma_i, p_i)}{\partial p_i} = \left(\frac{2g_{ii}}{I(p_{-i})}\right) \sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) - 2\left(\Gamma_i - \frac{p_i g_{ii}}{I(p_{-i})}\right) \left(\frac{\partial \gamma_i}{\partial p_i}\right) = 0 \tag{9}$$

Rearranging terms of Equation (9) yields

$$\gamma_i = \Gamma_i - \sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}\right) \tag{10}$$

Substituting for  $\gamma_i$  in Equation (3), we obtain the Nash equilibrium power formula as

$$p_i = \frac{I(p_{-i})}{g_{ii}} \Gamma_i - \frac{I(p_{-i})}{g_{ii}} \sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}\right) \tag{11}$$

According to Equation (11), when the interference detected by D2D user, we get

$$p_i^{(t+1)} = \begin{cases} \frac{I^{(t)}(p_{-i})}{g_{ii}}\Gamma_i - \frac{I^{(t)}(p_{-i})}{g_{ii}}\sigma\left(\frac{b_i I^{(t)}(p_{-i})}{c_i g_{ii}}\right), & \text{if positive.} \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

where  $p_i^{(t+1)}$  is the power of D2D user  $i$  at  $(t + 1)$ th iteration and  $I^{(t)}(p_{-i})$  is the interference received by  $i$ th D2D user at  $(t)$ th step iteration. We can also rewrite the power update algorithm in terms of previous power  $p_i^{(t)}$  and current SINR measurement  $\gamma_i^{(t)}$  as

$$p_i^{(t+1)} = \begin{cases} \frac{p_i^{(t)}}{\gamma_i^{(t)}}\Gamma_i - \frac{p_i^{(t)}}{\gamma_i^{(t)}}\sigma\left(\frac{b_i p_i^{(t)}}{c_i \gamma_i^{(t)}}\right), & \text{if positive.} \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

**Proposition 1.** A Nash equilibrium exists for the proposed game  $G$ .

**Proof.** For a game  $G$  the existence of equilibrium can be shown from extreme value theorem [34], A Nash equilibrium exists in a game  $G = [\mathcal{N}, P_i, J_i]$  for all  $i = 1, 2, \dots, N$  if the following two conditions are met,

1. The action set  $P_i$  is a nonempty, convex and compact subset of some Euclidean space  $\mathcal{R}^N$
2. The cost function  $J_i = p_i \rightarrow \mathcal{R}$  is continuous in  $p_i = [0, P_i^{max}]$ .

Note that the transmit power strategy space for each D2D user  $i$  in this game is defined by a minimum power, a maximum power, and all the power values in between these values. We also assume the maximum power is greater than the minimum power. Thus, the first condition of action set  $P_i$  is satisfied. To show that the function  $J_i(p_i, \gamma_i)$  is continuous in  $[0, P_i^{max}]$ , it is sufficient to show that the first order derivative  $\frac{\partial J_i}{\partial p_i}$  is defined in the interval  $[0, P_i^{max}]$ .  $\square$

$$\frac{\partial J_i(\gamma_i, p_i)}{\partial p_i} = \left(\frac{2g_{ii}}{I(p_{-i})}\right)\sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) - 2\left(\Gamma_i - \frac{p_i g_{ii}}{I(p_{-i})I_i}\right)\left(\frac{\partial \gamma_i}{\partial p_i}\right) \quad (14)$$

From Equation (2),

$$\frac{\partial \gamma_i}{\partial p_i} = \frac{g_{ii}}{I(p_{-i})} \quad (15)$$

Therefore, Equation (14) can be expressed as

$$\begin{aligned} \frac{\partial J_i}{\partial p_i} &= \left(\frac{2g_{ii}}{I(p_{-i})}\right)\sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) - 2\left(\Gamma_i - \frac{p_i g_{ii}}{I(p_{-i})}\right)\left(\frac{g_{ii}}{I(p_{-i})}\right) \\ &= \left(\frac{2g_{ii}}{I(p_{-i})}\right)\sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) - 2\left(\frac{\Gamma_i g_{ii}}{I_{ii}} - \frac{p_i (g_{ii})^2}{(I(p_{-i}))^2}\right) \\ &= \left(\frac{2g_{ii}}{I(p_{-i})}\right)\sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) - 2\left(\frac{\Gamma_i g_{ii} I(p_{-i}) - p_i (g_{ii})^2}{(I(p_{-i}))^2}\right) \end{aligned} \quad (16)$$

Since Equation (16) is real for  $[0, p_i^{max}]$ , equilibrium exist in  $G$ .

#### 4.2. Convergence

In [35], the author shows that if the algorithm  $p^{(k+1)} = f(p^{(k)})$  converges to a fixed point, the function  $f$  should satisfy the following three conditions:

1. Positivity  $f(p) \geq 0$
2. Monotonicity  $p \geq p' \Rightarrow f(p) \geq f(p')$

3. Scalability  $\forall \alpha \geq 1; \alpha f(p) \geq f(\alpha p)$

First, we prove the positivity condition, because

$$f_i(p) = \frac{I(p_{-i})}{g_{ii}} \Gamma_i - \frac{I(p_{-i})}{g_{ii}} \sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) \tag{17}$$

To make  $f(p) \geq 0$ , it needs

$$I(p_{-i}) < \frac{c_i g_{ii} \sigma^{-1}(\Gamma_i)}{b_i} \tag{18}$$

As  $\sigma^{-1}(\Gamma_i) \approx 1$ , if we choose a proper value for  $\frac{b_i}{c_i}$ , the positivity condition can be easily met.

The monotonicity condition can be proved by increasing best response function with respect to  $I(p_{-i})$  by differentiating Equation (15) with respect to  $I(p_{-i})$ , we get

$$g_{ii} \frac{\partial f_i(p)}{\partial I(p_{-i})} = \Gamma_i - \sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right) \left(\frac{b_i I(p_{-i})}{c_i g_{ii}}\right) \frac{\sigma\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}, a\right)}{\partial\left(\frac{b_i I(p_{-i})}{c_i g_{ii}}\right)} \tag{19}$$

Using inequalities  $\sigma(x, a) \leq x, a = 1$  and  $\frac{\partial \sigma(x, a)}{\partial x} \leq \frac{a}{2}$ , for monotonicity, we should have

$$\Gamma_i \geq \frac{(1 + 0.5a)b_i I(p_{-i})}{c_i g_{ii}} \Rightarrow I(p_{-i}) \leq \frac{c_i \Gamma_i g_{ii}}{b_i(1 + 0.5a)} \tag{20}$$

Finally, the condition of the scalability in our method can be written as

$$\begin{aligned} \alpha f(p) - f(\alpha p) &= \frac{\alpha p_i}{\gamma_i} \Gamma_i - \frac{\alpha p_i}{\gamma_i} \sigma\left(\frac{b_i p_i}{c_i \gamma_i}, a\right) - \left(\frac{\alpha p_i}{\gamma_i} \Gamma_i - \frac{\alpha p_i}{\gamma_i} \sigma\left(\frac{b_i p_i}{c_i \gamma_i}, a\right)\right) \\ &= \frac{\alpha p_i}{\gamma_i} \sigma\left(\frac{b_i \alpha p_i}{c_i \gamma_i}, a\right) - \frac{\alpha p_i}{\gamma_i} \sigma\left(\frac{b_i p_i}{c_i \gamma_i}, a\right) \end{aligned} \tag{21}$$

As  $\alpha > 1$ , we have  $\sigma\left(\frac{b_i \alpha p_i}{c_i \gamma_i}, a\right) \geq \sigma\left(\frac{b_i p_i}{c_i \gamma_i}, a\right)$ . Therefore, for the scalability, the positivity can be met and it is enough.

From the above analysis and according to the parameters, we can conclude that the power control function is standard function and the algorithm converges to a unique Nash Equilibrium point.

4.3. Dependence of Equilibrium on Distance

As the distance between the D2D transmitter and receiver is very small, we show that the Nash equilibrium also changes with the change of position of the cellular user in the system. The relation of the equilibrium with distance can be derived as follows. Consider one D2D pair (transmitter and receiver) and one cellular user changes his distance from base station. From Equation (20), the positivity and monotonicity condition for this case can be expressed as

$$A^2((r_0/r_2)^{\alpha/2})^2 P_2 \leq \frac{c_i \Gamma_i (A^2((r_0/r_1)^{\alpha/2})^2)}{b_1(1 + 0.5a)} \tag{22}$$

Thus,

$$r_1^1 \leq \left(\frac{c_1 \Gamma}{b_1 P_2}\right)^{1/\alpha} r_2^1 \tag{23}$$

where  $r_1^1$  and  $r_2^1$  are the distances between D2D transmitter and cellular user to the D2D receiver and base station, respectively. Equation (23) gives the upper bound of distance from the D2D transmitter to



its receiver for being in the equilibrium, and it is dependent on other cellular user’s power and the distance from its receiver.

### 5. Simulation Results

In this section, we evaluate the performances of the proposed game theory approach with two users, one D2D (Pair) and one cellular user. We performed the non-cooperative power control game simulation of the D2D underlying networks using GNU Octave simulation tool. The maximum transmit power of the D2D transmitters and cellular user are constrained as  $P_1^{max} = P_2^{max} = 1000$  mW, and the SINR target at the D2D and cellular user receivers is  $\Gamma = 5$ . The weighting coefficients  $b_i$  and  $c_i$  are chosen as 2 and 10, respectively, to satisfy the condition in Equation (18). The background noise is assumed to be  $8 \times 10^{-15}$  W [36]. The simulation of the underlying D2D system depends on the distance (path loss) between the cellular user and D2D receiver, so we consider fixed channel gain as  $g_{ik} = A(r_0/r_{ik})^{\alpha/2}$ , where  $A$  is a constant;  $\alpha$  is the path loss component;  $r_{ik}$  is the distance between the D2D transmitter,  $i$ , to the receiver of cellular user,  $k$ ; and  $r_0 = 100$  m is the reference distance. The simulation indicates that the Nash equilibrium of the proposed game of the D2D user depends on the action of the cellular user and its location.

In Figure 2, we illustrate the test of the cost function of both cellular and D2D users with respect to transmit power. In this part of the simulation, we consider a fixed distance between D2D user and its receiver (D2D receiver), as well as between the cellular user and base station, in which  $r_1 = r_2 = 200$  m. To achieve this test, we assume that the cellular user is transmitting at equilibrium power  $P_2^* = 100$  mW and we vary the transmit power of D2D user from 0 mW to 1 W. We observe that D2D user minimum value of cost function is at transmit power  $P_1^* = 500$  mW, as explained in the blue line of Figure 2, and this value represents the Nash equilibrium. By repeating the simulation with fixed value  $P_1$  at 500 mW and varying transmit power of the cellular user, we found that the minimum cost function  $J_2$  occur at transmit power 100 mW. Therefore, the Nash equilibrium solution of the system in the current set up for both cost functions to achieve the target  $\Gamma_i = 5$  is  $(P_1^*, P_2^*) = (500$  mW, 100 mW).

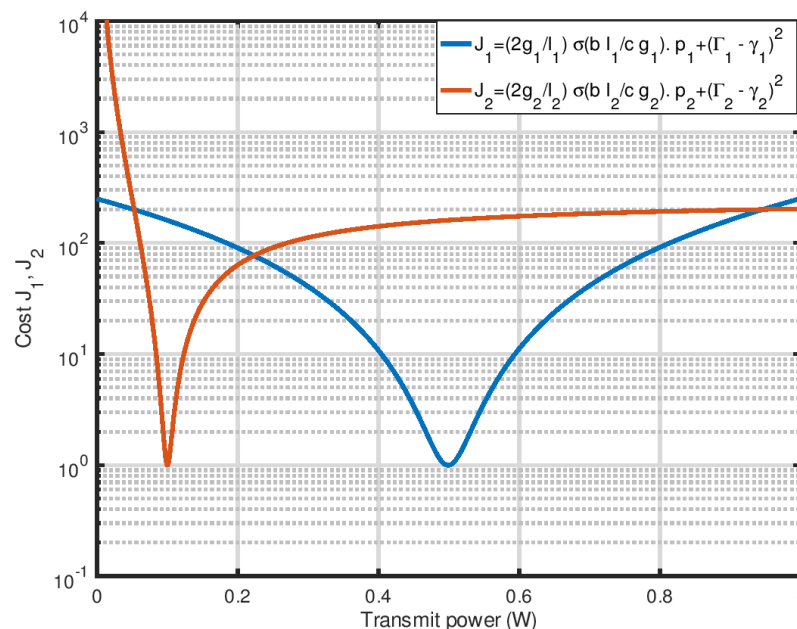


Figure 2. Cost function of D2D transmitter and cellular user as a function of transmit power.

The minimize points of cost function shown in Figure 2 are based on the cellular and D2D users' transmit power, but the Nash equilibrium is constrained by the interference as described in Equation (17).

In Figure 3, we test the interference effect to the D2D user caused by the change of transmitting power of the cellular user. We found that the Nash equilibrium of D2D user affected by cellular user transmits power, in which the Nash equilibrium increases by increasing the transmit power of the cellular user,  $P_k$ . We found that the Nash equilibrium and minimum of cost function, which exist in the case of transmitting power of the cellular user, do not exceed 170 mW. We also found that the Nash equilibrium does not exist in the system when the power of the cellular user is greater than 170 mW and within the limits of the D2D transmit power.

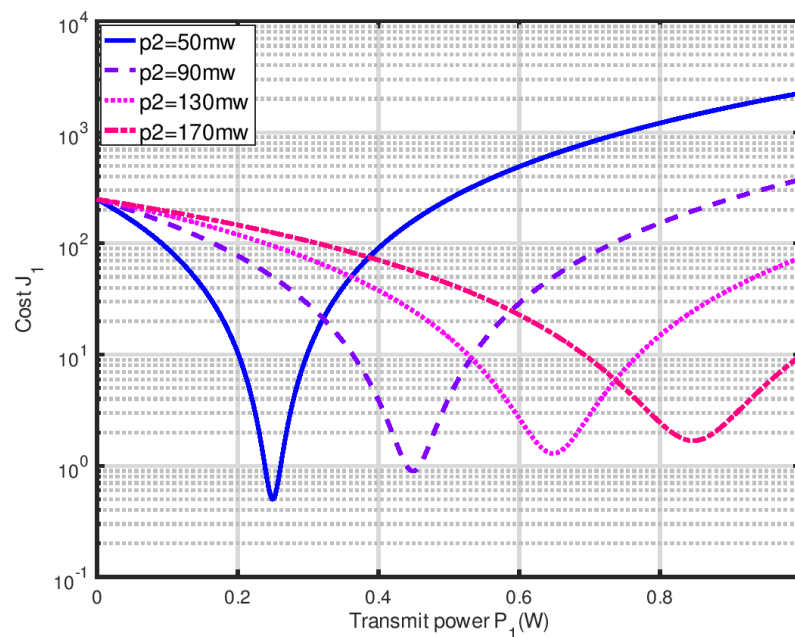


Figure 3. The Nash equilibrium of the D2D user depends on the transmit power of the cellular user.

In addition, we show the effect of distance between the D2D pair on the value of Nash equilibrium. Increasing the distance between the D2D pair with the fixed interference power from the cellular user is highly dependent on the distance between the transmitter and receiver (pair). Long distance encourages the transmitter to use high-power to maintain the target SINR  $\Gamma_i = 5$ , which will affect the value of the Nash equilibrium, as shown in Figure 4. By increasing the distance between the D2D pair greater than 350 m, the Nash equilibrium is no longer valid in the system within the limits of D2D transmit power.

We also show that the Nash equilibrium is dependent on the selected SINR target of system. Figure 5 shows the effect of target SINR to the Nash equilibrium, in which the increased of SINR target cause increasing in D2D transmit power. We found that with an increase in the target SINR value greater than 9, the Nash equilibrium is no longer valid in the system with the constraint of transmit power. Selecting a minimum value of target SINR that can guarantee the QoS of the system is important to reduce power consumption.

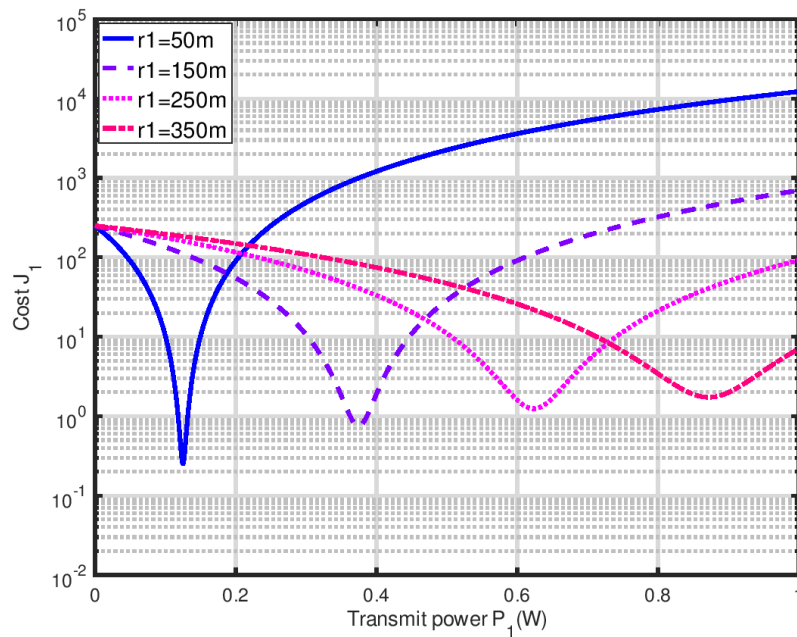


Figure 4. Nash equilibrium depends on the distance between the D2D transmitter–receiver pair.

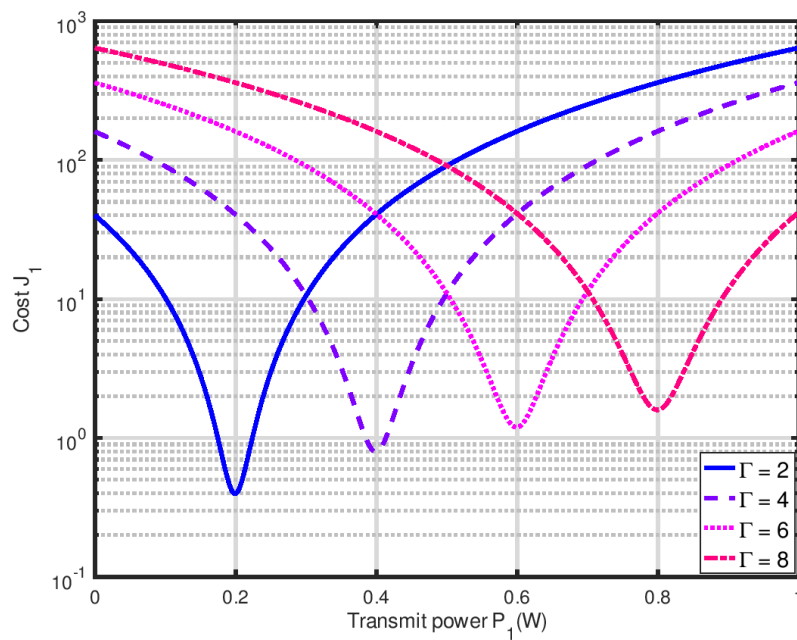


Figure 5. Nash equilibrium depends on the target SINR.

In Figure 6, we simulate the effect of a sigmoid parameter to the Nash equilibrium and the cost function. We fixed all parameters that can effect the D2D user from the cellular user to show the effect of a sigmoid parameter. The sigmoid parameter has an effect on the minimum value of cost function, but not the Nash equilibrium of D2D user. Increasing the sigmoid value causes a significant reduction in cost function with the same value of transmit power. Also, the sigmoid parameter has no effect on the existence of Nash equilibrium.

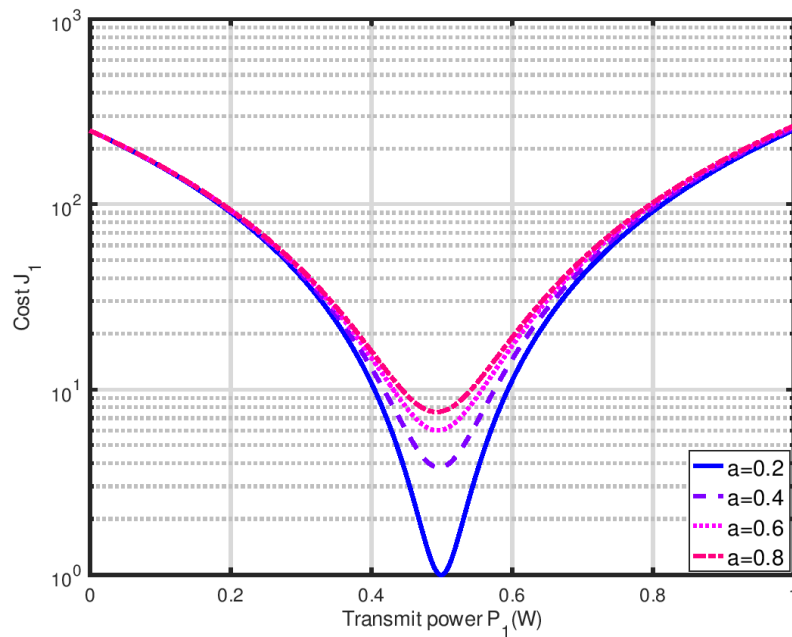


Figure 6. Nash equilibrium of D2D user depends on the sigmoid parameter  $a$

Finally, we simulate the effect of the pricing coefficients  $b_i$  and  $c_i$  to the Nash equilibrium and cost function values while keeping fixed all other parameters. Figure 7 shows that the increase in the ratio of  $b_i/c_i$  causes a decrease in the value of transmitting power (Nash equilibrium) of the D2D user, but with an increase in the cost function.

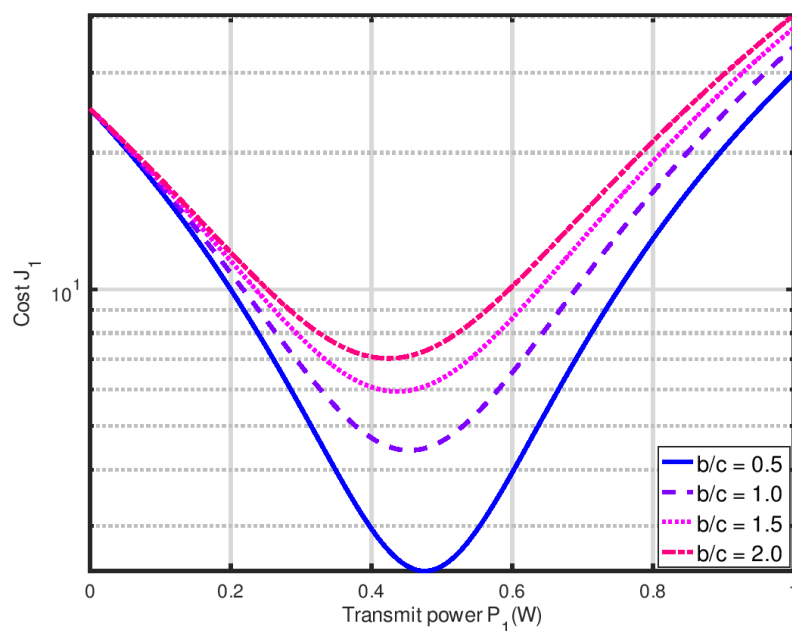


Figure 7. Nash equilibrium of D2D user depends on the pricing coefficients  $b_i/c_i$

### 6. Conclusions

A non-cooperative power control game has been proposed for the existence of a D2D network with cellular users, and the Nash equilibrium is derived. We show that the Nash equilibrium of the D2D user exists, is unique, and is constrained by the distance between the D2D transmitter and

receiver, the transmit power, received interference from the cellular user, and cost function parameters. We have shown that the Nash equilibrium occurs at the minimum value of cost function and at this point no user can improve its cost by deviating from Nash Equilibrium. The restriction of interference power caused by the cellular user, a distance between D2D transmitter and receiver, and the system target SINR on the Nash equilibrium has been verified with simulation. Moreover, the effect of the sigmoid and pricing coefficients parameters have also been verified in the simulation. The simulation results demonstrated that the sigmoid and pricing coefficients parameters do not affect the existence of Nash equilibrium, but instead change the value of cost function. This work is focused on the study of Nash equilibrium existence in a single D2D pair and a single cellular user using non-cooperative game model and with a variant system parameters. Based on this study, multiple users' networks can be modeled using the Stackelberg game model, in which the base station acts as a leader to coordinates the interference from D2D users to all transmitters. Future plan to extend this work is to consider a multi-user environment with admission and drop control algorithm to drop a user who cannot achieve the Nash equilibrium.

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## Abbreviations

The following abbreviations are used in this manuscript:

D2D	Device-to-Device
SINR	Signal-to-Interference-and-Noise-Ratio
QoS	Quality of service
CDMA	Code Division Multiple Access
MTC	Machine type communications
NOMA	Nonorthogonal multiple access

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