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An Integrated Replenishment and Transportation Model

Computational Performance Assessment

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13.1 Introduction

Transformation processes with multiple inputs typically exhibit nonlinearities in their output with respect to input usages. They have been traditionally modeled via production functions in the microeconomics literature (Heathfield and Wibe, 1987). One of the most common production functions is the Cobb–Douglas (C–D) production function. This production function assumes that multiple (n) inputs (also called factors or resources) are needed for output, Q, and they may be substituted to take advantage of the marginal cost differentials. In general, it has the form $Q = A\prod_{i=1}^{n}\left[x^{(i)}\right]^{\alpha_i}$, where A represents the total factor productivity of the process given the technology level, $x^{(i)}$ denotes the amount of input i used, and $\alpha i > 0$ is the input elasticity. The total elasticity parameter $r\left(=\frac{1}{\sum_{i=1}^{m}\alpha_i}\right)$ may be greater than (smaller than) or equal to 1 depending on whether there is diminishing (increasing) returns to resources, resulting in convex (concave) operational costs. The C–D production function was first introduced to model the labor and capital substitution effects for the US manufacturing industries in the early twentieth century (Cobb and Douglas, 1928). Despite its macroeconomic origins, since then, it has been widely applied to individual transformation processes at the microeconomic level, as well. For example, the C–D production function was employed to model production processes in the steel and oil industries by Shadbegian and Gray (2005) and in agriculture by Hatirli et al. (2006). Logistics activities associated with shipment preparation, transportation/delivery, and cargo handling also use, directly and/or indirectly, multiple resources such as labor, capital, machinery, materials, energy, and information technology. Therefore, it is not surprising that there is a growing literature on the successful applications of the C–D-type production functions to model the operations in the logistics and supply chain management context. Chang's (1978) work seems to be the earliest to construct a C–D production function to analyze the productivity and capacity expansion options of a seaport. Rekers et al. (1990) estimate a C–D production function for port terminals and specifically model cargo handling service. In a similar vein, Tongzon (1993) and Lightfoot et al. (2012) consider cargo handling processes at container terminals for their production functions. In a recent work, Cheung and Yip (2011) analyze the overall port output via a C–D production function. Studies on technical efficiency in cargo handling and port operations provide additional support for the C–D-type functional relationships, where output is typically measured in volume of traffic (in terms of twenty-foot equivalent unit—TEUs) and inputs may be as diverse as number or net usage time of cranes, types of cranes, number of tug boats, number of workers or gangs, length and surface of the terminals, berth usage, volume carried by land per berth, and energy (e.g., Notteboom et al. 2000, Cullinane 2002, Estache et al. 2002, Cullinane et al. 2002, 2006, Cullinane and Song 2003, 2006, Tongzon and Heng 2005). Comprehensive surveys can be found in

Maria Manuela Gonzalez and Lourdes Trujillo (2009), Trujillo and Diaz (2003), Tovar et al. (2007), and Gonzalez and Trujillo (2009). For land transportation, we may cite the evidence from Williams (1979) and for supply chain management, Ingene and Lusch (1999) and Kogan and Tapiero (2009).

Although multi-input activities in the area of logistics have received the attention of researchers for economic modeling and efficiency measurements, this body of knowledge has been only partially incorporated into decision making at the operational level. As Lee and Fu (2014) observed, the most commonly used transportation cost structures are tapering rates, proportional rates, and blanket rates (Lederer 1994, Taaffe et al. 1996, Ballou 2003, Coyle et al. 2008). Hence, scale economies are the most frequently made assumption. (See also Xu [2013] in a location context.) However, we believe that this assumption ignores the fundamental economic fact that output is typically nonincreasing in the input usage. That is, a C–D production function with total input elasticities being less than unity results in optimal input usage with usage costs being convex in the output level. Our work has been motivated by that the existing literature on the dynamic joint replenishment and transportation models lacks incorporation of the economic production functions. Incorporation of such functions of transportation/delivery activities into the existing logistics management models yields interesting theoretical and practical insights. First, these empirically supported functions, typically, result in the models to be nonlinear and convex in the decision variables for certain parameter settings. For such settings, the theoretical findings of the classical models do not hold any longer. Hence, these new settings are of theoretical interest. Second, the solution methodologies suitable and satisfactory for the classical models become less useful and, in some cases, even unusable. This necessitates the development of novel heuristics. (For a detailed discussion of both aspects in a dynamic lot-sizing framework, see Kian et al. 2014.) In this work, we focus on the suitability of the existing generic solvers and their computational performance for a logistics model with convex costs.

We envision a firm that produces a single product and delivers the production quantity to its vendor-managed inventory warehouse. We consider the dynamic joint replenishment and transportation problem for this integrated two-stage inventory system where the delivery times of the items from the production site to the warehouse and from the warehouse to a customer's site are negligible, but the logistical operations associated with shipment preparation, transportation/delivery, and cargo handling are nonlinear in the shipment quantity. In particular, we assume that the quantity transported requires multiple inputs whose usage is expressed by a C–D-type production function so that the resulting transportation costs are convex. Therefore, our work differs greatly from the existing models on replenishment and inbound/outbound logistics. Among the significant works in this area, we may cite Lippman (1969), Lee (1989), Pochet and Wolsey (1993), Lee et al. (2003), Jaruphongsa et al. (2005), Berman and Wang (2006), Van Vyve (2007), Hwang (2009), and Hwang (2010). Integrated replenishment and transportation problems have close similarity with the dynamic lot-sizing models in mathematical structure and analytical properties. A dynamic lot-sizing model with convex cost functions of a power form has been studied recently by Kian et al. (2014). It was shown that replenishment is possible even with positive on-hand inventory (contrary to the classical Wagner–Whitin model in Wagner and Whitin [1958]), and thereby, a forward solution algorithm does not exist. In lieu of the optimal solution, heuristics were designed and approximate solutions were investigated. For the related literature and the analytical intricacies of the particular lot-sizing model, we refer the reader to the aforementioned work.

The rest of the chapter is organized as follows. In Section 13.2, we present the assumptions of the model and provide three formulations. In Section 13.3, we provide a numerical study and discuss our findings.

13.2 Model

13.2.1 Assumptions

We consider a single item. The problem is of finite horizon length, T. The demand amount in period t is denoted by $d_t (t = 1,\ldots,T)$. All demands are nonnegative and known, but may be different over the planning horizon. No shortages are allowed. The amount of replenishment (production) in period t is denoted by q_t and is uncapacitated. Replenishment in any period t incurs a fixed cost (of setup) K_t (≥0) and unit variable cost, p_t. All units replenished in a period are transported to the warehouse; that is, dispatch quantity in a period is the same as the production quantity. Fixed costs associated with shipments are assumed negligible (or, equivalently may be viewed as subsumed in the fixed replenishment cost under the assumed dispatch policy). Each unit shipped in period t incurs a cost of τ_t. Additionally, the transportation and delivery use m (≥1) inputs with unit acquisition cost of input i in period t being $a_t^{(i)}$ for $1 \le i \le m$. It is assumed that there are no economies of scale in the acquisition of the inputs and that unit acquisition costs are nonspeculative over the problem horizon. These assumptions dictate that a lot-for-lot acquisition policy is optimal for the inputs needed. (A similar set of assumptions are implicitly made for the ingredients/raw materials needed for the replenishment that involves actual manufacturing.) The input usage for transporting qt units of the item in period t is determined through a stationary C–D function as $q_t = \prod_{i=1}^{m} \left[x_t^{(i)} \right]^{\alpha_i}$ with $\alpha_i \ge 0$ for all i. The stationarity of the function parameters are realistic in that the planning problem considered herein would be of very short term compared to the timeframe required for technological changes that would impact the values of the elasticity and total factor productivity parameters. The inventory on hand at the end of period t at the warehouse is denoted by I_t; each unit of ending inventory in the period is charged a unit holding cost of h_t. Without loss of generality, the initial inventory level, I_0, is assumed to be zero. Given that the short-term nature of the decisions, no discounting is assumed over the horizon although it can easily be incorporated into the model. The objective is to find a joint replenishment and transportation plan that determines the timing and amount of production and delivery (q_t) such that total costs over the horizon are minimized.

Before we proceed with the formulations of the problem, a few remarks are in order about the particulars of our problem setting. (1) In the presence of zero fixed costs of shipment, the assumed dispatch policy is optimal. However, with nonzero fixed costs, it would be suboptimal. This particular fixed cost structure has been studied by Jaruphongsa et al. (2005) with zero unit variable costs. Under nonspeculative (fixed and unit) costs, it has been established that the replenishment quantity in any period k needs to be either zero or equal to the sum of a number of future dispatch quantities. In our setting, we chose fixed shipment costs to be zero for the impact of the special nature of the variable costs to be brought to the foreground. (2) Since Lippman (1969), the shipments have taken into account cargo capacity of individual vehicles and considered stepwise cost structures. Again, for better exposition of the special cost function we assume herein, we ignore this aspect. Thus, our results may be viewed as a relaxation of this cargo capacity constraint. (3) The dynamic lot-sizing problems are special cases of the joint replenishment and transportation problems and, thereby, show close affinity with them under certain cost structures and policies. This is true in our setting, as well. The characteristics of the model herein are similar to those of Kian et al. (2014), and the two-echelon inventory system may be reduced to the single location lot-sizing model studied in the mentioned work. Therefore, in this work, we focus on the computational issues.

13.2.2 Formulations

We first formulate the problem as a mixed-integer nonlinear programming (MINLP) problem. We will consider two equivalent variants. In the first formulation, P_T^1, the decision variables are the replenishment (and shipment) quantities q_t, the binary variables y_t for replenishment setup, the input quantities $x_t^{(i)}$ for $i=1,\ldots,m$ with the intermediate inventory variables I_t for $1 \le t \le T$. The objective function is linear in the variables, but the constraints contain the nonlinear production function that relates the inputs to the replenishment/shipment quantity. In the second formulation, P_T^2, we first determine the optimal input usage for any replenishment/shipment quantity (which may be viewed as preprocessing) and incorporate the production function relationship into the objective function rendering the problem into a form with a nonlinear objective function with only linear constraints. In P_T^2, the decision variables are the replenishment (and shipment) quantities q_t, the binary variables y_t for replenishment setup with the intermediate inventory variables I_t for $1 \le t \le T$.

We state the first formulation P_T^1, which acts as a building block for the second formulation, formally as follows:

$$\min \sum_{t=1}^{T} \left[K_t y_y + \left(p_t + \tau_t \right) q_t + \sum_{i=1}^{m} \left(a_t^{(i)} x_t^{(i)} \right) + h_t I_t \right] \text{ s.t.} \tag{13.1a}$$

$$M y_t \geq q_t \quad t \in \{1, \ldots, T\} \tag{13.1b}$$

$$I_t = I_{t-1} + q_t - d_t \quad t \in \{1, \ldots, T\} \tag{13.1c}$$

$$q_t = A \prod_{i=1}^{m} \left[x_t^{(i)} \right]^{\alpha_i} \quad t \in \{1, \ldots, T\} \tag{13.1d}$$

$$y_t \in \{0,1\}, x_t^{(i)} \geq 0, \quad q_t \geq 0, \quad i \in \{1, \ldots, m\}, t \in \{1, \ldots, T\} \tag{13.1e}$$

where M is a sufficiently large positive number. The first set of constraints (13.1b) ensures that setups are performed only in the periods in which replenishment is positive, (13.1c) gives the evolution of on-hand inventories, (13.1d) represents the production function relating the inputs and the transported quantity, and (13.1e) are binary and nonnegativity constraints. We assume that the initial inventory is zero and these demands are net demands. The second formulation P_T^2 is obtained from P_T^1 by first deriving the optimal input allocations for a given shipment quantity. To this end, consider the subproblem where the input acquisition costs in period t are minimized given $q_t = Q$. As the input usage is uncapacitated, the first-order conditions imply that, for any i and j, $j \in \{1, \ldots, m\}$,

$$x_t^{(i)}(Q)^* = \frac{\alpha_i a_t^{(j)}}{\alpha_j a_t^{(i)}} x_t^{(j)}(Q)^* \tag{13.2}$$

where $x_t^{(i)}(Q)^*$ is the optimal usage of input i to transport Q units of the item. Hence, for $1 \leq i \leq m$,

$$x_t^{(i)}(Q)^* = \frac{\alpha_i}{a_t^{(i)}} A^{-r} \prod_{j=1}^{m} \left(\frac{a_t^{(j)}}{\alpha_j} \right) \alpha_j^r Q^r \tag{13.3}$$

(For details, see Heathfield and Wibe 1987.) Correspondingly, for a shipment quantity Q, the minimum transportation cost in period t, $C_t^*(Q)$, becomes

$$C_t^*(Q) = w_t Q^r + \tau_t Q \tag{13.4}$$

where

$$w_t = \left(\frac{1}{r}\right) A^{-r} \prod_{j=1}^{m} \left(\frac{a_t^{(j)}}{\alpha_j}\right)^{\alpha_j r}$$

The expression of $C^*(Q)$ enables us to rewrite the MINLP formulation as P_T^2 as follows:

$$\min \sum_{t=1}^{T} \left[K_t y_t + p_t q_t + C_t^*(q_t) + h_t I_t \right] s.t. \tag{13.5a}$$

$$M y_t \geq q_t \quad t \in \{1,\ldots,T\} \tag{13.5b}$$

$$I_t = I_{t-1} + q_t - d_t \quad t \in \{1,\ldots,T\} \tag{13.5c}$$

$$y_t \in \{0,1\}, \quad q_t \geq 0, \quad i \in \{1,\ldots,m\}, \quad t \in \{1,\ldots,T\} \tag{13.5d}$$

where M is as defined before. The constraints (13.5b), (13.5c), and (13.5d) perform the same function as in P_T^1, but we have been able to eliminate the input variables and to render all constraints linear at the expense of nonlinearizing the objective function. Clearly, the second formulation is more compact and has computational advantages as demonstrated in our numerical study. We can also formulate the problem as a dynamic programming (DP) problem. Define $J_t^T(I_t)$ as the minimum total cost under an optimal joint replenishment and transportation plan for periods t through T, where I_t is the ending inventory as defined before in the recursions (13.1c) or (13.5c). Then,

$$J_{t-1}^T(I_{t-1}) = \min_{q_t \geq \max(0, d_t - I_{t-1})} \left\{ K_t 1_{\{q_t > 0\}} + h_t I_t + p_t q_t + C_t^*(q_t) + J_t^T(I_t) \right\}$$
$$t \in \{1,\ldots,T\} \tag{13.6}$$

where $1_{\{q_t>0\}}$ indicates the existence of a setup in period t, with the boundary condition in period T being $J_T^T(I_T) = 0$ for any $I_T \geq 0$. The optimal solution is found using the earlier recursion, and $J_0^T(0)$ denotes the minimum cost over the problem horizon.

The main difficulty with this formulation is its high dimensionality. The memory requirements and the system state size become prohibitively large, and the solution times are too long. It is not suitable for problems of large sizes in terms of horizon lengths and/or demand values. For our work, this formulation is important in that it provides a guaranteed optimal solution and serves as the benchmark in our numerical study.

13.3 Numerical Study

For our numerical study, we constructed our experiment set in line with Kian et al. (2014).

We considered a problem horizon of $T=100$ periods. Period demands are generated randomly from three normal distributions with respective coefficients of variation, $cov=0.8$, 0.4, and 0.2 and standard deviation σ (=40) where negative demand values have been replaced with zero demands. We denote the three demand patterns by $D1$, $D2$, and $D3$, respectively. All other system parameters are stationary. Noting that unit replenishment cost p_t and unit transportation cost τ_t can be subsumed into h_t by simple transformations through inventory recursions, we assume them to be negligible over the entire problem horizon. We set unit holding cost rate, $h_t = h = 1$, and setup cost is selected as a function of the mean demand rate, $K_t = K = [J^2/2]\mu$, where J may be viewed as a proxy for the average size of a replenishment quantity under the simple EOQ formula. We have $J \in \{2, 3, 4, 5\}$. We considered $r=1.5$. This corresponds to the C–D-type economic production function with convex costs. To select the parameters for the nonlinear transportation/delivery component, we used the formulation P_2^T as the base. For this formulation, we set $w_t = w$ and considered the variable cost of transportation per unit when a dispatched quantity equals the average demand per period, $\bar{w}$ where $w = [w\mu^r]/\mu = w\mu^{r-1}$. Letting $a = h/\bar{w}$, we have $w = h\mu/(a\mu^r)$ with $a \in \{0.02,0.05,0.1\}$ so that the resulting variable cost for a shipment quantity of q units is given by $[h\mu/a](q/\mu)r$. Note that $\bar{w}$ is decreasing in a. The same sets of 10 demand realizations generated for each demand distribution were used for all experiment instances throughout the study. Overall, we have $120=(4\times 3\times 10)$ experiment instances for P_T^2. As part of our study, we also tested the efficacy of formulation P_T^2, which is structurally different from P_T^2.

For consistency, we selected the parameters for this formulation as follows. We considered three values of number of iso-elastic inputs, $m = 1, 2, 5$ and $\alpha_i = \alpha$ for $1 \leq i \leq m$ with $m\alpha = 1/r$. (All other parameters were selected as for P_T^2.) Overall, we have $360 = (3 \times 4 \times 3 \times 10)$ experiment instances for P_T^2. The optimal plan has been obtained by the DP algorithm discussed earlier. We tested the solvers AlphaECP, Baron, Bonmin, Couenne, LINDOGlobal, and KNITRO available online at the NEOS server (http://www.neos-server.org/neos/solvers/index.html). The server's goal has been described as specifying and solving optimization problems with minimal user input (Dolan et al. 2002). The solver defaults/options were set at their defaults except that the time limits on all have been set to 1500 s since lower time resources resulted in too many interrupts in preliminary tests.

In our numerical study, (1) we considered an overall assessment of the computational performances of the two formulations with respect to the demand patterns and the number of inputs using different optimizers, and (2) focusing on the formulation P_T^2, we used the ANalysis Of VAriance (ANOVA) to identify the factors that have statistically significant impact on the solution quality.

13.3.1 Overall Assessment

The performance measures are (1) the number of instances in which a feasible solution has been obtained by a solver, and (2) the percentage deviation from the optimal solution for the obtained solutions averaged over all 120 experiment instances for a particular demand distribution. Note that in the latter computation, the experiment instances in which a solver failed have been excluded.

We begin our analysis with our findings on formulation P_T^1. The overall performance summary with $m = 1, 2, 5$ for the entire experiment set for this formulation is presented in Table 13.1, where # denotes the first performance measure and % denotes the second. For the cases when no feasible solution was obtained, an m-dash (—) has been used to denote the unavailable second measure.

AlphaECP failed to obtain a solution in all experiment instances, whereas LINDOGlobal was able to obtain a solution in all experiment instances except for the demand distribution *D*2. However, for that pattern, it also resulted in a solution in the most number of instances.

Table 13.1 Overall Summary of Performance Measures

		ALPHAECP		BARON		BONMIN		COUENNE		LINDOGLOBAL		KNITRO	
		%	#	%	#	%	#	%	#	%	#	%	#
P_T^1	*D*1	—	0	119.37	118	0.00	30	63.65	96	3.51	120	56.25	19
$m=1$	*D*2	—	0	161.44	82	0.00	27	75.35	72	1.26	108	5.70	38
	*D*3	—	0	159.94	120	0.00	30	73.91	120	0.82	120	9.29	45
P_T^1	*D*1	—	0	—	0	0.00	27	86.91	72	18.74	120	6.48	87
$m=2$	*D*2	—	0	—	0	0.00	27	—	0	15.05	72	1.43	53
	*D*3	—	0	—	0	0.00	30	94.91	48	11.67	120	1.14	48
P_T^1	*D*1	—	0	—	0	0.00	27	71.62	84	231.66	120	23.81	52
$m=5$	*D*2	—	0	—	0	0.00	27	79.96	60	199.35	72	12.92	32
	*D*3	—	0	—	0	0.00	30	80.16	120	272.1	120	6.16	106
P_T^2	*D*1	1.45	120	15.57	120	2.90	120	1.28	120	5.44	120	6.20	120
	*D*2	0.53	120	4.03	120	0.86	120	0.67	120	1.54	120	1.23	120
	*D*3	0.38	120	1.36	120	0.53	120	0.45	120	1.02	120	0.81	120

Bonmin has low performance in obtaining a solution, but the quality of the obtained solution is very good (optimal in many instances). Regardless of the number of inputs in the system, it was able to get a near-optimal solution for $D1$. The distribution $D2$ seems to present the most difficulty for given m and other parameters except for Bonmin.

For LINDOGlobal, the number of inputs in the problem setting has a negative impact on the quality of the obtained solutions. For other solvers, the behavior may not be monotone (e.g., KNITRO, Bonmin). However, in a very general qualitative sense, we get the impression that solver performance (in both criteria) tends to worsen as the number of inputs increases in the problem setting. This observation has motivated us to construct the second formulation, P_T^2. For P_T^2, the performances of all solvers have improved significantly in terms of the number of instances for which a feasible solution was obtained; none of the solvers failed across the entire experimental bed. Also, the solution quality for all solvers except LINDOGlobal (for $m = 1$ case) has increased. These indicate that the formulation P_T^2 is more amenable to use on the available solvers.

13.3.2 ANOVA Assessment

The overall assessment presented earlier was based on the performances of the two formulations and the solvers in an aggregate sense. Next, we focus on the formulation P_T^2 and use the formal statistical tool ANOVA to identify the factors that impact the solution quality significantly in a statistical sense.

We considered a three-way ANOVA where the factors are (1) K (representing the fixed replenishment cost) considered in four levels K_i, $i = 1,\ldots, 4$; (2) W (representing the transportation cost coefficient, w) considered in three levels, $W_j, j = 1, 2, 3$ as given earlier in the experimental bed; and (3) the different solvers denoted by S with six levels, S_k, $k = 1,\ldots,6$ corresponding to the solvers in the order given earlier with $n = 10$ replications (corresponding to the demand realizations) at each experimental instance. The response variables y_{ijkl}, $i = 1,\ldots,4$; $j = 1,2,3$; $k = 1,\ldots,6$; and $l = 1,\ldots,10$ are taken as the percentage deviations of the solutions provided by the solvers from the optimal solution, which is obtained by DP. The ANOVA study was conducted for each demand distribution separately.

Table 13.2 ANOVA for *D*1

SOURCE	*DF*	SEQ SS	ADJ SS	ADJ MS	*F*	*P*
K	3	18285.12	18285.12	6095.04	1246.5	0
W	2	4611.81	4611.81	2305.9	471.58	0
S	5	17151.57	17151.57	3430.31	701.54	0
*K***W*	6	12221.28	12221.28	2036.88	416.56	0
*K***S*	15	7259.13	7259.13	483.94	98.97	0
*W***S*	10	2259.74	2259.74	225.97	46.21	0
*K***W***S*	30	6201.09	6201.09	206.7	42.27	0
Error	648	3168.54	3168.54	4.89		
Total	719	71158.26				

The ANOVA tables for the three distributions are given in Tables 13.2 through 13.4. The performance statistics for each factor level computed across the other experiment parameters are tabulated in Table 13.5 for each demand distribution. Finally, the interaction effects of the factor levels are provided in Figures 13.1 through 13.3 for the each distribution, respectively. The inspection of these results reveals the following findings.

Firstly, all the factors and the interactions have significant impact on the solution quality, which is indicated by very large *F* values and correspondingly very small *P*-values, implying that the hypothesis that states that all factor levels have the same effect on the response variable is rejected for all three distributions. A closer inspection of the results provides further information regarding (1) the relative impact of the factors, (2) direction of the factor-level impact, and (3) the interaction effect. We treat each demand distribution separately.

Table 13.3 ANOVA for *D*2

SOURCE	*DF*	SEQ SS	ADJ SS	ADJ MS	*F*	*P*
K	3	640.025	640.025	213.342	754.86	0
W	2	542.101	542.101	271.051	959.05	0
S	5	1020.997	1020.997	204.199	722.52	0
*K***W*	6	1163.260	1163.26	193.877	685.99	0
*K***S*	15	134.743	134.743	8.983	31.78	0
*W***S*	10	120.064	120.064	12.006	42.48	0
*K***W***S*	30	347.503	347.503	11.583	40.99	0
Error	648	183.140	183.14	0.283		
Total	719	4151.832				

Table 13.4 ANOVA for *D*3

SOURCE	*DF*	SEQ SS	ADJ SS	ADJ MS	*F*	*P*
K	3	451.116	451.116	150.372	3136.61	0
W	2	353.541	353.541	176.771	3687.26	0
S	5	86.901	86.901	17.38	362.53	0
*K***W*	6	855.188	855.188	142.531	2973.06	0
*K***S*	15	82.740	82.74	5.516	115.06	0
*W***S*	10	63.062	63.062	6.306	131.54	0
*K***W***S*	30	218.256	218.256	7.275	151.75	0
Error	648	31.066	31.066	0.048		
Total	719	2141.870				

Consider Table 13.2. Comparing the *F* values, we observe that the most important factors are, respectively, *K*, *S*, *W*, and the two-way *KW* interaction. From Table 13.5, we see that *K*4, *W*3, and *S*2 (Solver Baron) result in the worst solution quality on average. Next, inspecting the impact of average effect of different levels of factors from Table 13.5, we see that the largest deviation from the optimal results is observed when fixed cost is highest at *K*4 level, when *W* is at the *W*3 level, and the solver *S*2 is used. From Figure 13.1, we observe that the differential effect as *K* increases depends on the level of *W* implying a significant interaction of *K* and *W* with the worst performance occurring at *K*4*W*3 combination. Although not as significant, there is also some interaction of *K* with the solvers. As *K* level changes from 3 to 4, the performance deteriorates significantly with solvers *S*5 (LINDOGlobal) and *S*6 (KNITRO). A similar relation also holds regarding the interaction between *W* and the solvers.

Similar analysis for *D*2 and *D*3 reveals the following. For *D*2, the factors with the highest *F* values are ordered as *W*, *K*, *S*, and the two-way interaction *KW*. Table 13.5 shows that there are less drastic differences between the average solution quality corresponding to different levels of the factors. Figure 13.2 shows that the *KW* interaction is still significant, and the difference between the levels of *K* is highest for *W*3, where the interaction of solvers with *K* and *W* is reduced. The ordering of solver performances is similar to that of *D*1. For *D*3, we note that the factors with the highest *F* values are ordered as *W*, *K*, *KW*, and *S*. We again observe that the average solution quality corresponding to different factor levels generally becomes closer

Table 13.5 Response Variable Statistics for Different Factors

		MIN	MAX	AVE.		MIN	MAX	AVE.		MIN	MAX	AVE.
*D*1	*K*1	0	18.753	2.354	*W*1	0	18.753	2.979	*S*1	0	11.85	1.451
	*K*2	0	17.793	2.205	*W*2	0	33.234	4.578	*S*2	5.931	53.504	15.574
	*K*3	0	16.648	3.16	*W*3	0	60.361	8.87	*S*3	0	19.52	2.897
	*K*4	0.547	60.361	14.181					*S*4	0	6.934	1.283
									*S*5	0	50.718	5.438
									*S*6	0	60.361	6.204
*D*2	*K*1	0	5.44	0.878	*W*1	0	5.441	1.61	*S*1	0	4.088	0.531
	*K*2	0	5.167	0.656	*W*2	0	4.118	0.662	*S*2	1.4	14.351	4.029
	*K*3	0	6.166	1.316	*W*3	0	2.867	0.362	*S*3	0	6.423	0.858
	*K*4	0	14.35	3.057					*S*4	0	3.753	0.669
									*S*5	0	12.708	1.542
									*S*6	0	12.708	1.23
*D*3	*K*1	0	1.4	0.236	*W*1	0	1.86	0.28	*S*1	0	3.48	0.37
	*K*2	0	1.86	0.201	*W*2	0	1.56	0.244	*S*2	0.141	9.099	1.359
	*K*3	0	2.053	0.476	*W*3	0	9.09	1.748	*S*3	0	4.595	0.527
	*K*4	0	9.099	2.11					*S*4	0	3.024	0.45
									*S*5	0	8.848	1.017
									*S*6	0	8.848	0.811

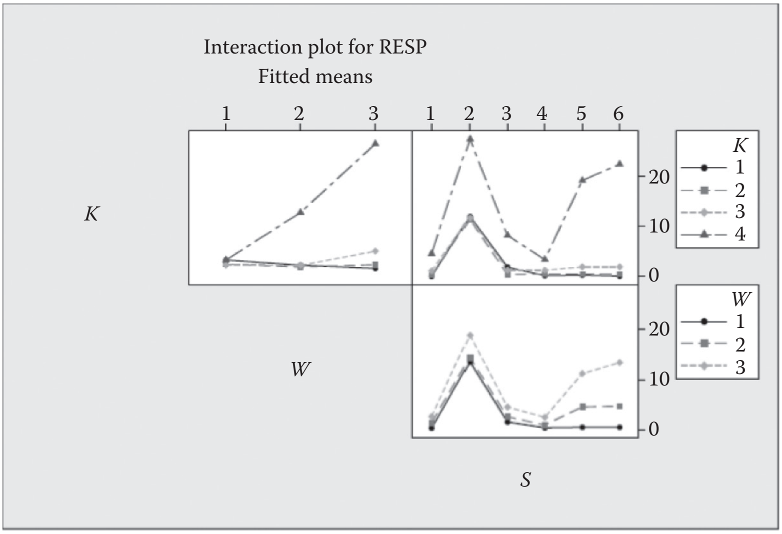

Figure 13.1 Factor interaction for *D*1.

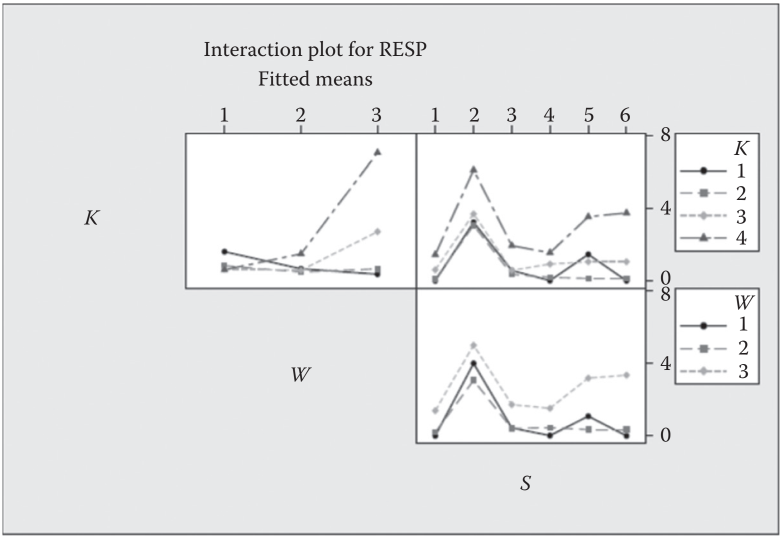

Figure 13.2 Factor interaction for *D*2.

to each other, while the *KW* interaction is still emphasized and the interactions with the solvers become less emphasized.

From the earlier analysis, we see that the solvers' performances get more and more closer to each other as the coefficient of variation of the demand distribution gets smaller and the worst performances

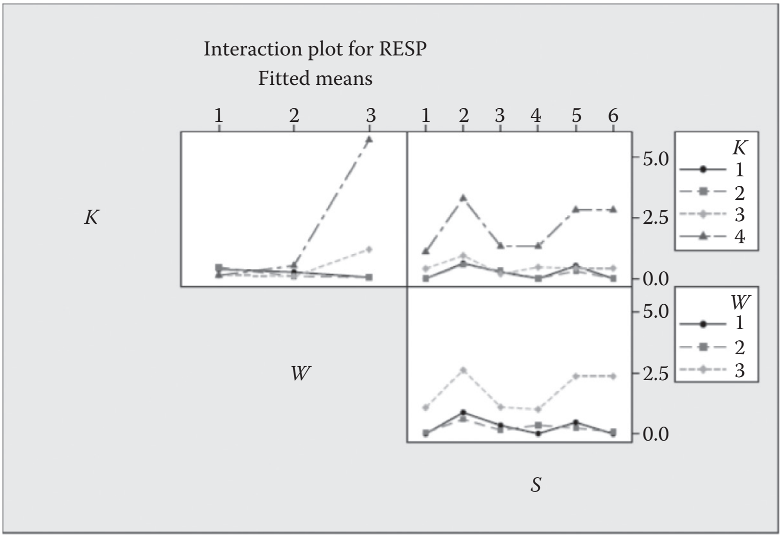

Figure 13.3 Factor interaction for *D*3.

are observed for the *K*4*W*3 large fixed cost and low transportation cost coefficient combination. Furthermore, *S*1, *S*3, and *S*4 (Solvers AlphaECP, Bonmin, and Couenne, respectively) are always among the best three performing solvers (although their ordering may change), whereas the worst performer is *S*2 in all three demand distributions. We observe that solver performances depend drastically on problem formulations as well as cost parameters. We should also mention that they may as well depend on possible user interventions such as initial point selections that were not imposed in our study.

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