Stability Analysis of Reinforced Slopes

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THE paper by Terry Ingold ("Highways and Transportation", March 1986) tackles important aspects of geotextile reinforced embankments over soft ground and provides some helpful design guidelines for the complex problems of soil and reinforcement interaction.

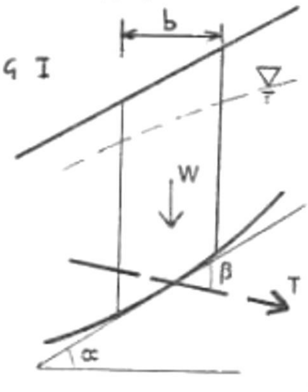
On the question of stability analysis the application of the 'Bishop' method is not straightforward because the factor of safety appears on both sides of the equation. The validity of applying a constant factor of safety to each slice of the analysis has been questioned by Chugh (1985) and the inaccuracies of the Bishop equation for deep slip surfaces (high negative values of α) have been reported previously (Skempton and Hutchinson 1969, Turnbull and Hvorslev 1967).

These problems are overcome if the Simple equation (1) is applied.
This equation based on conventional shear strength theory (Greenwood 1983, Turnbull and Hvorslev 1967) assumes resultant inter-slice forces are parallel to the slip surface. It gives sensible, consistent factors of safety for shallow and deep, circular or non-circular slip surfaces and with high or low water pressures.

The simple equation is readily adapted for reinforcement forces, i.e. equ (2)

The terminology is as used by Terry Ingold except that T is the available reinforcement force operating on the slice considered. This is illustrated in Figure 1. Ru or u may be used for water pressure as appropriate.

FIG I



- W — total weight of soil slice.
- c' ϕ' — effective strength parameters at the slip surface.
- u — pore water pressure at slip surface.
- Ru — pore water pressure ratio ($R_u = \frac{ub}{W}$)

$$F = \frac{1}{\Sigma W \sin\alpha} \Sigma\left[c'b\sec\alpha + (W - ub)\cos\alpha\tan\phi'\right] \qquad (1)$$

$$F = \frac{1}{\Sigma W \sin\alpha} \Sigma\left[c'b\sec\alpha + \left[(W - ub)\cos\alpha + T\sin\beta\right]\tan\phi' + T\cos\beta\right] \qquad (2)$$

- T — available reinforcement (or anchor) force operating on base of slice considered, (allowing for strain compatability, creep, weathering effects, anchorage lengths etc).

Equation (2) includes an enhanced normal stress component, T sin ß, in addition to the tensile restoring component T cos ß. The extent to which both the tensile and normal components will be mobilised depends on the strain characteristics of the reinforcement. Stiffer geomesh materials may rapidly develop both tensile and normal components but geotextiles requiring greater strain to develop their strength may not provide sufficient resistance before excessive straining of the soil occurs. For reinforcement placed without pretensioning it is suggested that the normal contribution is ignored, i.e. conservatively assume T Sin ß = 0, unless experimental evidence is available to demonstrate that it can be relied on.

When equation (2) is applied to pretensioned anchor type reinforcement both tensile and normal components may be included. Anchor reinforcement is predominantly at right angles to the slope and the initial benefit will be mainly in terms of increased normal stress on the potential shear surface (T sin ß). Only if failure commences will the anchor cable distort and provide a tensile restraining force, T cos ß.

The charts given by Terry Ingold are of limited practical value because they only cover particular cases and do not allow for water pressure or variable slope geometry. By applying equation (2) the designer can carry out his own 'simplified' analysis, as illustrated in the example, and begin to develop an understanding of the problem and its particular features.

Figure 2 gives an example of a 'computer assisted' hand calculation for the stability of a clay embankment with three layers of geomesh reinforcement at its base. The input data is obtained by measurement from the diagram. The computer program (written by Martin Wheeler) calculates the factors of safety of the unreinforced slope by five different methods. Input data may be edited for parametric studies. In this example the 'Bishop' factor of safety is shown to be somewhat optimistic in relation to the other methods. The reinforced factors of safety are calculated by the simple equation (2), firstly with the tensile contribution only, assuming T sin ß = 0, and secondly including both the tensile and normal contributions. The designer may then decide whether the additional benefits of the normal contribution can be relied on.

Slope stability is itself a complex subject and the addition of reinforcement to the soil requires careful consideration. For example the embedment lengths necessary to ensure that the required

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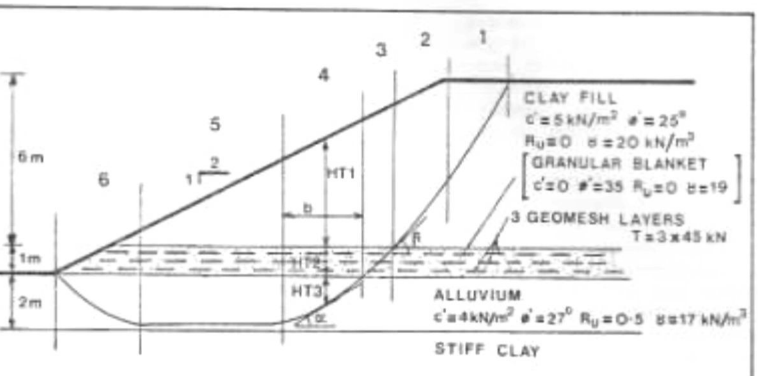
SLOPE STABILITY ANALYSIS PROGRAM SLIP ERD (T) BEDFORD



'Simple' equation for reinforced slopes

$$F = \frac{1}{\Sigma W \sin\alpha} \Sigma\left[c'b\sec\alpha + \left[(W - ub)\cos\alpha + T\sin\beta\right]\tan\phi' + T\cos\beta\right]$$

INPUT DATA FROM DIAGRAM
RU INITIALLY INPUT HW CALCULATED

Slice No.	Soils Types	HT1 M	DEN1 KN/M3	HT2 M	DEN2 KN/M3	HT3 M	DEN3 KN/M3	B M	ALFA DEG	C KN/M2	PHI DEG	RU	HW	K	T KN	BETA DEG
1	1	2.20	20.0	0.00	0.00	0.00	0.00	2.00	58.0	5.0	25.0	0.00	0.00	0.20	0.00	0.0
2	1	4.50	20.0	0.00	0.0	0.00	0.0	2.00	51.0	5.0	25.0	0.00	0.00	0.20	0.00	0.0
3	2	4.80	20.0	0.60	19.0	0.00	0.0	1.00	44.0	0.0	35.0	0.00	0.00	0.20	135.00	44.0
4	3	3.70	20.0	1.00	19.0	1.10	17.0	3.00	32.0	4.0	27.0	0.50	5.60	0.50	0.00	0.0
5	3	1.80	20.0	1.00	19.0	1.90	17.0	5.00	0.0	4.0	27.0	0.50	4.40	0.50	0.00	0.0
6	3	0.10	20.0	0.70	19.0	1.30	17.0	3.00	−31.0	4.0	27.0	0.50	1.90	0.50	0.00	0.0

OUTPUT DATA

FORCES (IN KILO NEWTONS) IN EACH SLICE

				SIMPLE	GREEN-WOOD	SWEDISH	BISHOP	JANBU	RESISTANCE FROM REINFORCEMENT	
SLICE	WEIGHT	DISTURB. FORCE	COHESIVE RESISTNCE	TOTAL RESISTNCE (K=O)	TOTAL RESISTNCE K as Input	TOTAL RESISTNCE	TOTAL RESISTNCE	TOTAL RESISTNCE (★)	TENSILE CONTRIBUTION (T COS β)	NORMAL CONTRIBUTION (T SIN β TAN ø)
1	88.00	74.63	18.87	40.62	51.75	40.62	60.37	115.82	0.00	0.00
2	180.00	139.89	15.89	68.71	84.82	68.71	102.28	166.06	0.00	0.00
3	107.40	74.61	0.00	54.10	64.19	54.10	67.91	96.17	97.11	65.66
4	335.10	177.58	14.15	86.55	100.68	58.28	91.56	111.39	0.00	0.00
5	436.50	0.00	20.00	131.20	131.20	131.20	131.20	137.76	0.00	0.00
6	112.20	−57.79	14.00	38.50	42.92	29.66	62.65	78.98	0.00	0.00
TOTAL	1259.20	408.91	82.91	419.57	475.68	382.56	515.97	706.19	97.11	65.66

(★) Janbu resistance not directly comparible with other solutions

FACTORS OF SAFETY (NO REINFORCEMENT)

Simple (K = O)	=	1.03
Greenwood (K as input)	=	1.16
Swedish	=	0.94
Bishop	=	1.26
Janbu (fo = 1.05)	=	1.16

FACTORS OF SAFETY (SIMPLE SOLUTION)

No reinforcement	=	1.03
Reinforced (tensile contribution only)	=	1.26
Reinforced (tensile & normal contribution)	=	1.42

FIGURE 2. Example analysis of a reinforced slope.

reinforcement force is available need to be checked together with the effect that the reinforcement has on shifting the location of the most critical slip surface. The simple analysis described in this note gives a basic guide to potential benefits of reinforcement and should give the designer confidence to tackle the more complex problems of interaction between soil and reinforcement.

ACKNOWLEDGEMENT

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