Experimental Design with Fuzzy Levels

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Abstract

In this paper, we have introduced a novel technique for planning experimental design employing fuzzy rule-based systems. The significant aspect of the proposed Experimental Design with Fuzzy Levels (EDFLs) is assigning a membership function for each level of variable factors. Consequently the design matrix and observed responses can be represented in a set of fuzzy rules. A number of examples are presented to clarify the proposed idea and the results are compared with the conventional Taguchi methodology. We have specifically planned a L_{18} orthogonal array EDFL for the solder paste printing stage of surface mount printed circuit board assembly to provide a model for the process and optimize the selection of variable factors.

Keywords: Experimental model, Taguchi method, fuzzy rule-based systems, solder paste printing

1 Introduction

The Taguchi quality control technique is an effective experimental design method for characterizing the optimal variable parameters and reducing performance variation for a manufacturing process (Bendell et al., 1989; Grove and Davis, 1992; Taguchi and Wu, 1980). The experimental design methodology of the Taguchi is distinguished by utilizing orthogonal arrays and the analysis of signal to noise (S/N) ratio. The orthogonal array design provides an economic method for studying the interaction of process variables on the mean and variance of a particular process response.

However, the traditional statistical design of experiment considers a number of factors at different levels which are measurable either qualitatively or quantitavely. The factor levels which the experiment has been carried out are representative of whole system functionality. In general, variable factors can be expressed with some linguistic terms such as *low* and *high* indicating uncertainty of their values. If some of the factor levels are not measurable, their values should be represented by equivalent fuzzy terms so that their importance is included in the system response.

This paper presents a new technique of experimental design with fuzzy levels. Applying this model, a functional equivalence between fuzzy rule-based system and experimental design will be shown. The functional equivalence enables us to apply what has been discovered for one to the other and vice versa.

As part of our investigation[1] to establish a closed loop control system for the solder

[1] Automatic Solder Paste Monitoring, Inspection and Control (ASPMIC) supported by EPSRC grant.

paste printing stage of surface mount printed circuit board (PCB) production, we are required to identify the quality and characteristic of solder paste deposit on PCB for disparate variables. It is essential that the printing variables be optimized so that the resultant solder paste deposit is uniform and with a proper geometry and shape. Therefore it is required to construct a model representing the relation between system response (height, area and volume of paste deposit) and variable factors (squeegee load, viscosity of paste etc.). We have employed the proposed method of EDFL to formulate a model for the process and optimize the variables.

The subsequent sections of this paper are organized as follows. In Section 2 experimental design with fuzzy levels will be introduced followed by explanation on fuzzy rule-based systems in Section 3. The functional equivalence between EDFLs and fuzzy rule-based systems is explained in Section 4. In Section 5 constructing a model for solder paste printing applying EDFL will be explained. Concluding remarks are then made in the final section.

2 Experimental Design with Fuzzy Levels

The goal of experimental design methodology is to obtain the best set of variable factors without exhaustive test. To achieve this goal, an *"experimental model"* is designed. It allows a mathematical estimation of the effect of each factor independently from the others through the definition of orthogonal arrays.

There is a well established theory of planning experimental design (Bendell et al., 1989; Grove and Davis, 1992; Taguchi and Wu, 1980). Primarily it is assumed that the levels of the variable factors are known. The experiment is usually designed for two or three levels and occasionally higher order levels. Considering the different levels of each factor a full factorial or orthogonal array fractional factorial combination of levels are advised for experimentation. Let us suppose that seven variable factors with three levels each have to be tested. Comprehensive test lead to $3^7 = 2187$ experiments; but using the orthogonal array, the minimum number of trial needed to perform is 18.

In order to estimate the optimal level of parameters, the effects of each individual variable factor is calculated. The principal of orthogonal array states that each value of a variable factor is opposed to each value of the other factors for the same number of experiments. The effects of each individual variable factor or their interaction are calculated to obtain the optimal level of parameters. Using a linear or nonlinear regressor, we can eventually construct a model representing the relation between variable factors and yield. The model can be used as a predictor to anticipate the yield for variable factors between the measured levels.

In this paper we propose to substitute the variable factors with *"linguistic variable factors"* (fuzzy variable factors). Linguistic variables are variables whose values are not numbers but words or sentences in a natural language. For example, the statement 'the temperature is hot' is not as exact as saying 'the temperature is $40^o c$'. The label 'hot' is a *"linguistic value"*(linguistic label) for the linguistic variable temperature with the understanding that 'hot' is similar to the numerical value $40^o c$ with less precision.

The linguistic values assigned to each linguistic variable are characterized by their

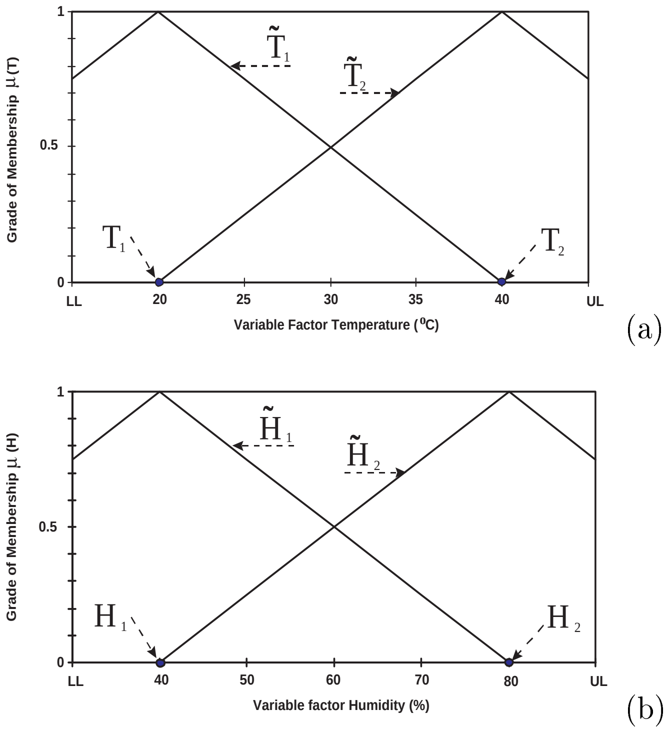


Fig. 1. Membership function of variable factors with two levels: (a) temperature T (b) humidity H.

membership functions (MFs). The MF is a function mapping the range of variable factor to values between and including 0 and 1 i.e. $\mu_{hot}(temperature) \in [0\,1]$.

The underlying idea of EDFL can be stated as follows:

- The judgement about the yield of product is usually made based on the designed trial for a few levels of variable factors. It would be useful if we can incorporate the information of a whole range of variable factors. For instance, consider the yield of a product which is affected by temperature, T, and humidity, H. If we plan a two-level experiment, we are only considering the effects of variable factors in those two levels i.e. $T \in \{T_1, T_2\}$ and $H \in \{H_1, H_2\}$. Instead of forcing the variable factors to be one of the two specific levels, we can determine it to belong to a whole range of variable factors with a degree of association i.e. $T \in [T_1, T_2]$ and $H \in [H_1, H_2]$.

- It is very difficult to involve the information related to the levels of factors which are not measurable precisely or where there are only qualitative expression available. To involve linguistic information in experimental design, we have to provide a proper model to represent this type of information. If some of the variable factors can not be measured precisely, but still need to use them in the design and creation of a model of the process, the ultimate approach is to use fuzzy variable factors.

If we again consider the yield of a product which was affected by temperature, T, and humidity, H. The levels of variable factors T_1, T_2 and H_1, H_2 can be fuzzified to $\tilde{T}_1, \tilde{T}_2$ and $\tilde{H}_1, \tilde{H}_2$. The superscript $\sim$ represents the fuzzy values in contradistinction to crisp values. The triangular MFs of $\mu_{\tilde{T}_1}(T)$, $\mu_{\tilde{T}_2}(T)$, $\mu_{\tilde{H}_1}(H)$ and $\mu_{\tilde{H}_2}(H)$ replaced with the known levels T_1, T_2, H_1 and H_2 are depicted in Fig. 1(a) and Fig. 1(b) for the variable factors T and H respectively. They are limited in two lower limit (LL) and upper limit (UL) bounds.

We assign the maximum grade of membership where the variable factors are in their known levels i.e. T_1, T_2, H_1 and H_2. The grade will be reduced from the maximum as the variable factor moves away from the measured level. Furthermore, if a linguistic term for the factor levels are available, they can be used in the same fashion. Fig. 2(a) and Fig. 2(b) illustrate the triangular MFs of fuzzy factors with three levels $\tilde{T}_1, \tilde{T}_2, \tilde{T}_3$ and $\tilde{H}_1, \tilde{H}_2, \tilde{H}_3$ for the variable factors T and H respectively.

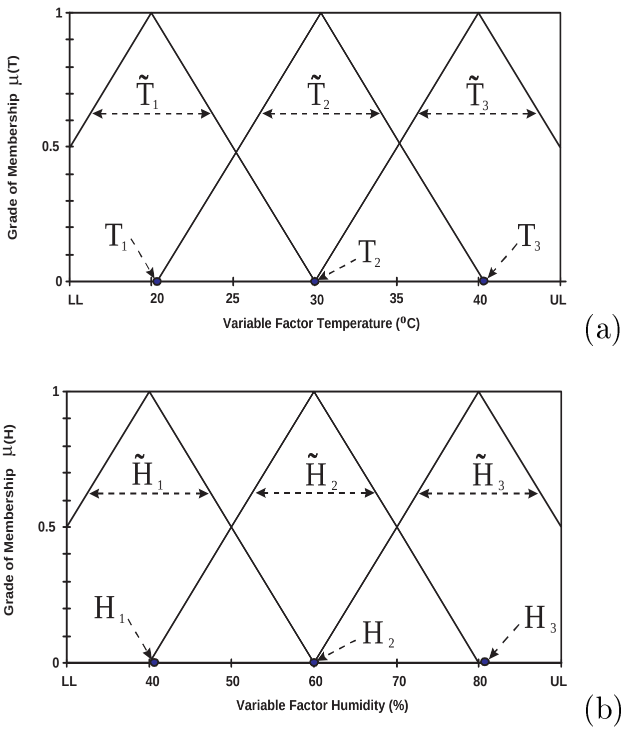


Fig. 2. Membership function of variable factors with three levels: (a) temperature T (b) humidity H.

3 Fuzzy Rule-Based Systems

To show the functional equivalence between fuzzy rule-based system and experimental design with fuzzy levels, in this paper, fuzzy rule-based systems of the following configuration are employed to represent the relation among linguistic information.

$$R^i\text{: If } x_1 \text{ is } \tilde{A}_1^i \text{ and ... } x_j \text{ is } \tilde{A}_j^i \text{ ... and } x_p \text{ is } \tilde{A}_p^i \text{ then } z \text{ is } B^i \qquad \star$$

where R^i is the label of i^{th} rule, $x_j : \ j = 1, 2, \ldots, p$ is the j^{th} variable factor and z is the output. $\tilde{A}_j^i$ $(i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, p)$ are fuzzy labels, and B^i are real numbers. n and p are the numbers of rules and variables, respectively. The number of individual MFs for a specific input value x_j $(\tilde{A}_j^1, \tilde{A}_j^2, ...\tilde{A}_j^n)$ is K_j. Different shapes of MF for fuzzy values, $\tilde{A}_j^i$, can be employed e.g. triangular or Gaussian. We assume that the universe of variable factors is limited to a lower limit (LL) and upper limit (UL) bounds, i.e. $x_j \in [LL_j, \quad UL_j], \quad j = 1, \ldots, p$.

We further assume that the MFs for each input are normal, i.e. $\sup_{x_j} \tilde{A}_j^i = 1, x_j \in [LL_j \, UL_j]$. Moreover for each variable factor:

$$\sum_{i=1}^{n} \mu_{\tilde{A}_j^i}(x_j) = 1, \; j = 1, 2, ..., p \tag{1}$$

The output, $z(t)$, at the t^{th} trial, as a function of variable factors $x_j(t) : \quad j = 1, 2, \ldots, p$, is given in the following equation (Wang and Mendel, 1992):

$$z(t) = \frac{\sum_{i=1}^{n} w^i B^i}{\sum_{i=1}^{n} w^i} \tag{2}$$

where B^i is the consequent parameters of rules and w^i is the rule firing strength given by:

$$w^i = \prod_{j=1}^{p} \mu_{\tilde{A}_j^i}(x_j(t)) \quad i = 1, 2, \ldots, n \tag{3}$$

It has been proved that fuzzy rule-based systems with the structure given in this section are universal approximators (Wang and Mendel, 1992), i.e. they are capable of capturing the nonlinear characteristics of any complex process with n variable factors. The reader is referred to (Mendel, 1995) for more details on fuzzy rule-based systems.

4 Functional Equivalence

From the information provided in last two sections it is clear that the EDFL explained in Section 2 and the fuzzy rule-based systems introduced in Section 3 are functioning in very similar ways. When MFs are defined for each level of variable factors, the design factor layout and response data can be presented in a set of fuzzy rules given in expression ($\star$). Where the variable factors are replaced with linguistic variables x_j, the output z can be substituted with any statistical terms such as average yield, $\bar{y}$, or signal to noise ration, S/N. The number of individual MF, K_j, is the number of levels considered for each variable factor.

In the following subsection a simple example is presented to clarify the functional equivalence between fuzzy rule-based system and experimental design with fuzzy levels.

4.1 Illustrative Example

A simple artificial example is presented to illustrate the EDFL (Lochner and Matar, 1990). Consider a chemical reaction where its yield, $\bar{y}$, was thought to be a function of three variables: Formulation (F), Mixer speed (S) and Temperature (T). An L_4 orthogonal array is selected to implement the design factors. The levels selected for the variable factors are listed in Table 1.

The combination of factor levels to be used for a L_4 Taguchi orthogonal array is given in Table 2. The experiment has been replicated 4 times. The last three columns in Table 2 are respectively the average of the four responses, y_1, y_2, y_3 and

Table 1: Levels selected for variable factors.

Factors	level 1	level 2
F :Formulation	I	II
S :Speed	50rpm	90rpm
T :Temperature	20^oc	40^oc

Table 2: L_4 design factor layout and response data.

Trial	F	S	T	FS	FT	ST	y_1	y_2	y_3	y_4	$\bar{y}$	s	S/N
1	1	1	1	2	2	2	12	12	10	13	11.75	1.26	-21.44
2	1	2	2	1	1	2	8	8	15	14	11.25	3.78	-21.38
3	2	1	2	1	2	1	18	16	14	12	15.0	2.58	-23.62
4	2	2	1	2	1	1	14	22	7	5	12.0	7.70	-22.75

y_4, standard deviation of samples, s, and signal to noise ration, S/N, by using the smaller the better formula. These statistical terms are clearly defined in the following equations:

$$\bar{y} = \frac{1}{N}\sum_{k=1}^{N} y_k \tag{4}$$

$$s = \sqrt{\left(\frac{1}{N-1}\sum_{k=1}^{N}(y_k - \bar{y})^2\right)} \tag{5}$$

$$S/N = -10\,log\left(\sum_{k=1}^{N} y_k^2/N\right) \tag{6}$$

where N is number of replication of experiment and y_k is the k^{th} response.

Variable factor F is a discrete level with two levels I and II. Variable factors S

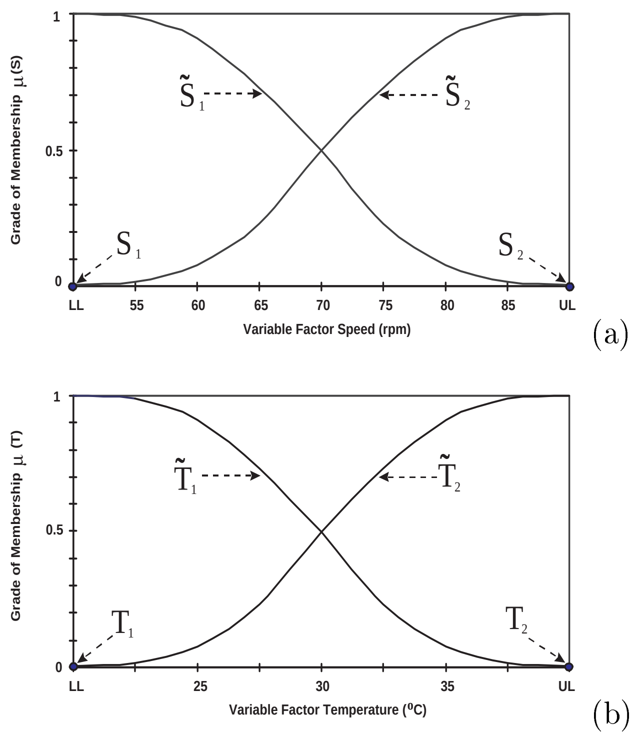


Fig. 3. Membership function of variable factors (a) speed (b) temperature.

and T are continous and they are measured in two levels S_1, S_2 and T_1, T_2. They can be considered as a fuzzy variable with two fuzzy labels $\tilde{S}_1, \tilde{S}_2$ and $\tilde{T}_1, \tilde{T}_2$ respectively. MFs are defined by Gaussian functions and they are depicted in Fig. 3(a) and Fig. 3(b) for variable factor S and T respectively. It should be noted that these MFs satisfy the normal condition given in Equation (1).

Upon defining the MFs of new linguistic variable factors, the design factor layout in Table 2 can be presented in a fuzzy rule-based system with 4 rules ($n = 4$), three inputs ($p = 3$) with two individual MFs for each input ($K_1 = K_2 = K_3 = 2$). Each row in the Table 2 represents a fuzzy rule, if we replace each level with its fuzzy value i.e. S_1 with $\tilde{S}_1$, T_1 with $\tilde{T}_1$ and so on. The following fuzzy rules are replaced with the L_4 design factor layout and average yield, $\bar{y}$, in Table 2.

If F is $Type\, I$ and S is $\tilde{S}_1$ and T is $\tilde{T}_1$ then $\bar{y}$ is 11.75

If F is $Type\, I$ and S is $\tilde{S}_2$ and T is $\tilde{T}_2$ then $\bar{y}$ is 11.25

If F is $Type\, II$ and S is $\tilde{S}_1$ and T is $\tilde{T}_2$ then $\bar{y}$ is 15.0

If F is $Type\, II$ and S is $\tilde{S}_2$ and T is $\tilde{T}_1$ then $\bar{y}$ is 12.0

The above rules can be repeated for standard deviation, s, and signal to noise ration, S/N, if the output average yield is replaced with the new term.

Employing these fuzzy rules, the effect of each individual factor or their combination on average yield, standard deviation or S/N ration can be studied easily. For instance, if we want to see only the effect of T (F, S) on the average yield, $\bar{y}$, the rules representing this experimental design must be rewritten in the following form:

If F and S are anything and T is $\tilde{T}_1$ then $\bar{y}$ is 11.75

If F and S are anything and T is $\tilde{T}_2$ then $\bar{y}$ is 11.25

If F and S are anything and T is $\tilde{T}_2$ then $\bar{y}$ is 15.0

If F and S are anything and T is $\tilde{T}_1$ then $\bar{y}$ is 12.0

where the values related to F and S are not important and their MFs are fixed to "anything". The MF of "anything" is a constant function with grade of 1 for the whole range of the variable factor.

If we again employ the decision Equation (2), this effect will be shown for different values of T. The effect of F, S and T on average yield are depicted in Fig. 4(a).

To visualize the relation between the average yield, $\bar{y}$ and fuzzy variables S and

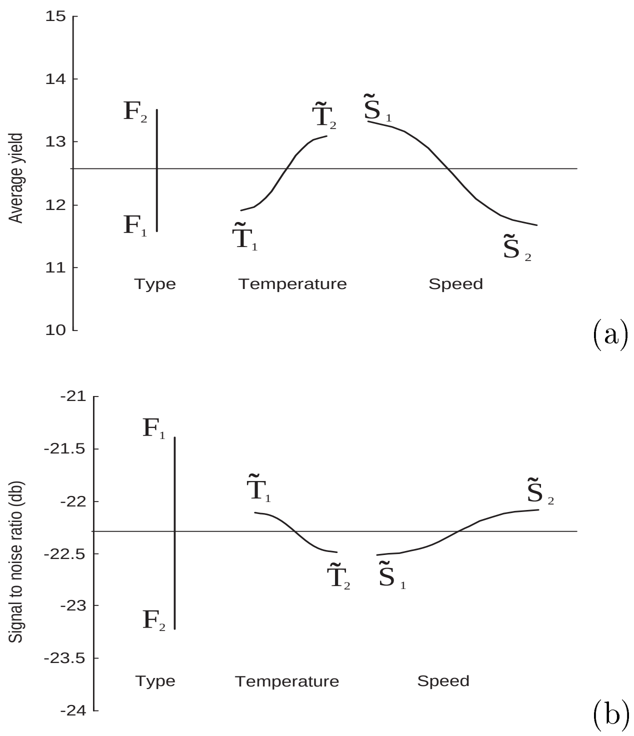
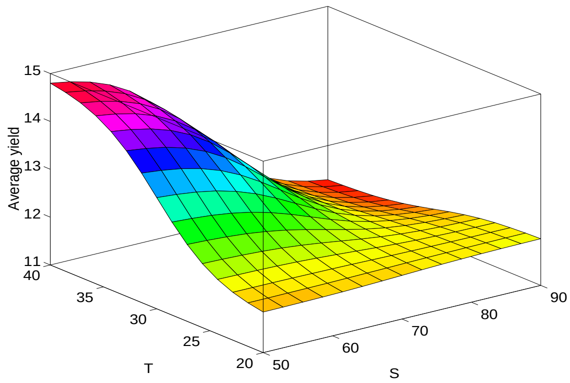


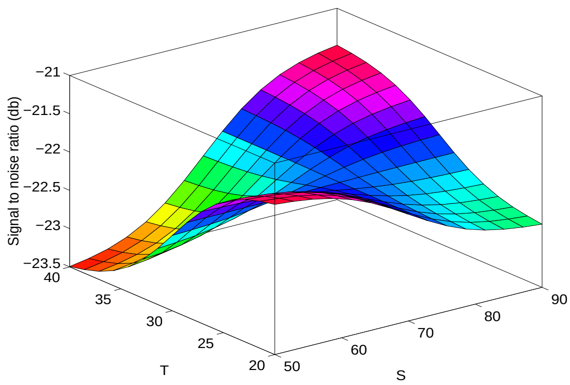
Fig. 4. Effects of F, S and T on (a) average yield $\bar{y}$ (b) signal to noise ratio.

T, a 3D surface is depicted in Fig. 5(a). The surface shows the interaction between two variable factors. The relation between the variable factors and S/N ratio can be studied in the same fashion. Fig. 4(b) shows the relationship between S/N ratio and variable factors F, S and T. The interaction surface between the signal to noise ratio and fuzzy variables S and T is shown in Fig. 5(b).

Fig. 4(a) predicts that variable factors F, S, and T should be set respectively at levels Type II, $\tilde{S}_1$ and $\tilde{T}_2$ to maximize the average yield. In order to minimize the effects of noise factors on performance characteristics, the S/N ratio must be maximized. Fig. 4(b) recommends levels Type I, $\tilde{S}_2$ and $\tilde{T}_1$. The results obtained from the EDFL analysis is exactly the same as the results obtained from the traditional methods.



(a)



(b)

Fig. 5. The nonlinear interaction between variable factors F, S, T and (a) average of yield $\bar{y}$ (b) S/N ratio.

5 Case Study: Solder Paste Printing

5.1 Process

This case study is an application of fractional factorial EDFL for the solder paste printing stage of Surface Mount Technology (SMT) (Chung et al., 1995; Ekere et al., 1994). The solder paste printing process starts by placing a metal stencil over the printed circuit board (PCB). Stencil openings (apertures) correspond to pad locations on the PCB where solder paste is required. A moving squeegee is located on top of the stencil to force the solder paste, rolling in front of the squeegee into the stencil

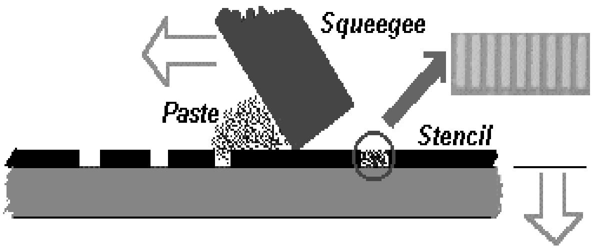


Fig. 6. A schematic of solder paste stencil printing.

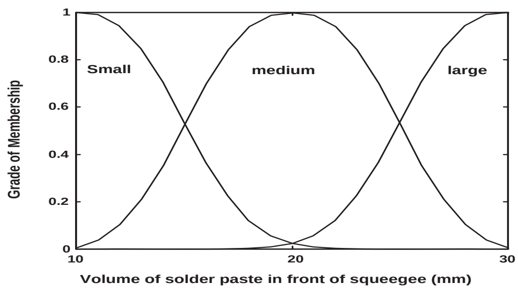


Fig. 7. Membership functions of factor D (volume of solder paste in front of squeegee).

openings. When the squeegee has traveled past all stencil openings, the stencil is removed and the PCB is ready for component placement. Fig. 6 illustrates a schematic diagram of solder paste stencil printing stage of surface mount PCB assembly.

The height, area and volume of the solder paste deposit is ideally, equal to the shape of the apertures. Solder paste deposit for 1.25 mm pitch[2] and higher is generally reliable and close to the ideal shape. The problem arises when fine-pitch elements ($\leq$ 0.6 mm pitch) are present on the PCB. Because of the increased height to width ratio and small aperture opening, it is more likely that incomplete filling will occur (skipping). The solder paste may also slump due to poor paste viscosity. For the stencil printing process to deliver the best results, a balance of interactive factors must be

[2]Pitch is the distance between two adjacent pads.

achieved. There are many factors which can influence stencil printing performances, either directly or through interaction with other factors. The list of dominant factors is provided in the next section.

5.2 Choice of Factors

In our experiment, six factors were chosen at three levels each. The factors are listed in Table 3 and they can be distinguished in different categories as follows:

- Printer related factors

  A: squeegee speed

  B: squeegee load

- Solder paste related issues

  C: viscosity of paste

  D: volume of solder paste in front of squeegee

- Environmental factors

  E: temperature

  F: humidity

The output of the experiment was measured by determining the percentage volume of solder paste in the stencil aperture ($\frac{paste\,volume}{aperture\,volume} \times 100\%$). The experiment used a L_{18} array with 4 replication (not shown for economy of space). The stainless steel

Table 3: Experimental factors and levels for stencil printing.

	Level		
Factors	Level 1	Level 2	Level 3
A :	10 mm/s	60 mm/s	110 mm/sec
B :	10 N	15 N	20 N
C :	650 Kcps	1000 Kcps	1350 Kcps
D :	small ($\approx$10 mm)	medium ($\approx$ 20 mm)	large ($\approx$ 30 mm)
E :	$21^o c$	$23^o c$	$25^o c$
F :	30%	45%	60%

stencil (thickness 0.6 mm) and metal squeegee were used. The experiment was repeated for three different rectangular apertures. They are 0.2×1.1 mm (small), 0.3×1.4 mm (medium) and 0.6×1.3 mm (large).

It is very difficult to measure the volume of solder in front of squeegee (factor D). We have asked the expert operator to specify three different levels and the measurement is performed only by operator's observation. This shows the importance of the EDFL which can easily incorporate the linguistic information. The MFs defined for these three levels are shown in Fig. 7. To convert the levels of other factors to their fuzzy values (fuzzify), Gaussian MFs are employed.

After forming the fuzzy rule-base using the L_{18} orthogonal array for three different aperture sizes, the effect of each factor is calculated. The affect of each individual variable factor for the three different apertures are depicted in Figs. 8(a-f). In each figure, there are three lines, each of which represents a different size of stencil opening.

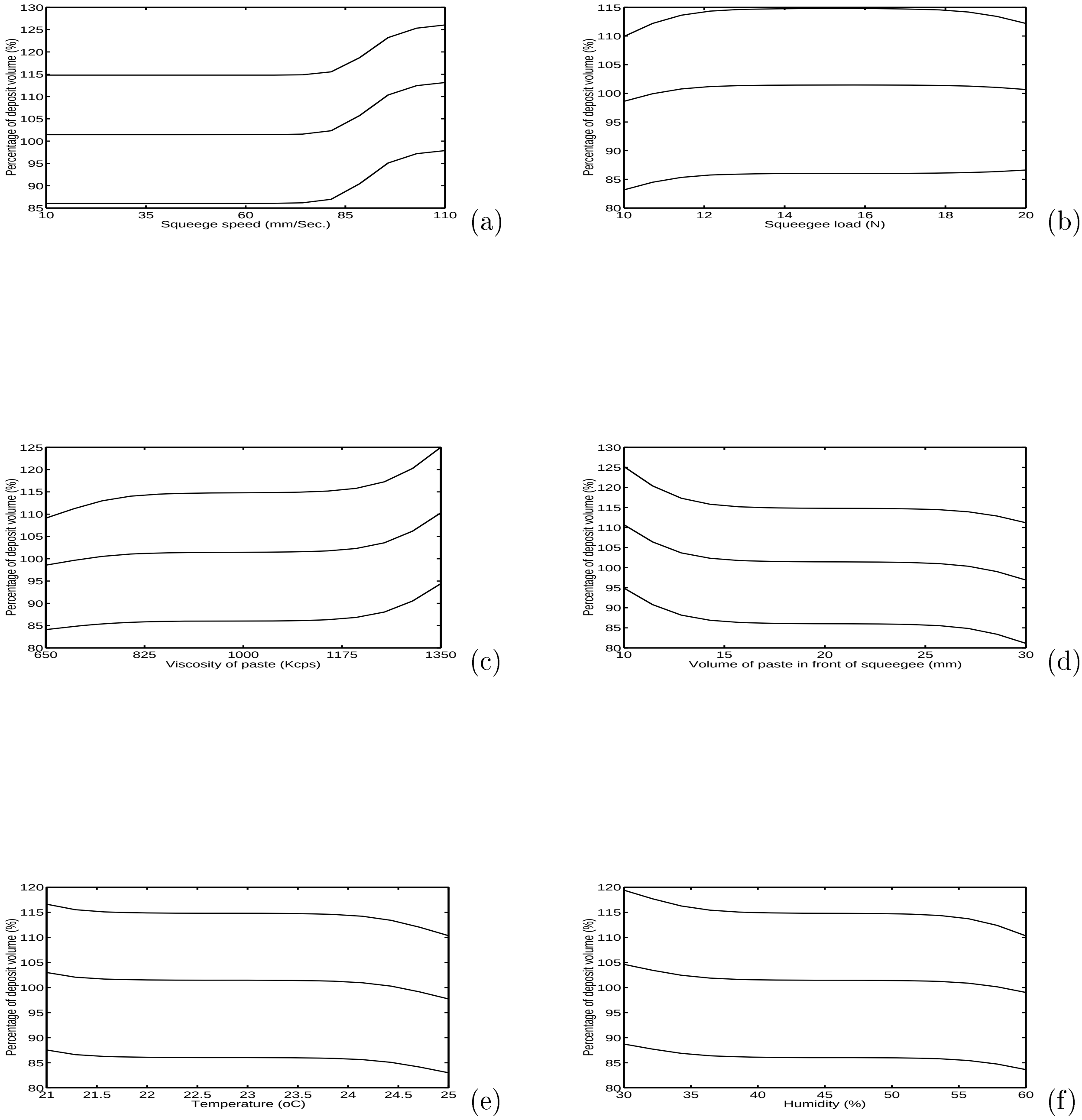


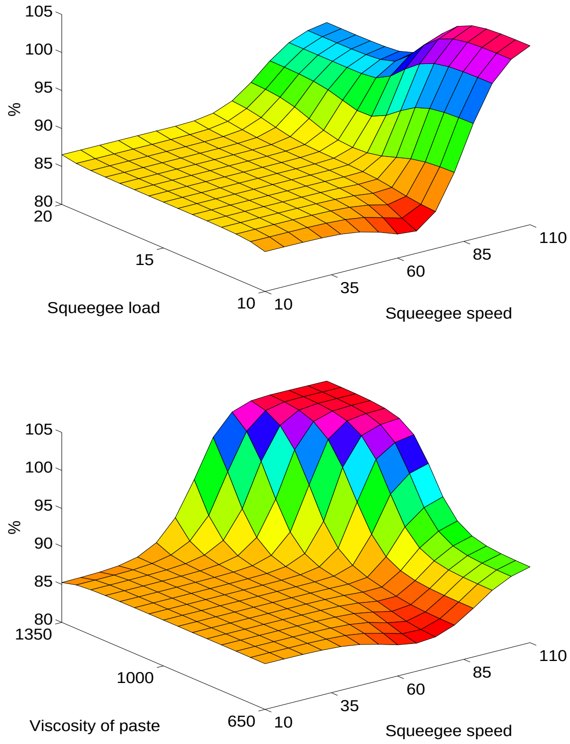
Fig. 8. Estimated effects of factors on the percentage of deposit volume for three different apertures: (a) squeegee speed (b) squeegee load (c) viscosity of paste (d) volume of solder paste in front of squeegee (e) temperature (f) humidity.

The lowest line is for the small aperture size (0.2×1.1), the middle one is for the medium size (0.3×1.4) and the top one is for the large aperture (0.6×1.3).

Fig. 8(a) is suggesting that to increase the percentage of deposit volume, the squeegee speed must be increased. A higher squeegee speed produces an increase in deposit volume for all three sizes of aperture. The effect of volume of paste in front of squeegee is shown in Fig. 8(d). It recommends that the volume of paste in front of the squeegee should be small to maximize the deposit volume.

If it is difficult to conclude a certain decision from the effect of individual variable factors, the interaction surface of two factors produces more useful information. For instance Fig. 8(a) identifies only the ideal squeegee speed. When information about the squeegee load is also required two sets of data should be combined. The interaction response surface between squeegee load and squeegee speed for 0.2×1.1 mm aperture is shown in Fig. 9(a) and suggests that the load should be kept at a low level and squeegee speed must be high. The result obtained for squeegee speed confirms the same decision obtained from the Fig. 8(a) considering only the effect of squeegee speed. Fig. 9(b) similarly shows the estimated interaction surface between squeegee load and paste viscosity. It shows that both the squeegee speed and the paste viscosity should be high to maintain a high volume of paste deposit.

When EDFL is used, it is not necessary to apply a linear or nonlinear regressor to predict the relationship between variable factors and process parameters, but that the relationship has been created automatically. The data formed in fuzzy rules can be used as a model for the process. To produce the model from which the conditions which maximize the paste deposit volume can be determined, the whole



(a)

(b)

Fig. 9. Estimated response surface for 0.2×1.1 mm aperture: (a) squeegee speed and squeegee load (b) squeegee speed and viscosity of paste.

data contained in Figures 8(a-f) should be combined.

6 Conclusions

The idea presented in this paper has introduced a generalization of the experimental design methods. It has shown that simply by substituting the levels of experimental design with their equivalent fuzzy labels, the experimental procedure can be formulated into a fuzzy rule-base system and a complete interaction surface between the different parameters generated. A number of examples has been presented which illustrate the underlining idea of EDFL.

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