Insight and search in Katona's Five-Square problem

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Abstract

Insights are often productive outcomes of human thinking. We provide a cognitive model that explains insight problem solving by the interplay of problem space search and representational change, whereby the problem space is constrained or relaxed based on the problem representation. By introducing different experimental conditions that either constrained the initial search space or helped solvers to initiate a representational change, we investigated the interplay of problem space search and representational change in Katona's five-square problem. Testing 168 participants, we demonstrated that independent hints relating to the initial search space and to representational change had little effect on solution rates. However, providing both hints caused a significant increase in solution rates. Our results show the interplay between problem space search and representational change in insight problem solving: the initial problem space can be so large that people fail to encounter impasse, but even when representational change is achieved the resulting problem space can still provide a major obstacle to finding the solution.

Most of us know the lovely experience when the solution of a difficult problem suddenly pops into the mind with a call of ‘AHA!’. For more than 100 years (Bühler, 1907; Duncker, 1935; Wertheimer, 1925) insight problem solving has fascinated, challenged, and often frustrated researchers, because the evasive nature of insight has made it a phenomenon that is very hard to capture by scientific methods (Öllinger & Knoblich, 2009; Sternberg & Davidson, 1995).

Early work in insight problem solving focused on Gestalt perceptual characteristics and functional fixedness (Duncker, 1945; Maier, 1930; Wertheimer, 1959) but more recently, insight research has mainly focused on two central positions, both of which are embedded in problem space theory (PST, Newell & Simon, 1972) and aim at extending the explanatory power of the PST to insight problems.

The first emphasizes the role of search and heuristics (Kaplan & Simon, 1990). In line with this, MacGregor and colleagues (2001) provided an elaborated cognitive process model suggesting that the application of two heuristics leads to success in solving insight problems. First, a maximization heuristic that implies that with each move the distance to the goal has to be reduced as much as possible; and second, a progress monitoring heuristic that keeps track of the current distance to the goal and the remaining available moves. Should the number of moves needed to solve the problem exceed the number of moves that are still available then this should trigger the search for new and promising states, the driving force for an insightful solution. This has been successfully demonstrated for some insight problems (Chronicle, MacGregor, & Ormerod, 2004; MacGregor, Ormerod, & Chronicle, 2001; Ormerod, MacGregor, & Chronicle, 2002).

The second theoretical account (Ohlsson, 1984a, 1984b, 1992) focuses on the importance of representational change. The basic idea is that problem elements are either encoded in an inappropriate way or that the problem representation is overly constrained. For example, in the

8-coin problem (Ormerod et al., 2002, see below), participants have to move two coins such that each coin touches exactly three other coins. The solution requires building two groups of coins so that one coin rests respectively on top of three others – a strong constraint being that participants search for coin arrangements in a flat 2D representation. The 2D constraint therefore needs to be relaxed (Öllinger, Jones, Faber, & Knoblich, 2012). In general, it is assumed that constraints arise because of prior knowledge (Keane, 1985; Knoblich, Ohlsson, Haider, & Rhenius, 1999; Montgomery, 1988; Ohlsson, 1984a, 1984b, 1992; Thevenot & Oakhill, 2008).

Recently, we suggested a model (Öllinger, Jones, & Knoblich, 2013, see also Jones, 2003) that elaborates that of Ohlsson (1992), characterizing insight problem solving as a dynamic search process where heuristics constrain the search space before and after an insight (Kaplan & Simon, 1990; MacGregor et al., 2001; Ormerod et al., 2002) and representational change provides the insight to enable a change in the problem representation (Knoblich et al., 1999; Ohlsson, 1992). The model explains insight via a strict sequence of events. First, the initial problem representation is incorrect, producing a problem space that is constrained to not include the solution path. Second, the problem solver should eventually either exhaust the problem space or realize that no further progress has been made after repeated failure with the current strategy, and hence reach impasse. Third, to overcome impasse a representational change is necessary. The problem solver will hopefully change the problem space in a way that now includes the solution path as well as additional paths that do not contain the solution (i.e. representational change usually increases the size of the problem space, although much of it may have already been searched prior to representational change). Fourth, efficient search of the subsequent problem space will enable solution of the problem unless representational change causes the problem space to become so large that exhaustive search is difficult.

It is this latter point that illustrates why solution rates in the famous nine-dot problem (Maier, 1930; Scheerer, 1963) are so resistant to helpful hints. Although hints to draw lines beyond the perceived 3 x 3 square and to turn at non-dot locations (see Kershaw & Ohlsson, 2004, for a more in depth summary) provide the necessary insights that are required to solve the problem, they also result in the problem space becoming almost infinite since lines can be drawn to any location on the paper (Öllinger, Jones, & Knoblich, 2013).

The current study

In this study we test the predictions of our model by applying it to another insight problem, namely Katona's (1940) Five-Square problem. As Figure 1a shows, the initial problem configuration presented to the solver consists of a symmetrical arrangement of five squares made of 16 sticks. The task is to reduce the number of squares from five to four by moving exactly three sticks. Figure 1b shows one of the viable solutions to the problem. Because the problem is symmetrical there are a total of four potential solutions, one on each side of the original figure.

The problem requires the insight of overcoming perceptual grouping: realizing that in order to solve the problem, one must create two separate groups of sticks.In terms of the four-stage model that was outlined earlier, the Katona problem normally begins with a problem space that – by figural integrity – does not include the solution path (stage one). The problem solver will therefore be making solution attempts that are fruitless in terms of solving the problem (stage two). With appropriate insight, the problem space will be extended to include the solution (stage three). Problem space search should then be able to locate a solution path (stage four). Two aspects of the problem are noteworthy: first, the initial problem space is still quite large, because there are many ways in which three sticks can be moved while maintaining the perceptual grouping; second, since representational change extends the problem space to include almost any location that a stick can be moved to, the problem space becomes much larger once insight has been achieved.

Search processes affect the selection of sticks and the potential locations that those sticks can be moved to. By analyzing the former, we can learn about the applied search strategy and the realization of important elements of the problem representation. By analyzing the latter, we can study whether a representational change occurs that overcomes perceptual grouping (Knoblich et al., 1999; Öllinger & Knoblich, 2009). Representational change and search are likely to both be factors in Katona's problem because the problem space changes after the decomposition of the grouping (see Mayer, 1995).

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In order to study the interplay between initial search space, representational change, and resulting search space we used two types of hints (see Figure 2). The representational change in the Katona problem is the realization that the five square figure needs to be separated into two groups. A key driver for this is to realize that the center sticks form the sides of more than one square. Highlighting center sticks (see Figure 2a, a-d) (S hint = start position of a move) should therefore reduce the initial search space and enable the problem solver to reach a representational change more quickly. However, once insight has been achieved this type of hint will not help the problem solver to constrain the number of locations that a stick can be moved to, since the potential end points of moves are almost limitless. Thus an E hint (end position of a move) that provides a possible location of where sticks need to be moved should constrain the post-insight problem space (Figure 2b, e-h). Note that this hint is also suggestive of a representational change, because it proposes that the perceptual grouping of the sticks needs to be broken. Varying the sequence of the hints (SE vs. ES) allows us to determine whether or not S and E hints are equally effective in triggering representational change.

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In a third condition we introduced a 4to5 version of the problem where participants had to increase the number of squares from 4 to 5 by moving three sticks; that is, the number of available moves were consistent across problems but for the 4to5 version participants were confronted with the solution configuration (Figure 1b) and were required to find a solution where this configuration was transformed into the initial state of the Katona problem (Figure 1a). If the main difficulty of Katona's problem relates to the breaking of a perceptual group, then solution of the 4to5 problem should be trivial in comparison to the 5to4 problems, since the former does not require the breaking of a perceptual group whereas the latter problems do.

Predictions of the model

Our first prediction is that there should only be a weak increase in solution rates in the SE and ES conditions that either address only the starting position or the ending position. Our model suggests that problem space search is critical both pre- and post-insight. The problem space remains relatively large when providing only the S or the E hint: revealing a starting position leaves the ending position to be in almost any location, and providing the ending position still leaves for the first move 16 sticks to select a move from. Only when both S and E hints are given should a dramatic increase in solution rates be observed, because the problem space is now adequately constrained.

Our second prediction relates to the emphasis on representational change within our model: changing the spatial grouping in the given problem configuration should be the main difficulty in all of the 5to4 problems. As a consequence, participants should select fewer moves that end at positions that lead to a fundamental change in grouping than moves that leave the grouping intact or lead to smaller changes. In support of this, the 4to5 variant should achieve very high solution rates because it does not require the breaking of a perceptual group.

Our third prediction concerns the use of heuristics in problem space search before a hint is given. For the control group and the experimental conditions, participants will initially use appropriate problem space heuristics to navigate the problem space. MacGregor and colleagues suggest that the most appropriate heuristic for problem space search in this type of problem is that of maximization (Chronicle et al., 2004; MacGregor et al., 2001; Ormerod et al., 2002). Participants should therefore select moves that reduce the number of shared sticks. Consequently, they will manipulate sticks having a high number of contacts (see Figure 3a): 6 contacts > 4 contacts > 2 contacts; however, the end point of moves involving these sticks will preserve the perceptual grouping of the five squares.

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Method

Participants. The 168 paid participants (48 males, mean age 25, range 18-35) were recruited by advertising at the University of Munich and in local newspapers and were randomly assigned to one of the four conditions (N=42). One participant from the SE condition was excluded from the data analysis because he did not make a single move.

Material. We used two different problem versions, 5to4 and 4to5. Three groups worked on different versions of the 5to4 problem and one group worked on the 4to5 problem (see Table 1). The control group and the 4to5 problem received no hints throughout the experiment. There were 16 round wooden sticks (length 50 mm; diameter 5 mm). Additionally, there were green and red colored rectangular shaped pieces of card (length 50 mm; width 5 mm) that were used to indicate the positions of hint 1 and hint 2. Due to the fact that the problem is symmetric and there are four potential sites where a solution can appear, we provided the hints randomly at one of the four possible locations. The position of the second hint always corresponded to the location of the first hint.

Procedure. Participants were tested individually in a quiet room and were allowed 15 minutes to solve the problem. In the three 5to4 conditions they received the following written instruction (in German).

"Your task is to move exactly three sticks so that you produce four squares of equal size."

In the 4to5 condition the instruction was: "Your task is to move exactly three sticks so that you produce five squares of equal size."

Participants were allowed to restart as often as they wanted. After an unsuccessful attempt participants were asked to rearrange the start configuration. A printed version of the start configuration was provided. Participants had to put the sticks on the printed version. The participants' workspace and hand movements were filmed using a camera. The camera was connected to a monitor behind a dividing wall that shielded the experimenter from the view of the participant. The experimenter encoded the start position and end position of each move as they were performed. This was done by clicking a mouse on the respective locations of the problem configuration displayed on a computer monitor. The computer recorded the start and end point of each move.

Hints: Two of the 5to4 conditions were given hints made of card that were laid on defined locations at specified times on the printed version of the problem (SE and ES, see Table 1). After 5 minutes, the first hint appeared for participants that had not solved the problem. After 10 minutes, for the remaining unsuccessful participants a second hint was provided. The first hint remained.

Condition	*Hint 1 (after 5 min)*	*Hint 2 (after 10 min)*
5to4, SE condition	A green card was put on a significant stick position (see Figure 2a) of the initial problem representation that had to be moved for a solution.	Hint 1 remains. A red card was put on a significant stick position of a potential end position of the solution (see Figure 2b).
5to4, ES condition	A red card was put on a significant stick position of a potential end position of the solution (see Figure 2 b).	Hint 1 remains. A green card was put on a significant stick position of the initial problem that had to be moved for a solution (Figure 2a).
5to4, Control Group (CG)	No Hint	No Hint
4to5 Group	No Hint	No Hint

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Results

Data Analysis: For the solution rate analysis, we conducted χ^2-tests that tested between groups pair-wise differences across the following categories: 0 = not solved, 1 = solved before the first hint, 2 = solved before the second hint, 3 = solved after the second hint and before the upper time limit. For the initial search analysis relating to the influence of the number of contacts on move selection, we used the schema illustrated in Figure 3. For the representational change analysis relating to whether moves broke the perceptual grouping, we examined the end point of moves before any hints were given, after the first hint was given, and after the second hint was given.

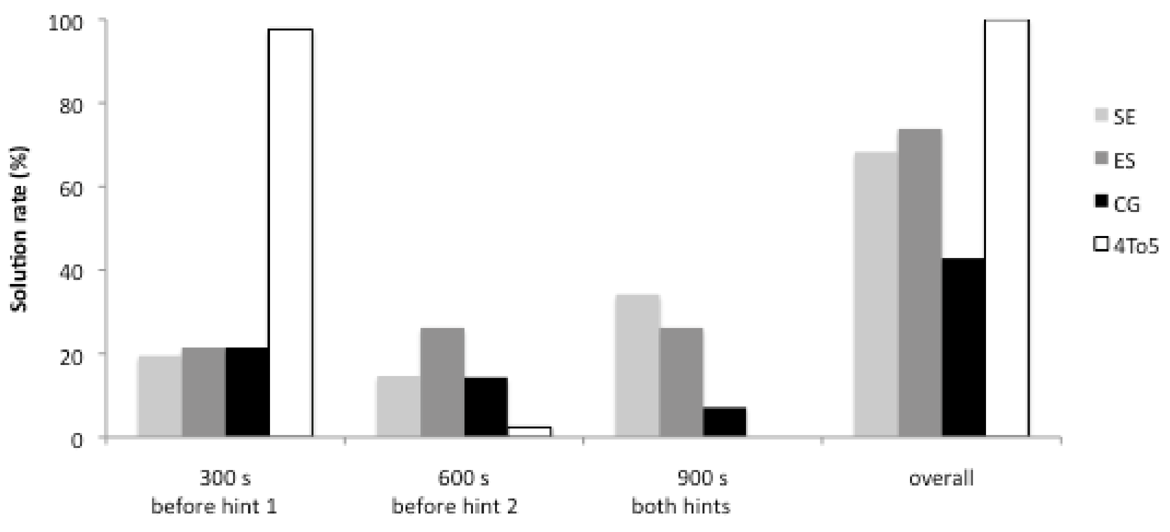
Solution rate: As Figure 4 illustrates, for all 5to4 conditions the increase in solution rate across the different time intervals was fairly similar in all conditions until the second hint appeared. The 4to5 group showed a completely different pattern. Participants solved the 4to5 problem almost immediately.

Prediction 1 states that only when providing a combination of hints will solution rates significantly increase. To evaluate the overall differences between the conditions, χ^2-tests were conducted. There was a significant difference between the SE (28 of 41 solved the problem) and the CG (18/42), $\chi^2(3, 83) = 10.44$, $p < .05$, $\Phi = .26$ and between the ES (31/42) and the CG, $\chi^2(3, 84) = 10.87$, $p < .05$, $\Phi = .31$, indicating that providing both hints increased the overall solution rate. There was no difference between the SE and ES, $\chi^2(3, 83) = 2.04$, $p > .56$, $\Phi = .16$.

We also analyzed whether there were differences between the 5to4 conditions across different time intervals, because our first predictions states that the provision of both hints will be far stronger than the provision of just one of the hints. As expected, before the first hint occurred there were no differences (all *p*s > .70). Between the first and second hint, there was numerically a stronger increase in the ES (see Figure 4), though this did not result in significance ($\chi^2(1,66) = 1.98$, $p > .15$, $\Phi = .15$, for ES vs. SE, and $\chi^2(1, 66) = 1.98$, $p > .15$, $\Phi = .15$ for ES vs. CG). There was also no difference between the SE and CG ($\chi^2(1,66)$ = ???, *p* > .??, Φ = .??).

As expected, single hints that either highlight a required start or end position had little impact on the solution rate. After the second hint, there were highly significant differences between the CG and SE, $\chi^2(1, 54) = 10.39$ $p = .01$, $\Phi = .44$, and between CG and ES, $\chi^2(1, 49) = 8.98$ $p = .01$, $\Phi = .42$. There was no difference between the SE and ES, $\chi^2(1, 49) = 0.17$, $p > .89$, $\Phi = .02$. Taken altogether, the analyses relating to prediction one support the emphasis on problem space search both before and after insight within our model.

Our second prediction that states that spatial grouping aspects are the main cause of problem difficulty. We therefore compare the 4to5 condition, where the grouping is broken from the beginning, with the 5to4 conditions. There was a highly significant difference between the 4to5 condition (42/42) and the CG, $\chi^2(3, 84) = 51.05$, $p < .01$, $\Phi = .78$. Similar effects were seen between the SE and ES conditions and the 4to5 condition (both *p*s < .01). The results support the emphasis on representational change within our model.



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Movement data: Our third prediction states that before any hints are given, participants will manipulate sticks that have a higher number of contacts (maximization strategy) while maintaining the perceptual grouping of the figure. For the movement analyses we excluded the 4to5 condition, since this condition had a different initial start configuration.

We first analyze whether a maximization strategy is adhered to. As shown above, the solution rate data clearly show that the conditions diverge after hint 2. However, it is still useful to examine move selections at every key time point (i.e. before any hints were given, after hint 1, and after hint 2) because this may reveal something about the relative strengths of the S and E hints. We therefore analyzed each time point separately by repeated measure ANOVAs. Note that the number of participants varies between time points.

For analyzing the start position of moves, we distinguished between moves that involved a different number of contacts between sticks. In the Five-Square problem there are three different categories of sticks: those that share 6, 4, or 2 contacts with other sticks. As Figure 3a illustrates there are four matchsticks that have two contacts, eight that have four, and four that share six contacts in the initial problem representation. We determined for each participant the frequency of selected matchsticks belonging to each category and weighted each number with the number of selected matchsticks in this category. For example, a participant who manipulated 10 matchsticks all having four contacts would receive the ratio 10/8, because there are 8 matchsticks with 4 contacts. In the next step, we determined for each participant the percentage of moves in each category, by dividing the weighted number of moves in each category by the sum of the three categories and multiplying the result by 100.

Move selection before hint: As Figure 5a shows, participants showed a general preference for the manipulation of sticks that share 6 contacts with other sticks. A repeated measures ANOVA with the factor Condition (SE, ES, CG) and the factor Number of Contacts (2 and 6 contacts) revealed a highly significant difference for the factor Number of Contacts, $F(1, 116) = 61.86$, $p < .01$, $\eta_p^2 = .35$. There was also a significant main effect for condition, $F(2, 116) = 3.53$, $p < .05$, $\eta_p^2 = .06$. Post hoc tests (Scheffé) revealed a significant effect between SE and ES only ($p < .05$). There was no significant interaction ($p > .84$). That is, before a hint was provided, participants in the CG and SE conditions showed a preference for manipulating sticks with a higher number of contacts. Unexpectedly, ES showed a less pronounced tendency, due to the fact that participants in this condition selected a higher amount of sticks with four contacts. The results predominantly support the view within our model that prior to insight, participants use relevant heuristics to explore the problem space.

Move selection after hint 1: As Figure 5b demonstrates the stick selection pattern differs between conditions after providing the first hint. Not surprisingly, for the SE condition that provided the initial stick to move, the manipulation of sticks with 6 contacts strongly increased. For the ES that emphasizes the end position of the solution there was a pronounced reduction in manipulating sticks with 6 contacts, and there was no preference for a particular type of stick. That is, after providing the ending position, participants are less sure about which sticks to move to the end location. As one would expect, the CG showed an almost unchanged pattern. A repeated measures ANOVA with the factor Condition (SE, ES, CG) and Number of Contacts revealed a highly significant main effect for the factor Number of Contacts, $F(1, 90) = 41.01$, $p < .01$, $\eta_p^2 = .31$ (selections involving sticks having 6 contacts being preferred) and no significant difference between the conditions, $F(2, 90) = 2.48$, $p = .09$, $\eta_p^2 = .05$. However, there was a significant interaction between the factors, $F(2, 115) = 11.36$, $p < .01$, $\eta_p^2 = .20$, illustrating the different patterns of move selections for the ES condition in comparison to the other conditions.

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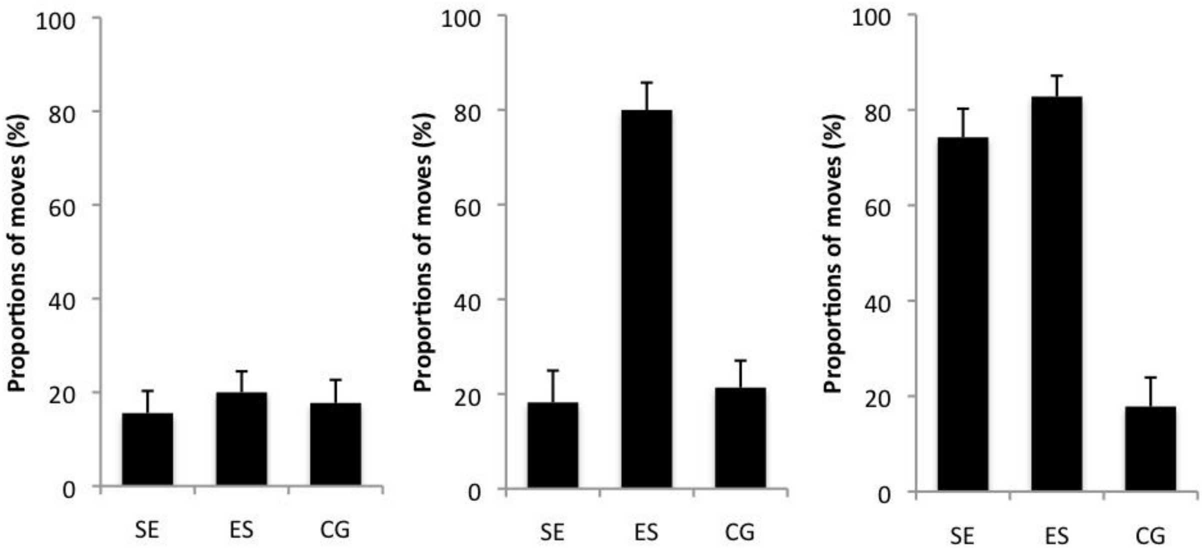
Move selection after hint 2: As Figure 5c illustrates, all conditions showed a strong preference for manipulating sticks with a large number of contacts, though this was less pronounced in the CG. A repeated measure ANOVA with the factors Condition and Number of Contacts revealed a significant main effect for the factor Number of Contacts, $F(1, 71) = 75.88$, $p < .01$, $\eta_p^2 = .52$ but no significant difference between the conditions ($p > .53$). However, there was a marginal significant interaction between the factors, $F(2, 71) = 3.06$, $p = .053$, $\eta_p^2 = .08$, which reflects the different pattern of move selections for the SE and ES in comparison to the CG.

Selection of end positions: We now examine whether the perceptual grouping was maintained (second part of prediction 3) by analyzing the end positions of moves. We distinguished between three different categories: 1) moves that respect the grouping of the given five square figure and indicate that no representational change appeared (e.g., position g1 and g2 in Figure 3b); 2) moves ending at positions that indicated the grouping had been broken (solution moves s in Figure 3b; it is important to note that an individual s-move does not necessarily break the grouping of the figure – for example, moving a center stick to position s1 or s3 results in loose ends within a still connected figure. However, loose ends are not part of a proper solution. Consequently, by closing those loose ends the grouping will be broken by the two consecutive moves); 3) all other positions where moves could end. Because only 6.1% of all moves were categorized as “others”, we restricted the reported analyses to the first (intact grouping) and second (solution moves) categories, but did not remove the “other” category from the data. As a consequence the first and second categories do not sum to 100%.

As per the starting moves, we focus on end positions before any hints were given, after the first hint was given, and after the second hint was given. Figure 6 plots the proportion of moves that break the perceptual grouping.



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End positions before hint: As Figure 6a illustrates, before a hint appeared, about 20% of moves ended at potential solution positions. An ANOVA with the factor Condition (SE, ES, CG) showed no significant effect for the factor Condition $F(2, 111) = .21$, $p > .80$.

End positions after hint 1: Figure 6b shows that after providing the E hint in the ES condition, the number of moves that break the perceptual grouping increased dramatically. In all other conditions, the end points of moves remain similar to what they were before a hint was given. An ANOVA with the factor Condition (SE, ES, CG) revealed a highly significant main effect, $F(3, 111) = 35.57$, $p < .01$, $\eta_p^2 = .44$. Post-hoc tests (Scheffé) showed reliable differences between ES and all other conditions (*p*s < .01). There was no difference between the CG and the SE condition.

End positions after hint 2: After the second hint (Figure 6c), the ES showed a high number of moves to solution positions (83%). The SE also showed a high number (74%). The CG remained at a rather low rate (18%). An ANOVA with the factor Condition showed a highly significant difference, $F(2, 69) = 38.87$, $p < .01$, $\eta_p^2 = .53$. Post-hoc (Scheffé) comparisons revealed SE differed significantly from CG ($p < .01$). The same holds true for ES ($p < .01$). There was no difference between SE and ES ($p > .56$).

Discussion

The study aimed at testing the predictions of the dynamic search and constraint model that was recently introduced by Öllinger, Jones & Knoblich (2013), by using the Five-Square insight problem introduced by Katona (1940). In two experimental conditions, we provided hints that highlighted the location of starting and ending moves that are crucial for solving the problem. This manipulation tested the prediction that constraining the problem representation and at the same time constraining the goal representation is necessary for a successful solution. In a third condition, we used a problem that reverses the moves that have to be performed in the original problem. In this condition, participants were asked to increase the number of the given squares from four to five. All experimental conditions were tested against a naïve control group.

Our data showed that both 5to4 experimental conditions had a significant increase in solution rates compared to the CG, but only when information about the start and the end configuration were both provided. This is because when only one piece of information is provided, the problem space is still too large to be efficiently searched and therefore no significant increase in solution rates is seen. This supports the view within our model that problem space search is important both pre-insight and post-insight. However, the key to the difficulty in the problem is the fact that the solution requires the perceptual grouping to be broken (the emphasis on representational change within our model). This was clearly illustrated in the 4to5 condition, which proved to be trivial. We now consider the implications for the solution rate results and the move selection results, together with other implications of our results.

Solution rate: The solution rate data indicates that there was a slight increase in solution rates over time. For the CG the solution rate was approximately 40% at the end. Until the second hint was given there was no significant difference between the two hint conditions and the CG. After the second hint the pattern changed completely. Irrespective of the order of the hints, there was a strong increase up to 68% in the SE and 73% in the ES condition. The data clearly demonstrates, as suggested by prediction 1, that for a solution, there needs to be a concerted interplay of problem space search and representational change that imply the decomposition of the perceptual grouping of the initial configuration. However, the data also illustrated the role that problem space search plays. Providing just the initial location of a move or the ending location of a move still left the problem space too large to be efficiently searched.

Move selection: The analyses of the start and end positions (Figures 5 and 6) provide more details on these findings. From the beginning, participants preferred to manipulate sticks with a higher number of contacts, as a heuristic account predicts (see prediction 2) (MacGregor et al., 2001; Ormerod et al., 2002). This can be taken as evidence that participants realized from the beginning that the central sticks that shared 6 contacts played an important role in reducing the number of squares (Mayer, 1995), even if they did not know why (since their moves did not break the perceptual grouping of the figure).

4to5 control condition: The 4to5 condition that required an increase in the number of the given squares from 4 to 5, showed completely different results to the 5to4 conditions. Almost all participants in the 4to5 condition solved the problem within the first 5 minutes. At the end all participants had solved the problem. Our model would predict (see prediction 3) that the problem is easy because the provided perceptual configuration defines a search space that already contains the solution path (i.e., no representational change is required). Therefore, a maximization heuristic can be applied, increasing the number of shared sticks from 0 to 4. An alternative explanation that was not explicitly tested in our study could be that perceptual grouping aspects driven by Gestalt principles like symmetry and pregnancy (Metzger, 1986) facilitate the move selection and result in a quick solution. That is, to close the gap and build a single and united figure (see also Lee & Johnson-Laird, 2013, for a more detailed analysis of perceptual aspects in matchstick tasks).

Strength of hints: Our model investigates the role of problem space search and representational change. The SE and ES conditions examine the strength of hints on subsequent solution of the problem. However, the hints differ in how they influence the problem space. In the SE condition, the first hint relates to the starting position (restricted search = constraining the problem space, Kaplan & Simon, 1990) – the end location could be almost any position. In the ES condition, the first hint relates to the ending position (breaking the perceptual grouping = representational change, Ohlsson, 1992) – the stick to move to this highlighted position could be still any one of the 16 sticks, and there are still two other moves to select. The problem space in the SE and ES conditions after the first hint is still too large (no exhausted search). Therefore, provision of the first hint did not facilitate solution rates in comparison with the CG. Although solution rates markedly increased after provision of the second hint, there was still no difference between the SE and ES conditions. This suggests that the strength of the S and E hints is equal, but there are two pieces of evidence that we believe support a view that the E hint is stronger than the S hint. First, there is a numerical advantage (albeit non-significant) in solution rates for the ES condition over the SE condition after provision of the first hint. This is because provision of the ending location (E hint) for a move leaves a finite number of starting locations to choose from (the 16 sticks) whereas provision of the starting location (S hint) leaves an almost “infinite” number of ending locations. Second, and more telling, is the CG move selection data. If one examines the sticks that participants in the CG move, the majority are sticks that share 6 contacts – and this is true at every key time point. Thus the majority of moves for participants in the CG involve the *correct* sticks. Compare this to the CG moves that break the perceptual grouping of the figure – only ~20% of moves break the grouping and this is consistent across key time points. That is, the majority of ending locations of moves are *incorrect*. Taken together, the data suggest that under no time restrictions, the ending location hint is likely to be stronger than that of the starting location hint. This assumption is in line with studies that showed that providing the key cue for a representational change can increase solution rates when the search space is adequately restricted (Kershaw & Ohlsson, 2004; Öllinger et al. 2012; Öllinger, Jones & Knoblich, 2013). Moreover, the comparison between the 4to5 and the 5to4 conditions convincingly demonstrated that the main source of problem difficulty is to break the perceptual grouping, which was fostered by the “E” hints and not by the “S” hints. In the current study therefore, the ES participants had to restrict the initial search space – the work that is done by the additional “S” hint – by themselves under time-limited conditions until the “S” hint was given; equally the SE participants still needed to overcome the main source of problem difficulty (breaking the existing grouping) under time-limited conditions until the “E” hint was given (see also Kershaw, Flynn, & Gordon, 2013; Kershaw & Ohlsson, 2004). Although a clear difference between the S and E hints is not seen in the current study, it seems plausible that if only one hint was provided (either “S” or “E”) that the “E” hint might be more efficient over time.

We suggest that the difficulty of the Five-Square problem and indeed other similar insight problems (see Öllinger, Jones & Knoblich, 2013 for a more detailed discussion) can be explained by the interplay of problem space search and representational change – and can be characterized by consecutive phases of restricting, changing, and restricting etc. of the problem space. The Five-Square problem is difficult because perceptual grouping aspects impose constraints on the move selection that dictate the selection of end positions. The selection of the start position is driven by a maximization heuristic as suggested by Ormerod et al. (2002), that locally reduces the number of shared sticks. As our movement data clearly showed, from the beginning participants manipulated sticks that were connected to a high number of other sticks, but their moves were very ineffective without a clear location to move the sticks to. Similarly, we also found that providing an ending position for a move disturbed the initial move selection process, and was only supportive after providing both the end and the corresponding start positions.

The results provide further evidence for our dynamic constraint and representational change model (Öllinger, Jones & Knoblich, 2013), that postulates that a representational change is sometimes not sufficient to solve insight problems due to the fact that modifying a problem representation often results in an even larger problem space that has to be constrained by efficient search strategies. Recently, Lee and Johnson-Laird (2013) provided a more general model for strategy changes in problem solving and tested their assumptions by using matchstick problems that do not necessarily require insight. Similar to our model they also assume phases of generation of different solution approaches and mechanisms that constrain those search spaces by additional assumptions so that the most efficient strategy can be selected. That is, the dynamic of search and constrain might be a fruitful and powerful process that can enlighten problem solving and worthy of further study. However Lee and Johnson-Laird's model provides no mechanism for representational changes, and therefore should have problems in explaining insight problem solving.

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Table Caption

Table 1: Experimental conditions and provided hints.

Figure Caption

Figure 1: Katona’s Five-Square problem. a) Start configuration. b) Solution of the problem. Black rectangles indicate the moved sticks.

Figure 2: Source and target positions of the perceptual hints. There are four locations where a hint indicates the starting position (a, b, c, d) (Figure 2a) and four corresponding locations where a hint indicates the ending position (e, f, g, h) (Figure 2b).

Figure 3: Analysis scheme for the selected chunks and end positions of the moves. a) the three colors indicate the number of contacts a stick shares with its neighbors (black = 6 contacts, gray = 4 contacts, white = 2 contacts). b) initial configuration and examples of potential end positions of moves divided into different categories. g = moves that obey grouping. s = moves that end at potential solution positions (e.g. the solution s1 to s3 in black).

Figure 4: Cumulative solution rate of the different experimental conditions and separated by the temporal presentation of hint 1, hint 2, and the upper time limit.

Figure 5a, b, and c: Selected sticks from different categories across the three conditions before the first hint (5a), after the first hint (5b), and after the second hint (5c). Standard error bars are displayed.

Figure 6: Proportion of moves that broke grouping. a) before a hint appeared, b) after providing the first hint, and c) after providing the second hint. Standard error bars are displayed.