Finite Element Modelling of interaction between surface and Darcy flow regimes through soils

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Abstract

The present work deals with the impact of surface flow on hydrodynamic conditions in saturated underground domains. A three dimensional finite element analysis of water flow has been used to obtain the required simulations. The results clearly show the effects of the surface flow on the hydrodynamic conditions of the subsurface porous regions. This analysis is an important prerequisite for the prediction of contaminant mobility in soils and hence provides a convenient tool for the prediction of environmentally important subsurface flow processes. For low permeability cases, considered here, governing equations consist of water continuity and Darcy equations. These equations are solved using a robust and reliable finite element procedure.

Key Words: Underground flow, Hydrodynamics, Darcy flow, Finite element modelling, Three dimensional porous flow.

1. INTRODUCTION

Seepage in soils is an essential topic of study in many civil engineering and environmental protection processes. For example, in the design of earth dams and retaining structures, that require quantification of drainage, amount of seepage need to be identified. Similarly seepage is the determining factor in the contaminants mobility as leachates in subsurface domains. Seepage flow models have been developed by many researchers (e.g. see Cedergren, 1994; Reddi, 2003). These investigations have shown that seepage regimes often have complicated characteristics mainly because of the heterogeneity of soils through which water flows. Heterogeneity of soil media arises for various reasons such as the presence of staggered layers of soil with different porosity. Therefore reliable mathematical description and modelling of seepage flow requires formulation of realistic features of the problem and the boundary conditions affecting the flow. Transport equations describing flow through porous media depend on the properties of the fluid and factors such as permeability and porosity of media (Bear, *et al*. 1991). In this regard it is important to distinguish between saturated and unsaturated domains. Seepage flow in unsaturated lands has been the subject of many studies (e.g. Troldborg, 2009; Uromeihy, 2007). However, unlike the unsaturated situations, the influence of surface flow on saturated domains has not been widely studied. This may be due to the common belief that surface flow makes only small contribution to the changes of hydrodynamic conditions under the ground in saturated domains. However as the results of this research show, there is a significant link between surface flow and subsurface conditions in saturated lands. Simulations obtained in this work can therefore be considered as quantitative analysis of the link between surface and subsurface flows for saturated cases.
The flow through porous soils is affected by hydraulic gradient and the coefficient of soil permeability. Permeability is a function of the range of grain size and shape, stratification, consolidation and cementation of the material. The rate of flow is commonly assumed to be directly proportional to the hydraulic gradient, however this is not always true under realistic

conditions. Theoretically, due to the increasing load, permeability of soils decreases with increasing depths. Therefore a layered heterogeneous strata is the common feature of flow domain in most environmental studies. In table 1 a typical range of soil permeability used in the present study is shown.

Table 1 Values of soil permeability

Degree of permeability	Range coefficient of permeability (m^2/s)	Soil Type
High	10^{-6}	Medium and coarse gravel
Medium	10^{-6}-10^{-8}	Fine gravel; coarse; medium and fine sand; dune sand; clean sand – gravel mixtures
Low	10^{-8}-10^{-10}	Very fine sand, silty sand, loose silt, loess, well fissured clays
Very low	10^{-10}-10^{-12}	Dense silt, dense loess clayed silt, poorly fissured clays
Impermeable	10^{-12}	Unfissured clays

Porous flow under the ground is governed by hydraulic head. Hydraulic head consists of the velocity, pressure and elevation heads. The velocity head in soils is usually negligible in comparison with the other heads, therefore here the main driving force is taken to be combined elevation and pressure heads, which for simplicity is represented by a single pressure term. Saturated flow in a porous medium should be modelled using either Darcy (Darcy, 1856); or Brinkman (Brinkman, 1947) equations depending on the permeability/porosity of the medium and flow Reynolds number (Wakeman and Tarleton, 2005). For very low Reynolds number (creeping flow) and porosity less than 0.6, the most suitable equation is Darcy's equation. This is the dominant situation in most types of seepage flow of water in soils. In a previous paper the validation of the Darcy's law for isotropic, homogenous, incompressible, saturated and isothermal porous media was considered (Kulkarni, et al. 2008). The Darcy equation inherently implies perfect slip conditions at domain boundary walls and does not include any wall effects (Ishizawa and Hori, 1966). Therefore an accurate solution scheme for this equation should be capable of yielding a slip velocity on the porous domains. The finite element scheme used in the present work can very effectively cope with such boundary conditions. This technique can also very effectively cope with irregular geometries. However, considering the large scale of lands where environmental phenomena needs to be studied, any irregularity of the domain walls can be ignored. In this work we have used a block as a representative section of a typical underground flow domain.

2. MODEL EQUATION

The governing model equations for the seepage flow of an incompressible Newtonian fluid such as water are represented as:

2.1. Equation of Continuity

The continuity equation (i.e. expression of conservation of mass) for an incompressible fluid is represented (using vector notations) as:

$$\nabla \cdot \vec{u} = 0 \qquad (1)$$

Where $\vec{u}$ is the velocity vector.

2.2. Equation of Flow

As mentioned earlier we have selected the Darcy equation to represent the flow equation (i.e. expression of conservation of momentum). Using vector notation this equation (Nield and Bejan, 1992) is written as:

$$\rho \frac{\partial \vec{u}}{\partial t} + \nabla p + \frac{\mu}{K} \cdot \vec{u} = 0 \qquad (2)$$

Where ρ is fluid density, p is the pressure, μ is viscosity of the fluid and K is the permeability of the porous medium. In its most general description K should be regarded as a second order tensor which is represented, in a matrix form, as:

$$K = \begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix} \qquad (3)$$

Where K_{xx}, K_{yy} and K_{zz} are the principle components of the permeability tensor along the x, y and z directions of a Cartesian coordinate system. Any anisotropy in a porous medium can hence be taken into account by assigning appropriate values to the components of the permeability tensor.
Conjunctive solution of equations (1) and (2) poses a mathematical problem as the first equation does not include a pressure term. Full mathematical analysis of the problem is somewhat obscure and requires lengthy explanations. However, it has been shown that a stable and accurate solution for these equations can be obtained provided that the solution scheme satisfies a condition known as the LBB condition (Reddy, 1986). A convenient way of satisfying this condition is to replace equation (1), which is the expression of incompressibility, with a modified form (Zienkiewicz and Wu 1991) as:

$$\frac{1}{\rho c^2} \frac{\partial P}{\partial t} + \nabla \cdot \vec{u} = 0 \qquad (4)$$

Where c is the speed of sound in the fluid. Equation (4) represents conservation of mass for a slightly compressible fluid.

3. BOUNDARY CONDITIONS

In the present work simulations of underground flow of water are obtained by the conjunctive finite element solution of equations (2) and (4) in a three-dimensional domain subject to the following boundary conditions:

1. Top surface: Constant velocity tangent to the surface.
2. Side walls: Perfect slip conditions.
3. Bottom surface: Perfect slip conditions.
4. Downstream surface: Zero pressure set as an arbitrary datum.

4. SOLUTION ALGORITHM

Numerical solution of equations (2) and (4) via the finite element method starts with the domain discretization into a computational mesh. Selection of a particular type of element is of importance in generating reliable results. In this work we have used 8 noded (brick elements) which generate tri-linear approximations for both pressure and velocity fields.

Application of the Galerkin finite element technique (Nassehi, 2002) generates a set of working equations which can be used according to the algorithm shown in Fig. 1 to obtain the required numerical results.

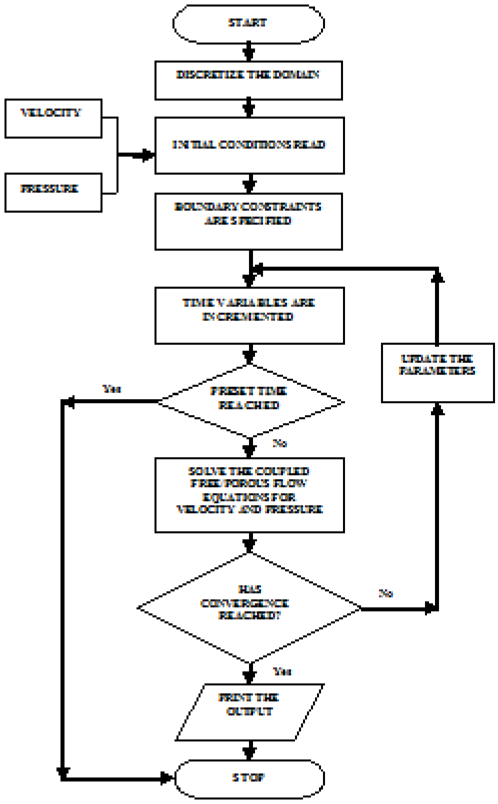

Figure 1: The solution algorithm implemented using the in-house developed program

5. COMPUTATIONAL RESULTS AND DISCUSSION

In this section the results obtained for a block domain (prism) subject to varying surface slope are presented and discussed.

In these simulations, the fluid under consideration is water with properties at 200 C, as viscosity= 0.001 $Kgm^{-1}s^{-1}$ and density = 1000 Kgm^{-3}. The velocity of sound in water is taken to be approximately as 10000 ms^{-1} .Depending upon the property of the permeable medium (which can be isotropic or anisotropic), appropriate values of permeability are used to cover a range of realistic situations. The time increment (Δt) used in the solution scheme is 20 seconds.

5.1. Block domain consisting of isotropic permeable medium

A block domain of 60 m width (W) x 60 m height (H) along x, y in one side of the domain and 60 m (W) x 30 m (H) along x, y in the other side of the domain and 200m length (L) along z axis is modelled. Therefore initially a surface slope of 8.53° is considered. The magnitude of surface velocity is 0.2 ms^{-1} .The computational grid used to simulate porous flow in this domain comprises of 5760 eight node brick elements and 6929 nodes, as shown in figure 2. The permeability coefficient for this homogeneous isotropic porous domain is taken as:

$(K_{xx} = K_{yy} = K_{zz} = 10^{-9} m^2)$ The schematic representations of the boundary conditions imposed over the different sides of this domain are shown in figures 3 and 4.

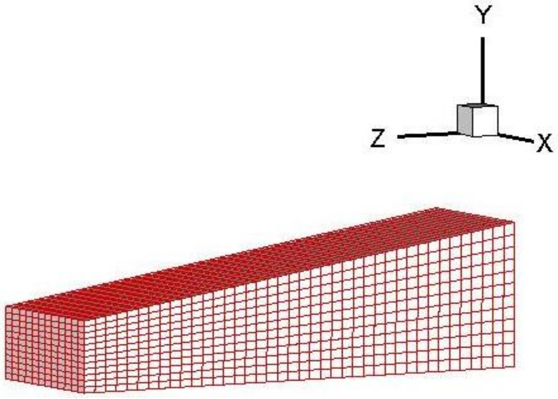

Figure 2: Finite element mesh for the homogeneous domain

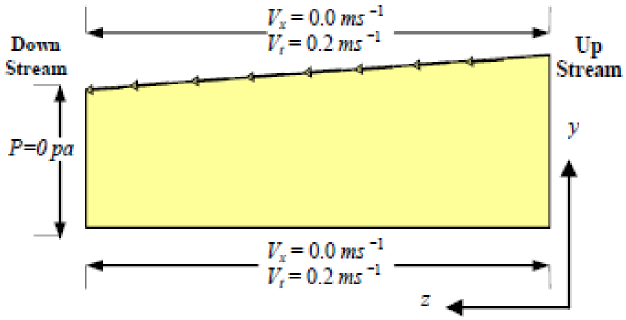

Figure 3: Schematic representation of the boundary condition in the yz plane

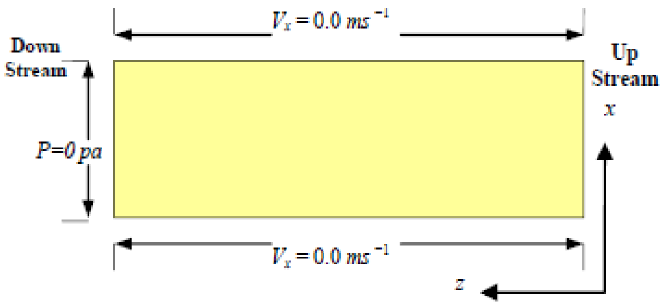

Figure 4: Schematic representation of the boundary condition in the xz plane

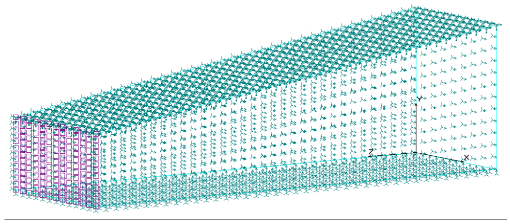

Figure 5: Three dimensional schematic representation of the boundary points at different faces of the domain

Figure 5 shows the position of the boundary nodes on the faces of the permeable domain under consideration. In addition to the top surface flow velocity (0.2 ms^{-1}) at the downstream exit a zero pressure datum is prescribed. Slip wall boundary conditions are imposed on the sides of the domain. This means that velocity components vertical to each wall is set to be zero whilst the other components are left to be free.

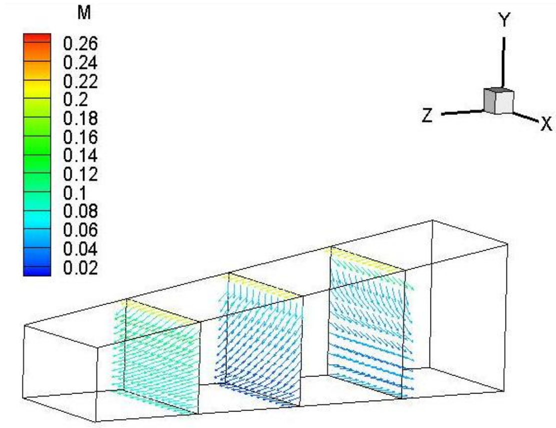

Figure 6: Velocity vector plot on three sample cross sections

To make the representation of the velocity field clearer, only the computed velocity vectors on three sample cross sections are shown in figure 6. The corresponding nodal pressures and magnitude of velocity are shown in figures 7 and 8, respectively. This result shows that, as intended, viscous stress associated with the bulk matrix of the fluid is transferred to the solid porous matrix.

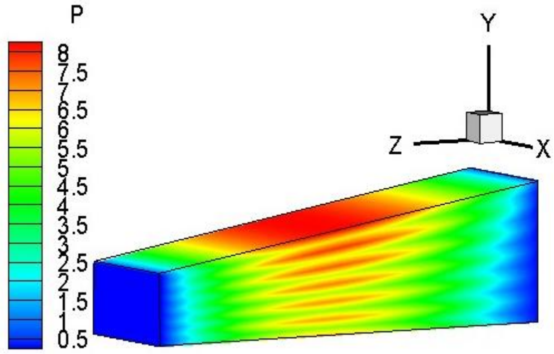

Figure 7: Pressure plot in the domain

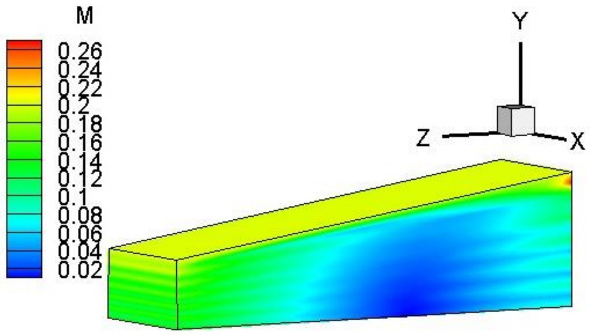

Figure 8: Velocity contour in the domain

Figure 9 clearly shows that despite the ground being saturated significant flow under the ground is generated by the imposed surface flow and, therefore, causing distribution of any existing contaminants in all directions.

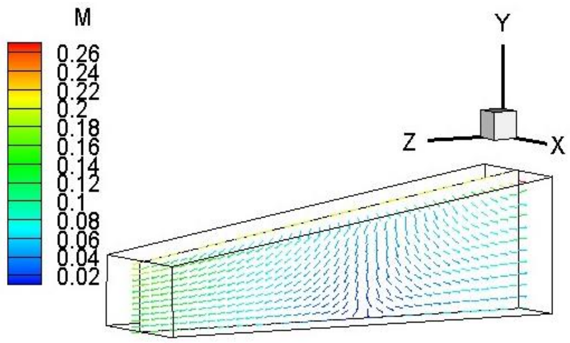

Figure 9: Velocity vector in the section parallel to the zy plane at x=30 m in the domain

To study the influence of the surface slope on the intensity of underground circulation by the surface flow, a second domain which has a different surface slope of 12.68˚ is also simulated. All other boundary conditions are kept to be similar to the previous case. Simulation results shown in figures 10 and 11 indicate the strong relation between the surface slope and the underground circulations caused by the surface flow. For example despite showing a pattern of circulation similar to the previous case, velocities have increased by as much as 38%. Velocity vector shown in Figure 12 indicate the same pattern of flow circulation compared to the previous case.

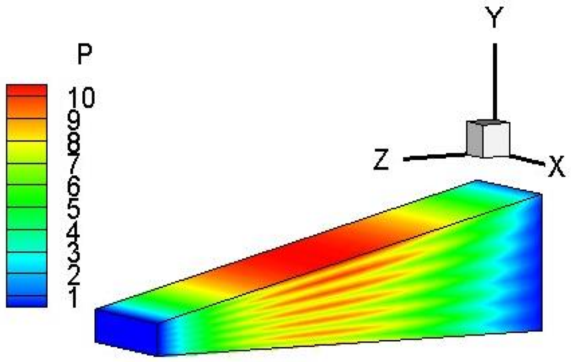

Figure 10: Pressure plot in the second domain

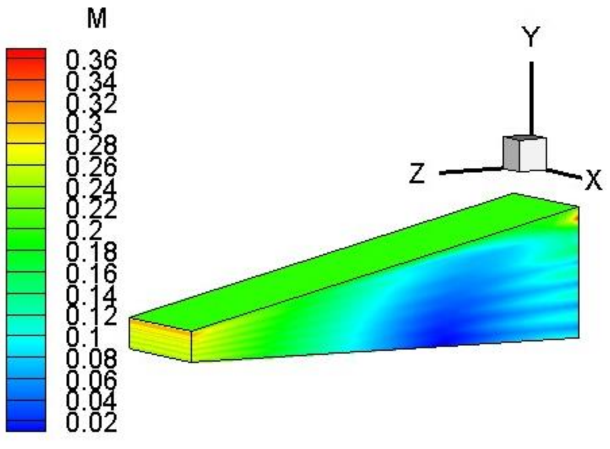

Figure 11: Velocity contour in the second domain

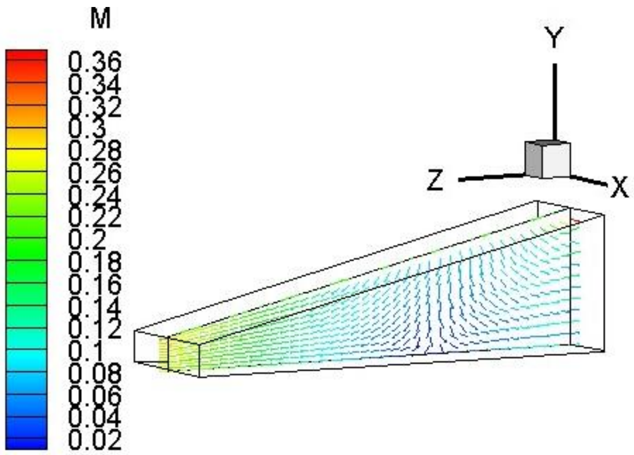

Figure 12: Velocity vector in the section parallel to the zy plane at x=30 m in the second domain

6. CONCLUSIONS AND RECOMMENDATIONS

The described simulation is based on the finite element solution of three dimensional Darcy and continuity equations. This provides a powerful means for the investigation of seepage flow of water in subsurface regions. The accuracy of the model is verified by its ability to preserve the agreement between the predicted velocity and pressure results with the theoretical expectations.

The simulations have shown that despite the land being saturated, rain water flow over the ground results in significant disturbance of the hydrodynamic equilibrium under the ground and hence causes movement of contaminants. It is also shown that water circulation under the ground is significantly affected by the surface flow velocity and the surface slope. As shown by these results underground flow direction is not dependent on the predominant direction of the flow over the surface and multi-directional flow established under the ground can occur in directions opposite to the direction of surface flow. A tentative conclusion is that flow under the ground can be in all directions and the reason for not detecting significant flow currents in the transverse directions in the simulations shown in this work is the imposition of solid wall conditions on the sides of the problem domains considered here. Closer examination of the described conclusions reveals the importance of the results obtained during the present project. For example considering that in many heavily industrialised regions of the world land is saturated, at least during winter months, a semi-circulatory flow under the ground caused by rainfall can cause contaminants migration in unexpected directions. Therefore costly underground reactive barriers which are increasingly used to stop migration of contaminants under the ground may become ineffective if underground flow patterns have not been correctly recognized. An additional problem which potentially has much more immediate impact on the civil environmental projects is the management of landfill sites. Landfill sites are usually underground domains of high permeability which can be easily saturated and by nature are unstable from physico-chemical and hydrodynamical points of view. An incorrect analysis of impact of rainfall on these sites can have devastating results on environmental safety.

The model can be used to consider a wider variety of boundary conditions than used in this work. Selection of different boundary conditions can be based on the actual situations in any given problem. The scheme is proven to be very flexible and there is no doubt that it can handle a large number of exterior boundary conditions. There is no theoretical difficulty that may prevent the extension of the methodology described in this paper to heterogeneous lands. Experimental work was beyond the scope of the present project, nevertheless, a simple methodology which can be used to obtain an ultimate verification for this model is proposed. A rig consisting of a reasonably large glass box can be filled with soil and water added to a level that it becomes saturated. A source of tracer, such as a dye, can be left at a point inside the soil surface. Experiments based on running water over the surface of such a rig and dismantling and observing the dispersion of the tracer within the soil matrix should readily establish the underground flow pattern caused by the surface flow.

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