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Statistical physics of fracture and earthquakes

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Manifestations of emergent properties in stressed disordered materials are often the result of an interplay of strong perturbations in the stress field around defects. The collective response of a long-ranged correlated multi-component system is an ideal playing field for statistical physics. Hence, many aspects of such collective responses in widely spread length and energy scales can be addressed by tools of statistical physics. In this theme issue some of these aspects are treated from various angles of experiments, simulations and analytical methods, and connected together by their common base of complex-system dynamics.

1. Introduction

Fracture, earthquakes and breakdown are some of the oldest yet unsolved problems in the physical sciences. The combination of some statistical regularities with what are often very costly and deadly events have encouraged many fertile lines of inquiry, which combine elegant and far-reaching ideas with practical consequences. Classical Greek religion associated earthquakes with the action of the sea-god Poseidon, while Aristotle described them as resulting from subterranean winds [1]. He also attempted to map earthquake risk, which he claims is higher in areas with plentiful hot-springs, but lower for islands far from the coast [1]. The need for accurate earthquake risk management strategy

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remains today, and will be the topic of several papers in this theme issue on fracture and earthquakes.

The prominent Renaissance figure, Leonardo da Vinci further observed that the tensile strengths of nominally identical metal wires tended to decrease with the length of the wire [2]. Similar observations were made by Galileo Galilei, also more than 400 years ago [3]. It was understood much later that these results are, in fact, manifestations of the extreme statistics of failure [4,5]. Due to microscopic fluctuations in material properties, the most extreme defect in a larger sample volume will tend to be more serious than any defect in a smaller sample volume, on average. The weaker point's ability to nucleate failure earlier implies a lower strength of the larger sample. The statistics of these extreme fluctuations are quite different in nature from those of average behaviour, or even critical fluctuations. Again, the statistical treatment of such rare events, and how much they can be anticipated (if not exactly predicted) will be a theme that we will return to repeatedly throughout this issue.

More formal investigations on fracture propagation began with the energy balance criterion proposed by Alan Griffith [6]. His seminal paper (which remains the second most cited paper in the long history of this journal) considered the energy balance required to activate an existing flaw or crack, and forms one footing of modern linear elastic fracture mechanics. The other is George Irwin's analysis of the stress intensification surrounding a single isolated crack [7]. Although Griffith and Irwin described abstract models of fracture, these were the result of important applied problems: Griffith worked at the Royal Aircraft Establishment during WWI, studying fatigue fracture in airplane wings, while Irwin was employed by the US Naval Research Laboratory during WWII, where he was charged with explaining the catastrophic fracture of the Liberty ships [8].

The deterministic fracture propagation, or stability analysis, of a perfect sample with an isolated defect, however, cannot explain the drastic effect that multiple defects have in a sample, which interact *via* the distorted stress field around them [9]. Such interactions can develop long-ranged correlations that lead to emergent properties in the response statistics of the samples, such as the so-called “crackling noise" of a scale-free avalanche size distribution, the formation of complex crack patterns and the roughness of real fractured surfaces [10–13].

In more recent years these observations have led to investigations along the path of statistical physics, which attempt to deal with emergent phenomena in complex systems through a minimal set of parameters. For example, the dimensionality of the space, the range of interactions of a system of elements on that space, and so on, have been used to explain the universal behaviour of both earthquakes, and large classes of related problems [14,15]. Considerable progress has since been made in understanding the nature of the fluctuations and the consequent emerging features of materials undergoing failure [16].

Scientists today are successfully investigating the effects of structural disorder and inhomogeneity on materials failure occurring from the microscopic level (fracture) to the tectonic scales (earthquakes). Advances have been made, approaching from a number of different directions: experimentalists, playing with simple configurations of cracking clay, or tearing sheets, have shown how the presence of disorder, or a symmetry-breaking field, can dramatically affect fracture phenomena, while theorists have studied similar processes in minimalistic statistical models, which have the power to bridge across multiple fields and applications [17–20]. Furthermore, earthquake researchers have discovered time-varying scaling statistics that have allowed insights into the more helpful characterisations of earthquake statistics and even to highlight dangerous areas having higher risks of major events [21–25].

This theme issue brings together these different developments concerning the extreme statistics of the failure of driven disordered materials. Intertwined with it is also the fascinating aspect of universality apparent in these statistics, which offers new applications of the concepts of phase transitions and criticality (in both the static and dynamic aspects of failures). Our aim has been to address these issues from a balanced perspective of experimental, numerical and analytical developments, reported by experts both in terms of original research papers and review articles. The contributions are divided into three groups. The first set of papers deal with various analytical, numerical and experimental approaches for considering fracture in disordered solids, the second set of papers deal with properties that are observed and analysed in laboratory scale but are transferable to tectonic scale of earthquakes, and finally in the third set are the papers dealing directly with earthquake models and forecasting techniques.

2. Summary

In the first set of eight papers the statistical properties of fracture in disordered solids are analysed from various approaches of experiments, numerics and analytical methods.

We start with a review [26], where the role of disorder in determining the failure mode of solids under stress is discussed. The mode of failure is always governed by nucleation when the system size is sufficiently large and the level of disorder is sufficiently low. However, for the very high limit of disorder, and also in the limit of a large range for the load redistribution following a local breakdown, the failure mode is governed by precursory avalanches or percolation-like failure. The scaling laws are discussed based on extreme statistics theory, as well as the fibre bundle and spring network models.

Continuing with quantifying the effects of strong disorder in fracture dynamics, in the next contribution a discrete element model simulation was made for the compressive failure (Brazilian test) of disordered solids [27]. The effect of disorder on the localisation of shear bands was studied. Particularly, the disorder was varied by changing the distribution from which the particle radii were chosen. With the increase in disorder strength, the width of the shear bands was shown to increase. The distribution of the fragment size was shown to be a power law, with very large as well as very small sized grains being present. In general, an increase in disorder has well-defined effects on the dynamics and fragmentation sizes of sample (*e.g.* suppression of precursory signals in quasi-brittle failures, percolation like structure of damaged region, and so on). This study is a step towards quantifying such scalings.

The intermittency of crack propagation is a result of the distortion of the stress field within a solid due to presence of heterogeneities. As such, we include an experimental study on how a single crack propagates through a brittle solid with tuneable disorder [28]. Here, the aim is to understand the effect of heterogeneities in the simplest possible laboratory scenario. The temporal organisations of the avalanches were compared with phenomenological laws of seismicity, and also to analytical predictions and simulations; a difference between global and local avalanche size distribution exponents was found. Further, a velocity dependence of the avalanche size distribution exponent was found, which may be useful for prediction of imminent failure and dangerous hot-spots in a threshold activated system. Finally, the temporal shapes of avalanches were discussed.

Next, the equation of motion of the velocity field of fracture front through a disordered solid is considered in analogy with the velocity field for turbulence in an incompressible fluid [29]. A generic equation of motion containing all terms permitted under the symmetry constraints of the system was written and the scaling exponents of the roughness profiles were calculated. Different limiting cases were found to map to known forms of driven surface dynamics, such as the Edwards-Wilkinson or the Kardar-Parisi-Zhang equations. The generalised form is of interest concerning the fracture dynamics in a wide variety of systems, where the constants of the equations can be taken from the specific properties of the sample under question.

As mentioned earlier, extreme statistics are intimately connected with fracture dynamics. In Ref. [30] the authors add to this connection with an experimental study on crack propagation along a weak plane between two PMMA blocks. The global as well as the local avalanches are noted, and the study explores the scaling of the extreme statistics in avalanches. They show how the exponents of various distributions are different in values from the mean-field predictions, but still maintain certain scaling relations between each other. An inverse square elastic line was also simulated and found to have similar statistics as the experiments, at least for a large enough length scale. For shorter scales, lower than the characteristic scale of the disorder present, the elastic line model is no longer valid, suggesting the need for a different approach, such as crack coalescence models of fibre bundles.

Ref. [31] deals with a similar crack propagation experiment to the preceding paper, but includes and highlights the effects of thermal noise. The thermal noise enables the propagation of a crack front, even when the driving force is below the critical limit given by Griffith's criteria, and follows Arrhenius dynamics. Their model, with its numerical simulations and analytical calculations of the morphology and velocity distribution of the crack front, is in agreement with the reported experiments.

Given the long range correlation developed during fracture, in quasi-static condition patterns can emerge that are characteristics of the constituents and even the history of movements of the material under study. Specifically, desiccation crack patterns emerge in various dried colloidal systems, from mud to paintings. The appearance of desiccation cracks while drying a paste or colloidal suspension can show a surprising variety of interesting behaviour. The next contribution considers how pastes can remember shear motions, which are then visualised when drying occurs [32]. An essential condition for the memory effect is the plastic nature of the paste, which is determined by the relative quantities of powder and water. In a water dominant or fluid state the memory is not stored and the same is valid when the power fraction is large and the system behaves as a semi-solid. For the state in between, however, the system has a finite yield stress and behaves as a plastic fluid. The effect of memory is seen in these cases. In the present work the authors present a systemic summary of the phase diagram of the memory effect, depending on the fraction of fluid and the amplitude of the vibration. They also discuss the rewriting of memory, where multiple vibration effectively erases the memory effect. A model of the colloidal suspension with linear compression and residual tension is used to explain these effects.

The desiccation crack patterns can also be influenced by external fields. In the review article of Ref. [33], the authors provide an overview of desiccation cracks in drying pastes and colloids, particularly in the symmetry-breaking presence of a strong DC or AC electric field.

The second set of articles also deal with the response statistics of disordered systems at a laboratory scale, but the physical principles are applicable to the much larger scale of earthquakes. Often, due to the scale-free nature of the stress field, or the critical correlation developed in a driven disordered system with an intermittent response, the dynamics do not depend strongly on the details of a system. This is a signature of self-organised criticality. In such cases, the dynamics observed in a system at a small scale will, in principle, retain its qualitative features when scaled up to, say, the tectonic scale. The next three papers explore such issues, and rescaling.

In Ref. [34], the remote triggering of seismic activities are addressed from the numerical model of a confined granular medium, representing the gouge layer found in faults, or separating plate boundaries. The so-called fluidisation at the resonance frequency of the gouge causes remote triggering of earthquakes. Other applications in terms of confined granular media are also discussed.

Segregation of the grain sizes in a granular gouge is a well-known phenomena. The mechanism for such a process is debated. One explanation comes from the just-mentioned fluidisation mechanism, and the subsequent segregation of the grains due to Brazil nut effect [34]. Such a effect requires a significant porosity of the system, and gravity. In Ref. [35], however, the authors show that neither fluidisation nor gravity is essential for the segregation effect. In their simulation of a dense bi-disperse system, the authors show that segregation occurs both with and without gravity, and that the relative positions of larger and smaller grains are interchanged in presence and absence of gravity. Indeed, by changing various aspects of the simulations, the authors conclude that it is the non-linear velocity profile of the system, which is responsible for the segregation effect.

In the third article of this section, the crack patterns of strike-slip faults are analysed in analogy with experiments on Plexiglas plates [36]. The plausibility of the scenario that piece-wise linear crack patterns are formed due to the bottom-up propagation of cracks in the earth's crust under mode-III loading is tested. Arguing that the basic principles of a crack induced by mode-III shear loading are valid over a wide range of length scales, experiments were conducted with Plexiglas plates under 4-point bending with cyclic and abrupt loading. The mode-III fracture advances could be observed for *in-situ* conditions due to transparency of the material. The observation of crack-front segmentations in the experiments were also verified by phase-field simulation models.

Finally, the last three papers of this theme issue deal with earthquake models and forecasting mechanisms. The forecasting mechanisms are essentially applicable to systems having the generic characteristics of intermittent dynamics.

In Ref. [37], the two dimensional Burridge-Knopoff model is simulated with a realistic rate and state friction law, in an attempt to reproduce the properties of large earthquakes. Particularly, the anisotropic characteristics of large avalanches are described. In spite of being a homogeneous model, the asperity like behaviour is dynamically generated in the system. The spatially anisotropic structure of the avalanches are unique for the two dimensional version of the Burridge-Knopoff model and only seen for the rate and state friction law.

Forecasting is a central question in catastrophic phenomena including in earthquakes. Due to their devastating consequences, there has been a vast effort in finding precursory signals for major earthquakes. A consensus, however, is that a prediction of the exact time and magnitude of a large quake is not possible. Nevertheless, there can be forecasting of a statistical form, *i.e.* the probability of a quake larger than a certain magnitude after a certain time interval. That gives a hazard map, which can be very useful for safety strategies in more earthquake prone areas. In Ref. [39], one such method is discussed, namely 'now-casting' the earthquake risks, which refers to the forecasting of earthquake risk at times very close to the present, based on past earthquake data. The method can deal with a variable rate of seismicity by the use of a natural time scale, where the sequence of the earthquakes is considered, rather than the real time separation of the events. Using the scaling of the productivity laws of earthquake sequences, the now-casting method was applied to three earthquake prone regions, *viz.* the Parkland earthquake of 2004, and induced seismicity in Groningen and Oklahoma.

In a similar forecasting problem to that described above, our final paper deals with a different method for such forecasting, *viz.* the change in the b-value of the Gutenberg-Richter like law with the local stress on the system [38]. The question here is to find the optimal time and length scale over which activity data is necessary to unambiguously determine a weak region in the system. As opposed to ref. [37], a weak region in this model is not dynamically generated but are embedded in the relative strength of the parts of the system, effectively creating an asperity like structure. It was found that given a rate of earthquake events, there exist an optimal scale up to which the spatial variations in the b-value and thereby the spatial variations of the stress profile can be determined within a limited error range. For other choices, the error range in such determinations become higher.

Data Accessibility. There are no additional data presented in the paper.

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