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A production planning problem with lost sales and nonlinear convex production cost function under carbon emission restrictions

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In this paper, a finite-horizon production planning problem with possible lost sales under several carbon emission restrictions is investigated. The studied model is deterministic with known demands which may not necessarily be met as lost sales are also allowed to provide reasonable flexibility with carbon emission restrictions. The problem is modeled as mixed integer nonlinear programming which has been reformulated in conic-quadratic form for convex cases. The problem is numerically investigated with respect to the costs incurred, the amount of carbon emissions and the magnitude of resulting demand losses. These issues are considered under different carbon restriction policies imposed over several block of periods over the planning horizon. Different carbon cap policies are defined and examined to with wide range of parameter sets to observe how different policies affect the amount of emission, cost and lost sales as the main KPI's set in this study. Numerical examples and their corresponding observations and managerial insights are provided accordingly.

keywords: *Carbon emission restrictions, Production planning, Nonlinear convex cost, Lost sales.*

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1. INTRODUCTION

In this study, the finite horizon production planning problem, also known as the lot sizing problem is revisited under carbon emission restrictions, where lost sales are allowed and a nonlinear production function is assumed. Production planning in a classical sense refers to the joint determination of the time and quantity of production so as to minimize the total cost consisting of the fixed costs of setup and the variable costs of holding, production and possibly shortage. It was first introduced by [1] and later, [2] introduced the capacitated version of the problem. The problem has been studied extensively from both model extension and solution approach perspectives since these pioneer works as will be briefly discussed in the literature review.

In the current work, it is aimed to enrich the literature of production planning by incorporating two aspects to the traditional problem, namely the sustainability issues, and the inclusion of a nonlinear production cost function. In particular, this paper (i) explicitly considers several carbon emission restriction policies and investigates their impact on the performance measures, (ii) allow lost sales in our model.

The motivation behind these extensions arises from the increasing attention of the scientists and the society regarding the alarming levels of carbon emissions, which is considered to be a major environmental problem. As a consequence, national and international authorities set certain standards and restrictions to the amount of carbon emitted from the industrial processes. As such the carbon emissions consciousness and the formal measures taken for the reduction of greenhouse gases in general and carbon dioxide emissions specifically bring new paradigms to the statement and solutions of the industry related problems. The operations research community has already begun to reconsider the traditional problems under carbon sensitive settings and this literature has been growing rapidly. However, to the best of our knowledge, the lot sizing problem under carbon emissions considerations has not been studied sufficiently.

Regarding the carbon emission restrictions, it is now well known that there are several measures taken to reduce the carbon emissions, among which strict carbon caps, cap and trade mechanisms, and carbon taxes are the most widely known tools so far, which may be applied separately or in combination. It is reported in several studies that it is still not clearly known which of these tools are the most effective ones in reduction amounts and the resulting losses in general welfare. On the other hand, some studies, especially in the fuel of aircraft transportation shows that the increase in the demand due to both the increase in human population and the technology becoming more available to individuals with lesser incomes, makes it very difficult to keep the carbon emissions at the desired levels. Hence it is expected that more stringent regulations for carbon restrictions are on their way. This issue is incorporated in this work by considering several types of restrictions on carbon emission levels. In particular, carbon restrictions imposed on periods of various lengths over the planning horizon, are examined. Based on the regulations, the restrictions may apply in cumulative, periodic, rolling or seasonal patterns. They are referred to as *cap patterns* which are explicitly addressed in this article. Further, cap patterns are allowed to change over the planning horizon. That is even if the total allowed emission quantity over the planning horizon is fixed, the imposed caps may follow a certain trend. For instance, the industrial emission caps may be gradually decreased towards the winter months where emissions for heating increase. This issue is referred to as *cap trend*, where three alternatives are investigated as increasing, constant and decreasing trends.

As mentioned above, another aspect of our model is the assumption of lost sales. In a system of product supply, an unmet demand may either be backlogged or lost completely with possible penalties. Lost demands occur either due to the stochasticity of demand process which may result in a demanded quantity that exceeds the available supply, or due to the limited resources or capacity of the manufacturer even if precise demand estimates may be available. Capacity constraints would be encountered in the forms of physical capacities, budget limits or legislative restrictions such as the carbon emission restrictions that are addressed in the current study. If the production quantity fails to satisfy the demand due to carbon emission restrictions, the manufacturer may lose a part of potential demand despite the ample capacity and the resources in order to adhere the environmental regulations or to keep the production cost at a profitable level. Lot sizing problems with lost sales have been addressed in the literature before, however with linear costs. Here it is observed that some structural properties that hold for the models with linear costs do not hold for the current model that involves nonlinear convex cost. A counter example and also a formal proof for the traditional case are provided for the case where carbon restrictions do not apply. The nonlinearity of the mathematical model prevents efficient use of the available general nonlinear solvers and therefore a second-order conic formulation for the problem is used to conduct the numerical study.

Regarding the practical aspects, it is observed in the numerical study that different carbon cap/tax policies may drastically affect the selection of appropriate carbon cap policy depending on the problem parameters and the preference of the policy-maker.

In summary, this study contributes to the literature of production planning and sustainability by *(i)* considering several integrated carbon emission/tax restriction structures over the planning horizon, *(ii)* providing a detailed numerical study that investigates the impact of system parameters and carbon restriction structures on performance measures as the total cost, the amount of carbon emissions, and the loss of welfare with respect to these restrictions.

The remainder of this article is organized as follows: In the next section, some related studies are reviewed highlighting our contributions. In Section 3, the problem assumptions are given with some analytical results for the special case of unrestricted carbon emissions. The solution approach is discussed in Section A while Section 4 provides the results of an extensive numerical study that illustrates the impacts of the system and carbon restriction parameters on the performance measures. The paper concludes in Section 5.

2. RELATED LITERATURE

As mentioned before, the introduction of lot sizing problems to the literature goes back to several decades and since then an extensive literature has been built. Therefore only a few review papers for interested readers are mentioned. Single level lot sizing problems are reviewed by [3] with both exact and heuristic solutions. [4] review single item lot sizing problems with focus on uncapacitated ones. They identify polynomially solvable cases and discuss four different mathematical programming formulations. Various solution methods are also discussed. [5] inspect capacitated lot sizing and scheduling models and classify them as big and small bucket models. Big bucket models refer to long but small number of periods while models with short periods are called small buckets. They focus on the big bucket type capacitated lot sizing problems and their extensions and discuss the solution algorithms. Among other recent reviews, [6, 7] focus on modeling and solution approaches while [8–10] provide classifications for inventory lot sizing models. Only a few of recent papers address the production planning model with nonlinear convex function (see [11–13]) and the problem considered in this article, has such a configuration. The operations research literature with carbon emission considerations have accumulated significantly in recent years, where the work of [14] is among the earliest ones. Regarding the lot sizing problems [15] consider an uncapacitated model with multi-mode (different production modes with different unit cost and emission) under periodic, cumulative and rolling carbon emission constraints. Linear cost functions and carbon emissions are assumed. They establish that the periodic constrained problem can be solved via existing DP algorithms while the cumulative constrained problem is an NP-hard problem. In another work, [16] consider a lot sizing problem with cumulative carbon emission capacity. They demonstrate that the problem is NP-hard and provide a polynomial approximation algorithm for a concave production cost case. In their model, carbon emission is a linear function of setup, inventory, and production quantity at each period. They assume a general concave production cost and provide a fully polynomial-time approximate algorithm and a Lagrangian heuristic which rests on Lagrangian relaxation of the carbon emission constraint and using Wagner-Whitin algorithm for the relaxed problem. [17] develop a deterministic optimization model that incorporates carbon emissions in a multi-echelon production-inventory model with lead time constraints. They consider both carbon tax and cap in separate cases and compare base stock and fixed order inventory policies. [18] present three optimization models to determine the re-manufacturing quantity optimization to maximize the total profit under three common carbon emission regulation policies: emission cap, carbon tax, and cap-and-trade. [19] considers a production-inventory problem with abatement possibility where production costs are non-decreasing and convex functions of the production level, and the costs of investment. He compares the optimal strategy before and after emission permit banking and pollution abatement. [20] studies a stochastic lot sizing model under cap-and-trade regulation and investigate the system parameter effect in a numerical study. [21] examines a EOQ-based production planing under carbon tax and cap-and-trade regulations. [22] study an inventory problem under non-stationary stochastic demand conditions with emission and cycle service. They consider carbon cap-and-trade regulatory mechanism and investigate the effects of emission parameters on the supply chain performance. [23] study the integration of the carbon emission constraint into the single item uncapacitated lot sizing problem (ULSP) under the cap-and-trade policy. They show that the problem is polynomially solvable when the budget is unlimited while it is NP-hard when the budget is limited. [17] study a multi-echelon inventory model and compare the carbon emission tax and cap to provide some insights to the policy-makers and [24] study an inventory model to analyze impacts of carbon footprint and low-carbon preference on the production decision in the cap-and-trade system. [18] propose a max—min approach for an inventory model with limited information on demand distribution and investigate the analytical effect of emission policies: carbon emissions capacity, carbon tax, and cap-and-trade.

[25] propose a mixed integer programming model to minimize cost and meet the customer's demand under different manufacturing constraints and under carbon emission constraint. They develop two hybrid heuristic approaches for their solution methods.

Here, a brief review of some of the existing papers in production planning which address lost sales in their assumptions is provided. [26] generalize the single item Wagner-Whitin lot-sizing problem with lost sales. They prove several optimality conditions and develop a forward polynomial-time recursive algorithm. [27] study a lot sizing model with bounded inventory in which the unmet demand is lost. They characterize optimality conditions and propose a polynomial time solution algorithm. [28] consider an inventory model with a continuous review policy for perishable products with lost sales. They use a Markov chain analysis and compare (Q, r) policy with the time-based benchmark.
[29] use a linear regression to approximate fill-rate to determine the safety stock level in an inventory model with lost sales. [30] deal with the multi-item capacitated lot-sizing problem with setup times and lost sales. They propose an adaptive neighborhood search meta-heuristic algorithm as a solution approach. [20] studies an inventory model to determine the safety stock by using a regression model to approximate the lost sales instead of back-ordering. The reader may refer to the review paper of [31] which provides a comprehensive classification of the production planning models with lost sales based on the inventory and replenishment policies; and to [32–34] for more recent studies in this research area.

For better positioning, our work in the related literature a classification of production planning problems considering sustainability is provided in Table 1. Inventory models and finite horizon dynamic production planning (lot sizing) problems are distinguished as stated in the third column denoted as *INV* and *LOT* respectively. The fourth column indicates the carbon emission policies assumed in the studies that have appeared in the literature. The three main carbon regulations: cap-and-trade, carbon tax, and restricted cap are respectively denoted by Trade, Tax and Cap. The cap patterns are classified as Cumulative C, Periodic P, Rolling R or Seasonal S . The column entitled production cost shows the structure of the cost function while the next column indicates whether lost sales (LS) are allowed or demand is fully satisfied (FS).

3. PROBLEM STATEMENT

In this section, the system parameters are introduced and the optimization problem is formally stated. The solution approach will be discussed in the following section after an elaboration of a special case.

Table 1. classification of the similar studies in the literature. LS: lot sizing, INV.: inventory, Trade: Cap-and-Trade, Cap (P,C,R,S): emission constraint (Periodic, Cumulative, Rolling, Seasonal)

Article	Inventory vs. Lot-sizing	Cabon Regulation	Production cost	Lost Sales
[15]	LOT	Cap: P, C,R	Linear	FS
[16]	LOT	Cap:C	Concave	FS
[18]	INV	Cap,Tax,Trade	Linear	LS
[19]	INV	Trade, Tax	Linear	FS
[20]	INV	Cap	Linear	FS
[21]	INV	Tax, Trade	Linea	FS
[22]	INV	Trade	Linear	FS
[23]	LOT	Trade	Linear	SM
[24]	INV	Trade	Linear	FS
[35]	IN	Trade	Linear	FS
Our Work	LOT	Cap & Tax: P, C, R,S	Convex	LS

The firm manufactures a single product with known demands over the planning horizon. There are T periods of production, $t = 1, ..., T$; and n different resources, $i = 1, ..., n$. The demand in period t is denoted as d_t. The fixed unit setup cost for any positive amount of production in period t is K_t. Carrying a unit product from period t to the next period costs h_t.The production cost follows lacks the economies-of-scales and follows a nonlinear convex function as $c_t X_t^r$ where X_t is the production quantity at period t. Each of the setup, resource usage and storage operations has a linear contribution in carbon emission possibly with different coefficients. In our model, emission induced from production operation is a linear function of consumed resources. It is assumed that a setup for production at period t results in ζ_t amounts of carbon emission. Storage of each unit of product at period t has γ_t amount of carbon emission and producing a unit of product in period t results in β_t amount of carbon emission. As mentioned before, a novel feature of our study is the incorporation of carbon emission restrictions that are imposed by regulatory bodies. The total carbon allowance over the planning horizon permitted by the regulations is κ. The aforementioned parameters and variables are summarized in Table 2.

In order to capture various carbon restriction policies, further bounds on the usage of this total amount are assumed. The particular constraint structures assumed in our study are as follows: *(i)* There may be single cumulative constraint κ over the entire planning horizon; *(ii)* the constraints can be imposed in a rolling manner over periods of fixed lengths; *(iii)* can be imposed over a certain number of consecutive periods which is referred to as seasonal; *(iv)* finally, constraints can be imposed on each period. These alternative patterns can be represented by the index ℓ and the set J, where ℓ represents the number of consecutive periods over which a certain cap is imposed, which will be referred to as blocks and the set J consists of the starting periods for the consecutive blocks. Although it is possible to have each block having different lengths, in the current study each block has the same length. For instance, for the cumulative cap, $\ell = T$ and for rolling caps the blocks could take an arbitrary length l. The choices studied in our study are presented in

Table 2. Summary of the notations used in the mathematical model

Symbol	Definition
Parameters	
T	Total number of periods
d_t	Demand in period t
k_t	Fixed setup cost in period t
h_t	Holding cost per unit of product in period t
p_t	Penalty cost for unit lost sales.
ζ_t	Emission amount due to a setup in period t
γ_t	Emission amount due to unit product holding in period t
β_t	Emission amount per unit product i in period t
r_1	convexity of variable production cost function
r_2	convexity of variable production emission function
κ	Overall emission permit
κ_t	Emission permit in period t
Variables	
X_t	Production amount in period t
I_t	Inventory level at the end of period t
L_t	Lost sales amount i period t
Y_t	0-1 variable which takes 1 if production occurs in period t

Table 3. Constraint (5) represents carbon cap policies which is characterized by parameter ℓ and indices set J as shown.

Table 3. Cap policy characterization

Policy	ℓ	J
Cumulative	T	$\{1\}$
Rolling	l	$\{1, 2, \ldots, T-l+1\}$
Seasonal (Cluster)	l	$\{1, l+1, 2l+1, \ldots, kl+1\}$ where $k = \lfloor \frac{T}{l} \rfloor$
Periodic	1	$\{1, \ldots, T\}$

The primary objective is to find the optimal production plan that minimizes the total cost over the planning horizon under carbon emission restrictions. The mathematical formulation of our problem (PLC) is stated as follows.

$$\text{(PLC)} \quad \min \sum_{t=1}^{T} \left(k_t Y_t + h_t I_t + p_t L_t + c_t X_t^{r_1} \right) \quad \textbf{(1)}$$

s.t.

$$I_0 = 0, \quad \textbf{(2)}$$

$$I_t = I_{t-1} + X_t - d_t + L_t, \quad t = 1, ..., T \quad \textbf{(3)}$$

$$L_t \le d_t, \quad t = 1, ..., T \quad \textbf{(4)}$$

$$\sum_{t=j}^{j+l-1} \left(\zeta_t Y_t + \gamma_t I_t + \beta_t X_t^{r_2} \right) \le \kappa_j, \quad \forall j \in J \quad \textbf{(5)}$$

$$X_t \le M Y_t, \quad t = 1, ..., T \quad \textbf{(6)}$$

$$X_t, I_t, L_t \ge 0, Y_t \in \{0, 1\}, \quad t = 1, ..., T \quad \textbf{(7)}$$

In the formulation above L_t denotes the amount of demand loss in period t. Each unit of loss costs p_t, and M denotes a sufficiently large number (i.e., $M = \sum_{i=1}^{T} d_t$).

Although our main objective is the cost minimization, the major side-effects such as the amount of unsatisfied demands and the amount of carbon emissions under optimal plan are important to be investigated. Hence, in our numerical study, three response variables including total cost, total emission and unmet lost demand are introduced to study and gain some insights for policy-makers by comparing the effect of several carbon cap restrictions on the optimal response variables.

The main optimization problem is stated as (PLC) above. However before providing our solution approach the special case where there are no carbon constraints is elaborated. First, some counter examples showing that the optimality resulted established in the literature for linear costs does not necessarily hold for our case with a convex costs function. Several structural results for the optimal lost sales amount are also provided.

For the linear counterpart of the problem, [26] shows that, in an optimal solution, the following results hold:
(i) $I_{t-1}^* \times X_t^* = 0, \forall t.$
(ii) $X_t^* \times L_t^* = 0, \forall t.$
(iii) $L_t^* \times (d_t - L_t^*) = 0, \forall t.$

As a notable finding, it is shown by a counter example that these results do not necessarily hold when convex costs are involved.

Example 1 *Consider a 2-period problem of (PLC) form with the parameters given in Table 4. The optimal production and loss amounts are* $(X_1^*, X_2^*, L_1^*, L_2^*) = (40, 50, 100, 10)$ *with the total cost of 345 units. In both periods,* $X_t \times L_t \neq 0$*; In period 2,* $L_t(d_t - L_t) \neq 0$ *and* $I_{t-1} \times X_t \neq 0$ *and therefore, none of the optimality properties of linear cost problems holds here.*

Table 4. Parameter of the counter example

(d_1, d_2)	r_1, r_2	(k_1, k_2)	(c_1, c_2)	(p_1, p_2)	κ
$(100, 100)$	2	(0,0)	$(0.05, 0.05)$	$(0.5, 5)$	∞

Note that in an optimal solution of the linear case, the Wagner-Whitin property holds, i.e., the demand should be either completely met or completely lost. Moreover, it must be met at periods with positive production. In contrast, in the case of convex production cost, demand may be lost partially at each period of optimality. Moreover, it may be optimal to produce and stock the product in a period without using it in that period in order to benefit from incurring less cost of production or lost sales in future periods.
To formalize the above finding, a common definition is recited here which is followed by some structural results.

Definition 1 *A consecutive number of periods,* $\langle u, v \rangle$*, is called a regeneration for a production plan in which* $I_{u-1} = I_v = 0$.

Lemma 1 *Inventory decomposition property: Any optimal production comprises regenerations which can be solved independently.*

Proposition 1 *Suppose that* $p_i + \sum_{s=i}^{j-1} h_s > p_j$ *for all* i, j, $1 \le i < j \le T$. *Then, in an optimal solution of (PL), the following holds within each regeneration* $u \le t < s \le v$:

(a) Partial lost sales can be occurred at most in one period.

(b) The period with positive lost sales, if exists, takes place in the last period of the regeneration, v.

(c) If $u = v$, *then* $L_v \in \{0, d_v, d_v - (\frac{p_v}{rc_v})^{\frac{1}{r-1}}\}$. *Otherwise, if* $u < v$ *then* $L_v \in \{0, d_v - (\frac{p_v}{rc_v})^{\frac{1}{r-1}}\}$.

The assumption of this proposition is called the non-speculative motives property in the lot-sizing literature.

Proof (a) Suppose to the contrary that, in an optimal solution, $\exists i, j$, $u \le i < j \le v$ such that $0 < L_i < d_i$ and $0 < L_j < d_j$. Consider the new solution: $L'_i = L_t - \epsilon$, $L'_{t+1} = L_{t+1} + \epsilon$, $I'_t = I_t - \epsilon$. Then the cost change in the new solution is $(p_{t+1} - p_t - h_t)\epsilon$ which is negative due to the assumption and it implies that the solution is improved and it contradicts with the optimality assumption. Hence, if we have lost sales in more than a single period, then all except the first one should be complete loss.

(b) Suppose to the contrary that in an optimal solution $\exists i < v$ such that $0 < L_i \le d_i$. Following the same procedure as in the previous part, a new solution in which $L'_v = \epsilon$ and $L'_i = L - \epsilon$ has $(p_i + \sum_{s=i}^{v} h_s - p_v)\epsilon$ amount less cost. This implies that the solution in which $L_v = 0$ is not optimal. Using part (a) implies that only one of them, L_v, can be positive.

(c) Let S be the set of production periods within this regeneration. Replacing the variable I_t in the objective function with its equivalent as $I_t = \sum_{i=u}^{t} X_i - d_i + L_i$ and then, by using the Lagrangian relaxation of the constraints we come up with the following Lagrangian function:

$$\mathcal{L} := \sum_{t \in S} \left(K_t + w_t X_t^r\right) + \sum_{t=u}^{v} \Big[p_t L_t + (h_t - \lambda_t)\Big(\sum_{i=u}^{t} X_i - d_i + L_i\Big) + \mu_t (L_t - d_t) - \gamma_t L_t - \theta_t X_t\Big]. \quad (8)$$

Consequently, from the first order optimality rules, we must have

$$\frac{\partial \mathcal{L}}{\partial L_i} = \sum_{s=i}^{v} (h_s - \lambda_s) + \mu_i + p_i - \gamma_i = 0, \quad u \le i \le v, \quad (9)$$

and

$$\frac{\partial \mathcal{L}}{\partial X_i} = \sum_{s=i}^{v} (h_s - \lambda_s) + r w_i X_i^{r-1} = 0, \quad u \le i \le v. \quad (10)$$

Combining (9) by (10) results in

$$r c_i X_i^{r-1} = \mu_i + p_i - \gamma_i \qquad u \le i \le v. \quad (11)$$

In addition, since $I_i > 0$ for $u \le i < v$ the complementary slackness condition (C-S) necessitates $\lambda_i = 0$, $u \le i < v$. From parts (a) and (b) we know that $L_i = 0$ for $i < v$, and for period v, due to (C-S) property L_v is either in boundary, $L_v = 0, d_v$, or it has a partial solution, i.e., $0 < L_v < d_v$ with zero values for the Lagrangian multipliers: $\mu_t = \gamma_v = 0$. For the latter case, when $i = v$, (11) reduces to

$$X_v^* = (p_v / r c_v)^{1/(r-1)}. \quad (12)$$

If $u < v$ in the last period of the regeneration we have $I_{v-1} + X_t + L_t - d_t = I_v = 0$ which implies that $L_v = (d_v - X_v - I_{v-1}) < d_v$. This indicates that we cannot have a complete demand loss in the last period of the regeneration unless $I_{v-1} = 0$ or equivalently $u = v$. □

A. Solution Method

The mathematical model given in (PLC) is a nonlinear mixed integer optimization problem and if it is fed directly into mixed integer nonlinear (MINLP) solvers the obtained outputs might usually be sub optimal with less reliability and consistency when the problem parameters change (see [36]). Here, a second order conic quadratic reformulation with minimum auxiliary cone constraints, described in [37], is applied to the problem at hand in (PLC), and therefore, it can be solved via MIQP optimization packages with faster and more reliable outputs.

4. NUMERICAL STUDY: COMPARISON OF CAP RESTRICTION POLICIES

The results of the numerical study, aiming to investigate how the total cost, total emission and total lost sales change in an optimal production plan under different carbon cap patterns, are presented in this section. Different settings are considered for the system parameters such as the demands over the planning horizon, set-up costs, and input elasticity parameters. All the optimizations models within our numerical experiments are coded in the conic quadratic form in C++ and solved via CPLEX solver using Concert Technology. The CPU times were quite fast due to the size of the models with an average of 17 seconds so they are ignored and the managerial insights from the results are focused instead.

Six demand realization vectors were drawn with elements, d_t, $t = 1, ..., T$ from a uniform distribution. All costs are stationary. Setup costs are based on the economic order interval as $k_t = k = \frac{\mathcal{T}^2}{2}\bar{d}$ where $\mathcal{T}$ is used as an order interval and $\bar{d}$ as the average of demand realizations.

The output elasticities of the resources are chosen in a way to provide convexity level $r = 1.5$. The resources with higher elasticity are assumed to be more expensive and the expensive resources are in turn assumed to have lower emission rates. In order to set the carbon cap restriction parameters, the model first solved with no emission constraints which does not allow for lost sales, i.e., $L_t = 0, \forall t$ and $\kappa_j = \infty, \forall j$ in (PLC); then the total cost TC_0 is computed and the unit penalty for demand loss has been set as $p = \eta TC_0 / D$ where η is a constant coefficient, TC_0 and D denote the total cost and total demand of the corresponding case, respectively. Using the obtained penalty for the lost demand, the uncapacitated model is re-solved with permission to demand loss and the obtained emission amount, TE_0 is used as a base for setting the total carbon emission cap with a tightness factor δ as $TE_0(1 - \delta)$ for the corresponding instance. The Cap trend is set based on the ratio of the cap in the first block to the last block, s, keeping the total cap constant. The levels and values of the parameters used in our study are summarized in Table 5.

Both rolling and seasonal carbon emission patterns were considered. For each, 6 different lengths for carbon emission blocks

Table 5. List of parameters and their values

Name	Description	Values
Horizon length		$T = 24$
Demand	number of realizations=6	$d \sim U(0, 100)$
Setup Cost	$k_t = \mathcal{T}^2/2\bar{d}$ $\mathcal{T} \in \{2, 6\}$	$\in \{100, 900\}$
Unit holding cost		$h = 1$
marginal cost ratio	$a := \frac{h_t}{rc_t\bar{d}^{r-1}}$ ratio of holding to production cost	$\{0.2, 0.05\}$
Convexity factors	r_1, r_2	$r_1, r_2 = 1.5$
Resource cost	$c_t \sim U(0, 1)$	
Product emission	$\beta \sim U(0, 1)$	
Block length	$\ell \in \{1, 2, T/8, T/6, T/3, T\}$	$\{1, 2, 3, 4, 8, 24\}$
Cap	$TE_0(1-\delta)$ set for each instance separately	
Cap tightness		$\delta \in \{5\%, 15\%, 25\%\}$
Policy/pattern:	rolling, block	
	decreasing, constant, increasing	$s := \frac{\text{cap in the first block}}{\text{cap in the last block}}$
	rolling	$s \in \{1\}$
	block: $\ell \neq T$	$s \in \{\frac{1}{8}, 1, 8\}$
Lost sales penalty:	$p = \eta(TC_0)/D)$	$\eta \in \{1.5, 3\}$

were set. For the rolling emission cap, the policy cap trend is constant, for the seasonal emission policy, except for $L = T$, 3 forms of emission cap trends were considered: *(i)* decreasing *(ii)* constant, and *(iii)* increasing. The slope of the emission cap is denoted by s in Table 5 and defined as the ratio of the emission cap on the first block to that on the last block. In total, 846= $(2 \times 2 \times 6 \times 3 \times 2 \times 6 \times 1)$=$(k \times c \times Rep \times \delta \times \eta \times l \times s)$ problem instances for the rolling policy, and 2160 $= (2 \times 2 \times 6 \times 3 \times 2 \times 5 \times 3)$=$(k \times c \times Rep \times \delta \times \eta \times l \times s)$ problem instances were tested for the seasonal policy are tested.

Summary of Figures

- Fig. 1 illustrates the effect of the cap policy on production cost components.
- Fig. 2 illustrates the trend of the total, per capita and unused carbon emission allowance over different block length.
- Fig. 3 reflects the trade-off between total cost and emission via Pareto analysis.
- Fig. 4 reflects the trade-off between total lost sales and emission via Pareto analysis.

The ratios of the optimal objective value, the emission amount, and the lost sales amount in the carbon capacitated model to their corresponding values in the uncapacitated model are calculated. Then they are averaged over the instances and summarized in Tables 6–8 with labels TC, TE, and LS respectively. Label l_i, $i \in \{1, \ldots, 6\}$, in the rows indicates the block length (L), and the columns δ_i, $i \in \{1, 2, 3, \}$, nominate the cap tightness (δ) levels. Furthermore, the decreasing, constant and increasing cap patterns among the blocks are denoted by s_1, s_2, s_3, respectively in the corresponding columns in Tables 7-8. Also, p_1 and p_2 indicate the lost sales penalty (p) levels. In Table 6, as expected in each region TC is increasing in δ, p and decreasing in L. TE is increasing in L and p but decreasing in δ. LS is increasing in δ but decreasing in p and L. The same pattern appears in Tables 7–8 for s_2.

The effect of cap slope over the blocks is interesting: The constant uniform cap (s_2) leads to the lowest cost, the highest emission and the lowest lost demand. The decreasing cap (s_1) results in the second rank for all response variables while the increasing cap (s_3) results in the highest cost, the lowest emission and the highest lost sales among the policies. Additionally, some violations of the expected trend in columns s_1 and s_3 are observed. For example, TC increases from l_4 to l_5.

Table 6. Average sensitivity of response variables to cap policies: rolling

		p_1			p_2		
		δ_1	δ_2	δ_3	δ_1	δ_2	δ_3
TC	l_1	1.156	1.200	1.257	1.319	1.450	1.758
	l_2	1.044	1.072	1.110	1.056	1.112	1.190
	l_3	1.027	1.049	1.075	1.038	1.082	1.150
	l_4	1.022	1.038	1.064	1.032	1.066	1.130
	l_5	1.012	1.028	1.051	1.016	1.042	1.098
	l_6	**1.001**	**1.010**	**1.030**	**1.001**	**1.012**	**1.043**
TE	l_1	**0.670**	**0.572**	**0.464**	**0.742**	**0.701**	**0.587**
	l_2	0.819	0.762	0.690	0.830	0.772	0.713
	l_3	0.851	0.789	0.710	0.851	0.790	0.716
	l_4	0.864	0.797	0.717	0.865	0.799	0.717
	l_5	0.876	0.805	0.726	0.877	0.807	0.723
	l_6	0.950	0.850	0.750	0.950	0.850	0.750
LS	l_1	0.190	0.307	0.444	0.093	0.132	0.281
	l_2	0.038	0.068	0.118	0.002	0.014	0.034
	l_3	0.029	0.057	0.102	0.003	0.008	0.030
	l_4	0.029	0.052	0.103	0.001	0.009	0.026
	l_5	0.019	0.039	0.080	0.001	0.003	0.017
	l_6	**0.004**	**0.015**	**0.050**	**0.000**	**0.000**	**0.000**

A. Sensitivity to Cap Policy parameters

Finding out the sensitivity of the optimal plans to the cap restriction policies such as the cap tightness, the length of cap blocks and the pattern of the cap over the blocks consisting of rolling, decreasing, constant patterns, might be of the readers' interest. Thus, the purpose of these analyses is twofold: first, to observe how the cost components proportions alter under different policies, and second, to see how the trend of the per unit emission, the unused emission cap, the total emission, and the lost sales behave under the aforementioned policies.

The graphs on the left side of Figs. 1-2 depict the effect of block length while the ones on the right side depict the effect of cap tightness. They are classified into rolling and seasonal

Table 7. Average sensitivity of response variables to cap policies: seasonal, p_1

		s1			s2			s3		
		δ_1	δ_2	δ_3	δ_1	δ_2	δ_3	δ_1	δ_2	δ_3
TC	l_1	1.193	1.223	1.258	1.156	1.200	1.257	1.224	1.250	1.282
	l_2	1.100	1.120	1.146	1.038	1.064	1.100	1.139	1.159	1.184
	l_3	1.076	1.091	1.112	1.018	1.036	1.061	1.115	1.132	1.155
	l_4	**1.071**	**1.086**	**1.108**	1.014	1.029	1.052	**1.115**	**1.131**	**1.151**
	l_5	1.086	1.098	1.112	1.008	1.021	1.043	1.135	1.146	1.161
	l_6	-	-	-	**1.001**	**1.010**	**1.030**	-	-	-
TE	l_1	**0.598**	**0.543**	**0.476**	**0.667**	**0.573**	**0.464**	**0.534**	**0.491**	**0.426**
	l_2	0.751	0.698	0.644	0.833	0.767	0.699	0.671	0.625	0.574
	l_3	0.797	0.754	0.698	0.871	0.806	0.726	0.704	0.667	0.619
	l_4	0.799	0.750	0.696	0.898	0.823	0.734	0.707	0.671	0.623
	l_5	0.789	0.761	0.709	0.912	0.832	0.739	0.676	0.638	0.601
	l_6	-	-	-	0.950	0.850	0.750	-	-	-
LS	l_1	0.301	0.360	0.436	0.194	0.305	0.444	0.381	0.425	0.493
	l_2	0.149	0.191	0.237	0.034	0.063	0.106	0.232	0.274	0.321
	l_3	**0.118**	**0.147**	0.186	0.021	0.044	0.085	**0.202**	**0.236**	**0.272**
	l_4	**0.118**	0.149	**0.185**	0.020	0.038	0.083	0.212	0.243	0.280
	l_5	0.150	0.166	0.194	0.014	0.031	0.066	0.257	0.276	0.301
	l_6	-	-	-	**0.004**	**0.015**	**0.050**	-	-	-

Table 8. Average sensitivity of response variables to cap policies: seasonal, p_2

		s1			s2			s3		
		δ_1	δ_2	δ_3	δ_1	δ_2	δ_3	δ_1	δ_2	δ_3
TC	l_1	1.567	1.675	1.812	1.319	1.451	1.758	1.725	1.823	1.940
	l_2	1.259	1.324	1.410	1.049	1.095	1.165	1.440	1.508	1.601
	l_3	1.197	1.246	1.315	1.023	1.057	1.112	1.384	**1.437**	**1.514**
	l_4	**1.183**	**1.238**	**1.308**	1.020	1.044	1.100	**1.382**	1.443	1.519
	l_5	1.243	1.282	1.335	1.011	1.031	1.077	1.497	1.528	1.569
	l_6	-	-	-	**1.001**	**1.012**	**1.043**	-	-	-
TE	l_1	**0.661**	**0.613**	**0.548**	**0.742**	**0.700**	**0.587**	**0.582**	**0.536**	**0.480**
	l_2	0.785	0.744	0.681	0.845	0.786	0.722	0.684	0.641	0.597
	l_3	0.823	0.782	0.718	0.871	0.807	0.727	0.703	0.672	0.627
	l_4	0.838	0.794	0.717	0.899	0.823	0.738	0.712	0.678	0.630
	l_5	0.841	0.796	0.732	0.911	0.831	0.741	0.676	0.640	0.605
	l_6	-	-	-	0.950	0.850	0.750	-	-	-
LS	l_1	0.228	0.277	0.345	0.093	0.133	0.281	0.319	0.366	0.421
	l_2	0.087	0.113	0.151	0.002	0.008	0.025	0.190	0.220	0.262
	l_3	0.062	0.082	0.111	0.001	0.007	0.018	0.171	**0.191**	**0.224**
	l_4	**0.056**	**0.076**	**0.109**	0.001	0.004	0.017	**0.167**	0.196	0.231
	l_5	0.080	0.098	0.123	0.000	0.002	0.008	0.236	0.248	0.263
	l_6	-	-	-	**0.000**	**0.000**	**0.000**	-	-	-

with decreasing, constant and increasing cap from top to bottom. The horizontal axis denotes the block lengths in the left side and tightness level in the right side graphs. Fig. 1 illustrates the cost component proportions while Fig. 2 depicts the other response variables (total and per unit emission, lost sales, unused cap).

A.1. Effect of Cap Patterns

To compare the rolling policy with seasonal cap policy, the left (right) side of Table 6 should be set against to the middle part of Table 7 (Table 8). The following general results are observed in these tables.

Observation 1 *The rolling cap block policy is close to but slightly more environment-friendly policy than the seasonal cap policy with a constant cap.*

Observation 2 *Cap policies have similar effects on the cost and emissions for both high and low value-added manufacturer (i.e., for both p).*

Comparing column s_3 with the other columns, in Table 8 also graphs in Fig. 2-(g),(h), the following are noticed.

Observation 3 *For a fixed total emission allowance, seasonal emission cap with an increasing cap trend tends to be more environment friendly .*

From the cost perspective, comparing different rows in Fig. 1 indicates the following.

Observation 4 *The changes in cost components for the increasing and the decreasing cap trends are the same over the block length or tightness levels. Furthermore, the production cost has the highest proportion in all cap patterns except in the periodic one. In the periodic cap pattern, lost sales are the dominant component (see Fig. 1).*

Observation 5 *The rolling and seasonal policies with constant cap include the highest portion of production costs, whereas the seasonal ones either with increasing or decreasing cap, reduces the production and setup costs while increasing the lost sales proportion (see Fig. 1).*

A.2. Effect of Block Length

Here, the effect of the block length on the response variables is summarized.
Looking at the left side of Fig. 1 the following patterns are revealed for the cost components proportions in the optimal objective values.

Observation 6 *As the block length increases, the setup cost proportion first increases, then decreases for seasonal cap both with increasing and decreasing trends, whereas for constant seasonal cap it behaves in the opposite way. For the rolling cap policy, it does not exhibit any special pattern.*

Observation 7 *For the seasonal cap with both the increasing and decreasing trends, the production cost proportion first increases and then decreases with respect to the block length, while it has a monotone increasing trend for the constant seasonal cap. For the rolling cap, there is a nondecreasing trend in general.*

Observation 8 *For the seasonal cap with both the increasing and decreasing trends, the lost sales proportion first decreases then increases with respect to the block length, while it has a monotone decreasing trend for the constant seasonal and rolling caps. These are almost the opposite of the behavior w.r.t. the production costs.*

One may realize that some of the observed patterns are counter intuitive when seasonal cap trends are not constant. For instance, as the block length increases, this allows for a more flexible production plan and hence lower lost sales proportions are expected, however, it has not been observed here.

Also, as we see in the left side of Fig. 2, the trends of the lost sales and the unused emission cap change similarly. This indicates that the lost sales arise from the imposed cap policy. As the block length increases, the per-unit carbon emission increases; however, the total emission increases in a concave form. The unused carbon emission cap and the lost sales are generally decreasing in block length. Their values are higher in the seasonal policy with the increasing cap trend compared to the other cap trends and interestingly, their amounts increase in this policy when the block length increases from 4 to 8, which is also unexpected (see Fig. 2-(g)).

A.3. Effect of Cap Tightness

As we can see in the right side of Fig. 1, as the tightness of carbon cap increases (i.e., less emission allowance), the lost sales proportion increases while the production and setup costs decrease for all cap patterns. In the right side of Fig. 2, we observe that as the tightness level of the cap increases, the per unit and total emission both decrease (but with different rates). Looking at the other two variables leads to the following interesting observation which illustrates how the carbon policies make a double effect on total emission amounts.

Observation 9 *As the carbon emission tightness increases, the unused carbon emission cap increases as well.*

This seemingly counterintuitive result may be explained as follows. As the tightness increases, the value of carbon emission (i.e., shadow price) also increases which renders lost sales more beneficial (less costly). As the amount of demand to be satisfied gets smaller, it becomes less desired to do production for a little demand. This generates savings from not doing setups and production, hence, there remains further unused carbon allowances.

For instance, in Table 6, for δ_1 and l_4, the TE value is 0.864 which is less than $1 - \delta_1 = 0.95$.

5. CONCLUSION AND FUTURE RESEARCH DIRECTION

In this paper, a dynamic production planning problem with a nonlinear production cost function, and with the possibility of lost sales was studied. The problem was investigated under several carbon emission cap policies either gained from the literature or proposed by the authors.

It is demonstrated that convex production cost which lacks economies-of-scales brings about the violation of optimality structures corresponding to the linear counterpart of the problem. For instance, Wagner-Whitin demand integrality property and all-or-nothing lost sales optimality properties of [26] do not hold under the convexity assumption and therefore, an optimal lost sales may be partial. In addition, it enabled us to deploy a conic quadratic reformulation.

Besides, Carbon cap policies was classified by their pattern: (periodic, cumulative, seasonal), their trend (decreasing, constant, increasing), and tightness (emission allowance in total). Then, their effect were examined on the total cost, total emission and also total lost sales. Since these objectives are conflicting, the selection of a proper cap policy hinges on the targeted levels for each criterion. Several insights from managerial viewpoint to make policies for cost minimizer manufacturers were provided based on our observations in the numerical test:

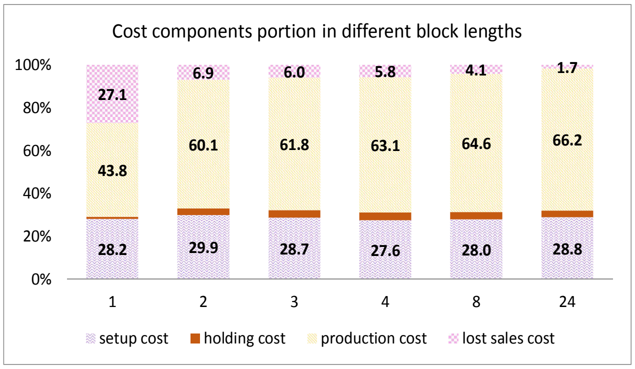
- Different cap policies have a similar effect on the trend and behavior of profit seeker manufacturer regardless of their margin.
- regulating the emission level in such a way that starts with strictly limited allowance and gradually increases and becomes less strict (i.e., more emission allowance later), has a more contribution to the environment than a policy in the opposite way around.
- By regulating seasonal or rolling emission restriction basis, especially with shorter time horizons, the policy-makers can achieve more tangible results as a reaction to the global warming problem.
- A tighter emission regulation, may lead to excess of emission allowance which is caused by avoiding the production, which in turn, may drive them towards the carbon market to sell them rather than using for extra production.

In the unreported further investigations of the cap policies in this study, it was found that almost half of the policies are dominated by the others and among the dominant ones, a multi-objective ranking with equal weights revealed that the periodic and/or seasonal cap patterns with shorter block lengths are the best ranked policies when setup costs are high while cumulative or seasonal patterns with longer block lengths are the best ranked ones for problems with low setup costs regardless of the lost sales penalty and the production factor.

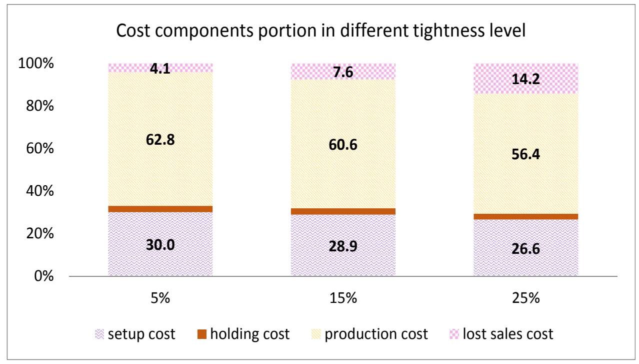
This study can be extended to other carbon emission frameworks (cap-and-trade and tax) to obtain insights on the effect of carbon cost (or fine) which may provide interesting results for policy-makers.

Block Length

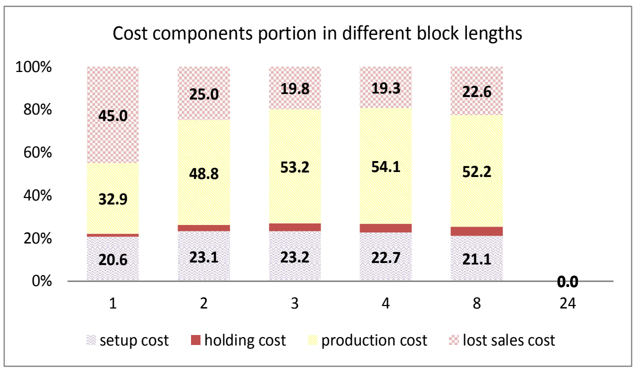
Tightness



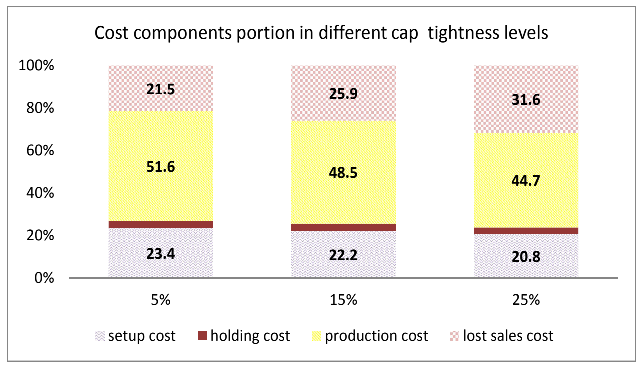
(a) rolling cap, constant



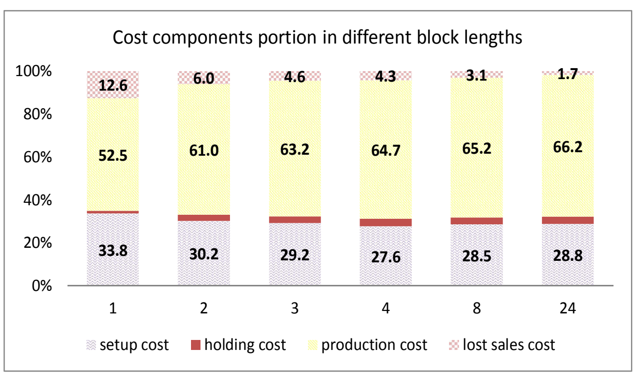
(b) rolling, constant cap



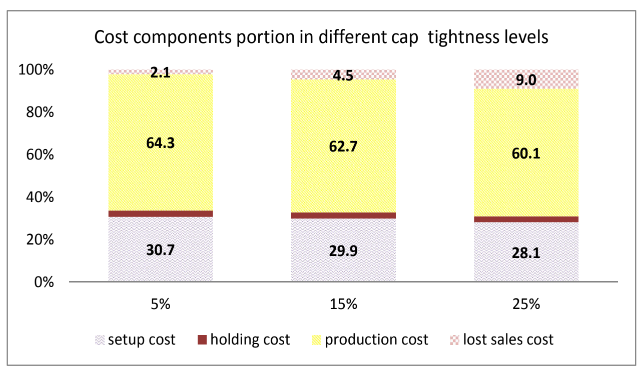
(c) seasonal, deceasing cap



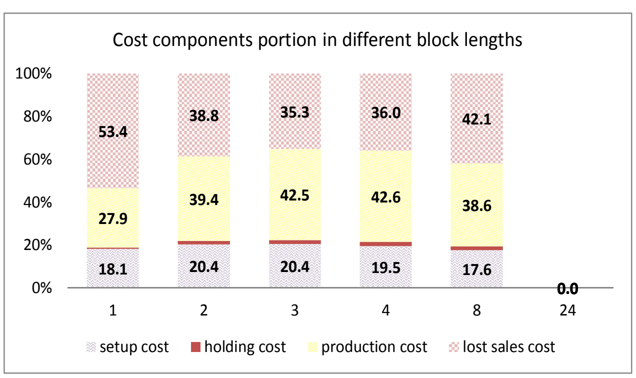
(d) seasonal, decreasing



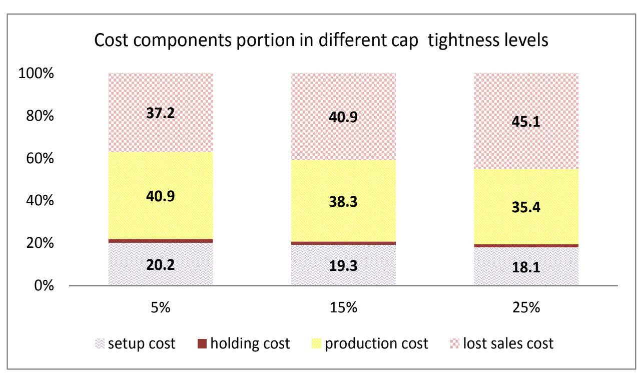
(e) seasonal, constant



(f) seasonal, constant cap



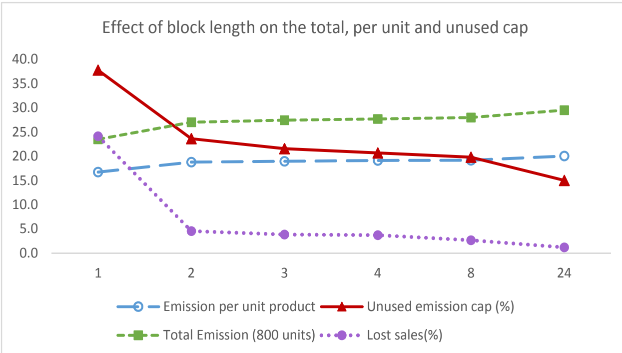
(g) seasonal, increasing



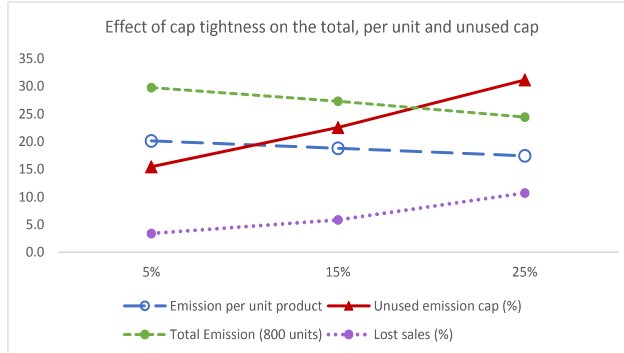
(h) seasonal, increasing cap

Fig. 1. Sensitivity of cost components variables to the cap policy.

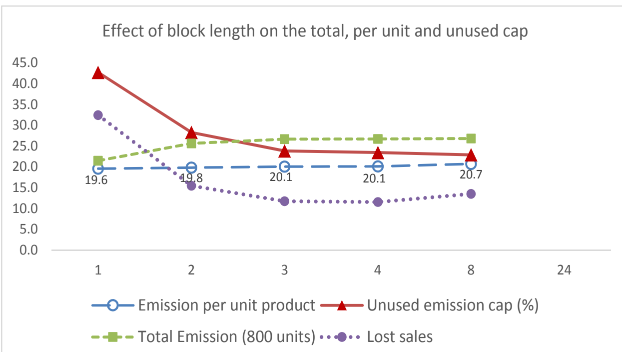
Block Length | Tightness



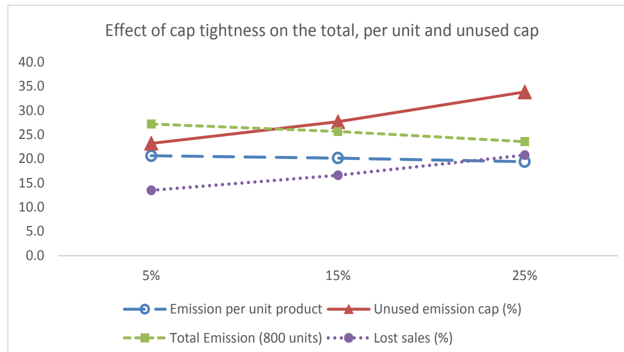
(a) rolling cap, constant



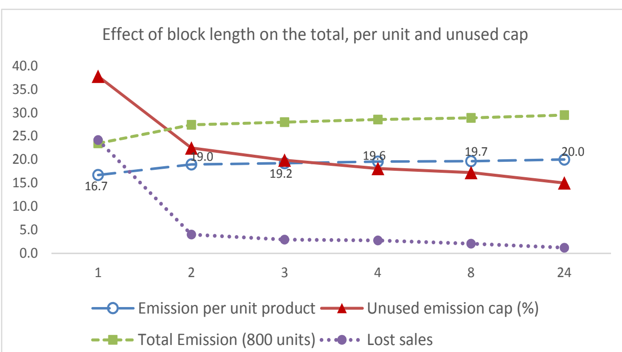
(b) rolling, constant cap



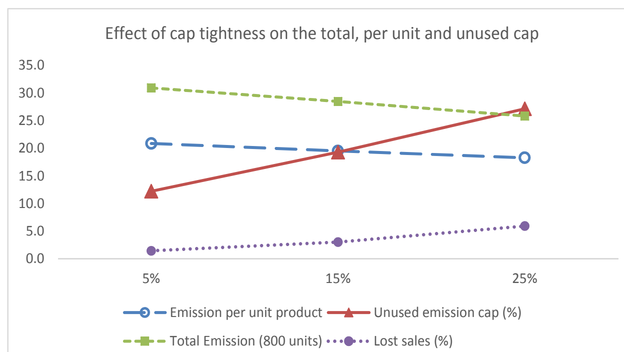
(c) seasonal, deceasing cap



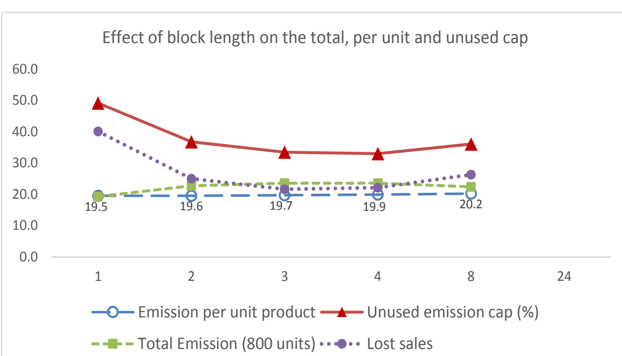
(d) seasonal, decreasing



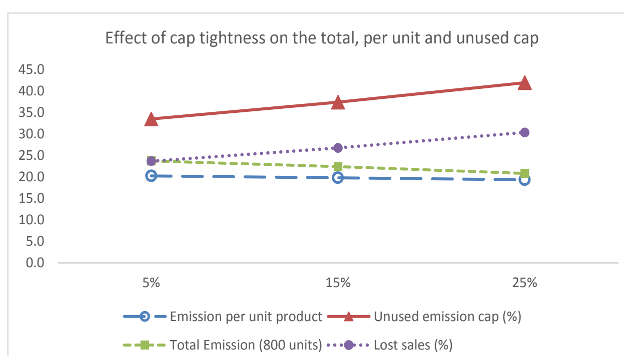
(e) seasonal, constant



(f) seasonal, constant cap



(g) seasonal, increasing



(h) seasonal, increasing cap

Fig. 2. Sensitivity of the total and per unit emission, the unused emission cap and the lost sales amount to the cap policy.

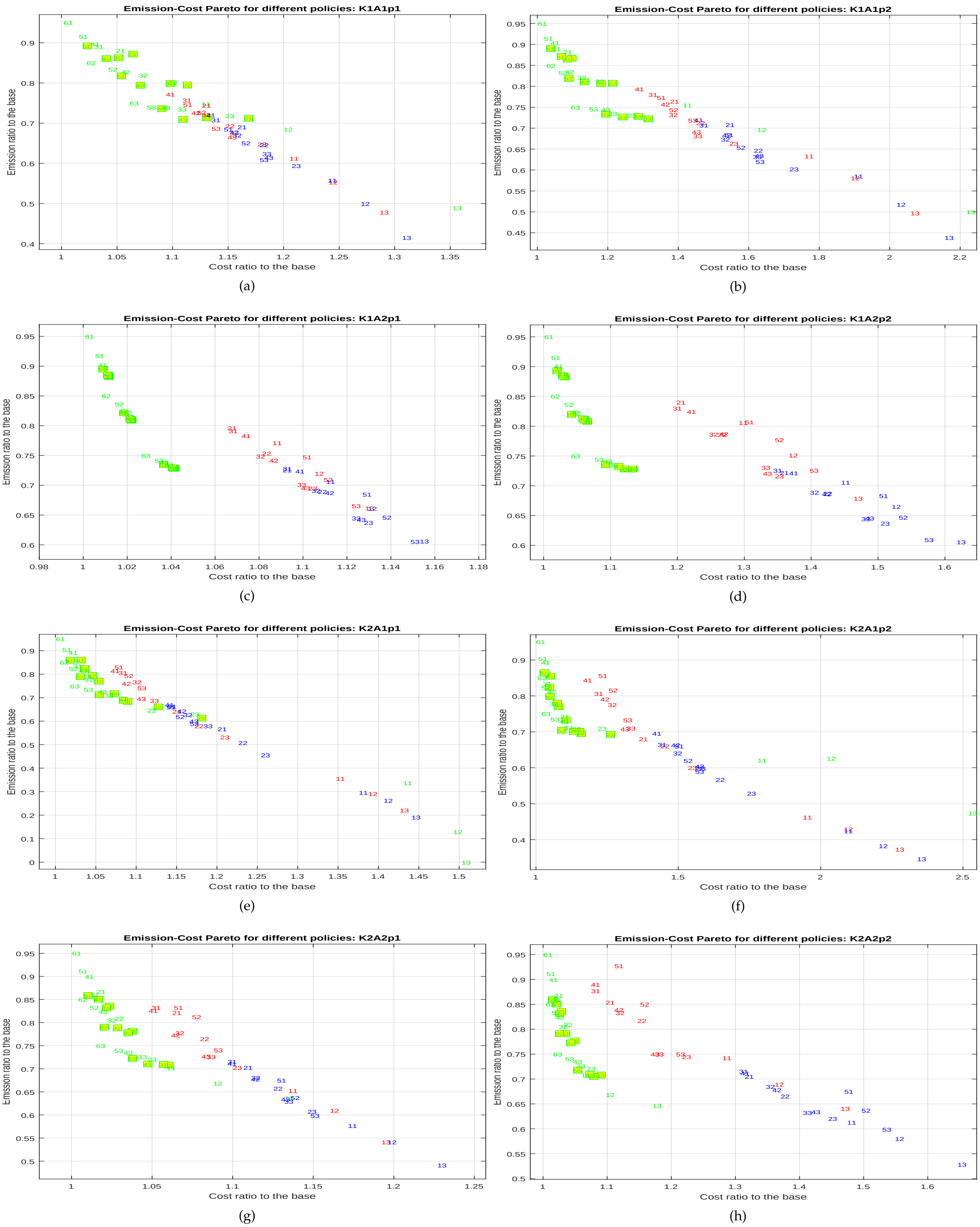


Fig. 3. Cost-Emission Pareto analysis of the cap policies.

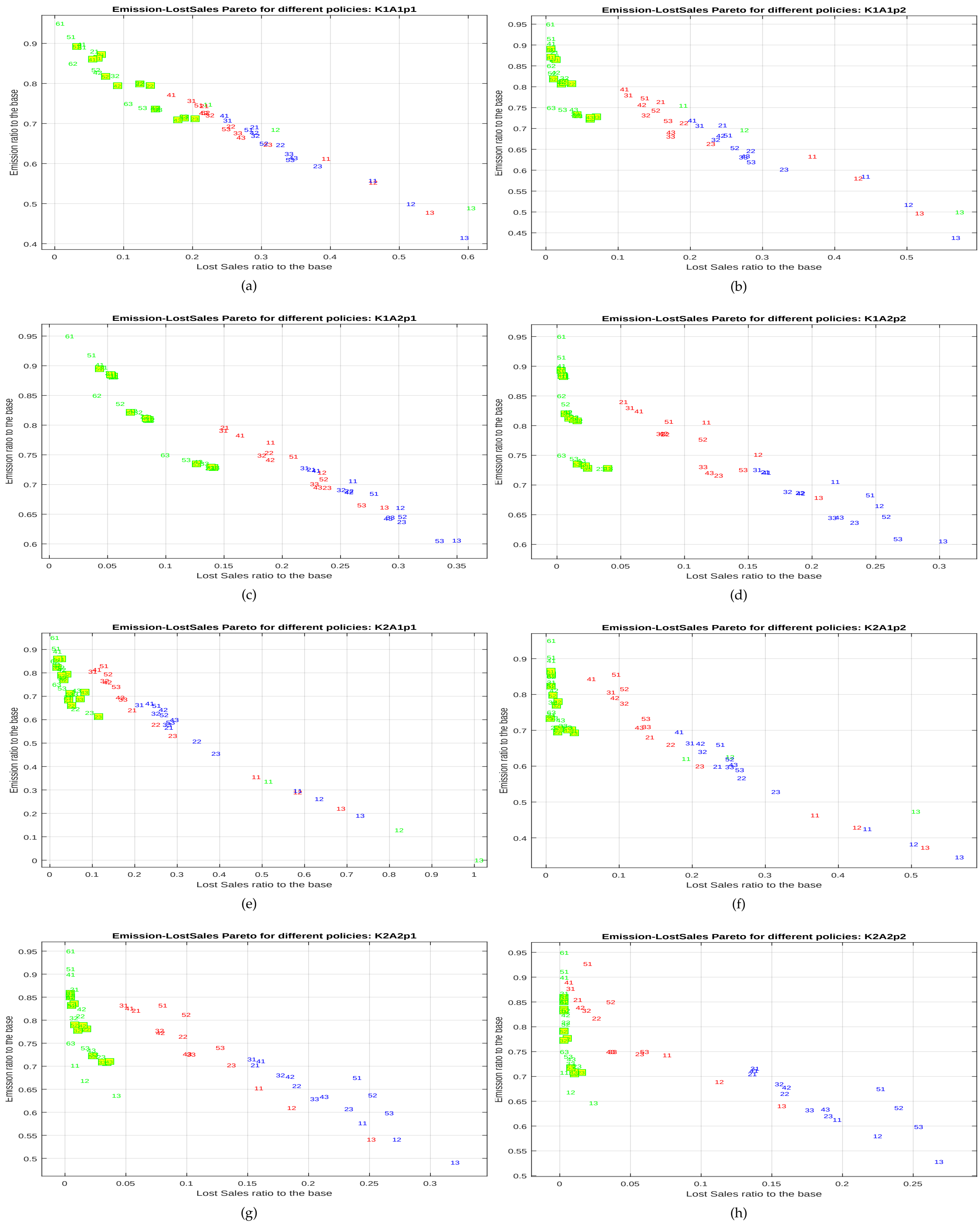


Fig. 4. Lost sales-Emission Pareto analysis of the cap policies. The first digit is the level of block length, the second digit denotes the tightness level. Rolling policies are marked with filled boxes.

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