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#### DYNALIC RESPONSE OF FOUNDATIONS

by

#### Eliathamby Sivapalan

A thesis submitted to the Council for National Academic Awards in partial fulfilment of the requirments for the degree of

Doctor of Philosophy.

#### March 1981

Sponsoring Establishment: Trent Polytechnic, Nottingham.

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#### DYNAMIC RESPONSE OF FOUNDATIONS

#### by E.Sivapalan

#### ABSTRACT

The thesis describes a field investigation into the dynamic response of shallow concrete foundation pads constructed on a natural soil deposit. Particular emphasis is placed upon the response of 'passive' foundations excited by vibrations transmitted through the soil from an adjacent 'active' foundation subjected to periodic excitation derived from a rotating mass type vibrator.

A test rig incorporating the vibrator and ancillary equipment for independent control of operating speed and amplitude of dynamic load was constructed. Instrumentation for the measurment of foundation and ground surface accelerations in the range 0.000001g to 1g was assembled from commercially available units. A series of field tests involving two sets of active and passive foundation pads was conducted.

The transmission of vibrations through the soil from one foundation to another is examined by considering the ratio of the vertical displacement of the passive foundation. It is shown that the transmission ratios are substantially independent of both the mass ratio of the active foundation and the intensity of excitation. The test results are used to demonstrate the influence of soil adhesion along the sides of a passive foundation on its dynamic response. Also, the influence of the passive foundation on the response of the active foundation, the nonlinear behaviour of the supporting soil and the interaction between two closely spaced passive foundations are illustrated.

To examine the practical application of existing theories, the results of published theoretical parametric studies are used to derive equations of motion of both the active and the passive foundations, and vertical displacements calculated using the measured in situ soil parameters are compared with the field results. The differences between predicted and measured responses are discussed. The problems which require further study are identified and listed as suggestions for further research.

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#### <u>NOTATION</u>

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- A Amplitude of displacement of rigid circular footing on an elastic halfspace.
- $a_{o} = \frac{wr}{V_{s}}$  , dimensionless frequency
- $B = \frac{h \ G_s}{r \ G} , \ \text{Ratio of the product of shear modulus of side}$  layer and depth of embedment to the product ofshear modulus of underlying medium and radius of foundation.}
- B Half-width of foundation.
- b Mass ratio of foundation.
- C Dynamic compliance of halfspace.
- C Dynamic compliance of stratum.
- $C_r$  Dynamic compliance of rock underlying stratum.
- C1.2 Stiffness and damping parameters of halfspace.
- c Damping constant of single degree of freedom analogue.
- D Depth of soil adhesion along the sides of embedded foundation.
- d Distance from centre of active foundation.
- e Eccentricity of rotating mass.
- $e_{E}$  Equivalent eccentricity for four-shast layout.

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 $F_{1.2}$  - Lysmer's modified displacement functions.

f<sub>1.2</sub> - Reissner's displacement functions.

- f Frequency of excitation
- f(X) Raleigh frequency equation
- G Shear modulus of soil beneath footings.
- G<sub>s</sub> Shear modulus of soil layer adjacent to sides of embedded foundation.

- g 9.81 m/sec<sup>2</sup>, Acceleration due to gravity.
- H Thickness of soil layer.
- h Depth of embedment .
- i = -1 , Imaginary unit.

 $K_{1.2}$  - Average values of  $(C_1 - BS_1)$  and  $(C_2 - BS_2)/a_0$ 

k - Spring constant of single degree of freedom system .

- $L_{p}$  Length os surface waves.
- 1 Lenth of rectangular base.
- M Mass of foundation

M \_ M \_ P Mass of active and passive foundations, respectively.
 m - Eccentric mass on rotating shafts of vibrator.

 $N_{z}(t)$  - Vertical dynamic reaction along sides of embedded footing.

 $N_{av}(t)$ ,  $N_{pv}(t)$  - Vertical dynamic reactions along sides of active and passive foundations, respectively.

 $N_{pH}(t)$  - Horizontal soil reactions acting on passive foundation due to embedment. -xvii -

- $R_{z}(t)$  Vertical dynamic reaction at base of rigid footings on halfspace.
- R<sub>za</sub>(t) Vertical dynamic reaction at base of single active foundation.

 $R_{av}(t)$ ,  $R_{pv}(t)$  - Vertical dynamic reaction at bases of active and passive foundations, respectively, of a two-foundations system.

$$R_{pH}(t)$$
,  $M_{pp}(t)$  - Horizontal soil reaction and moment at base  
of passive foundation.

 $r,r_o$  - Radius of cylindrical footing; also equivalent radius of square or rectangular foundation.

 $r_a$ ,  $r_p$  - Radii of active and passive foundations, respectively. P(t) - Vertical excitation force at time t.

 $P_{-}$  - Amplitude of excitation force.

- P: Amplitude of vertical dynamic force on ground surface directly beneath the active foundation.
- Stiffness and damping parameters due to embedment.
- s(t) Vertical side dynamic reaction per unit depth of embedment.

t - time

U(t) - Horizontal (radial) displacement of rigid foundation.  $V_R$ ,  $V_s$ ,  $V_p$  - Velocities of surface waves, shear waves and compression waves through the soil

w - Circular frequency of excitation .

X - Root of Raleigh frequency equation.

- Z(t) Vertical displacement of rigid circular footing.
- $Z_{a}(t)$  Complex vertical displacement of active foundation.

Z\_ - Amplitude of vertical displacement.

- Z Complex vertical displacement of passive foundation in free vibration.
- Z Free field vertical displacement at location of passive foundation.

- $Z_{pgo}$  Amplitude of free field motion.
- z Vertical coordinate.

P - Bulk density of soil

 $\mu$  - Poisson's ratio .

 $\phi$  - Angle of rotation of passive foundation in the vertical plane.

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#### CHAPTER I

#### INTRODUCTION

#### 1.1. GENERAL

The behaviour of foundations subjected to dynamic loads and the effects of such motions on the environment have constituted important problems since the earliest forge hammers caused settlement and vibration problems. The necessity for developing effective and economical design for foundations subjected to dynamic loads has become more important in recent years. This has been caused, primarily, by the increase in capacity and number of industrial installations that are intensive sources of vibration, and, partly by the requirements of safety and stability of structures in regions affected by blasts or earthquakes.

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#### 1.2. DESIGN REQUIREMENTS

Dynamically loaded foundations can be classified into the following categories according to their design requirements:

- (i) Foundations subjected to steady state vibration originating from rotating or reciprocating machinery such as compressors, pumps, internal combustion engines, etc.;
- (ii) Foundations subjected to intermittent loads from the operations of punch press, forging hammers, etc.;
- (iii) Foundations supporting sensitive equipment such as electron microscopes, calibration test stands, etc., which require isolation from ambient vibrations at the site ;
- (iv) Foundations for special equipment such as precision tracking radar which are designed to resist dynamic loads developed by the supported unit and also to protect the entire structure

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from dynamic energy that may be transmitted through the ground from external sources.

 (v) Foundations of tall structures such as multistorey buildings which are sensitive to large scale dynamic energy arriving from blasts or earthquakes.

In each case, the design requirements are dictated by the criteria for satisfactory operation of the supported unit. For foundations supporting machines which produce sustained vibrations, this design criterion is no longer interpreted simply as meeting the specifications recommended by the manufacturers of the supported machinery. The effects on the environment of vibrations, generated by the machine and passed through the foundation into the soil, are also taken into consideration. Dynamic energy transmitted from a vibrating foundation may be detrimental to sensitive equipment in the vicinity or cause annoyance to people living nearby. As a result, there is an increasing tendency to regard ground vibrations induced by industrial activity as a health hazard and a form of pollution of the natural environment. Thus, it

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has become necessary to establish design criteria for machine foundations based on limiting levels of vibration some distance away from the foundation. Furthermore, the design of twin-plant installations involving two or more independent machine foundations built on separate base slabs require an assessment of the effects of through-soil coupling on the dynamic response of the foundations.

#### 1.3. ANALYSIS OF SOIL-FOUNDATION INTERACTION

Soil-foundation interaction has received considerable theoretical study in connection with the design of machine foundations. The widely accepted method of analysis treats the soil as a semi-infinite, elastic, isotropic, homogeneous medium and the foundation as a rigid circular body resting on the surface of the medium. In recent years, approximate solutions have been formulated to deal with embedded footings, footings of rectangular or square base, layered soils, etc. The analysis usually requires the determination of a set of foundation stiffness functions which relate the forces and moments at the foundation to the relative

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motion between the foundation and the soil at a point well removed from the structure. These functions are then used, together with equations, for the dynamic equilibrium of the structure to analyse the soil-structure system. For practical computations these frequency dependent functions are usually replaced by equivalent springs, dashpots and masses.

Most of these theoretical solutions have been confined to single-structure configuration and little is known about the transmission of vibrations between adjacent foundations. Relatively few experimental studies checking the validity of these theories in practical applications have been carried out. The reasons for this deficiency appear to be the difficulty in creating a suitable environment in the laboratory and the difficulties in conducting field tests with control over the system's parameters.

#### 1.4. AIM OF INVESTIGATION

With the intention of providing more experimental data on the dynamic behaviour of foundations embedded in natural soil deposits and on the transmission of vibrations between adjacent foundations,

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an experimental programme was designed and carried out as described in the following chapters. The programme was designed to study the dynamic behaviour of a system consisting of an embedded foundation excited by a rotating mass-type vibrator; a natural soil deposit (weathered Keuper Marl) and a foundation receiving vibrations at some distance away from the excited foundation.

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#### CHAPTER II

#### LITERATURE SURVEY

#### 2.1. INTRODUCTION

Vibrations of soils and foundations have received a considerable amount of scientific attention since the 1930s. As a result, the relevant literature consists of numerous publications most of which describe analytical studies. Early attempts at analysing foundation vibration problems were based on the assumption that the foundation and the soil formed a simple mass-spring-dashpot system. The mass of the system was considered to be a sum of the mass of the foundation and the mass of a portion of the soil which was supposed to be oscillating in phase with the foundation. Different rules for evaluating the 'in-phase' mass of the soil have been suggested by Lorenz (1934), Crockett and Hammond (1949), Pauw (1953) and Rao and Nagaraj (1960). However, a series of controlled tests performed by DEGEBO (German Society for Soil Mechanics) revealed that the 'spring-mass' theory was inadequate to explain the test

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results. The in-phase mass of soil that would make the theory and the test results agree was not a constant for a particular foundation but varied with the frequency of excitation, the static load and the dynamic load applied to the footing.

The unreliability of the simple 'spring-mass' theories led to the development of the 'elastic halfspace' theory which was first introduced by Reissner (1936). This approach treated the soil as a semi-infinite, elastic, uniform, homogeneous, isotropic medium, or as it is commonly denoted an elastic halfspace, and the foundation as a rigid circular body placed on the surface of the halfspace. Although the application of such a theory to analyse vibrations of actual foundations is not quite realistic, it formed the basis of nearly all further analytical studies of foundation vibration problem because it permitted evaluation of many parameters that had previously seemed to play an important part in the problem, but which could not be completely defined, such as Poisson's ratio, shear modulus, bulk density of the soil, mass and size of foundation and pressure distribution over the contact area.

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Mathematically the oscillation of a rigid circular footing on an elastic halfspace constituted a mixed boundary value problem which was first solved approximately by assuming a convenient form of stress distribution beneath the footing. Development of mathematical techniques, later enabled derivation of exact solutions, but as shown by Warburton (1973), the difference between the results given by the approximate method and the exact method were negligible except for the horizontal translational mode of vibration. A full description of the various techniques employed to obtain solutions for the theoretical problem can be found in Barkan (1962), Eycroft (1956) and Gladwell (1968).

The final expression derived for the amplitude of displacement of the footing is usually written as,

$$A = \frac{P_{\bullet}}{Gr_{o}} \sqrt{\frac{f1^{2} + f2^{2}}{(1 + ba_{o}^{2}f1)^{2} + (ba_{o}^{2}f2)^{2}}}$$
(2.1)

in which

A - Amplitude of displacement of the footing.  $$^{\circ}$$  P<sub>o</sub> - Amplitude of external periodic force

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G - Shear modulus of the elastic halfspace.

r - Radius of the footing.

a - A dimensionless expression of the frequency of excitation, often described as frequency factor,

defined as:  $a_0 = \frac{2\pi fr_0}{V_s}$ 

 $V_s$  - Velocity of shear wave through the medium. f - Frequency of excitation in Hz.

b - Mass ratio of the rigid footing defined as:

$$b = \frac{m}{r_o^3}$$

(2.1b)

(2.1a)

m - Mass of the footing.

f - Bulk density of the halfspace.

and,  $f_1$  and  $f_2$  - are two complicated functions of frequency factor  $a_0$  and Poisson's ratio (.) ) of the halfspace being effectively the inphase and out of phase components of displacement per unit force of a rigid massless plate of the same dimensions and shape as the rigid footing. Expressions for  $f_1$  and  $f_2$ 

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vary with the form of stress distribution assumed beneath the footing. The analysis is mainly directed towards the derivation of expressions for these two displacement functions.

#### 2.2. DYNAMIC RESPONSE OF 'SURFACE' FOUNDATIONS

#### 2.2.1 Theoretical Development

Reissner's (1936) solution for the vertical oscillation of a rigid footing on the surface of an elastic halfspace was extended into more useful and more accurate forms by several investigators notably, Sung (1953), Arnold, et al (1955), Hsieh (1962) and Awojobi and Grotenhuis (1965). The main differences between these studies are the modes of vibrations and the boundary conditions considered. Sung (1953) presented solutions for displacement functions  ${\bf f}_1$  and fo corresponding to three different types of stress distribution over the circular contact area on the surface of the halfspace. The response curves obtained by Sung indicated that the dynamic response of a foundation can be influenced by controlling the flexibility of the foundation. None of the assumed stress distributions, however, could yield rigid body displacements of the foundation. Arnold et at (1955) and Bycroft (1956) presented solutions for different

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modes of vibrations of the rigid body. They assumed relevant static stress distribution between the rigid body and the halfspace for each of the four modes of vibration (i.e. vertical translation, horizontal translation, rocking about a horizontal axis and rotation about a vertical axis), evaluated the weighted average of the displacements beneath the footing and thus established more accurate values for displacement functions  $f_1$  and  $f_2$  than those presented by Sung (1953).

By reorganising Reissner's basic equations, Hsieh (1962) showed that all modes of vibrations of a rigid circular footing could be represented by equations which have the same general form as the equilibrium equation for a damped single degree of freedom spring-mass-dashpot system. The major difference was that the damping term and the spring reaction term were functions of  $f_1$  and  $f_2$  which, as described earlier, varied with frequency of excitation and Poisson's ratio ( $\mu$ ) of the medium. Lysmer and Richart (1966) extended this treatment by introducing modified displacement functions  $F_1$  and  $F_2$ 

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$$F_1 = \frac{\mu f_1}{1 - \mu}$$
 and  $F_2 = \frac{\mu f_2}{1 - \mu}$  and developed a

spring-mass-dashpot-analogue with constant parameters which closely approximated the behaviour of a rigid footing on the elastic halfspace. The equation of equilibrium for this analogue is given by:

$$M\ddot{Z} + \left\{ \frac{3.4 r_0^2 \sqrt{\rho} G}{1 - \mu} \right\} \dot{Z} + \frac{4G r_0}{1 - \mu} \qquad Z = P(t)$$

$$(2.2)$$

where

M - Mass of the footing.

Z - Vertical displacement.

P(t) - Vertical force applied to the footing.

ro - Radius of the footing.

and P, G, and  $\rho$  are the properties of the elastic halfspace as defined for equation 2.1.

Lysmer's analogue is applicable for the vertical translational mode only. Similar analogues for other modes of vibration of the footing were presented by Hall (1967). Nonlinear response studies (Funston and Hall,

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1967) have made extensive use of these simplified analogues.

Awojobi and Grootenhuis (1965) attempted to provide an 'exact' solution to the problem by adhering to the physical requirement of a uniform displacement under the rigid footing and zero normal and shear stresses at the surface away from the footing. This approach led to a set of dual integral equations which were solved by a numerical method. The authors found that the stress distribution beneath the footing changes with frequency of excitation, and the assumption of static stress distribution is valid only for very low values of the frequency factor a .. With further developments of mathematical techniques more accurate solutions for  $f_1$  and  $f_{2}$  have been presented by Gladwell (1968), Richardson (1969) and Clemmet (1974). Clemmet (1974) considered the coupling between horizontal motion and rocking motion of the foundation and derived solutions for the coupled mode of vibration.

Lysmer and Kuhelemeyer (1969) developed a general

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method through which an elastic halfspace could be replaced by a finite model with a special viscous boundary condition. They used the finite element method to analyse the dynamic response of a rigid footing on the finite model and showed that the results agreed with the results given by the classical elastic halfspace theories. Wass and Lysmer (1972) developed an alternative finite model which included transmitting boundaries and an infinitely rigid base, to represent the elastic halfspace. They claimed that the utilization of this model, in the analysis of dynamic response of foundations, was less expensive and more accurate than the use of the viscous boundary model. Although further development of the dynamic finite element method (Kausel et al (1975)) has made possible the study of the response of footings on soils other than an homogeneous halfspace, the assumption of an infinitely rigid base at a relatively shallow depth is an unavoidable requirement which can be justified only in the case of a shallow layer overlying a very stiff rock.

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Because of the simplification arising from cylindrical coordinates, all of the aforementioned theories were developed for rigid footings with circular bases. Kobori and Thomson (1962), and later, Elorduy et al (1967) studied vertical vibration of rigid rectangular footings on an elastic halfspace. Richart et al, (1970) compared the results obtained by Elorduy et al, for a rectangular base of length to width ratio 2 with results obtained by Bycroft (1956) for a circular footing having the same area as the rectangular base and found that the difference between the displacement functions given by the two methods were negligible.

### 2.2.2 Applications of the theory.

The application of elastic halfspace solutions to actual foundation vibrations problems and the implications of the theoretical predictions regarding the significance of different parameters of the foundation-soil system were discussed by Richart (1962), Whitman (1965), Barkan (1962) and Richart et al (1970). It was found that qualitatively, the results given by the elastic halfspace theory agreed with observed behaviour of foundations provided that the soil broadly resembled an elastic halfspace. When the theory was applied to practical size foundations, the results indicated that foundations experiencing translational modes of vibration would be heavily damped due to loss of energy by the propagation of waves, whereas this radiation damping would be small in systems experiencing rotation, especially rocking. Richard (1962) studied the influence of the mass ratio of the footing on its dynamic behaviour and presented a family of design curves illustrating the decrease in resonant frequency and increase in resonant amplitude with increase in mass ratio. Analysis of the influence of the parameters of the soil (i.e. shear modulus and poisson's ratio) revealed that small

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variations in the shear modulus affected the results considerably, but variations in poisson's ratio within the range 0.25 to 0.4, which is reasonable for natural soil deposits, had very little effect on the displacements predicted by the theory. Richart (1974) recommended that for calculation purposes, poisson's ratio may be taken as 0.33 for sands and 0.4 for clays with little loss of accuracy. The theory obviously, was inadequate to deal with nonlinear response and Whitman (1965) recommended that the application of the theory should be limited to situations where the intensity of excitation did not result in accelerations exceeding 0.5g.

### 2.2.3 Experimental Studies on Surface Foundations.

Fry (1963) carried out a series of vibration tests on different circular footings from 5 to 16 feet in diameter which were excited in the vertical, torsional and rocking modes of vibration. All of the footings were constructed in situ on the surface of the soil at two test sites, one of which consisted of a silty clay soil and the other consisted of a sandy soil. Fry reported that both soil deposits were uniform and homogeneous to depths several times greater than the diameter of the largest footing tested; thus simulating an elastic halfspace. The results of these tests were compared by Richart and Whitman (1967) with displacements calculated using the elastic halfspace theory. Different types of stress distributions beneath the footings were considered in the calculations. Richart and Whitman found that the assumption of static stress distribution beneath the footings gave the closest agreement with the measured results but, even then, the calculated maximum amplitudes were onethird to one-half of the observed maximum amplitudes. Richart and Whitman attributed this large difference between theory

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and test results to inaccurate determination of shear modulus of the soil, variation of stress distribution with frequency of excitation and nonlinear response of the supporting soil. The variations of maximum amplitude and frequency of maximum amplitude with mass ratio of the footing, however, followed the trend predicted by the elastic halfspace theory.

Drenvich and Hall (1966) conducted transient loading tests on a 0.3m diameter model footing resting on a finite bed of sand placed in a container. They measured the pressure distribution beneath the footing and found that the total stress distribution during transient loading was somewhat similar to static stress distribution. They showed that displacements computed using Lysmer's analogue agreed closely with that part of the test results which were not affected by waves reflected from the walls of the container.

Moore (1971) carried out field vibration tests on 15in. diameter, one inch thick steel plates of different mass ratios placed on different soils. By comparing measured maximum amplitudes with those computed from elastic halfspace solutions formulated by Sung (1953) with uniform, parabolic rigid base

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stress distributions beneath the footings, Moore found that for clayey subsoils the closest prediction of the observed results was obtained with rigid base pressure distribution. For the sandy subsoil, they stated that their test results indicated a parabolic stress distribution at low mass ratios and rigid base stress distribution at high mass ratios. An examination of their results suggests that different interpretations are possible. Nevertheless, a general conclusion that can be drawn from these test results is that calculated and observed amplitudes of displacement do not maintain a consistent relationship in all cases.

MacCalden and Matthiesen (1973) carried out a series of tests on a 4 foot diameter concrete foundation resting on a uniform silty clay. The authors used Bycroft's (1956) halfspace solution which assumes static stress distribution to calculate the displacements of the footing and found that the measured displacements are 60% to 80% higher than the predicted displacements but the frequency of maximum amplitude calculated was very close to that measured in the field.

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### 2.3. DYNAMIC RESPONSE OF FOUNDATIONS ON STRATIFIED SOILS

Theoretical study of vibrations of foundations constructed on stratified soils has been confined to the analysis of the relatively simple case of a rigid footing oscillating on the surface of a single elastic layer overlying an infinitely rigid rock. Arnold et al (1955) assumed that the stress distribution beneath the footing was the same as the static stress distribution in the case of a rigid footing on a uniform halfspace and presented solutions for all four modes of vibration of the footing in a form similar to equation 2.1. It was found that the displacement function  $f_1$  and  $f_2$  depended on the ratio of the thickness of the stratum (H) to the radius of the footing (r). Warburton (1957) gave a more detailed account of the solution for vertical mode of vibration and presented numerical. values  $f_1$  and  $f_2$  for different values of mers ratio. It was shown that, unlike in the case of a uniform halfspace, true resonance with amplitude of motion approaching infinity occurred for a massless rigid footing oscillating on the stratum. The frequencies at which resonance occurred were given by,

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(2.3)

in which

 $V_s$  - Velocity of shear waves through the stratum.  $\mu$  = Poisson's ratio of the medium.

and, n = 1, 2, 3, ...

Warburton identified the resonant frequencies as the natural frequencies of the stratum. For footings with mass, Warburton found that the resonant frequencies varied with the mass ratio of the footing. He presented a family of curves each corresponding to a different mass ratio and giving the variation of dimensionless frequency of maximum amplitude with mass ratio of the foundation. In general, frequency of maximum amplitude decreased with increasing mass ratio. The author compared his theoretical predictions with experimental results obtained for model footings on an artificial layer and recommended that the application of 'stratum-on-rigid base' theory should be restricted to foundations on shallow starta (i.e. H/r < h). For a foundation placed on a deep startum (i.e. H/r > h) elastic halfspace assumption was recommended.

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An important observation that emerged from this study was that the frequency response of a small footing (i.e. mass ratio <5) on a stratum would reach a sharp peak at a relatively low frequency compared to the almost flat response of the same footing on an equivalent elastic halfspace. The practical use of this concept in determining whether a natural, heterogeneous soil deposit should be represented by an elastic halfspace model or by a stratum on rigid rock model was described by Warburton (1965).

Whitman (1969) presented a general study on the response of a foundation subjected to horizontal vibration on the surface of an elastic stratum. He also considered several cases involving different mass ratios and stated that for (H/r) > h, soil amplification due to the oscillation of stratum and soilstructure interaction due to the oscillation of the footing can be considered separately. He showed that the displacement function  $f_1$  and  $f_2$  for an elastic halfspace model and those for a stratum on rigid rock model with (H/r) > h, are nearly equal at all frequencies except those closer to the fundamental natural frequency of the stratum. At this frequency, the dis-

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placement function for the latter model approaches infinity. For the analysis of foundation-stratum systems with ratios less than four, Whitman proposed an equivalent, two degrees of freedom system in which the stratum was replaced by a mass and a spring.

Wass and Lysmer (1972) used finite element techniques to study vertical vibrations of a footing resting on a deep (i.e. H/r equal to 10) viscoelastic stratum. An infinitely rigid base beneath the stratum was assumed. The authors presented a set of response curves each corresponding to different mass ratios of the foundation, which are reproduced in figure 2.1. Each response curve showed two peaks, the first at a frequency very close to the natural frequency of the stratum and the second at a frequency same as the frequency of maximum amplitude of the same foundation on an equivalent elastic halfspace.

Gazetas and Rosett (1979) derived solutions for vertical vibrations of a long strip footing of halfwidth  $\overline{B}$  resting on a two layered medium consisting of an elastic stratum of compliance C<sub>s</sub> overlying a compliant rock, of dynamic compliance

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FIG. 2.1

RESPONSE SPECTRA FOR VERTICAL MOTION OF CIRCULAR FOOTINGS ON HOMOGENEOUS LAYER ( $\frac{H}{T}$  = 10, 5% critical damping) ( AFTER WASS AND LYSMER, 1972) - 27 -

Cr . They presented a number of response curves corresponding to different mass ratios of the footing, different values of

 $\frac{C_s}{C_r}$  ratio and different depths of the stratum. By comparing  $C_r$ 

these response curves with similar response curves obtained for foundations on a stratum overlying a rigid base and foundations on an elastic halfspace, the authors concluded that for foundation vibration problems, a layer of soil overlying a compliant rock could be replaced by either a single stratum overlying an infinitely rigid rock or by an elastic halfspace, depending on the mass ratio of the footing, the ratio of depth of the top layer to the width of the footing, compliance ratio of the two layers and the type of excitation employed. For foundations of high mass ratio, small errors in modelling the soil were found to be unimportant. For small mass ratios ( b <20), stratum on rigid rock model was appropriate for ( $\frac{C}{r}/C_s$ )>2,  $\frac{H}{R}$  4 and a constant force type of excitation.

When dealing with rotating machinery type of excitation, a halfspace model was found to be more appropriate, especially so for  $\frac{H}{R} > 4$  or  $(C_r/C_s) < 2$ .

### 2.4. DYNAMIC RESPONSE OF EMBEDDED FOOTINGS

Kaldjian (1969) used the finite element method to study the effect of embedment on the stiffness of a foundation-soil system. He found that the stiffness increased with increase in depth of embedment and the stresses developed along the vertical faces of the embedded footing were mainly responsible for this effect.

Novak and Beredugo (1972) and Beredugo and Novak (1972) have described an extensive study on forced vibrations of embedded footings. They employed an approximate analytical approach formulated by Barnov (1967), which treated the soil on the sides of the footing as a layer separate from the soil beneath the base of the footing. With this assumption, Novak and Beredugo developed solutions for the foundation in vertical and coupled horizontal translation and rocking modes of vibration. The authors found that embedment increased the stiffness of the footing-soil system and resulted in an increase in resonant frequency and decrease in resonant amplitude. Comparison of the theoretical predictions with experimental results from field tests on footings embedded

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in undisturbed soils revealed that the theory and test results agreed qualitatively but experimental resonant amplitudes were two to three times higher than those predicted. Experiments conducted with backfilled foundations showed that the increases in resonant amplitude with embedment were much smaller than those predicted by the theory. It was found that the effect of embedment depended on the density of the backfill and the quality of the bond between the soil and the footing.

Stoke and Richart (1974) conducted a series of model tests on 200mm. diameter concrete footings to study the effect of embedment on the dynamic response of the footings. Each footing was first tested as a cast in place footing, afterwhich, soil contact along the embedded depth was removed and tested again. The results showed that the damping ratio and the damped natural frequency of the footing increased significantly with embedment for vertical, horizontal and rocking modes of vibration of the footing. The authors presented two well documented case studies which described the steady state response of two prototype machine foundations. One of the foundations was embedded in a cohesionless soil and the other in a cohesive

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backfill. Both foundations were set into rocking and sliding motions. It was found that the actual response of the foundation embedded in the cohesive soil agreed more closely with the response predicted by the theory for a surface foundation than with the response predicted by the theory for an embedded foundation. For the foundation with cohesionless backfill, however, actual displacements were about 20% of the displacements predicted by the theory for surface foundations. Based on these observations, Richart and Stoke concluded that the design of foundations embedded in a cohesive backfill and subjected to rocking and sliding motions, should not rely upon the beneficial effects of embedment.

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# 2.5. <u>SURFACE RESPONSE IN THE VICINITY OF A HARMONICALLY EXCITED</u> FOUNDATION

Analytical solutions for the surface motions in the vicinity of a vibrating foundation were presented by several investigators (Arnold et al (1955), Barkan (1962), Richardson et al (1971)). These solutions were obtained by treating the soil as an elastic halfspace and the foundation as a rigid circular body. Richardson et al (1971) considered an elastic halfspace with poisson's ratio zero and found that for each of the four modes of vibration of the foundation, the surface displacements decayed very rapidly with distance from the foundation. The rate of decay of displacements in the vertical translation, torsion and rocking modes decreased with increasing frequency of excitation.

Barkan (1962) conducted an extensive series of experimental investigation on propagation of surface waves through different soils and compared the results with elastic halfspace solutions. It was found that for vertical vibration of the source, the experimental orbits of motion of the soil particles differed considerably from the elliptical orbit predicted by

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the theory, especially, for relatively small distances from the source. The amplitudes of vertical component of displacement of the surface, calculated from the theory, agreed with measured amplitudes at small distances from the source but at large distances from the source, calculated amplitudes were much higher than the measured amplitudes. With increase in distance from the source, attenuation of soil displacement were much more rapid than the inverse proportion to distance predicted by the theory. Frequency response measurements on the surface of natural soil deposits resulted in response curves different in shape to the response curve for the source foundation. The frequency response of the ground surface had several maxima whereas the response of the source had only one maximum. Earkon did not give a satisfactory explanation for this difference.

Warburton et al (1972) presented a comparison between free surface response determined from the theoretical solution given by Richardson et al (1971) and experimental results obtained by Palloks (1964). The vertical and horizontal components of the free surface displacement were considered in the comparison. A qualitative agreement between theory and test

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results was achieved but quantitatively considerable differences were observed.

MacCalden and Matthiesen (1973) presented results of field tests in which displacements were measured for a range of points on the soil surface responding to vertical and horizontal harmonic excitation of a nearby footing. The soil considered was described as a silty clay. Contrary to the findings of Barkan (1962), the authors found that the experimental response was greater than their theoretically determined response. Clemmet (1974) compared these test results with his theoretical solution which included material damping and found better agreement between predicted and measured response. 

### 2.6. COUPLED RESPONSE OF TWO FOUNDATIONS

A theory for the response of two, periodically excited, rigid, geometrically identical circular footings attached to the surface of an elastic halfspace was presented by Richardson (1969). He used the principle of superposition to combine the displacements of the free surface and the response of a single massless disc oscillating on the surface to obtain the response of the whole system when one of the footings was subjected to vertical harmonic excitation. It was shown that the presence of the receiving footing affected the primary response of the excited footing and introduced relatively small rocking and horizontal translational displacements. The response of the receiving footing was studied by considering/transmission ratio of the system which was defined as the ratio of the amplitude of a component of displacement of the second footing to the vertical amplitude of the excited footing. These ratios had maxima at frequencies associated with resonances of the receiving foundation. For vertical excitation of the first foundation, it was shown that the second footing underwent vertical, horizontal and rocking motions, and under certain

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circumstances, the peripheral amplitude of displacement of the second foundation exceeded the vertical displacement of the excited footing. The solutions for other forms of excitation of the first footing were presented by Warburton et al (1972).

Lee and Wesley (1973) extended Richardson's theory for two footings to cover any number of footings attached to a common elastic base. MacCalden and Matthiesen (1973) also employed a similar approach and developed equations for the motions of a passive foundation placed some distance away from an active foundation which was subjected to dynamic loads. The authors considered all four modes of excitation but neglected the influence of the passive foundation on the response of the active foundation in order to simplify the They presented the results of an experimental problem. investigation of the transmission of harmonic vibration from one circular footing to another geometrically identical foundation at 5.5m. away from the former. Both footings were of 1.22m. diameter and constructed in situ on the surface of a uniform silty clay deposit. The experimental results

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confirmed that vertical, excitation of the active foundation resulted in vertical, horizontal radial and rocking motions of the passive foundation and the horizontal and rocking motions were coupled. Vertical displacements of the passive foundation measured on the edge nearest to the active foundation were of the same order as those predicted but displacements measured at the far edge were about four times less than those predicted by the theory. As measurements were taken only at two points on the surface of the passive foundation, it was not possible to define clearly the response of the passive foundation.

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### CHAPTER III

## DEVELOPMENT OF EQUATIONS OF MOTION FOR COUPLED RESPONSE OF TWO EMBEDDED FOUNDATIONS

### 3.1. GENERAL

The experimental work described in Chapter VI investigated the dynamic response of two foundation pads embedded in a natural soil deposit when one of the foundations was excited by a vertical sinusoidal (with respect to time) force, the amplitude of which was proportional to the square of the frequency of excitation. A rigorous analytical investigation of the problem involving all the variables, especially , the variation of elastic properties of the soil with depth; is extremly difficult and is possible only with further development of the finite element techniques described in Section 2.2.1. However, Warburton et.al (1972) and MacCalden et al (1973) have derived approximate solutions for coupled vibrations of two rigid circular masses attached to the surface of an elastic halfspace, and this theory may be - 38 -

further extended to the case of two foundations embedded in an elastic halfspace. The effect of embedment can be included by the approximate analytical approach formulated by Barnov (1967) and developed by Novak and Beredugo (1972).

Figure 3.6 shows the active foundation-soil-passive foundation system considered. The foundation subjected to external dynamic load is referred to as the 'active foundation' and the foundation receiving vibrations through the ground is termed as the 'passive foundation'.

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### RESPONSE OF ACTIVE FOUNDATION

#### 3.2.1 Embedment in Elastic Halfspace

The external force and soil reactions acting on the active foundation in the absence of the passive foundation are shown in figure 3.1 The soil is treated as an ideal elastic halfspace. The equation of motion for steady state vibration is:

$$P(t) - R_{z}(t) - N_{z}(t) - MZ(t) = 0$$
(3.1)

in which

 $P(t) = P_{o} e^{iwt}$  is the complex vertical excitation.  ${\tt R}_{_{\rm Z}}$  (t ) - Dynamic vertical reaction at base of the foundation.

 $N_{\pi}$  (t) - Dynamic vertical reaction along the sides of the foundation.

In order to solve equation 3.1, the following assumptions are made:

- (i) The footing is a rigid body with a circular base of radius ra
- (ii) There is a perfect bond between the sides of - 40 -

3.2.



FIG.3.1. EXTERNAL FORCE AND SOIL REACTIONS ACTING ON EMBEDDED FOUNDATION.



FIG. 3.2 VARIATIONS OF STIFFNESS PARAMETERS C<sub>1</sub>,S<sub>1</sub> AND DAMPTNG PARAMETTRS C<sub>2</sub>,S<sub>2</sub> FOR HALFSPACE AND SIDE LAYER WITH FREQUENCY FACTOR a<sub>0</sub>

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the footing and the soil.

(iii) The soil underlying the foundation is an elastic halfspace and the overlying soil is an independent elastic stratum composed of a series of infinitesimally thin independent elastic layers.
This approach was first formulated by Barnov (1967) and later developed by Novak and Beredugo (1972).

Under these assumptions, the relationship between the displacement of the base of the foundation and the force acting on the surface of the elastic halfspace can be written as:

$$Z(t) = \frac{R_{z}(t)}{Gr_{a}} \frac{(f_{1} + if_{2})}{(3.2)}$$

in which:

G - Shear modulus of the underlying medium.
f<sub>1,2</sub> - are the well known displacement functions.
A description of f<sub>1</sub> and f<sub>2</sub> is given in Section 2.2. Expressions for f<sub>1</sub> and f<sub>2</sub> can

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be obtained from Bycroft (1956), Hsieh (1962) or Gladwell (1968).

and, ra - Radius of the foundation.

From 3.2,

$$R_{z}(t) = G^{r}a(C_{1} + C_{2}) Z(t)$$

(3.3)

in which:

$$C_1 = \frac{f_1}{f_1^2 + f_2^2}$$
 and  $C_2 = \frac{f_2}{f_1^2 + f_2^2}$  are the

dynamic compliance of the elastic halfspace. The dynamic reaction acting on the vertical faces of the foundation may be expressed as:

$$N_{z}(t) = \int_{0}^{h} s(t) dz \qquad (3,4)$$

in which: s(t) is the reaction per unit depth of embedment which is assumed to be independent of depth Expression for s(t) was formulated by Barnov (1967) as:

$$s(t) = G_{s} (S_{1} + iS_{2}) Z(t)$$
(3.5)

in which:

 $G_{s}$  - is the shear modulus of the soil adjacent - 43 -

to the vertical faces of the foundation. For  $\mathcal{X}$  foundation embedded in media which can be represented by an elastic halfspace  $G_s$  is equal to G.

$$S_{1} = 2 \quad a_{0} \quad \left\{ \frac{J_{1}(a_{0}) J_{0}(a_{0}) + Y_{1}(a_{0}) Y_{0}(a_{0})}{J_{0}^{2}(a_{0}) + Y_{0}^{2}(a_{0})} \right\}$$

$$(3.6)$$

in which:

a is the frequency factor given by  $a = \frac{wr_a}{v_s}$ 

 $S_2 = \frac{4}{J_0^2(a_0) + Y_0^2(a_0)}$ 

(3.7)

 $J_{o}(a_{o}), J_{1}(a_{o})$  - Bessel's functions of the first kind of order zero and one

respectively.

 $Y_0(a_0), Y_1(a_0)$  - Bessel's functions of the second kind of order zero and one res-

pectively.

Substituting for s(t) in equation 3.4,

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Total side reaction  $N_z(t) = \int_0^h G_s (S_1 + iS_2) Z(t) dz$ =  $G_s h (S_1 + iS_2) Z(t)$  (3.8)

Substitution of equations 3.3 and 3.8 into equation 3.1 yields,

$$P(t) - Gr_{a}(C_{1} + iC_{2}) Z(t) - G_{s}h (S_{1} + iS_{2}) Z(t) = MZ(t)$$
(3.9)

For steady state vibrations,  $\tilde{Z}(t) = -w^2 Z(t)$ . Therefore equation 3.9 can be written as;

$$P(t) - Gr_a(C_1 + iC_2) Z(t) - G_sh (S_1 + iS_2) Z(t) = -w^2 M Z(t)$$

From which,

$$Z(t) = \frac{P(t)}{G r_a \left[ C_1 + \frac{hG_s}{r_a G} S_1 - \frac{Mw^2}{r_a G} \right] + i \left[ C_2 + \frac{hG_s}{r_a G} S_2 \right]}$$
(3.10)

The real part of the complex displacement Z(t) is given by  $Z = Z_0 \cos(wt + \beta)$ , in which,  $Z_0$  is the real amplitude of displacement.

From equation 3.10,

$$Z_{o} = \frac{P_{o}}{Gr_{a} \left[ \left( (C_{1} + BS_{1}) - \frac{Mw}{Gr_{a}} \right)^{2} + (C_{2} + BS_{2})^{2} \right]^{\frac{1}{2}}}$$
(3.11)

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and the phase shift 
$$\oint = \tan^{-1} (C_2 + BS_2)$$
  
 $(C_1 + BS_2) - \frac{m tr^2}{Gr_a}$   
(3.12)

in which:  
B = 
$$\frac{hG_s}{F_sG}$$
 (3.12a)

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### 3.2.2 Embedment in an Elastic Stratum.

The expression for the amplitude of displacement of a foundation embedded in an elastic stratum underlain by a rigid base will be of the same form as the expression derived in Section 3.2.1 for the amplitude of displacement of a foundation embedded in an elastic halfspace. However, appropriate values of displacement functions  $f_1$  and  $f_2$  must be used to calculate the dynamic compliances  $C_1$  and  $C_2$ .

As described in Section 2.3., the displacement functions  $f_1$  and  $f_2$  for a foundation on a stratum depend on an additional parameter,  $H/r_a$ , the ratio of the thickness of the stratum to the radius of the foundation. Appropriate values of  $f_1$  and  $f_2$  may be obtained from Warburton (1956) or Novak and Beredugo (1972). For a deep stratum (i.e.  $\frac{H}{r_a} > 4$ ), however, the dynamic compliances  $C_1$  and  $C_2$  are nearly the same as those for an elastic halfspace provided that all the other parameters are the same in both cases.

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### 3.2.3 Simple Analogue for Active Foundation Motion

The equation of motion for the embedded foundation (i.e. equation 3.10) is similar to the equation of motion of a single degree of freedom system with spring factor k, expressed in the form

$$k = Gr_a \left( C_1 + \frac{hG_s}{r_a G} S_1 \right)$$
 (3.13)

and damping factor , expressed as,

$$c = (C_2 + \frac{hG_s}{r_a G} S_2) \qquad (3,14)$$
  
Using, B=(hG\_s)/(r\_a, G),  $a_0 = \frac{Wr}{V_s}$   
and  $V_s = (G/p)^{\frac{1}{2}}$ 

The expressions for k and c can be written as,

$$k = Gr_{a} (C_{1} + BS_{1})$$
 (3.13a)

and, 
$$c = (\rho G)^{\frac{1}{2}} r_a (C_2 + BS_2) / a_0 (3.14a)$$

The variations of  $C_1$ ,  $C_2$ ,  $S_1$  and  $S_2$  with the frequency factor  $a_0$  are given by Novak and Beredugo (1972) which are reproduced in Figure 3.2. It can be observed that  $C_1$  decreases and  $S_1$  increases with.  $a_0$ and  $C_2$  and  $S_2$  vary almost linearly -48-
with a o

In Figure 3.3, the variations of  $(C_1 + BS_1)$  and  $(C_2 + BS_2)$  with a are shown for different values of B.  $\frac{a_0}{a_0}$ It is evident that for a particular value of B, both

 $(C_1 + BS_1)$  and  $(C_2 + BS_2)$  vary very little with  $a_0$  in  $a_0$ the range 0.2  $< a_0 < 1.5$ .

Thus the spring factor k and the damping factor cin equations 3.13(a) and 3.14(a) can be approximated to constant values given by:

$$\mathbf{k} = \mathbf{Gr}_{\mathbf{a}} \cdot \mathbf{K}_{\mathbf{l}} \tag{3.15}$$

(3.16)

and,

in which:

 $K_1$  and  $K_2$  are the constant values obtained from Figure 3.3, for  $(C_1 + -BS_1)$  and  $(C_2 + BS_2)$  respectively.  $\frac{a_0}{a_0}$ 

 $c = (\rho G)^{\frac{1}{2}} r_a K_2$ 

Thus, the embedded active foundation-soil system can be represented by the single degree of freedom spring-mass-dashpot

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system shown in Figure 3.4.

A comparison of the frequency response of one of the active foundations considered in this investigation, computed using equation 3.11, with the frequency response computed using the simple spring-mass-dashpot analogue is shown in Figure 3.5.



В	K = (C + BS )	$K_{2}^{=}(C_{2}^{+BS})$
		a <sub>o</sub>
1/2	7.4	8.8
3/4	8.0	13.5
1	8.7	12.2
1.2	9.3	. 13.6

FIG.3.4a Single Degree of Freedom Analogue for Vertical Vibration of Active Foundation FIG.3.4b Average Values of Stiffness and Damping Parameters Obtained from Fig.3.3.



#### 3.3 RESPONSE OF TWO FOUNDATIONS SYSTEM

# 3.3.1 Equations of Motion

The external force and soil reactions acting on the two-foundations system are shown in Figure 3.6. All the simplyfing assumptions considered in Section 3.2 for a single foundation system apply for each of the two foundations considered in fig.3.6.



In fig.3.6, the location of the active foundation is referred to as Position (A) and the location of the passive foundation is referred to as Position (B). For the forces and reactions shown in fig.3.6, the first subscripts 'a' and 'b' denote that the variable is related to the active foundation or the passive foundation respectively. The second subscripts v, H,  $\emptyset$  indicate the mode of vibration as vertical, horizontal, radial and rocking, respectively.

- $P(t) = P_o e^{iwt}$  is the complex harmonic excitation in the vertical direction.
- $R_{av}$  (t) Soil reaction at the base of the active foundation.
- $N_{av}$  (t) Soil reaction along the sides of the active foundation.

 $R_{pv}$  (t),  $R_{pH}$  ,  $M_{p}\phi$  - Soil reactions at the base of the passive foundation.

Z<sub>a</sub> (t) - Complex displacement of the active foundation in the vertical direction.

 $Z_p$  (t), (  $U_p(t)$  - Complex displacements of the passive foundation in the vertical and radial

directions respectively.

p (t) - Rotation of the passive foundation about an axis passing through the centre of the base of the foundation.

 $M_a, r_a$  - Mass and radius of the active foundation. and,  $M_p, r_p$  - Mass and radius of the passive foundation.

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Under the assumptions described in Section3.2, equation of motion for vertical vibration of the active foundation is given by :

$$P(t) - R_{av}(t) - N_{av}(t) = M_{a} Z_{a}$$
 (3.19)

The vertical motion of the active foundation induces vertical and horizontal(radial) motions of the elastic halfspace, and, in consequence, gives rise to vertical, radial and rocking motions of the passive foundation which, in turn, induces some rocking and radial motions of the active foundation.

Considering vertical motions of the passive foundation, the equation of motion is given by,

$$O - R_{pv}(t) - N_{pv}(t) = M_{p} \tilde{Z}_{p}(t)$$
 (3.20)

For steady state harmonic excitation:

$$\ddot{Z}_{p}(t) = -w^{2} Z_{p}(t)$$
  
 $\ddot{Z}_{a}(t) = -w^{2} Z_{a}(t)$ 
  
(3.21)

The displacement  $Z_p$  of the passive foundation located at position'B'is determined by the superposition of the displacements caused by the surface forces at position'B' (i.e.  $Z_{po}$ ) and the free field displacement

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at (B) caused by the forces acting on the surface beneath the active pad.

i.e. 
$$Z_{p} = Z_{po} + Z_{pg}$$
 (3.22)

Similarly,

 $Z_{a} = Z_{a0} + Z_{ag}$  (3.23)

whene 
$$Z_{ao}$$
 - Displacement of the active pad due  
to surface forces at (A).  
and,  $Z_{ag}$  - Displacement of the active pad due

to surface forces at (B).

The solution can be considerably simplified if the influence of the passive pad on the displacement of the active foundation is ignored.

The  $Z_{pg}$  in equation 3.22 is a free field displacement at (B) due to the surface forces at (A) and it does not give rise to any soil reaction at(B).

Thus, the soil reaction acting on the passive pad are associated only with  $\rm Z_{po}$  and may be expressed by:

$$R_{p}(t) = G_{r_{p}}(C_{1} + iC_{2}) Z_{po}(t)$$
 (3.24)

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Where, r is the equivalent radius of the base of the passive foundation.

The soil reaction along the sides of the passive foundation is given by, \_

$$N_{p}(t) = G_{s} h_{p} (S_{1} + iS_{2}) Z_{po}(t)$$
 (3.25)

in which,

h - depth of embedment of passive foundation,

G<sub>s</sub> - Shear modulus of soil adjacent to the sides of the foundation.

Substituting for  $R_{pv}(t)$ ,  $N_{pv}(t)$  and  $Z_p^{*}(t)$  from equations 3.24, 3.25 and 3.21, equation 3.20 can be written as:

 $\operatorname{Gr}_{p}(\operatorname{C}_{1} + \operatorname{iC}_{2}) \operatorname{Z}_{po}(t) + \operatorname{G}_{s}\operatorname{h}_{p}(\operatorname{S}_{1} + \operatorname{iS}_{2}) \operatorname{Z}_{po}(t) = \operatorname{w}^{2} \operatorname{M}_{p} \operatorname{Z}_{p}(t)$ 

i.e. 
$$Gr_{p}(C_{1} + iC_{2})(Z_{p}(t) - Z_{pg}(t)) + G_{s}h_{p}(S_{1} + iS_{2})(Z_{p}(t) - Z_{pg}(t))$$
  
=  $w^{2}M_{p}Z_{p}(t)$  (3.26)

Using 
$$B_p = \frac{G_s h_p}{G r_p}$$
,  $a_p = wr_p (\rho/G)^{\frac{1}{2}}$ ,  
 $b_p = -\frac{M_p}{\rho r_p}$  and

rearranging the terms of equation 3.26, an expression for the vertical displacement of the passive foundation , in terms of

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the free field displacement at the location of the passive foundation ( $\rm Z_{pg}(t))$  , can be written as:

$$Z_{p}(t) = \frac{\left[ (C_{1} + B_{p}S_{1}) + i (C_{2} + B_{p}S_{2}) \right] \cdot Z_{pg}(t)}{(C_{1} + B_{p}S_{1} - b_{p}a_{p}^{2}) + i (C_{2} + B_{p}S_{2})}$$
(3.27)

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## 3.3.2 Expression for Surface Displacements.

An expression for the free field vertical displacement  $\mathbf{Z}_{pg}$  in terms of the surface force beneath the active pad can be extracted from the solutions for wave propagation given by Bycroft (1956), Barkan (1962) and MacCalden et al (1974).

In current notation,  

$$Z_{pg} = \frac{P_{as}}{Gr_{a} d^{\frac{1}{2}}} \left( \begin{array}{c} < \\ \\ \\ \\ \end{array} \right) \frac{k^{2}}{8 f'(X)} \left( \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \right) \frac{1}{2} e^{i(wt - X_{1}d - \pi/4)} \\ (3.28)$$

where:

- Centre to centre distance of the passive pad. ' d  $\underline{\mathbf{k}} = \frac{\mathbf{W}}{\mathbf{V}}$  where  $\mathbf{V}_{\mathbf{s}}$  - velocity of shear wave.  $f^{1}(x_{1}) = df(x)$  at  $x = x_{1}$ dx  $f(x) = (x^2 - 1) k^2 - \gamma k^2$ 

frequency equation.

is the Raleigh

 $\boldsymbol{\prec} = (\mathbf{x}^2 - \underline{\mathbf{h}}^2)^{\frac{1}{2}} \qquad \boldsymbol{\triangleleft} = (\mathbf{x}_1^2 - \underline{\mathbf{h}}^2)^{\frac{1}{2}}$  $\boldsymbol{\beta} = (x^2 - \underline{k}^2)^{\frac{1}{2}} \qquad \boldsymbol{\beta} = (x_1^2 - \underline{k}^2)^{\frac{1}{2}}$  $h = W / V_P$ 

 $X_1$  - is a root of the Raleigh frequency equation

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and is usually expressed as a specified constant times  $\frac{\mathbf{k}}{\mathbf{k}}$ . The value of the specified constant depends on the poisson's ratio of the halfspace.

$$P_{as} e^{iWt} = -R_{av}(t) \qquad (3.29)$$

From equation 3.19

$$P(t) = R_{a}(t) \rightarrow N_{a}(t) - w^{2}M_{a}Z_{a}(t)$$
 (3.30)

From equation 3.3

$$Z_{a}(t) = \frac{R_{a}(t)}{Gra(C_{1a} + iC_{2a})} \qquad (3.31)$$
(3.31)
(3.31)
(3.32)

compliances for the active pad.

From equation 3.8 and 3.3  $N_{a}(t) = \frac{G_{s} h (S_{1a} + S_{2a})}{G_{r_{a}} (C_{1a} + iC_{2a})} R_{a}(t) \qquad (3.32)$ 

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By substituting equations 3.29, 3.31 and 3.32 into equation 3.30 and rearranging the terms,

$$P_{as} = \frac{-P_{o}(C_{1} + iC_{2})}{\left[(C_{1a} + iC_{2a}) - \frac{Mw^{2}}{Gr_{a}} + \frac{G_{s}h_{a}}{Gr_{a}}(S_{1a} + iS_{2a})\right]}$$
(3.33)

Substitution of equation 3.33 into equation 3.30 gives the complex vertical displacement of the surface beneath the passive foundation as:

$$Z_{pg} = \frac{-\underline{k}^{2} \operatorname{Sin}(X_{1}r_{a})}{8G r_{a}d^{\frac{1}{2}} f'(X)} \left[ \frac{2}{\Pi X_{1}} \right]^{\frac{1}{2}} \left[ (C_{1a} + iC_{2a}) - \frac{Mw}{Gr_{a}}^{2} + \frac{C_{3}h_{a}}{Gr_{a}} S_{1a} + iS_{2a} \right]$$
(3.34)

# 3.3.3 Displacement of the Passive Foundation.

Introducing:

$$\overline{K} = \frac{\underline{k}^2 \operatorname{Sin}(X_1 r_a)}{8 \ \mathrm{G} \ r_a(\mathrm{d})^{\frac{1}{2}} \ \mathrm{f}^{\,\prime}(X)} \left[ \frac{2}{\Pi X_1} \right]^{\frac{1}{2}} \cdot \prec_1$$

$$B_a = (\mathrm{G}_{\mathrm{sh}_a})/\mathrm{Gr}_a \quad \text{and} \quad a_o = \frac{\operatorname{wr}_a(\boldsymbol{\rho})^{\frac{1}{2}}}{(\mathrm{G})^{\frac{1}{2}}}$$
and substituting equation 3.34 into equation 3.27

the complex vertical displacement of the passive pad is:

$${}^{Z}_{p} = \frac{\left[ \left( {}^{C}_{1p} + {}^{B}_{p} {}^{S}_{1p} \right) + i \left( {}^{C}_{2p} + {}^{B}_{p} {}^{S}_{2p} \right) \right] \cdot \overline{k} \cdot P_{o} \left( {}^{C}_{1a} + i {}^{C}_{2a} \right) e^{i \left( wt - X_{1} d - \frac{\pi}{4} \right)}}{\left[ \left( {}^{C}_{1p} + {}^{B}_{p} {}^{S}_{1p} - {}^{b}_{p} {}^{a}_{p} \right) + i \left( {}^{C}_{2p} + {}^{B}_{p} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{a}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{a}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{a}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{a}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{a}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{a}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{a}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{S}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{S}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{b}_{a} {}^{S}_{o} \right) + i \left( {}^{C}_{2a} + {}^{B}_{a} {}^{S}_{2p} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{C}_{a} {}^{S}_{a} \right) + i \left( {}^{C}_{a} {}^{S}_{a} + {}^{C}_{a} {}^{S}_{a} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} - {}^{C}_{a} {}^{S}_{a} \right) + i \left( {}^{C}_{a} {}^{S}_{a} + {}^{C}_{a} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} \right] \right] \left[ \left( {}^{C}_{1a} + {}^{B}_{a} {}^{S}_{1a} \right) + i \left( {}^{C}_{a} {}^{S}_{a} + {}^{C}_{a} {}^{S}_{a} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{C}_{a} {}^{S}_{a} \right] \right] \left[ \left( {}^{C}_{1a} + {}^{C}_{a} {}^{S}_{a} \right) \right] \left[ \left( {}^{C}_{1a} + {}^{C}_{a}$$

(3.35)

The real part of the expression on the righthand side of equation 3.35 will give the real displacement of the passive pad. The amplitude of displacement is inversely proportional to the square root of R, the distance from the centre of the active pad. Displacement of different points on the passive pad predicted by equation 3.35) will vary

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according to  $\frac{1}{d^2}$  and are not compatible with the rigid body motion assumed. However, the displacement at the centre of the pad can be taken as the average displacement of the pad when the dimensions of the pad are small compared to the distance of the pad from the active footing. For larger pads an averaging procedure given by Richardson et al (1972) can be employed to obtain the average displacement of the passive pad.

# 3.4. <u>SIMPLIFIED ANALYSIS</u>

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From equation 3.27 , the vertical displacement of the passive pad in terms of the free field displacement  $Z_{pg}(t)$  can be written as:

$$Z_{p} = \frac{(A_{1} + iA_{2})}{(A_{1} - b_{p}; a_{p}^{2}) + iA_{2}} Z_{pg}^{(t)}$$
(3.36)

here 
$$A_1 = (C_1 + \frac{G_sh_p}{G_r}S_1)$$

and, 
$$A_2 = (C_2 + \frac{G_s h_p}{G_r} S_2)$$

The real part of  $Z_p$  is given by  $Z_0 \cos(wt + \phi)$ where  $Z_0$  is the real amplitude and  $\phi$  is the phase difference between ground

motion and passive pad motion.

$$Z_{0} = \frac{(A_{1}^{2} + A_{1}^{2})^{\frac{1}{2}} Z_{pg0}}{\left[(A_{1} - \frac{M_{p}}{G} \frac{w^{2}}{r_{p}})^{2} + A_{2}^{2}\right]^{\frac{1}{2}}}$$

where, Z - real amplitude of ground motion.

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 $Z_{o} = \frac{\left[ (Gr_{p}A_{1})^{2} + (Gr_{p}A_{2})^{2} \right]^{\frac{1}{2}} Z_{pgo}}{\left[ (Gr_{p}A_{1} - Mw^{2})^{2} + (Gr_{p}A_{2})^{2} \right]^{\frac{1}{2}}}$ (3.37)

Equation 3.37 is identical to the solution for the displacement of a single degree of freedom mass-spring-dashpot system excited by a support displacement  $Z_{pg}$ . The spring and damping parameters of such a system would be,

spring constant 
$$k_p = G r_p A_1$$
  
=  $G r_p (C_1 + \frac{G_s h_p}{G r_p} S_1)$  (3.38)

damping constant

$$c_{p} = G r_{p} A_{1} / w$$
  
=  $\frac{G r_{p}}{W} (C_{2} + \frac{G_{s} h_{p}}{G r_{p}} S_{2})$   
=  $(\gamma G)^{\frac{1}{2}} r_{p}^{2} \cdot \frac{1}{a_{0}} (C_{2} + \frac{G_{s} h_{p}}{G r_{p}} S_{2})$   
(3.39)

Introducing  $B_p = \frac{G_s h_p}{G_r r_p}$ , equations 3.38 and

3.39 can be written as:

$$k_p = G r_p (C_1 + B_p S_1)$$
(3.38(a))

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and,

$$c_{p} = (\rho^{c}G)^{\frac{1}{2}} r_{p}^{2} (\frac{c_{2} + B_{p}S_{2}}{a_{0}})$$
 (3.39(b))

Although  $C_1$ ,  $C_2$ ,  $S_1$  and  $S_2$  are functions of the frequency factor  $a_0$ , Figure 3.3 shows that  $(C_1 + B \cdot S_1)$  and  $(C_2 + B \cdot S_2)$  vary very little with  $a_0$  within the

frequency range 0.2  $< a_0 < 1.5$ .

Thus, the simple spring-mass-dashpot system shown in Figure 3.7 can be used as a simple model of the passive foundation-soil system.

With reference to Figure 3.7, the displacement amplitude of the passive foundation is given by,

$$^{Z}_{o} = \frac{(k_{p}^{2} + c_{p}^{2} w^{2})^{\frac{1}{2}} Z_{pgo}}{\left[(k_{p}^{2} - M_{p} w^{2})^{2} + c_{p}^{2} w^{2}\right]^{\frac{1}{2}}}$$
(3.40)

Equation (3.40), together with free field displacement at the location of the passive foundation, measured in the absence of the passive foundation, can be used to calculate the displacements of the passive foundation.

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#### CHAPTER IV

#### DESCRIPTION OF EQUIPMENT AND INSTRUMENTATION

#### 4.1. GENERAL

Experimental investigations into dynamic characteristics of foundations and soils can be approached in two ways. Experiments can be performed either with steady state, sinusoidal excitation for which a purpose built vibration generator has to be acquired, or with transient excitation which can be developed by relatively simple arrangements such as blast, falling weights, etc. For steady state excitation the interpretation of data will be a simple process, whereas, rigorous analysis of data, often in the frequency domain, will be necessary to obtain the required information from tests conducted with transient excitation. The quality of data obtained from either approach will depend on the efficiency of the measuring instruments. The first approach was adopted for the current investigation.

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### 4.2. EXCITATION EQUIPMENT

The excitation equipment used in the current study consisted of a 'rotating mass type' vibrator, a diesel engine to drive the vibrator and ancillary equipment for controlling the operating speed and the dynamic load developed by the vibrator. The prime mover and the ancillary equipment were mounted on a mobile test rig. The vibrator was rigidly attached to this test rig only when it was moved from place to place.

## 4.2.1 Vibrator:

The basic vibrating unit of the excitation equipment was a mechanical vibrator in which the excitation force was derived from the rotation of out of balance shafts. The force developed was vertically oriented and was proportional to the square of the speed of rotation of the shafts. The vibrator was originally designed by Morgan (1974) as a part of a 'vibrating table apparatus'. Only a brief description of the vibrator is given in this section as full details of the design and construction of this unit can be found in Morgan (1974).

Plate 1 shows a view of the vibrator assembled and ready for testing. Figure 4.1 is an end elevation drawing of the vibrator and Figure 4.2 is a transverse cross section. An isometric view of the vibrator and the drive layout is given in Figure 4.3. With reference to these figures, the vibrator consisted of a box frame of welded construction (1) with an access panel (2) made of transparent perspex bolted to one of the vertical faces. The box frame carried eight heavy duty taper roller bearings (7) in which the vibrator

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shafts (3), (4), (5) and (6) rotated. The excitation force was derived from the eccentric masses (8), which were bolted to flats machined on the vibrator shafts. The upper pair of rotating shafts (i.e. shafts (3) and (4) in Fig. 4.3) were driven from one side of the box by the pulley (9) through the gear cluster (10). The lower pair of rotating shafts was driven from the other side by the pulley (11) and the gear cluster (12). By this arrangement it was possible to drive each pair of shafts independently of the other pair of shafts. Lubrication for the gears and bearings was provided by an oil splash bath in the bottom of the vibrator box.

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The two pairs of shafts were geared to rotate in opposite directions so that the horizontal component of the centrifugal force developed by one pair of rotating shafts cancelled out the corresponding horizontal component developed by the other pair of shafts. Thus, the force generated by the vibrator was vertically oriented and the force-time history was sinusoidal. The excitation force was proportional to the eccentric mass and to the square of the speed of rotation of the shafts. The capacity of the bearings were such that the

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maximum dynamic load that could be generated by the vibrator was limited to  $\pm 53$  kN -

#### 4.2.2 Drive Arrangement.

Figure 4.3 shows an isometric view of the drive layout. The selection of a suitable prime mover depended on the power required to drive the vibrator over the desired frequency range. Calculation of the power necessary to drive the vibrator was hampered by lack of data regarding power losses in the transmission, the bearings and in rotating the gear clusters through the lubrication bath inside the vibrator box. In order to get some information on the power requirement, the vibrator was driven in the laboratory by a three phase 3.73 kw (5 H.P.) motor at different speeds up to 1400 r.p.m. and the power consumed for the drive was measured. During these trial runs, oils of different viscosities were tested as the lubricating fluid in the splash bath, to find a suitable lubricant that ensured a reasonable working life for the bearings and gear clusters with a minimum of power less.

These measurements indicated that the power required increased with increase in operating speed and a light oil (Tellus 23) could be used as the lubricant instead of the heavy motor oils. Extrapolation of results revealed that a prime

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mover capable of producing at least 6 kw would be required to drive the vibrator over the proposed speed range. As electric motors of this capacity required a three phase power supply which was not available at the test site, it was decided to make use of a two cylinder diesel engine, that was available in the laboratory, as the prime mover for the excitation equipment. The maximum power output available from this engine was 8.2 kw and the BHP rating was 6.72 kw (9 H.P.). The recommended operating speed of the engine was 1500 r.p.m. A serious disadvantage in using the diesel engine as the prime mover was that it developed considerable vibrations during operation which interfered with the dynamic loads generated by the vibrator. The procedure adopted to overcome this problem is described in Section 5.2.5.

An important requirement of the drive arrangement was a speed control device that should enable variations of the frequency excitation and steady maintenance of any particular frequency of excitation for a period long enough (about five minutes) to obtain permanent records of relevant data for subsequent analysis in the laboratory. The device which was

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eventually chosen for this purpose, was an infinitely variable hydraulic gear box commercially known as 'Carter' gear box. The maximum input speed for the hydraulic gear box (detail (17) in Fig. 4.3) was 1500 r.p.m. and it was possible to obtain any speed in the range 50 r.p.m. to 1440 r.p.m. at the output shaft. The variation of speed was achieved by the rotation of a valve (Plate 2) which controlled the amount of fluid displaced inside the gear box from the input compartment to the output compartment.

Power transmission from the diesel engine to the vibrator through the hydraulic gear box was achieved by the timing belt and pulley drives shown in Figure 4.3 and Plate 3. The two timing belts pulleys 9 and 11 (Fig. 4.3) on either side of the vibrator box driving the upper and lower pairs of vibrator shafts were driven by the timing belt drives 15 and 16, from a common transfer shaft (19). The common transfer shaft (Plate 3) was driven by the timing belt drive (14) from the output shaft (20) of the hydraulic gear box (Plate 4). The hydraulic gear box was driven by the main drive belt (13) from the diesel engine. As the maximum rotational speed of the output shaft of the

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hydraulic gear box was 1440 r.p.m., a 'step-up' arrangement in the timing belt drive(15,16) from the common transfer shaft to the vibrator, was adopted to give vibrator shaft speeds up to 2400 r.p.m. After the preliminary tests, the step-up ratio was increased to 2.5 to give vibrator shaft speeds ranging from 125 r.p.m. to 3600 r.p.m.

The prime mover and the drive arrangement were mounted on a 4.26m. long 1.22m. wide test rig fitted with a pair of wheels. The test rig (Plate 4) was a box frame divided into four compartments. The first three compartments (Fig. 4.4) supported the engine, the hydraulic gear box and the transfer shaft. These three components were mounted on slotted platforms so that they can be moved to and fro to give the required amount of tension for the timing belts. The fourth compartment of the test rig was provided to carry the vibrator box during transport from the laboratory to the test site. When the tests were in progress, the vibrator was not supported by the test rig, but stood clear of the test rig on a pedestal which was attached to the test foundation. In order to counteract the horizontal pull of the timing belts on the vibrator box, two

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dummy belts passing over two dummy shafts on the sides of the vibrator and a tensioning frame at the far end of the test rig were provided as shown in Figure 4.4. Both the driving belts and the dummy belts were of the same length and material and were initially tensioned to the same amount. Further details of the setting up procedure are given in Section 5.2.2.

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# 4.2.3 Control of Excitation Force.

The dynamic load generated by the vibrator could be altered by changing either the eccentric masses on the rotating shafts or the frequency of operation. The proposed testing programme included a series of tests where different excitation forces would be applied to a foundation at a constant frequency. To accomplish this by opening the vibrator box and changing the eccentric masses would have been a cumbersome and time consuming process which would have considerably slowed the proposed test schedule. The four shaft vibrator layout, however, lent itself to a simple and effective method of force amplitude control without having to alter the eccentric masses. いてきるいのないのであったい

The two timing belt pulleys on either side of the vibrator box driving the upper and lower pairs of shafts respectively, were driven by the common transfer shaft shown in Figure 4.3. With reference to Figure 4.3, the transfer shaft (19) was made of two half-shafts (21) and (22), joined by a variable coupling (23), which is shown in Plate 5. The variable coupling consisted of two discs each welded to one of the half-shafts. Eighteen equally spaced holes were drilled on a 15cm. diameter of one disc

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and twenty equally spaced holes were drilled on the other disc. In this arrangement, two diametrically opposite holes could be aligned on each disc in 180 different positions. Thus, by using two bolts to couple the disc and by appropriate indexing, the relative angular position of the two half-shafts could be adjusted in steps of 2<sup>0</sup>. Since timing belt drive could not slip, and, therefore, provided positive angular locations, a change in the position of the coupling on the transfer shaft resulted in a change in the relative angular position of the upper and lower pairs of rotating shafts inside the vibrator. The effect that this adjustment had on the net excitation force generated by the vibrator is illustrated in Figure 4.5, when the relative angular position of the upper and lower pairs of rotating shafts were set to some angle  $\theta$ as shown in Figure 4.5, a relative phase difference  $\theta$ introduced between the vertical forces generated by each pair of shafts. The net force generated by the vibrator at some frequency w was the vectorial sum of the vertical components of the centrifugal force developed by each pair of shafts and was equal to:

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FIG4.5 ADJUSTMENT OF AMPLITUDE OF DYNAMIC LOAD DEVELOPED BY VIBRATOR.

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$$P = 4 \text{mew}^2 \cos \theta/2 \cdot \cos(\text{wt} + \theta/2)$$
 (4.1)

which can be written as:

$$P = 4me_{E} \cdot w^{2} \cos(wt + \theta/2) \qquad (4.2.)$$

where:

P - net vertical force generated by the vibrator. m - eccentric mass on each shaft. e - radius of gyration of each eccentric mass. and,  $e_E = e \cos \theta/2$  is hereinafter referred to as the equivalent eccentricity of each

#### rotating mass.

Thus, the force developed by the vibrator was a time dependent sinusoidal force of amplitude 4 me w<sup>2</sup>Cos $\theta/2$ . Setting the angle  $\Theta$  at different values resulted in different force amplitudes at the same frequency of rotation. Therefore, the simple operation that had to be carried out to change the excitation force was to stop the machine and set the relative angular position of the two half-shafts to the desired value (Plate 5). The holes in the variable coupling were numbered to facilitate selection and the relative angular position of the two

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half-shafts could be adjusted in steps of 2° which, with a step-up pulley ratio of 2.5 between the common transfer shaft and the vibrator, would result in a change of angle  $\Theta$  in steps of 5°.

4.5.0

## 4.2.4 Trial Tests:

When the assembly of the excitation equipment was completed, a few trial tests were run to check its performance. Figure 4.6 shows the force developed by the vibrator when the relative phase difference between the upper and lower pairs of rotating shafts was set to 150°. The vibrator reacted against the weight of the test rig and rested on the pedestal shown in Figure 4.4. Load developed was measured by three equally spaced load cells at the base of pedestal. A few low frequency tests were also carried out with the vibrator reacting against its own weight to determine the suitable tension that should be applied to the timing belts and the dummy belts so that thore would be neither horizontal movement of the vibrator nor difference between the estimated and measured loads. By trial and error, it was found that if the initial tension in the belts was such that a 10 Kgf. load placed at the centre of the span produced a deflection not less than 14mm., the performance of the vibrator was satisfactory. This tension was less than the tension recommended by the timing belt manufacturers for maximum efficiency in power transmissions, but the diesel engine had enough excess

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AMPLITUDE OF VERTICAL FORCE DEVELOPED BY VIBRATOR

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power to accommodate this loss. A typical u.v. trace of the force-time history is shown in Figure 4.7.

The speed holding capacity of the hydraulic gear box was checked against measurements taken on the vibrator shaft using a calibrated revolution counter. It was found that after every change in speed, at least two minutes should be allowed for the output speed of the hydraulic gear box to stabilise. For output shaft speeds less than 1000 r.p.m., the hydraulic gear box held the speed constant within 10 r.p.m. for a period of 10 minutes. At high speeds, the output speed was held constant for a period of 4 minutes but a drop in speed by about 2% was observed when measurements were taken over a period of 10 minutes. As steady state vibration data monitored over a short period at any particular frequency would be adequate for analysis, the performance of the hydraulic gear box was considered satisfactory.

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# (b) WAVE-FORM OF VERTICAL LOAD AT 48 Hz FREQUENCY

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FIG. 4.7 TYPICAL WAVE-FORM OF FORCE DEVELOPED BY VIBRATOR: (a) AT LOW FREQUENCIES (b) AT HIGH FREQUENCIES

#### 4.3. INSTRUMENTATION

#### 4.3.1 Basic Requirements:

The instrumentation chain used in this investigation to monitor vibrations was assembled from commercially available units to meet several basic requirements dictated by the conditions at the test site and the capacity of other equipment. The first and the most important requirement was the ability to measure vibrations varying in intensity from 0.5g acceleration at the source of vibration to 0.0001g at some distance away from the source over a frequency range of 1 Hz to 100 Hz. A second requirement was that the system should provide for recording field data for future analysis in the laboratory. A third requirement was that it should be possible to obtain at least two simultaneous vibration measurements recorded on a common time base in order to verify phase relationships. As a final requirement, the system must be robust, portable and capable of being operated by batteries.

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# 4.3.2 Instrumentation Chain.

The basic set up of the vibration measurement system consisted of an accelerometer as the sensing transducer which converted mechanical motion into an electrical signal, a charge amplifier for signal conditioning and an F.M. magnetic tape recorder for signal recording and storage. Different combinations of accelerometers and charge amplifiers were used depending on the intensity of vibration monitored.

# (a) <u>Accelerometers</u>:

Piezo-electric accelerometers were used to monitor ground and foundation vibrations. In general, a typical piezoelectric accelerometer consists of two or more piezoelectric discs supporting a relatively heavy mass which is preloaded by astiff spring. The whole assembly is mounted on a metal housing with a rigid base, as shown in Figure 4.8. When the base is rigidly attached to a vibrating surface, the mass will exert a variable force on the piezo-electric discs and a variable charge proportional to the force and, therefore, the acceleration of the mass in the principal direction of motion, is

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FIG.4.8. BASIC CONSTRUCTION OF PIEZOELECTRIC ACCELEROMETERS

developed across the piezo-electric discs. The spring-mass system is usually designed for very high resonant frequency, and for frequencies much lower than the resonant frequency, the acceleration of the mass will be virtually the same as the acceleration of the base. Therefore the charge built up across the piezo-electric discs will be proportional to the acceleration to which the accelerometer is subjected. The sensitivity of an accelerometer is usually given in terms of the charge developed per unit acceleration of the base along the principal axis of the accelerometer. Most accelerometers are slightly sensitive to accelerations in a plane normal to the principal axis and this transverse sensitivity is usually expressed as a percentage of the principal axis sensitivity. A more detailed description of the different types of accelerometers and the advantages and disadvantages of their use in comparison with other types transducers can be found in Skipp (1977), Kuhen (1977) or Broch (1973).

Two different groups of piezo-electric accelerometers were used in the current investigation. The first groups of accelerometers were small and hence, less sensitive, AQ 40 accelerometers manufactured by Environmental Equipment Ltd.

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The nominal charge sensitivity of a typical AQ  $40^{-1}$ accelerometer was 40 pc/g in the principal direction and its transverse sensitivity was less than three per cent of the principal axis sensitivity. The recommended frequency range over which the accelerometer could be used was 2Hz to 10 kHz. In conjunction with the rest of the instrumentation chain, the lower limit of acceleration that could be measured by a typical AQ 40 accelerometer 0.005g. To measure low intensity vibrations B & K 8306 accelerometers (Plate 13) were used. Α typical 8306 accelerometer had a built in preamplifier which increased its nominal charge sensitivity in the principal direction to 10,000 pc/g. The transverse sensitivity of the B & K 8306 accelerometer was about five per cent of the main axis sensitivity. The lowest level of vibration that could be measured by this accelerometer was 10<sup>-5</sup>g but the maximum acceleration was limited to 1g. A third type of accelerometer used in this investigation was the B & K 4345 accelerometer which had a nominal charge sensitivity of the order of 100 pc/g.

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## (b) Signal Conditioning Preamplifiers.

As piezo-electric accelerometers are of high output impedance, they are normally used in conjunction with impedance matching preamplifiers which transform the high output impedance of the accelerometer to a lower value suitable for measuring or recording instruments. Also the preamplifiers amplify the relatively weak output signals from accelerometers so that less sensitive instruments can be used to measure the strength of the signals. The preamplifiers used in this investigation were charge amplifiers which were preferred over voltage amplifiers because they have negligible sensitivity to the length of the cable carrying the charge from the accelerometer. Furthermore, the charge amplifiers had a very small d.c. offset voltage compared to that of a voltage amplifier.

The AQ 40 accelerometers were normally used in conjunction with a modular QM series (Environmental Equipments) charge amplifier system which consisted of six modules, each of which houses a charge amplifier and a switched low pass filter (Plate 6). The low pass filter could be set in different positions varying from 25HZ to 1000HZ. A ten turn gain potentiometer was provided with each charge amplifier module which enabled gain variation

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from one to 12.5 mV/pC. A calibrating oscillator and a monitoring meter were built in so that each charge amplifier can be calibrated separately. The outputs from any charge amplifier could be monitored on the meter without interrupting measurement or recording with another instrument. The dynamic range of the charge amplifiers was one pC to 5000 pC(pk) and the frequency response was 1Hz to 100 kHz. The modular system was portable and could be operated from 245V A.C. or 28 to 32 volts D.C.power supplies. It was possible to make use of these preamplifiers for the 8306 acclerometer as well but the maximum accleration that could be monitored by this combi--nation was limited to 0.5g.

Apart from the QM series modular system, two, B & K 2635 type charge amplifiers (plate 6) were also used in the instrumentation. The type 2635 charge amplifier was a portable equipment with self contained battery supply. These charge amplifiers had built in active integration networks for conversion of vibration acceleration time history monitored by the accelerometers to either velocity or displacement time history. Also available was a high pass filter set at 1 Hz to reduce interference from signals -98 - developed by accelerometers in response to low frequency temperature fluctuations. Type 2635 charge amplifier consisted of a selectable gain giving outputs in 10 dB steps from 0.01  $mv/ms^2$  to 10,000  $mv/ms^2$ . It could be used over a wide range of frequencies from 0.1 Hz to 200 kHz in the acceleration mode and 1 Hz to 1000 Hz in the displacement mode. It was equipped with a push button activated test oscillator, from which output signals varying from 10 mv (pk) to 1 V (pk) at 160 Hz frequency could be obtained for calibration of other instruments.

# (c) Data Storage.

As the test programme included multi-directional measurements and simultaneous measurements of different quantities, online processing of data became difficult and it was necessary to store field data for later processing in the laboratory. The instrument used for data storage was an eight channel FM magnetic tape recorder (Plate 6). The maximum input voltage to each channel was limited to 1 V (pk) and a step down variable gain amplifier was provided of the input to ensure that it was not overloaded. An inhibit switch was provided with each channel

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which enabled recording on different channels at different times. It was also possible to transfer information from one channel to another channel without erasing the original. As the tape recorder was an FM recorder, it offered frequency linearity from D.C. The upper frequency limit was a function of the tape speed. The different tape speeds available were 15/16 in/sec,  $3\frac{3}{4}$  in/sec and 15 in/sec and the corresponding upper frequency limits were 312 Hz, 1.25 kHz and 5 kHz, respectively. The input impedance of the tape recorder was not sufficient for direct recording with piezo-electric accelerometers. The tape recorder was portable and could be powered by either 240 V A.C. or 32, 4 Ah capacity batteries. With this battery operation, the continuous running time was limited to four hours.

Figure 4.9 shows a line diagram of the different combinations of accelerometers and charge amplifiers used in this investigation. Also given in Figure 4.9 are the maximum and minimum levels of vibration that could be measured by each combination.

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(i) FOR MEASURMENT OF ACCELERATIONS IN THE RANGE 0.01 m/s<sup>2</sup> to 1.2 X 10<sup>3</sup> m/sec<sup>2</sup>



QM Charge Amplifier with

FM tape





recorder Low Pass Filter

. filter



AC Voltmeter



FIG 4.9 DETAILS OF DIFFERENT COMBINATIONS OF INSTRUMENTS USED FOR FIELD VIBRATION MEASUREMENT.

# 4.3.3 Calibration of Instruments.

#### (a) Calibration of Accelerometers.

The calibration of the different instruments described in Section 4.3.2, was carried out in two stages. Firstly, each item of the instrumentation chain was calibrated separately and then the entire chain of equipment was calibrated both in the laboratory and in the field.

Each acceleromoter was calibrated against a RA10M type reference accelerometer, the calibration of which is traceable to the British Calibration Service. An 18 kgF, purpose built, electrodynamic exciter (Plate 7) provided the necessary steady state harmonic excitation over a wide frequency range. The reference accelerometer was attached to the vibrating table of the exciter and the accelerometer to be calibrated was screwed on to the reference accelerometer with a threaded steel stud. The QM series charge amplifier (Section 4.3.2 (b)) which had a built in calibrator was used to condition the signals from the accelerometers and measurements were taken by a calibrated digital volt meter. Figure 4.10 shows the frequency response (i.e. the variation

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of charge sensitivity with frequency of excitation) of a typical AQ 40 accelerometer. It can be observed from Figure 4.10, that the variation of charge sensitivity is negligible within the frequency range 0 to 100 Hz. Therefore, the charge sensitivity at 30 Hz frequency was taken as the fundamental charge sensitivity of the accelerometer. A similar procedure was adopted for field calibrations but the small exciter shown in Flate11 provided excitation.

It is well known that the frequency response of an accelerometer is influenced by the method adopted for mounting the accelerometer to the vibrating surface. As multi-directional measurements were included in the proposed test programme, it was decided that accelerometers will be mounted on an aluminium block with threaded steel studs and the mounting block would be attached to the surface of the test foundation either by foundation bolts or with plastic steel (Plate 10). Hence it was necessary to calibrate the mounted natural frequency of the accelerometers, especially for the relatively heavy 8306 accelerometers, as the upper frequency limit for an accelerometer is usually taken as one third of the mounted natural frequency. The calibration process was repeated first with the aluminium block screwed to

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FIG. 4.10.b FREQUENCY RESPONSE CHARACTERISTICS OF 8306 ACCELEROMETER MOUNTED ON ALUMINIUM CUBE ATTACHED TO REFERENCE ACCELEROMETER.

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the reference accelerometer and the accelerometer to be calibrated mounted on top of the block. For the second series of calibrations, a metal plate was rigidly fixed to the reference accelerometer with counter sunk studs and the mounting block was attached to the metal plate with plastic steel. The second series of tests gave the lowest mounted natural frequency (730 HZ) which was about 13 times greater than the highest frequency of excitation (60 HZ) employed in the investigation. Figure 4.10(b) shows the frequency response of an 8306 accelerometer when the mounting block was rigidly attached to the reference accelerometer. For the AQ 40 accelerometers, mounted natural frequencies, being of the order of 6 kHz, were well above the highest frequency of excitation considered for the field tests.

The transverse sensitivity of each accelerometer was measured by attaching the accelerometer to the mounting block with the principal axis of the accelerometer perpendicular to the main axis of the calibration exciter. It was found that the transverse sensitivities of both types of accelerometers were negligible compared to the corresponding principal axis sensitivity. The maximum transverse sensitivity measured for an 8306 accelerometer

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was less than six per cent of its principal axis sensitivity and the maximum transverse sensitivity for an AQ 40 accelerometer was less than eight per cent of its principal axis sensitivity.

## (b) Calibration of Charge Amplifiers.

As mentioned in Section 4.3.2(b), the QM series charge amplifier system had a built in oscillator for the calibration of each charge amplifier module. Whenever the variable gain of a charge amplifier was altered to regulate the amplitude of the output signal, the gain setting was calibrated by injecting a standard 70 Hz signal into that module from the calibration oscillator and measuring the output of the module. The 'type 2635' charge amplifiers had fixed gain settings which were calibrated against a QM charge amplifier using a signal from the reference accelerometer as the common input. The output of the push button activated test oscillator built in the 'type 2635' charge amplifiers was used as a reference signal to calibrate the input gain of the tape recorder.

#### (c) Calibration of the Instrumentation Chain.

The entire chain of equipment (Fig. 4.9) was calibrated by - 107 -

subjecting each transducer to a known acceleration. Once again, the reference accelerometer was used to measure the input acceleration. The output given by the tape recorder was measured by a volt meter, and the charge sensitivity of the accelerometer was calculated by accounting for the input attenuation of the tape recorder and the amplification of the charge amplifier. This exercise was repeated in the field with the use of a portable electrodynamic exciter. In the field calibrations, only the fundamental charge sensitivity of each accelerometer was measured.

It was found that, for the AQ 40 accelerometer, the fundamental charge sensitivity was independent of the type of preamplifier used. For the 8306 accelerometers, slight variations ( <1% of nominal charge sensitivity) of charge sensitivity were obtained when they were used in conjunction with different modules of the QM charge amplifier system.

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## 4.4. COMMENTS

The instrumentation used for measuring vibrations proved to be satisfactory. The method of mounting accelerometers for foundation vibrations measurement in the frequency range 2 Hz to 60 Hz did not appear to be critical. Accelerometers on mounting blocks attached to the surface of a foundation pad either by plastic steel or by foundation bolts gave the same results as the accelerometers on mounting blocks cast as integral parts of the foundation pad. For field vibration measurement, it appeared that low pass filters are a necessity as spurious signals developed frequently. The accelerometers proved to be fairly robust and durable. In general, the charge sensitivities of the accelerometers decreased by about six per cent over a period of twelve months. Improper grounding of instrumentation chain and short circuiting of the miniature coaxial cables carrying the accelerometer output caused considerable difficulties during the early stages of the investigation, when due to inclement weather, considerable surface water covered the site. These problems were overcome by the use of electrically isolated studs for mounting the accelerometers

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and by providing a waterproof seal at the accelerometer output sockets.

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PLATE 2 HYDRAULIC GEAR BOX FOR ADJUSTMENT OF OPERATING SPEED.



PLATE3 TRANSFER SHAFT AND TIMING BELT DRIVE ARRANGEMENT.



PLATE 4 TEST RIG



VARIABLE COUPLING FOR ADJUSTMENT OF VIBRATOR EXCITATION FORCE PLATE 5



PLATE 6 ELEMENTS OF VIBRATION MEASUREMENT SYSTEM.



#### CHAPTER V

#### TESTING PROGRAMME AND TEST PREPARATIONS

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#### 5.1. TESTING PROGRAMME

The main objective of the experimental investigation is to study the dynamic response of a foundation located some distance from a source of vibration. It was proposed that the source of vibration would be a foundation pad subjected to steady state, time dependent, sinusoidal force excitation derived from the vibrator described in Chapter IV. The foundations would be fully embedded, cast in situ and their bases would be either rectangular or square sections. The mass ratios of the foundations would be comparable to those of machine or structural foundation pads.

Figure 5.1 shows the dimensions and locations of the three foundations tested. For the sake of clarity, a foundation is hereafter described as 'active foundation ' when it is subjected to excitation by the vibrator and 'passive foundation' when it is excited by vibrations transmitted through the soil from another active foundation. The test programe was designed to obtain the following

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information:

- (i) Frequency response of active foundation A before casting either foundation B or foundation C;
- (ii) Frequency response of ground surface at five different positions;
- (iii) Frequency response of active foundation A with foundation B in position;
  - (iv) Frequency response of passive foundation B when it was fully embedded;
  - (v) Frequency response of both active foundation A and passive foundation B for three different mass ratios of foundation A;
  - (vi) Frequency response of passive foundation B after removing the soil surrounding the sides of the footing in steps;
- (vii) Frequency response of passive foundation C;
- (viii) Frequency response of passive foundation B after installing foundation C;
  - (ix) Frequency response of active foundation B;
  - (x) Frequency response of passive foundation A;
  - (xi) Frequency response of active foundation B with the vibrator set at four different levels of excitation;

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(xii) Displacements of active foundation A at three different frequencies of excitation with the vibrator set at seven different levels of excitation.

A brief description of the different tests that were performed to collect the required data are given in Chapter V with the test results.

### 5.2. TEST PREPARATIONS

#### 5.2.1 Construction of Test Foundations.

The site of the tests was an undeveloped open field approximately 10 km south-west of Nottingham. The first test foundation constructed on this site was foundation A, the dimensions of which are shown in Figure (5.1). After clearing vegetation and top soil over a 4m by 3m testing area, an excavation of size 0.56m by 0.56m was taken out to a depth of 0.56m in the The base of the excavation middle of the testing area. and the vertical faces on all four sides up to a height of 150mm were carefully trimmed with a trowel to bring the cross section at the base to 0.61m x 0.61m and the total depth of the hole to 0.61m. The base was levelled and steel arrows were pushed into the ground to mark off an area of 0.46m x 0.46m about the centre of the base. Concrete was poured in to form the first 150mm of foundation A. A mould of internal section 0.46m by 0.46m was then lowered onto the wet concrete guided by the arrows, and an assembly of four 200mm long, 12mm diameter foundation bolts tied by a 3mm thick steel plate at 50mm from the 122 -

threaded end of the bolts was suspended from a bar running across the mould in such a way that the centre of the steel plate was vertically above the centre of the base of the excavation. Concrete was then poured into the mould to complete the foundation, and the surface levelled flush with the aforementioned steel plate. About 50mm length of each foundation bolt was left protruding above the plate in order to attach the exciter described in Section (4.2). Immediately after pouring, 130mm long M12 bolts with accelerometer mounting blocks screwed to their threaded ends were inserted into the wet concrete at positions about 100mm from the edges on the mould in such a way that the underside of each accelerometer mounting block was in contact with surface of the concrete.

The bar holding the foundation bolts and the steel plate was removed about 20 minutes after pouring was completed, and the level of the steel plate and the mounting blocks were checked with a spirit level and adjusted accordingly. The mould was removed after allowing sufficient time for the concrete to set and the gap between the sides of the excavation and concrete was filled with soil taken out of the excavation. The backfill was tamped uniformly around the sides of the foundation by a steel disc. The density of the backfill measured

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at the end of the testing programme was 85% of the density of the undisturbed soil which was in contact with the bottom 150mm of foundation A.

The second foundation (i.e. foundation B in Figure 5.1) was formed by pouring concrete directly into an excavation taken out to the required dimensions. As foundation B was larger and thicker than foundation A, a wire mesh reinforcement was provided and it was proposed that four, 12mm diameter screw rods running the entire depth of the foundation would be used to attach the vibrator. The screw rods were tied at the base and at about 50mm from the top ends by 3mm thick steel plates, and the whole assembly was suspended from a bar supported on two pegs which were driven into the ground at equal distance from the centre of the excavation. When pouring concrete into the excavation was completed, the bar and the upper steel plate were removed and an accelerometer mounting block with M12 bolts attached to its base was partially embedded in the wet concrete at the centre of the base. A few accelerometer mounting blocks were placed at different points each at least 100mm from the nearest edge of the foundation.

The third foundation (i.e. foundation C in Figure (5.1) was a

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partially embedded foundation which was cast after completing a series of tests involving the first two foundations. The embedded part of foundation C was cast directly into an excavation as in the case of foundation B and the exposed part was cast using a mould. As there was no plan to excite this footing by the vibrator, there was no necessity to locate and mark the centre of the base on the top surface. In order to ensure that the dynamic response of each foundation would correspond to that of a rigid footing, no foundation was subjected to dynamic tests within 28 days after casting.

## 5.2.2 Setting up of Vibrator on Foundations.

The mechanical vibrator (Plate 1) was attached to the foundations through the pedestal shown in Plate 8. The pedestal consisted of a 20mm thick base plate which was of the same dimensions as the  $\frac{1}{2}$ " thick steel seating on the surface of The base plate had four 12mm drilled holes aligned foundation A. with the holes in the steel seating on the foundation through which the foundation bolts passed. In order to ensure correct alignment the holes had been drilled with the base plate and the steel seating clamped together. The pedestal was first lowered centrally on to the seating guided by the protruding foundation bolts and locked in place by four pairs of lock nuts. The test rig with the vibrator suspended on the frame was towed to the testing area and positioned such that the vibrator was directly The weight of the test rig was then taken up over the pedestal. by six scissor jacks welded to the underside of the test rig and the wheels were removed. By manipulating the six jacks, the vibrator was lowered on to the pedestal, guided by two angle brackets, the horizontal legs of which were attached to the top surface of the pedestal. When the vibrator was about 20mm above

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the pedestal, rollers were introduced beneath the jacks and the vibrator was moved until the three tapped holes on either side of the vibrator box were aligned with three similar holes in the vertical legs of the angle brackets. The jacks were lowered and the vibrator was locked in place on the pedestal by three studs. The bolts holding the vibrator on the frame of the rig were removed and the rig was further lowered so that the vibrator stood clear of the frame. The angle brackets, the tapped holes on the sides of the vibrator and the pedestal had been carefully machined to ensure that the geometric centre of the underside of the vibrator box was vertically above the centre of the base plate of the pedestal. Any eccentricity of the dynamic load applied to the foundation was thus limited to tolerances allowed in machining the components of the vibrator and error in the vertical alignment between the centre of the top surface of the footing and the centre of the base of the footing.

After setting up the vibrator on the foundation, vibration isolation mountings were introduced between the jacks supporting the test rig and the ground to minimise the transfer of vibrations generated by the engine and other rotating parts of the drive into

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the ground. The isolation mountings (Plate 9) were made of bonded 'square grip' pads which were designed to give a resonant frequency of 11Hz for the test rig supports and thereby achieve 60% vibration isolation when the engine was operated at 1500 r.p.m. The efficiency of these mountings were checked during the preliminary tests described in Section 5.2.4.

## 5.2.3 Method of Attachment of Accelerometers.

To measure the acceleration time histories of the test .foundations, accelerometers were mounted on aluminium blocks attached to the foundation (Plate 10). The aluminium blocks which were cubical blocks of side 63mm were either cast as integral parts of the foundations or screwed to threaded ends of M12 bolts buried in the foundation and tightened against the concrete surface. Trial measurements (Plate 11) showed no difference in the acclerations measu--red using either form of mounting. For approximate measure--ments at points where foundation bolts were not provided. accelerometer mounting blocks were attached to the found--ation by plastic steel or a cynoacrylate adhesive. The accelerometers were rigidly attached to the mounting blocks by threaded steel studs. Drilled and tapped holes were provided on each face of the accelerometer mounting blocks for multi-directional measurements (Plate 10).

Previous reseachers (Kuhen (1977), Skipp (1977) have found that for vibration measurements on rigid surfaces such as concrete, the method of mounting is not critical provided that the frequency of the signal measured was less than one fifth of

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the mounted natural frequency of the accelerometer. The maximum frequency of excitation considered in this inves--tigation was twelve times greater than the lowest mounted charge sensitivity measured during calibrations of differ--ent accelerometers with different types of mountings (Section 4.3.3.a)

Unlike foundation Vibration measurements, ground vibration measurements were sensitive to the method adopted for attaching the accelerometer to the ground surface. The effect of ground coupling variations on the response of an accelerometer placed on the surface of the ground has been noted by several previous researchers (Skipp, 1977; Warburton, 1965) and various methods have been suggested to obtain 'true' ground motion.

Ideally, the measurement of ground vibrations using any type of transducer should fulfil two requirements. The placement of the transducer should not alter the free field motion of the ground and the response of the base of the transducer should be the same as the free field motions. In practice, both conditions are impossible to fulfil, but the distortion of free field motions can be kept to a minimum by using a light transducer such as an accelerometer with a rigid but not heavy base and the

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base of the transducer or the mounting block, if one is used, could be designed such that the natural frequency of the base-soil system will be much higher than the frequency of vibration to be measured. Such a transducer will limit the differences between the motions of the transducer base and the true motions of the ground to negligible levels. Apart from satisfying these basic requirements, it was proposed that the method selected for attaching accelerometers to the ground should enable multi-directional measurements at any particular position and should be reproduceable without difficulty. Different methods of attachment such as burying the accelerometer in the soil, mounting accelerometers on small concrete benches cast on the ground and mounting the accelerometer on a large thin plate with spikes at the four corners driven into the ground, were tried and checked for reproduceability. These checks were carried out in the labo--ratory with the accelerometers attached to the surface of sand placed in the container shown in Plate 11. An electrodynamic vibrator was used to excite the sand.

The method that was eventually selected to attach the accelerometers to the soil made use of an aluminium cube of side dimensions 70mm with a 150mm long spike

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attached to the centre of its base (Plate 12). The spike was pushed into the ground until the base of the alumi--nium cube rested firmly on the soil. Accelerometers , were attached to the cube in a triaxial configuration with threaded steel studs. This method offered a reproduceability of  $\pm$  6% (i.e. accleration amplitudes during repeated measurements did not vary by more than 6% of the arithmetic mean). In the field, the reproduceability deteriorated to  $\pm$ 8% for repeated measurements taken on the same day. The mass and dimensions of the block were such that the resonant frequency of the mounting block soil system, estimated using Lysmer's simplified analogue (Section 2.2.1) was 430 Hz which was seven times the maximum frequency of excitation employed.

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# 5.2.4 Testing Procedure.

After setting up the vibrator on a foundation accelerometers were attached to the mounting blocks on the foundation and the exciter was set into operation. When a test was conducted for the first time on a foundation, the vibrator was run at a constant frequency for about fifteen minutes to allow for any settlement of the foundation due to compaction. Then a rapid sweep through the available frequency range was carried out to note the frequen--cies at which fluctuations in the response occurred. This was achieved by continuously monitoring the accelerometer output while the infinitely variable speed control was adjusted from zero to maximum output speed. When this process was completed, the engine speed was set at 1500 r.p.m., and the operating speed of the vibrator was increased in predetermined steps, starting from 125 r.p.m At each speed setting. the vibrator was allowed to run for about two minutes for the speed to stabilise, afterwhich, the signals given by the accelerometers were conditioned and recorded in the tape recorder. In general, data was recorded either at 15 in/sec for a period of three minutes or at  $3\frac{3}{4}$  in/sec for a period of five minutes. - 133 -

The speed of rotation of the vibrator shafts were measured using a revolution counter before and after each recording. The gains of the charge amplifiers and the tape recorder input channels were adjusted after each speed alteration to keep the level of the recorded signals close to 1 V peak, and after each gain alteration, appropriate cali--brations were carried out as described in Section 4.3.3.

The processing of the data was carried out in the laboratory. The output from each tape recorder channel was passed through a narrow band pass filter tuned to the frequency of excitation to remove unwanted components of the signal. The filtered signal, a sinusoidal waye repre--senting the acceleration-time history, was fed into either an a.c. voltmeter or a u.v. recorder to measure the period of the signal. As the frequency of excitation computed from the period of the recorded signals was more accurate than the coarse measurements taken with the revolution counter in the field, the former was used in the conversion of amplitudes of acceleration to amplitudes of displacement.

5.2.5 Comments on the Performance of the Vibrator.

Preliminary tests were carried out in the field with the excitation equipment set up on the small concrete footing. The vibrator was attached to the footing as -134 - described in Section 5.2.2. The footing was set into vibration and accelerations of the footing as well as the accelerations of the vibrator box were monitored. It was found that the acceleration-time history measured on the footing was equal in magnitude and phase to acceleration-time history measured on the top surface of the vibrator box. This indicated that the bolted connection between the vibrator, the pedestal and the footing were rigid. With the tension in the timing belts adjusted to the values chosen from trial tests conducted in the laboratory (Section 4.2), there was no significant horizontal transverse (i.e. perpendicular to the direction of the belt drive) motion of the footing. A significant horizontal longitudinal component of acceleration was measured but it was found that this component of motion was caused not by the vibrator but by the engine used to drive the vibrator.

The major problem encountered was the isolation of vibration generated by the engine. Although, the engine was a two cylinder engine which, in theory, should not have any unbalanced forces at its operating speed, it was found that the engine developed vertical and horizontal vibrations at its operating speed which was 1500 r.p.m.

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The vibrations developed by the engine were transmitted through the jacks supporting the test rig into the ground. The footing which was about twelve feet from the jack closer to the engine responded to the vibrations trans--mitted through the ground.

The dynamic response of the footing monitored after disconnecting the vibrator from the drive showed that the engine induced vertical and horizontal longitudinal (i.e. parallel to the main axis of the test rig ) motions of the foundation. Frequency analysis of the accelerometer signals revealed that the vibrations transmitted from the engine consisted of components at the operating frequency of the engine (i.e. 25 Hz.) and its higher harmonics. Introduction of the vibration isolation mountings, shown in Plate 9, beneath the jacks reduced the level of the higher harmonics to negligible levels but the principal component, although reduced by 40%, still remained significant. The response of the ground surface was also measured at different distances from the foundation. It was found that beyond 3m, the ground motions induced by vibrations from the engine were negligible. Measurements were repeated with the vibrator connected to the drive but set at zero dynamic loading. The results obtained for horizontal

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motions of the foundation were very much the same as those obtained in the previous test, but it was observed that the vertical component of motion induced by engine vibrations decreased with increase in the rotating speed of the vibrator shafts.

Thus, the output signals given by accelerometers attached to the active foundations during the main series of tests consisted of two frequency components; the major component at the frequency of excitation and a relatively small component at the operating frequency of the engine. The level of the second component was, in general, negligible compared to the level of the major component and the wave forms obtained from the accelerometers were sinusoidal (Fig.5.2) except at very low frequencies of excitation. Nevertheless, all signals recorded were passed through a narrow band pass filter to separate the components before amplitude of phase difference measurements were taken. By this procedure, it was possible to obtain the true response of the footings to dynamic loads applied by the vibrator as long as the rotating speed of the vibrator shafts were kept at least 3% greater or lower than the operating speed of the engine. The engine was operated most of the time at its optimum speed of 1500 r.p.m. When response

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TYPICAL WAVE FORM OF ACCELERATION-TIME HISTORY MEASURED IN FIELD TESTS: (a) FOR Wave form of passive foundation response at 50 Hz frequency Nave form of active foundation response at 50 Hz frequency LOW FREQUENCY EXCITATION (b) FOR HIGH FREQUENCY EXCITATION Wave form of output of acclerometer on passive foundation B for 26 Hz frequency Nave form of passive foundation response when processed through a narrow band pass(3% band width) filter FIG. 5.2

measurements were taken at frequencies closer to 25 Hz, the engine speed was reduced to 1000 r.p.m.

Although the aforementioned procedure gave satis--factory results, the interference caused by engine vibrations made on-line processing of data difficult. Furthermore, the necessity to isolate the engine vibrations resulted in a lateral drive which made the initial setting up procedure difficult and time consuming. For future experimentation using this vibrator, a vertical drive from an electric motor mounted on top of the vibrator box or a hydraulic drive from a pump some distance from the testing area is recommended.



FLATE 8 ATTACHMENT OF VIBRATOR TO TEST FOUNDATION.

















MEASUREMENT OF VERTICAL MOTIONS OF PARTIALLY EMBEDDED PASSIVE FOUNDATION B at THREE NONCOLLINEAR POINTS ON THE SURFACE. PLATE 14



ARRANGEMENT OF VIBRATOR AND ACCELEROMETERS FOR MEASUREMENT OF VERTICAL RESPONSE OF ACTIVE FOUNDATION B. PLATE 15

#### CHAPTER VI

#### DESCRIPTION OF TESTS AND PRESENTATION OF TEST RESULTS.

#### 6.1. GENERAL

The series of tests described in this chapter were performed at a site located 10 km south-west of Nottingham. Published information on the geology of the area suggests that the test site is underlain by Keuper Marl. The thickness of the Keuper Marl deposit in this area is estimated to be between 200m and 225m. A brief description of the broad geological features of Keuper Marl is given in Section 7.1.

In this chapter the test results, together with brief descriptions of the tests, are presented in the same sequence as they were collected in the field. A general discussion of the test results is given in Chapter VII.

#### 6.2. MEASUREMENT OF RELEVANT SOIL PROPERTIES

6.2.1 Preliminary Site Investigation.

A number of boreholes were put down at the test site to investigate the subsoil conditions. The locations of the boreholes and the boreholes data are shown in Figure 6.1. A hand auger was used to advance the boreholes denoted by HA in Figure 6.1, and a boring rig consisting of a derrick and a winch unit was used to put down the boreholes denoted by BH. The boreholes PA<sub>1</sub> and PA<sub>2</sub> were advanced by a powered auger.

As shown in Figure 6.1, the test site was covered with a black/brown top soil varying in thick--ness from 200mmto 400mm. The top soil was followed by a firm to stiff reddish brown clay interspersed with thin bands of greyish green silty clay and blue siltstone fragments. The thickness of the clay layer varied between 3m and 4m, decreasing in a southerly direction. The clay was underlain by a soft rock comprising mudstone and pale green sandstone bands. These soft rocks offered resistance to hand augering but were penetrated by the powered auger up to a depth of 8m below the surface. The depth of the water table below the surface varied from season to season

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and the main series of tests were carried out when the water table was about 2m below the ground surface.

Figure 6.2(a) shows the variations of index properties with depth. Undrained triaxial tests were carried out on both 38mm and 100mm samples taken from the boreholes, and the results are presented in Figure 6.2(b). Figure 6.2(c) shows the variation with depth of the number of blows required to advance a 'Macintosh probe' into the soil at a position 3m from borehole HA 9.

## 6.2.2 Measurements of Dynamic Soil Properties.

The velocity of shear waves through the soil was measured by the 'surface wave method' introduced by Jones (1959). A description of this standard seismic procedure is given in Appendix I. The mechanical vibrator described in Section 4.2, was used to propagate waves through the soil and the force developed by the vibrator was such that the dynamic stresses exerted on the soil surface were of the same order as those developed for the main series of tests. As shown in Figure 6.3, the velocity of shear wave through the soil increased with increase in depth. Measurements were repeated on four different occasions corresponding to different weather conditions. The variations in the measured values of shear wave velocity, at any particular depth. during these repeated measurements were limited to  $\pm$  12% and the percentage of variation was of the same order irrespective of whether the depth considered was above or below the water table. In Figure 6.3, average values from the four measurements are presented. Also shown in Figure 6.3, are the values of shear wave velocity, measured at different depths by the 'cross-hole seismic survey'

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technique developed by Stoke and Wood (1972). A description of the technique is given in Appendix I. Bearing in mind that the cross-hole method involved a high degree of personal error in identifying the time of arrival of shear waves, the agreement between the values of shear wave velocity obtained by the two methods can be considered as reasonable.

The velocity of P-waves through the soil was also measured using a geophone and a seismograph. The 'directarrival-survey' technique (Richart, et al (1970)) was employed. From the measured values of P-wave velocity and shear wave velocity, the Poisson's ratio of the soil was computed as described in Appendix I. It was found that the Poisson's ratio varied between 0.34 and 0.41. As noted by Richart (1974), the accuracy of these values , are questionable because the computation involved small differences of rather large numbers. However, previous researchers (Funston and Hall(1968), Richart, et al (1970) have shown that for cohesive soils which are capable of supporting block type foundations, Poisson's ratio, in general, varied from 0.35 to 0.45, and for the evaluation of dynamic response of foundation, using available theoretical solutions, any value within this 152 -

range can be used without significant loss of accuracy. Thus, an average value of 0.37 was taken as the Poisson's ratio of the soil for the computations described in Section 7.3. The bulk density of the soil measured from 38mm diameter samples varied from 1760 kg/m<sup>3</sup> to 2050 kg/m<sup>3</sup>. Measurement by Sand Replacement method at a position close to the test foundations resulted in a bulk density of 1805  $\pm$  85 kg/m<sup>3</sup> at depths nearly equal to the depth of the bases of the foundations below surface, and this value was used to compute the mass ratios of the foundations.

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The shear modulus (G) of the soil was computed from measured values of shear wave velocity ( $V_s$ ) and bulk density ( $\rho$ ) using the following equation:

$$G = V_{s}^{2} p$$
(5.1)

As both V<sub>s</sub>and *p* increased with depth, G also increased with depth and the variation is shown in Figure 6.4. The range of values of G, shown in Figure 6.4, is of the same order as that measured by Cunny and Fry (1973) for firm to stiff clayey soils.

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## 6.3. PRELIMINARY TESTS.

Preliminary tests were carried out on a small concrete pad having plan dimensions 0.6m by0.6m and thickness 0.3m. The design of the foundations tested in the main series of tests were based on the results obtained from these tests.

For the first series of preliminary tests the pad was set into vibration by the small electrodynamic vibrator shown in Plate 11. The vibrator developed constant amplitude periodic sinusoidal forces. The maximum force amplitude available from the exciter was 44.4 N. The vibrating table of the exciter was strengthened by an additional leaf spring and attached to the foundation through four bolts. The bolts were buried in the concrete when the footing was cast. The mass ratio of the pad, with the electrodynamic vibrator in place.was 4.1.

The pad was set into vibration and the resulting motions were monitored at three noncollinear points on the surface of the pad. The amplitude of the dynamic load was kept constant throughout the test by maintaining the current supplied to the exciter at a constant level of 4 amperes. Accelerations measured

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at different points on the surface of the pad were in phase and equal in magnitude. As the accelerometer outputs were sinusoidal, the amplitude of displacement at any particular frequency was obtained by dividing the corres--ponding amplitude of acceleration by the square of the frequency of excitation.

The variation of displacement amplitude of the pad with frequency of excitation is shown in Figure 6.5 by a full line. The frequency response of the pad is a steadily increasing curve with a fluctuation in the form of a minor peak at 23 Hz. Comparison with theoretical response curves presented by previous researchers for foundations of mass ratio similar to that of the test footing, suggests that the frequency response of the footing is similar to the theoretical response obtained by Wass and Lysmer (1972) for a foundation oscillating on the surface of a deep stratum (Fig.2.1).

For the next series of tests, the mechanical vibrator described in section 4.2 was attached to the footing. The performance of the vibrator was checked and improvements were made as described in Section 5.2.5. The phase differ--ence between the upper and lower pairs of rotating masses was set at  $160^{\circ}$  and the foundation was set into vibration and the resulting motions were monitored by accelerometers -155 - placed at three non-collinear points on the surface of the footing. The maximum frequency that was available from the vibrator at that time was 42 Hz.

It was found that the accelerations monitored by all three accelerometers were in phase and the differences between their magnitudes were negligible. This indicated that the pad did not undergo any significant rocking motions and any eccentricity of loading inherent in the procedure on attaching the vibrator to the foundation pad (see Section 5.2.2) was negligible. The displacementfrequency curve computed from measured acceleration amplitudes is shown in Figure 6.5 as dotted lines. The shape of this frequency response curve was steeper than that of the frequency response obtained in the previous test because the amplitude of the force developed by the vibrator was not a constant but increased with the square of the frequency of excitation and the mass ratio of the pad was increased to 10.5 by the addition of the mechanical vibrator. However, the fluctuation in the response at 21 Hz is somewhat similar to the minor peak at 23 Hz in the response obtained from the previous test. The appearance of minor peaks at nearly equal frequencies in the response curves corresponding to two different

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mass ratios of the same pad suggests that the dynamic behaviour of the soils beneath the pad may be responsible for these fluctuations. Further discussion on the implications of this phenomenon is given in Section 7.1.2.

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#### 6.4. MAIN SERIES OF TESTS

6.4.1 General

Three foundations designated as A. B and C were tested in the main series of tests. The dimensions and locations of these foundations are shown in Figure 5.1. The dynamic response of two of these . foundations (i.e. foundation A and foundation B) were monitored both in the active state (i.e. excited directly by the vibrator) and in the passive state (i.e. excited by vibrations transmitted through the soil from a nearby foundation). Foundation C was tested only as a passive foundation. As the vibrator was of mass 520kg, the mass ratio, and, in consequence, some of the dynamic characteristics of a foundation depended on whether it was tested as an active foundation or a passive foundation.

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# 6.4.2 Frequency Response of Active Foundation A.

Foundation A was a fully embedded foundation of plan dimensions 0.61m by 0.61m and thickness 0.61m. The dynamic response of active foundation A was measured in a series of tests carried out before casting either foundation B or foundation C. A full description of the test preparations are given in Section 5.2.

With the vibrator attached, the mass ratio of active foundation A became 12.4. The phase difference between the upper and lower pairs of eccentric masses inside the vibrator was set at 165° to give an equivalent eccentricity  $e_{\rm F}$  of 0.0058m for each eccentric mass. The response of the foundation was monitored by accelero--meters attached to its surface at three noncollinear points. From multi-directional measurements, it was found that the predominant mode of vibration of the foundation was vertical translation. Vibrations generated by the diesel engine, which drove the vibrator, and transmitted through the ground to the test foundation induced a horizontal longitudinal (i.e. parallel to the longitudinal axis of the test rig) motion of the found--ation at the operating frequency (i.e. 25 Hz) of the engine. The horizontal component was about 15% of

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the vertical response induced by the vibrator at this frequency. No significant horizontal transverse or horizontal longitudinal components of motions corres--ponding to other frequencies of excitation were observed. Different types of accelerometers were used for simul--taneous measurement of accelerations at different points on the surface of the foundation. The output signals given by the vertically oriented accelerometers were in phase but the amplitudes of accelerations differed by about ± 6% from point to point. However, these variations were in such a random manner that they did not indicate rocking of the foundation. When the same accelerometer was shifted from point to point, the difference in the outputs given by the accelerometer at different points did not exceed ± 2% of the first measurement.

The amplitudes of vertical displacements of active foundation A computed from measured acceleration ampli--tudes are presented in Figure 6.6, in the form of a displacement vs. frequency of excitation curve. The amplitudes of displacements shown in Figure 6.6 are average values obtained from four measurement sessions

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repeated on different days. The reproduceability of the maximum amplitude of the active foundation, in repeated tests was, within  $\pm$  10%, and the reproduceability of the frequency of maximum amplitude was  $\pm 2\%$ . Considering average values, the frequency of maximum amplitude of active foundation A was 53 Hz and the maximum amplitude was 0.062mm. Another noticeable feature in the response was the minor peak at 21 Hz which was similar to the fluctuation in the response of the pad used in the preliminary tests.

The dynamic load developed by the vibrator was monitored from time to time by three equally spaced load cells attached to the base of the pedestal of the vibrator (Plate 8). A typical u.v. recorder trace of the load-time variation is shown in Figure 4.7. The amplitude of the dynamic load measured at relatively low frequencies, for which the motions and the inertia force of the vibrator are negligible, are shown in Figure 6.7, in comparison with the amplitude of dynamic load evaluated by equation 4.1.

# 6.4.3 Measurements of Surface Motion of Soil.

Soil surface motion induced by vertical vibrations of active foundation A were monitored at different points along the XX axis (Fig.5.1) of the foundation. A description of the procedure adopted for ground motion measurements is given in section 5.2.3.

Vertical excitation of active foundation A resulted in ground motions comprising not only vertical components of displacement but also hori -zontal(radial) components. Horizontal transverse components were negligible compared to the vertical and horizontal radial components. The reproduce--ability of ground displacements at points which were either too close (i.e. d < 3m) to or too far i.e. d > 8m) from the source foundation was poor. especially, at low frequencies of excitation (f<30Hz). Measurements taken at the same location on different days but with the same accelerometer varied by as much as ± 18%. Displacements at points located within 3m and 8m from the source were reproduced to an accuracy of  $\pm$  10%, which was of the same order as the reproduceability of the active

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foundation displacements. However, the reproduceability of the ratio between displacements measured at two different points was within <u>+</u> 4% irrespective of the distance between the points.

The ground motions monitored simultaneously at five different positions are presented in Figure 6.8 and Figure 6.9. Figure 6.8 shows the variation of the vertical components of displacement with distance from active foundation A and Figure 6.9 shows the variation of the horizontal components of displacements. Both components of displacement decreased with increasing distance from the active foundation but the attenuation with distance was more severe than that predicted for an elastic halfspace. To illustrate this difference, the attenuation of displacements in an elastic halfspace, calculated using the soil surface displacement measured at 50 Hz at a position 3m from the active foundation as the datum, is also shown in Figure 6.8.

The relative phase difference between vertical and horizontal components of accelerations of the gournd surface varied from about  $60^{\circ}$  at 1.65m to  $90^{\circ}$  at 4.8m and  $120^{\circ}$  at 12m distance from the source.

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The frequency response of the ground monitored at four positions along the XX axis is shown in Figure 5.10. The shapesof the vertical response curves were not similar to that of the active foundation response. The ground response consisted of several minor peaks, some of which did not appear consistently in repeated measurements, whereas, the corresponding response (Fig.6.6) of the active foundation consisted of only one fluctuation at 21 Hz. It can be observed from Figure 6.10, that the frequency response of the ground surface tends to reach a plateau as the frequency of excitation is increased beyond 45 Hz. This effect becomes more pronounced with increase in the distance from the active foundation.

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6.4.4. DYNAMIC RESPONSE OF PASSIVE FOUNDATION B

6.4.4(a) Frequency Response with Full Embedment.

The first passive foundation tested (i.e in Fig.5.1) was cast in situ at a distance of 7.31m from the centre of active foundationA. It was of rectangular base of dimensions 0.91m by 0.68m and height 0.8m. The mass ratio of passive foundation B was 7.94. It was fully embedded and was formed by pouring concrete directly into a hole excavated to the required dimensions. Further details of test preparations are given in Section 5.2.1.

Vertical excitation of the active foundation resulted in vertical as well as horizontal radial motions of the passive foundation. The motions of the passive foundation were measured by two 8306 accelerometers (Plate 13) connected to QM series charge amplifiers and a B & K 4345 accelerometer connected to a type 2635 charge amplifier set at a high gain. Measurements taken by different accelero--meters at the centre of the foundation varied

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by about  $\pm$  5% and measurements recorded by the same accelerometer in repeated tests varied by  $\pm$  10%.

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Figure 6.11 shows the variation of vertical displacement amplitudes of the passive foundation with frequency of excitation. The three frequency response curves shown in Figure 6.11 were measured at three noncollinear points on the surface of the foundation at the same time. The outputs of the accelerometers were in phase and the differences between the displacements measured at different points were negligible. The frequency response curves reached a plateau at 23 Hz and a well defined peak at 40 Hz. Horizontal radial components of the displacements of the passive foundation measured at the aforementioned noncolinear points are plotted against frequency of excitation in Figure 6.12. No evidence of twisting motion of the foundation was detected.

The dynamic response of the active foundation was also monitored during these tests to study the influence of the presence of the passive foundation on the response of the active foundation. Accelerometers attached to the active foundation in a triaxial configuration showed that the influence of the passive

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foundation on the response of the active foundation was insignificant except for a fluctuation in the vertical response at 40Hz frequency. The vertical frequency response of active foundation A is shown in Figure 6.24, together with its primary response measured before constructing passive foundation B.

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# 6.4.4(b) Transmission Ratio.

In order to describe, quantitatively, the transmission of vibrations from the active founda--tion to the passive foundation, the transmission ratio, defined as the ratio of vertical displace--ment of the passive foundation to the corres--ponding vertical displacement of the active foundation, was computed for different frequen--cies of excitation. The variation of transmission ratio with frequency of excitation is presented in Figure 6.13. The transmission ratio vs. frequency curve shows two well defined peaks. the first of which occurs at 23 Hz. It appears that this peak is related to the fluctuations in the response of both the active foundation and the passive foundation at frequencies closer to 23 Hz. The second peak in the transmission ratio vs. frequency curve appears at 40 Hz, which, as shown in Figure 6.11, is the frequency of maximum ampli--tude of the passive foundation. At the frequency of maximum amplitude of the active foundation (i.e. 53 Hz.) no discontinuity in the variation of transmission ratio could be observed. - 168 -

A series of tests were carried out to study the influence of the mass ratio of the active foundation on the transmission ratios of the two foundations system. The mass ratio of the active foundation was increased by attaching additional weights to the top plate of the vibrator, and the frequency response of both foundations were monitored for three different mass ratios of the active foundation. The results are presented in Figure 6.14. The external force applied to the active foundation at any particular frequency was kept the same in all three tests. From Figure 6.14, it can be observed that the transmissio ratios for the system are substantially independent of the mass ratio of the active foundation. - 169 -

# 6.4.5. EFFECT OF SOIL ADHESION ON RESPONSE OF PASSIVE FOUNDATION

The influence of soil adhesion along the vertical faces of the passive foundation on its dynamic response was investigated in a series of tests. The depth of soil adhesion on the sides of the foundation was reduced by removing the soil surrounding the found--ation in several steps. The mass ratio of the active foundation and the relative phase difference between the two pairs of rotating masses inside the vibrator were kept at the same level as those for the tests in which the response of the fully embedded foundation was measured. This series of tests were completed within a week when there was no appreciable change in the weather conditions. During each test, accelerations of the passive foundation were measured at three or more noncollinear points on the surface of the foundation, and, wherever possible, the same accelero--meters were kept at the same points.

#### (a) First Test.

The first test in this series was conducted after removing the top 170mm of soil surrounding the passive foundation. Thus 630 mm of the sides of the foundation

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was in contact with the soil. The vertical response of the foundation is shown in Figure 6.15. The positions where the accelerometers were attached to the foundation are marked in the sketch shown in Figure 6.15, with the same symbols as those used to mark the co-ordinates of the respective frequency response curves. The vertical response curves shown in Figure6.15 are similar to the vertical response curves obtained for the fully embedded passive foundation (Fig.6.11). The displacement amplitudes measured by the accelerometer at Position'a' (see Fig.6.15) were slightly greater than the amplitudes measured at the other two positions, but the relative phase difference between the outputs of all three accelerometers were zero, as was in the case of the fully embedded passive foundation.

The horizontal radial motion of the foundation is shown in Figure 6.21. Only the measurements taken at the centre of the foundation are presented because there was no significant twisting motion of the foundation in any of the tests in this series, and, in each test, the horizontal radial components measured at different points on the surface of the foundation were almost equal in magnitude and were in phase. Horizontal transverse motions

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were negligible compared to the vertical and the horizontal (radial) motions of the foundation.

## (b) Second Test

The second test in this series was carried out after removing another 200 mm of soil from the sides of foundation B, so that 430mm of the sides of the foundation was in contact with soil. The frequency response measured by vertically oriented accelerometers (Plate 14) is shown in Figure 6.16. Vertical displacements measured at the centre of the foundation and at position 'b' were equal in magnitude and phase but relatively larger displacements were recorded by the accelerometer at position a. A fourth accelerometer was placed at the middle point of the edge of the passive foundation furthest from the active foundation (i.e. position'd') and the test was repeated. The displacements measured at position'd' were much less than the displacements measured at the centre of the foundation. Since vertically oriented accelerometers measure both vertical translation motion and rocking motion, it appears that, in this test, the passive foundation developed significant rocking motions in addition to vertical translation. The

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combined vertical translation and rocking motions reached a peak at 40Hz at position a' and at 43 Hz at the centre of the foundation. The displacement of the top surface of the foundation determined from measurements taken at the four positions is shown in Figure 6.17 for two different frequencies of excitation. Figure 6.17 shows that the rocking motion is significant at 40 Hz but, at 43Hz, the predominant motion of the passive foundation is vertical translation.

The horizontal radial components of the displacements of the passive foundation are shown in Figure 6.21.

#### (c) Third Test.

The third test in this series was carried out after removing the surrounding soil to the full depth of the foundation so that the sides of the foundation were completely free of soil adhesion. As rocking motions were anticipated, vertical vibrations of the passive foundation were monitored by at least four accelero--meters at any instant during this test. The location of the accelerometers and the frequency response curves given by these accelerometers are shown in Figure 6.18. Vertical displacements measured at centre'c'

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and at Position 'b' were almost equal in magnitude and were in phase, and the frequency response curves reached a peak of 45 Hz. However, the frequency response measured at Position'a' (Figure 6.18) started to deviate at 27 Hz from the frequency response measured at the centre and reached a peak at 36 Hz. The displacements measured at Position 'd' were smaller than the displacements measured at other positions up to a frequency of 45 Hz. For frequencies of excitation greater than 45 Hz, displace--ments measured at all four points were nearly equal in magnitude and were in phase.

From Figure 6.18, it appears that the passive foundation underwent considerable rocking motions when the frequency of excitation was between 25 Hz and 45 Hz. Beyond 45 Hz frequency, the predominant motion of the foundation appears to be vertical translation. In order to define the response of the passive foundation more clearly, displacements were measured at three more positions on its surface, and the results are presented in Figure 6.19 as peak displacements of the top surface of the passive foundation at the instant of maximum motion. The results are given for three different frequencies of excitation representing the three - 174 - different types of response developed during a frequency sweep. The arrows shown in Figure 6.19 indicate the posi--tions on the surface of the foundation where the measure--ments were taken. The height of an arrow represents the amplitude of displacement and the direction represents the phase of the displacement-time history.

The angle made by the top surface of the passive foundation with the horizontal plane can be considered as a measure of the rocking motions of the foundation. Figure 6.20 shows the variation of the angle of rocking with frequency of excitation. It can be observed that when the frequency of excitation increases beyond 25 Hz, the angle of rocking increases sharply to a peak value of 13.3 X 10<sup>-7</sup>radians at 32.5 Hz and then falls rapidly to a tenth of this value at 45 Hz.

Horizontal radial motions of the passive foundation, measured at its centre are shown in Figure 6.21. The horizontal response curve shows two peaks, the first of which appears at 32.5 Hz and the second peak at 50 Hz. It is interesting to note that the frequency of the first peak coincides with the peak frequency of rocking motion shown in Figure 6.20. This suggests that the horizontal radial motions were coupled with the rocking motions of -175 - the passive foundation.

In order to illustrate the effect of soil adhesion on the response of the foundation, the frequency response measured by vertically oriented accelerometer at positions 'a' and 'c' for different depths of soil adhesion are presented in Figure 6.22 and 6.23, respectively. These figures show that the total displacement of the foundation at any particular frequency, decreases with increase in depth of soil adhesion. The comparison up to 45 Hz frequency does not lead to any definite conclusions because different modes of vibrations are involved, but at frequencies of excitation greater than 45 Hz. the predominant mode of vibration of the foundation was vertical translation and it can be observed that for the same external excitation, an increase in the depth of soil adhesion on the sides of the passive foundation results in a reduction of the displacements of the foundation in vertical translation.

The displacements of the active foundation were also monitored throughout this series of tests and the results are presented in Figure 6.24, together with the primary response measured before casting foundation B.

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#### 6.5 SUPPLEMENTARY TESTS

## 6.5.1 Frequency Response of Passive Foundation C.

A third foundation (i.e. foundation C in Fig 5.1) was introduced in between foundation A and foundation B at a position 5.3m from the centre of foundation A. Foundation C was of square base of side 0.56m and height 0.71m. It was a partially embedded foundation and was cast in situ as ... described in Section 5.2.1. A series of tests were carried out to investigate whether vertical excitation of active foundation A induced any rocking motions in the partially embedded passive foundation C. The frequency response measured by vertically oriented accelerometers are presented in Figure 5.2.3. The force applied to the active foundation was kept the same as that during the previous series of tests. From Figure 6.25, it can be observed that the partially embedded passive foundation undergoes vertical translation as well as rocking motions when the active foundation was excited vertically. The rocking motions evaluated from measurements taken at position f and position g (see Fig. 6.25) are presented in Figure 6.26 as a plot of angle of rocking vs.frequency of excitation. In order to measure horizontal radial motions, accelerometers were attached to the foundation at the positions shown in Figure 6.27. As shown in Figure 6.27, the horizontal displacements measured at Position 'k', which was 25 cm below the top surface of foundation C, were smaller than the horizontal displacements measured on the surface of the foundation. This suggests that the horizontal motion of the foundation is coupled with its rocking motion.

The response of passive foundation B was also measured in the vertical and horizontal radial directions after placing foundation C in position. The frequency response of the passive foundation B in the vertical direction is given in Figure 6.28 and the response in the horizontal radial direction is shown in Figure 6.29.

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### 6.5.2. Frequency response of Active Foundation B

In order to study the dynamic response of foundation B in the active state, and that of foundation A in the passive state, the mechanical vibrator was removed from foundation A and attached to foundation B. The vibrator was set up so that the longitudinal axis of the machine was perpendicular to the line joining the centres of foundations A and B (Plate 15). The vertical faces of foundation B at this stage were completely free of soil adhesion. The mass ratio of active found--ation B was 11.47. For these tests, the phase difference between the upper and lower pairs of rotating masses inside the vibrator was set at 155° to give an equivalent eccentricity of 0.0096m.

The response of the foundation was measured at different points on its surface, as shown in Plate 8 and Plate 14. As in the case of active foundation A, the outputs given by the accelero--meters placed with their axes perpendicular to the longitudinal axis of the vibrator were negligible and the outputs of accelerometers

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placed parallel to the longitudinal axis of the vibrator were limited to about 8% of the corresponding outputs of vertically oriented accelerometers. Vertical displacements measured at different points on the surface of the foundation are shown in Figure 6.30. The frequency of maximum amplitude of active foundation B was 40 Hz and the vertical response reached a minor peak at 18.5 Hz.

#### 6.5.3. Frequency Response of Passive Foundation A.

The motions of passive foundation A, induced by vertical excitation of active foundation B were monitored in the vertical, horizontal radial (i.e. parallel to line joining the centres of both foundations) and horizontal transverse directions. The passive foundation was fully embedded and its mass ratio was 5.04. のないので、「ないない」ので、

Figure 6.31 shows the vertical components of displacement of passive foundation A and Figure 6.32 shows the response in the horizontal radial direction. Horizontal transverse motions were negligible. Vertical displacements measured at different points on the surface of the pad were in phase and were nearly equal in magnitude suggesting that no significant rocking motions were developed. Also, no evidence of any significant twisting motion was detected. Thus, the dynamic response of passive foundation A was similar to the dynamic response of passive foundation B when it was fully embedded.

The vertical response of passive foundation A reached a peak at the same frequency as the frequency of maximum amplitude (i.e.40 Hz) of active foundation B.

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However, the variation of the transmission ratio of this system with frequency of excitation, which is shown in Figure 6.33, does not show a peak at 40 Hz. This suggests that the peak at 40 Hz in the response of the passive foundation is simply a reflection of the peak in the vertical response of active foundation B. Further evidence in support of this conclusion is available in Figure 6.34 and Figure 6.35, which show a set of vertical response curves for active foundation B and the corresponding response curves of passive foundation A. and the second of the standard of the second of the second of the

#### 6.5.4. Tests on Non-linear Response.

(a) Non-linear Response of Active Foundation B-Soil System.

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Theoretical studies of vibrations of foundations assume linear elastic behaviour of the supporting soil but previous researchers who have carried out experimental investigations into dynamic response active foundations (Novak and Beredugo, 1972, Ehrler, 1968) have consistently found wide scatter in the experimental results and have attributed the scatter in the results to non-linear behaviour of the supporting soil. A series of tests were included in the current study to investigate the significance of this non-linearity.

The frequency response of active foundation B was monitored in four tests, each conducted with a different setting of the phase difference between the upper and lower pairs of eccentric masses inside the vibrator. Thus, the external force applied to the foundation at any parti--cular frequency was changed in each test. The vertical response curves obtained from the four tests are shown in figure 6.34. It can be observed that the frequency of maximum amplitude decreases with increase in the intensity of excitation. This behaviour is similar to that of a mass-spring-dashpot system with a

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non-linear strain softening type spring. The maximum amplitude of displacement increased disproportionately with increase in the applied load. Considering peak values, a 42% increase in the amplitude of dynamic load (from 3.3 kN to 4.7kN) resulted in 77% increase in the peak amplitude of displacement, and 5% reduction in the peak frequency.

The vertical response of passive foundation A was also monitored during these tests, and the response curves are presented in Figure 6.35. Figure 6.36 shows the variation of transmission ratio of the two foundations system with frequency of excitation in each of the four tests.

#### (b)Non-linear Response of Active Foundation A-Soil System.

The vibrator was shifted to foundation A and the vertical displacements of active foundation A were measured at 30, 31 and 32 Hz frequencies for seven different settings of the phase difference between the two pairs of eccentric masses inside the vibrator. The variations of the amplitude of displacement at 32 Hz frequency with the amplitude of applied load and that with the amplitude of dynamic stress, calculated assuming uniform stress distribution beneath the foundation, are shown in Figure 6.37. It can be observed - 184 - that, in a strict sense, the assumption of linear behaviour for this system is valid only for small displacements of the order of 0.03mm. The corresponding amplitude of dynamic stress is  $4.1 \,\mathrm{kN/m^2}$ , and the amplitude of acceleration is 0.12g. Above this limit, a 50% increase in the amplitude of dynamic load resulted in 73% increase in the amplitude of displacement at 32 Hz, although the amplitudes acceleration are less tha than 0.5g, an acceleration level which is recommended in some publications as the safe limit below which the effect of non-linearity can be safely ignored.



FIG 6.1 BOREHOLES LAYOUT AND SOIL PROFILE



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FIG. 6.20 ROCKING MOTION DEVELOPED BY PASSIVE FOUNDATION B AFTER COMPLETE REMOVAL OF SOIL ADHESION ALONG ITS SIDES. MAXIMUM ROCKING ANGLE Vs. FREQUENCY OF .' EXCITATION RELATIONSHIP.





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### CHAPTER VII

## DISCUSSION OF TEST RESULTS

# 7.1 GEOTECHNICAL PROPERTIES AND DYNAMIC CHARACTERISTICS OF SOIL

### 7.1.1 Description of Subsoil Conditions

The location of the test site and the subsoil conditions disclosed by borings at the test site are described in Section 6.2. Geolo--gical map and published information on the geology of the area (Elliot, 1961) indicate that the outcrop at the test site is Keuper Marl. The broad geological features of the Keuper Marl are well known, and a consider--able amount of information on the extent, nature and engineering properties of the Keuper Marl can be found in Chandler(1967), Chandler and Davis(1973) and Meigh(1976). Only a brief summary need be given here. The Keuper Marl is a triassic deposit. It comprises mainly reddish brown mudstones and silty mudstones with subordinate bands of sandstone and siltstone, formed from the wind and water borne products of desert weathering -223-

and laid down in a low lying basin of shallow water lagoons and mud flats. The blocky and relatively homogeneous mudstones on exposure to weathering acquire a profile grading downwards from rock completely weathered to clay near the surface to unaltered rock at depth. Chandler and Davis (1973) studied the effect of weathering on the strength and index properties of Keuper Marl and found that the rock mass may be conveniently classified into four zones according to the degree of weathering. They reported that, in general, the liquid limit and natural water content increase while the strength decreases from zone I (i.e. unweathered Marl) through to Zone IV (i.e.fully weathered Marl). Meigh(1976) studied the deformation characteristics of Keuper Marl in terms the variation of drained modulus of elasticity with depth, and found that the modulus often increasing linearly with depth. The rate of increase was found to be significantly influenced by the depositional environment.

At the test site, the Keuper Marl occurs immediately beneath the black/brown topsoil. The fully weathered zone near the surface is a reddish brown silty clay with a general change in consistency from firm to stiff with - 224 - depth. At isolated locations, the reddish brown clay was interspersed with thin bandsof grey green silty clay(Figure 6.1). The thickness of the clay layer varied between 3m and 4m. The index properties and natural water content of the soil varied with depth, and from position to position, but the measured values of these properties fall within the range of values presented by Chandler and Davis(1973) for zone IV Marl. The results of undrained triaxial tests carried out on both 38mm and 100mm diameter 'undisturbed' samples of the clay showed considerable . scatter (Fig.6.2) but the variations within successive sets of strength data from increasing depths indicated that, in general, the undrained strength increased with depth.

The clay layer was underlain by partially weathered Keuper Marl comprising of brown mudstone and grey sandstone bands. The exact details of these soft rocks are of minor importance to this investigation as they lie at depths several times greater than the halfwidth of the largest foundation tested.

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#### 7.1.2. VARIATION OF SHEAR MODULUS WITH DEPTH

The fundamental soil parameter for the design of of foundations subjected to dynamic loads is the rigidity of the subsoil, expressed usually in terms of the shear modulus of the soil. As described in Section 6.2, the shear modulus of the soil was measured by a standard seismic procedure which gave average values of the shear modulus of the soil over a wide area. The shear modulus measured in the field was not a constant but varied with depth, as shown in Figure 6.4. This variation appears to be compatible with the changes in soil conditions with depth disclosed by the borings. Up to a depth of about 3.5m below the surface, the shear modulusof the soil increases monotonically with depth , and the absence of any significant discontinuity in the variation suggests that the firm to stiff reddish brown brown silty clay within the top 3.5 m can be treated as a single non-homogeneous layer, in which, the nonhomogeneity is characterised by a linear variation of shear moduls with depth.

The discontinuity in the shear modulus-depth depth profile at a depth of 3.5m indicates the change -226-

of strata from clay to mudstone and suggests that the average thickness of the clay layer can be taken as 3.5m. As the increase in the shear modulus at the discontinuity is only about 60% and the measured values of the shear modulus are finite at depths greater than 3.5m, it is evident that the mudstone cannot be considered as infinitely rigid compared to the overlying clay. Thus, the variation of shear modulus with depth indicates that the soil deposit at the test site is a multi-layered continuum consisting of a non-homogeneous stratum overlying a compliant rock. Existing theories for the design of foundations subjected to dynamic loads , however, are based on the assumption that the underlying soil can be represented by relatively simple soil models, in which, the elastic properties remain constant with depth. In the past, two such models representing extreme categories of actually encountered soil profiles have been studied extensively. They are the elastic halfspace model and the elastic stratum overlying an infintely rigid (i.e. non-compliant) rock model \*. Analytical procedures have almost exclusively investigated the first soil model, whereas,

<sup>\*</sup> Awojobi(1972) and Gazetas, et.al,(1979) have formulated theories applicable to foundations vibrating on more complex media such as 'elastic layer overlying a compliant rock and 'Gibson Soil', but even these rigorous solutions are of approximate nature, and not applicable to the more general case of non-homogeneous layer overlying a compliant rock. - 227 -

finite element solutions assume generally an infinitely rigid boundary placed at some depth(see Section 2.2.1). As the soil deposit at the test site does not fulfil all the requirments of either model, the preliminary tests described in Section 6.3 were carried out to assess the dynamic beha--viour of the soil deposit.

# 7.1.3. Dynamic Behaviour of the Soil Deposit

The value of simple field vibration tests in assessing the dynamic behaviour of a natural soil deposit was described by Warburton(1965). He outlined an experi--mental procedure, which involved in situ vibration tests on a foundation of small mass ratio, to determine whether, with a mass on the ground, the latter behaves as a semiinfinite medium or as a shallow stratum of finite depth. The characteristic that differentiates between the two models is the shape of the response curve for a low value of mass ratio, as the response contains no sharp peaks for a semiinfinite medium but attains a marked resonance for a stratum (see Section 2.3).

The preliminary tests carried out at the test site are described in Section 6.3 and the results are presented in Figure 6.5. The full line in Figure 6.5 shows the

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vertical frequency response of a small foundation of mass ratio 4 subjected to periodic, constantforce amplitude excitation. Except for the minor peak reached at 23 Hz. the response is similar to that of a foundation of similar mass ratio vibrating on an elastic halfspace. However, the response of the same foundation but with an increased mass ratio of 10.4 also reached (dotted line in Figure 6.5) a . minor peak at a frequency closer to 23 Hz. As there was no significant rocking motion of the foundation and no fluctuation in the applied load at or near 23 Hz frequency in both tests, it appears that the minor peak in the response of the foundation must be attributed to some pheno--menon associated with the dynamic behaviour of the underlying soil. Further evidence in support of this suggestion is available from the response curves obtained for the active and passive foundations during the main series of tests. A fluctuation in the form of either a minor peak or a plateau appeared in the vertical response of all of the foundations at frequencies close to 23 Hz, but the frequency at which the irregularity appeared in the response of a passive foundation was not the same as the frequency of the minor peak for the corresponding active foundation.

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It has been noted by several researchers (Whitman, 1969, Warburton, 1957) that the frequency response of a foundation on a deep elastic stratum (i.e. thickness of stratum grater than four times the halfwidth of the foundation ) overlying a rigid base is similar to the frequency response of the same foundation on a uniform halfspace except at frequencies closer to the natural frequency of the stratum. At these frequencies, the resonance of the stratum, caused by total reflection of waves at the boundary leads to very large amplitudes of motion of the foundation when the stratum is per fectly elastic and the boundary is infinitely rigid. However, it has been shown by Wass and Lysmer (1972) and Gazetas, et al, (1979) that the introduction of material damping or flexibility of the underlying base resulted in a minor peak, similar to the minor peak in the response curves shown in Figure 6.5, with finite amplitude displacement at a frequency close to the natural frequency of the stratum. Wass and Lysmer (1972) showed that the relationship between the natural frequency of the stratum, the thickness of the stratum and its dynamic properties is given by equation 2.3 (see Section 2.3).

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From equation 2.3, the thickness of a stratum which has a natural frequency at 23 Hz varies between 2.44m and 3.83m depending on the representative value of shear wave velocity chosen from Figure 6.3. Thus, it appears that the dynamic behaviour of the soil mass at the test site can be approximated to the dynamic behaviour of an elastic stratum, the thickness of which may vary between 2.44m and 3.83m. The actual thickness of clay varied between 3m and 4m but it is of interest to note that the average depth of the boundary between the clay and the mudstone, indicated by the discontinuity in the shear modulus-depth profile (Figure 6.4), falls within the range of values given by equation 2.3 for the thickness of the equivalent stratum. As described in Section 2.3, when the dynamic behaviour of a soil deposit is similar to that of an elastic stratum, the dynamic response of foundations constructed on the soil depend on the ratio of the thickness of the stratum (H) to the radius or halfwidth of the foundation (r). For  $\frac{H}{r}$  greater than 4, the 'stratum effect ' (i.e. amplification due to resonance of the stratum) is significant only at frequencies close to the natural frequency of the stratum and the

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dynamic soil-foundation interaction is similar to that in the case of a uniform halfspace. For  $\frac{H}{r} < 4$ , the stratum effect and the dynamic soil-foundation interaction cannot be separated. Although the variation of shear wave velocity precluded the accurate determination of the thickness of the equivalent stratum, it is reasonable to assume that for a foundation of halfwidth less than one fourth of the lower limit of H given by equation 2.3 (i.e. 2.44m), the stratum effect will be confined to a narrow band of frequencies close to 23 Hz. Hence, in the design of the test foundations attempts were made to ensure that the resonance of the foundation due to dynamic soil-foundation interaction was at frequencies greater than 23 Hz.

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# 7.2. <u>DYNAMIC RESPONSE OF ACTIVE FOUNDATIONS</u> (Measured Response.)

Two active foundations were tested in this investigation. Active foundation A, the frequency response of which in translational mode is shown in Figure 6.6, was a fully embedded foundation of mass ratio 12.4 and plan dimensions o.61m by 0.61m. The frequency response of active foundation B, which was of mass ratio 11.47 and base area 0.91m X 0.68m is shown in Figure 6.30. The base of foundation B was 0.80m below the ground surface but the vertical faces were not in contact with the soil. The general shape of the response curves of both foundations are similar to the theoretical response curves (Fig.2.1) presented by Wass and Lysmer (1972) for foundations resting on the surface of a deep stratum overlying a rigid base. The frequency of maximum amplitude of foundation B in vertical translation is 40Hz which when expressed in terms of the dimensionless frequency factor a (see equation 2.1a) falls

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between 0.685 and 1.07 depending on the representative value of shear wave velocity (V $_{\rm s})$  selected from Figure 6.3. This range is of the same order as the natural frequencies predicted by elastic halfspace solutions for foundations of mass ratio approaching 12. The frequency of maximum amplitude of active foundation A in vertical translation is 53 Hz which falls between 0.716 and 1.12 when expressed in terms of frequency. factor a. Although the mass ratio of foundation A is higher than the mass ratio of foundation B, the dimensionless frequency of maximum amplitude of foundation A is greater than that of foundation B because the former was fully embedded and the effect of embedment, as shown by Novak and Beredugo (1972), is to increase the natural frequency of a foundation.

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A feature common to both response curves is the appearance of a minor peak near 20 Hz which can be attributed to the stratum effect discussed in Section6.1. It is interesting to note that the dimensionless frequency of the minor peak decreases with increase in mass ratio, somewhat similar to the trend predicted by Warburton (1957) for a foundation resting on a shallow elastic stratum.

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# 7.3 COMPARISON OF MEASURED AND PREDICTED RESPONSE OF ACTIVE FOUNDATIONS.

7.3.1. General.

Considerable attention has already been given to the development of theoretical solutions for evaluating the dynamic response of active foundations but relative--ly few researchers have reported comparison between theoretical response and experi--mental results. Even in these few cases. the purpose of the comparison has been to check the methods used to arrive at the theoretical solutions and hence, the experimental conditions were deliberately chosen to suit, as much as possible, the ideal conditions required by the theory. The footings used were of circular base and rested on the surface of soil. The soil media considered were either homogeneous layers prepared in laboratories or uniform soil deposits with no significant variation of shear modulus with depth. In contrast, the the active foundations considered in the

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current investigation were of rectangular or square bases and were placed below the surface of a natural soil deposit. Thus, the soil and the footings are similar to those encountered in engineering practice and the comparison between frequency response measured in the field and frequency response calculated from theoretical solutions, which is shown in Figure 7.1, can be expected to give useful information on the application of available solutions to practical problems. 

### 7.3.2 Evaluation of Active Foundation Response.

The equations of motion of the active foundations were extracted, as described in Section 3.2, from the classical theory for a rigid disc oscillating on an elastic halfspace and the extensions to this theory to include elastic strata (Warburton, 1957, Whitman, 1969) and embedded footings (Novak and Beredugo, 1972). Equation 3.3 was used to calculate the vertical displacements of the embedded active foundation A. As the vertical faces of active foundation B were not in contact with soil, it was treated as a 'surface foundation' and the displacements were calculated using equation 2.1. The radius 'r' used in the computations was the radius of a circular area equal to the base area of the foundation considered. As described in Section 2.2.1, Elorduy, et at, (1967) and Richart, et al. (1970) have shown that this assumption involves little loss of accuracy for rectangular bases of length to width ratio less than two.

The soil at the test site consisted of a

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non-homogeneous stratum overlying a compliant rock. As discussed in Section 7.1, the dynamic behaviour of the soil mass can be approximated to that of an elastic stratum overlying a rigid base but the thickness of the stratum cannot be determined exactly. The dimensions of both active foundation, however, are such that the ratio of the lowest possible value that can be attributed to the thickness of the equivalent stratum to equivalent radius of either foundation was greater than six. Hence, the displacement functions  $f_1$  and  $f_2$ necessary to compute the stiffness and damping para--meters of the soil medium can be taken directly from available elastic halfspace solutions. An alternative approach is to assume an average value of the thickness of the stratum and evaluate f<sub>1</sub> and f<sub>2</sub> using the finite element method outlined by Wass and Lysmer (1972). Both methods would lead to inaccurate displacements at frequencies closer to 23 Hz, the natural frequency of the stratum but at frequencies greater than 23 Hz, the 3 difference between the values of displacement functions evaluated by the two methods are negligible. The dis--placement functions  $f_1$  and  $f_2$  used in the computations were taken from the elastic halfspace solutions given 238 -

by Robertson (1966). An added difficulty in the comparison between theory and test results was the variation of shear modulus with depth, shown in Figure 6.4. Any value between the upper limit 52.5 X  $10^3$  kN/m<sup>2</sup> and the lower limit 19.5 X  $10^{3}$ kN/m<sup>2</sup> can be used in the calculations, and each value would yield a different response curve. Awojobi (1972) suggested that for translational mode of vibrations. a heterogeneous medium with linear variation of shear modulus with depth is equivalent to homogeneous halfspace of shear modulus same as the shear modulus of the hetero--geneous medium at a depth equal to the foundation half--width, whereas, Richart, et al, (1970) and Whitman (1972) recommended that the shear modulus at a depth equal to 1.5 times the halfwidth of the foundation can be taken as the representative value. No experimental evidence has been presented in support of these suggestions.

A convenient method to check whether the uncertainty in estimating a representative value for the shear modulus or other factors are responsible for any difference between theoretical and experimental results is to evaluate both the the upper bound of the response, using the lower limit of the shear modulus values measured in the field, and the lower bound of the response, using the upper limit of shear modulus, and compare them with the response measured in the field. -239 -

#### 7.3.3 Comparison of Theory and Test Results.

Figure 7.1 shows the comparison between calculated and measured vertical displacements of active foundation A. At lower frequencies of excitation, the experimental response fall within the upper and lower bounds predicted but at frequencies closer to the measured frequency of maximum amplitude, the actual response lies outside the estimated limits. Similar differences between calculated and measured response can be observed in Figure 7.2 which shows the comparison between theory and test results for foundation B. It is of interest to note that for both foundations, the theoretical maximum amplitudes are lower than the actual maximum amplitudes which justifies the omission of any attempt to account for material damping in adapting the elastic halfspace theory to the problem considered in this investigation.

Although the theory underestimates the maximum amplitudes of displacements of the foundations, the frequencies of maximum amplitude measured in the field fall within the limits predicted by the theory.

Previous experimental investigations (Richart and Whitman, 1967, Novak and Beredugo, 1972, MacCalden et.al, 1973) conducted with circular foundations constructed

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on the surface of silty clay deposits which were described as uniform and boundless, and, hence, more closely approximated to the ideal elastic halfspace than the soil deposit considered in the current investigation, have consistently resulted in maximum amplitudes of displacement two to three times greater than those predicted by the elastic halfspace theory. In the current investigation the differ--ence between the measured maximum amplitude and the lower limit of the calculated maximum amplitude is 85% for active foundation A and 60% for foundation B. It appears that the simplifing assumptions - made to account for the variation of soil properties with depth: the effect of embedment and the shape of the bases of the foundation have not contributed significantly to the discrepancy between theory and test results.

The discrepancy between theory and test results has been attributed by previous researchers to factors such as : presence of fissures and other discontinuities in the soil; inaccurate measurement of dynamic properties of the soil; non-linear behaviour of the soil; deviation of actual stress distribution beneath the foundation from the rigid base stress distribution assumed by the theory

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and rocking motions caused by undetected eccentricity of the applied dynamic load. For the current investigation, attempts were made in the setting up of the tests (Section 5.2.2) to ensure that the eccentricity of the dynamic load applied to the foundations was negligible and no significant rocking motions of the active foundations were developed. For the calculation of the displacements of the foundations, displacement functions  $f_1$  and  $f_2$  were taken from the exact solutions given by Robertson(1966). This solution did not assume a parti--cular form of stress distribution beneath the foundation but adhered to the physical requirments of uniform displacement of the rigid base. As shown in Figures 7.1 and 7.2, the differences between theory and test results are more pronounced at high frequencies than at lower frequencies of excitation. Two factors which may have contributed to the large differences between theory and test results are: (i) non-linearity of the soil and(ii) over-estimation of damping of the soil-foundation system by the theory.

(i) Non-linearity of the soil

The theory assumes a linear elastic behaviour of the supporting soil. This assumption may -244 -

give negligible errors at relatively low frequencies when the excitation force and the response of the soil are small enough to expect reasonably linear elastic behaviour of the soil. However, for the rotating mass type excitation, the dynamic force increases with increase in frequency, and, in consequence, the errors in the theory also increase as the combined dynamic and static load becomes large enough to induce non-linear behaviour of the supporting soil. The non-linear behaviour of both soil-foundation systems was investigated in a series of tests and the results are presented in Figures 6.34 and 6.37 and discussed in Section 6.5.4. In Figure 7.1, the measured response curve crosses the upper limit of the ... theoretical response at 43 Hz. The amplitude of dynamic load at 43 Hz is 2.3 kN which, assuming uniform stress distribution, gives an amplitude of dynamic stres equal to 6.2 kN/m<sup>2</sup>. From Figure 6.36 , it is evident that, at this stress level, the behaviour of the soil-foundation system is non-linear. As the non-linearity of the soil is of the strain softening type, displacements of the foundation calculated by assuming linear behaviour would be less than the actual displacements.

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### (ii) Over-estimation of Damping.

It is well known that the damping possessed by a vibrating system has the greatest effect on the amplitudes of motion of the system when the frequency of vibration is in the neighbourhood of of the resonant frequency. As, for both soilfoundation systems, large differences between predicted and measured displacements were at frequencies closer to the frequency of maximum amplitude of the systems, it is probable that the damping considered by the theory is greater than the actual damping in the soil. Although the theory assumes perfectly elastic behaviour of the soil, the effect of damping is introduced through the loss of wave energy radiated from the foundation thoroughout the medium. Since the medium is considered uniform and infinite in extent. there can be no reflection of waves; thus they must continue to propagate outward while carrying off the input energy. However, the loss of wave energy in the Keuper Marl deposit at the test site may be less than that in an ideal elastic halfspace because of the reflection of elastic waves from

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discontinuities in the soil mass, such as, fissures and layer interfaces.

### 7.3.4. Representative Value of Shear Modulus.

In order to determine a representative value of the shear modulus that can be used in the calculation of the displacements of the passive foundation (Section 7.4.2) the response of each active foundation was calculated for different values of the shear modulus and compared with the corresponding measured response. in Figures 7.3 and 7.4. The different values of shear modulus used in the computations are: the shear modulus measured at a depth equal to 1.5 times the halfwidth of the foundation; the shear modulus at a depth equal to the halfwidth of the foundation and the arithmetic mean of the upper and lower limits of shear modulus shown in Figure 6.4. Examination of Figures 7.3 and 7.4 reveals that, in both cases, the shear modulus measured at a depth equal to 1.5 times the halfwidth of the foundation gives the best agreement with the response measured in the field.

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### 7.3.5 Summary.

The comparison between theoretical and experimental results, discussed in this section shows that it is possible to adapt the classical theory for the oscillation of a rigid disc on the surface of an elastic halfspace to determine the response of a foundation embedded in a natural soil deposit and obtain reasonable estimates of the frequency of maximum amplitude (i.e. 'reasonant' frequency). The amplitudes of displacement at frequencies lower than the frequency of maximum amplitude can also be estimated with reasonable accuracy but the maximum amplitude predicted by this method can be as low as half the actual resonant amplitude. - 250 -

#### 7.4 DYNAMIC RESPONSE OF PASSIVE FOUNDATIONS

### 7.4.1 Dynamic Response of Fully Embedded Passive Foundations

The motions of two fully embedded passive foundations, resulting from vertical excitation of the corresponding active foundation, were monitored. The dimensions and mass ratios of the two passive foundations and those of the corresponding active foundations are shown in Figure 5.1. 

### (a) Response of Passive Foundation B

The first set of tests on an active foundation-soil-passive foundation system consisted of foundation 'A' as the active foundation and found--ation B as the passive foundation. Both foundations were fully embedded. Vertical excitation of the active foundation induced vertical and horizontal radial motions of the ground which, in turn, set the passive foundation into vertical and horizontal radial motions. Vertical displacements measured at three noncollinear points on the surface of the passive foundations were in phase and of nearly equal magnitude (see Figure 6.11) suggesting that no significant rocking motion

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was developed. Thus, the displacements measured by vertically oriented accelerometers on the passive foundation can be directly attributed to vertical translation of the foundation. As both active and passive foundation undergo uncoupled vertical translation, a comparison of the vertical displacements of the active foundation with corresponding displacements of the passive foundation should give some information about the factors influencing the transmission of vibrations from the active foundation to the passive foundation. As shown in Figure 6.11, the vertical displacements of the passive foundation increases with increase in frequency and reaches a minor peak at 23 Hz and a second well defined peak at 40 Hz. The corresponding active foundation response (Fig.6.24) shows a minor peak at 21Hz, a fluctuation at 40 Hz, the implications of which are discussed in Section 7.4.4, and a maximum at 53 Hz. The first minor peak in the response of each foundation can be attributed to the 'stratum effect' discussed in Section (7.1.3), and the frequency at which the peak appears is ' the natural frequency of the stratum modified by the presence of the foundation. The transmission ratio  $\left(= \frac{\text{vertical displacement of the active pad}}{\text{vertical displacement of the active pad}}$ 

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of excitation curve (Fig. 6.13) also shows two peaks, the first one at 23 Hz and the second peak at 40Hz which appears to be the frequency of maximum amplitude of foundation B in damped free vibration. When the frequency of excitation increases beyond 40 Hz, thè passive foundation response enters anti-resonance phase while displacements of the active foundation increases as it approaches resonance. Consequently, the trans--mission ratio then decreases with increase in frequency. The same trend was found to continue at frequencies greater than 53 Hz when the active pad's response decreases with increase in frequency of excitation.

### (b) Response of Passive Foundation A.

The second active foundation-soil-passive foundation system consisted of foundation B as the active foundation and foundation A as the passive foundation. Passive foundation A was fully embedded but active foundation B had no contact with soil on its sides. The mass ratio of foundation A without vibrator mass was 5.04, and back calculations from the results obtained for the same footing when it was tested as an active foundation showed that the

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damped natural frequency of passive foundation A is 96 Hz which is well outside the range of operating frequencies available from the vibrator. Under these conditions, the passive foundation response can be expected to follow the response of the active foundation, and any deviation from this trend can be attributed to the influence of the transmission characteristics of the soil medium.

Figures 6.30 and 6.31 show the vertical frequency response curves of active foundation B and passive foundation A respectively. From Figure 6.31. it is evident that passive foundation A did not undergo any significant rocking motions. The frequency at which the first minor peak appears in Figure 5.28 is not exactly the same as that in Figure 5.29 because the mass ratios of the active and passive foundations are not equal, and, hence, the modification of the natural frequency of the stratum will not be the same for both foundations (Whitman. 1969). As the frequency increases beyond 25 Hz, the response of the passive foundation increases with increase in the response of the active foundation and reaches the maximum response at the same frequency as the active foundation. Thus, it appears that, at frequencies greater than 23 Hz, the response of the passive

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foundation follows the response of the active foundation However, the transmission ratio vs. frequency curve shown in Figure 6.33 indicates that the transmission characteristics of the soil also has some influence on the response of the passive foundation at frequencies greater than 43 Hz. As shown in Figure 6.33, the transmission ratio for the two-foundations system reaches a sharp peak at 23 Hz and falls to a value which remains almost constant from 30 Hz to 43 Hz. Above 43 Hz, the transmission ratio decreases with increase in frequency of excitation. As mentioned earlier, the resonant frequency of passive foundation A in damped free vibration is estimated as 96 Hz. Hence, it is expected that the transmission ratio of this system should attain a further increase at the natural frequency of the passive foundation, as in the case of the first two-foundations, system discussed in Section 7.4.1(i). However, the transmission ratio measured decreases between 43 Hz and 55 Hz. This indicates that the soil medium between the two foundations act like a low pass filter transmitting less energy as the frequency of vibration increases above 43 Hz. The amplitudes of displacement of the soil surface measured

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at different points along a radial line from active foundation A (Fig.6.8) lend support to this suggestion since the attenuation of vertical displacement with distance is more severe at frequencies above 43 Hz than at lower frequencies. Considering the results of both two-foundations systems, it can be noted that the maximum value of transmission ratio for each system was at a frequency very close to 23 Hz, which, as discussed in Section 7.1, is considered as the natural frequency of the clay layer. At first sight, it appears that the layered soil deposit transmits maximum dynamic energy when the frequency of excitation is close to the natural frequency of the top layer. However, the transmission ratios at these frequencies are greatly influenced by the difference between the mass ratios of the active foundation and the passive foundation. In both systems, the mass ratio of the active foundation was greater than that of the passive foundation. In consequence, the natural frequency of the stratum modified by the passive foundation is slightly higher than that in the case of the active fourdation. Thus, the response of the passive pad reaches the

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minor peak (Fig. 6.11) when the response of the active foundation has passed its minor peak, resulting in a high value of transmission ratio. Such high value is unlikely to develop if the mass ratio of the passive foundation is either equal to or greater than that of the active foundation. 

## (c) <u>Influence of Mass ratio of Active Foundation and</u> Applied Load on Transmission Ratio.

The influence of the mass ratio of the active foundation on the transmission ratios for 'active foundation A-soil-passive foundation B' system is shown in Figure 6.14. Figure 6.14 indicates that the trans--mission ratio is substantially independent of the mass ratio of the active foundation at all frequencies of excitation except those close to the fundamental natural frequency of the stratum. Because of the practical limitations in increasing the mass ratio of the active foundation, only three different mass ratios were considered in this series of tests, but the results obtained are in agreemint with the predictions of Warburton et al (1971) who found that the transmission ratio for two cylindrical masses attached to the surface of an elastic halfspace is independent of the mass ratio

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of the excited mass.

Figure 6.36 shows the transmission ratios for the second active foundation-soil-passive foundation system for different levels of excitation. As discussed in Section 6.5.4 and Section 7.3, the active foundation displayed non-linear response at frequencies greater than 40 Hz and,hence, the variation of transmission ratio with the amplitude of external load applied to the system at these frequencies is complicated. But, for frequencies of excitation less than 40 Hz, the transmission ratio for any particular frequency appears to be largely independent of the amplitude of dynamic load applied to the active foundation at that frequency. Further experimentation involving different sets of active and passive foundations would be worthwhile because the independence of transmission ratio on the mass ratio of the active foundation or the dynamic load applied to the system is of considerable practical value. The phenomenon can be used to estimate, prior to installation, the damage that a potential source of vibration may cause to existing foundations in the vicinity.

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# 7.4.2. COMPARISON OF MEASURED AND PREDICTED RESPONSES OF

### PASSIVE FOUNDATION B

An exact theoretical solution for the disp--lacements of a fully embedded passive foundation, in terms of the external force applied to the active foundation - soil - passive foundation system is not available and isdifficult to formulate. However, an approximate solution can be obtained, as shown in Chapter III, by introducing the approximate analy--tical approch, proposed by Barnov(1967) to account for the effect of embedment on the response of a single active foundation, into the theoretical solutions given by Richardson (1969) and MacCalden et al(1973) for forced vibrations of two cylindri--cal masses attached to the surface of an elastic halfspace. The solutions consider the displacement of the passive foundation to be the vectorial sum of the free field displacements at the location of the passive foundation and the displacements caused by the forces developed at the interface between the passive foundation and soil.

The errors involved in representing the soil at the test site by an elastic halfspace for

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the evaluation of active foundation motions are discussed in Section 7.3. For the active foundations, consideration of soil conditions could be limited to a zone of depth equal to four times the halfwidth of the foundation. The passive foundation motions, however, are caused by surface waves propagated from the active foundation, and, hence, variations in subsoil conditions within the zone through which the surface waves travel must be taken into consideration. From field investigations, it has been found (Heukelom and Foster, 1960, Fry, 1963, 1965) that the bulk of the surface waves travel through a zone of about one-wavelength deep.As described in Appendix I, the wave length of the surface waves propagated through the soil at the test site varied from 6.7m for 27Hz frequency to 2.5m for 56Hz frequency of excitation. Thus, the zone of soil which influences the motions of the passive foundation includes not only the clay layer (Fig. 6.1) but also a part of the mudstone and sandstone bands underlying the clay. Hence, it is expected that the error involved in replacing the soil deposit by an elastic halfspace model for the cal--culation of the response of the passive foundation will be greater than that in the case of the active foundation.

Fig.7.5 shows a comparison of the vertical displacements

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of the embedded passive foundation B measured in the field with the vertical displacements calculated using equation (3.35) in Chapter III. For the calculation of displacements, the shear modulus of the soil is taken as 25.9 X  $10^3$ kN/m<sup>2</sup>, the value of shear modulus which gave the closest agreement between the theory and test results in the case of active foundation A (Section 7.3.4). The displacement function fland foused to compute the stiffness and damping parameters C1 and C2 were taken from Robertson (1966). Vertical trnaslation mode was chosen for the comparison because it is widely regarded as the fundamental mode of vibrations for checking a theory. Also. the test results showed (Figure 6.11) that the vertical response of the fully embedded passive foundation was not coupled with any other mode of vibration.

From Figure 7.5, it can be observed that the amplitudes of displacement predicted by the theory are about two times the measured amplitudes of displacement. The estimated frequency of maximum amplitude is about 20% greater than the measured frequency of maximum amplitude. Also shown in Figure 7.5 is the vertical frequency response of the passive foundation calculated using the simple mass-springdashpot model described in Section 3.3.3 with the

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free field displacements measured at the location of the passive foundation as the support motions of the model. Since the displacements computed by this approach agreed closely with the measured displacements of the passive foundation, it appears that the large differences bet--ween the response predicted by the more exact theory (i.e. equation 3.35) and the measured response are mainly due to over-estimation of the free field displacements induced by active foundation A at the location of passive foundation B. The comparison between measured and predicted responses of active foundation A indicated (Section 7.3.3) that the input energy carried away from the active foundation by elastic waves propagated into the soil medium is less than that estimated for waves propagated into an elastic halfspace. Furthermore, the variation of transmission ratio for the second two-foundations system (see Section 7.4.1.b) with frequency of excitation showed that the transmission characteristics of the soil at the test site, unlike those of an ideal elastic halfspace, are frequency dependent. These differences between the transmission characteristics of the soil and the trans--mission characteristics of an ideal elastic halfspace may have contributed to the discrepancy between computed. and measured free field displacements.

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### 7.4.3. Effect of Soil Adhesion on Response of Passive Foundations.

A series of tests were carried out on passive found--ation B to study the influence of soil adhesion along its vertical faces on its dynamic response. The tests are described in Section 6.4.4 (b) and the frequency response curves for passive foundation B corresponding to different depths of soil adhesion along its sides are presented in Figures 6.11, 6.15, 6.16, 6.18 and 6.21.

The dynamic response of passive foundation B with full embedment is described in Section 7.1.1(a). It was found that vertical excitation of active foundation A induced vertical and horizontal (radial) motions of the fully embedded passive foundation but no significant rocking motions were developed. Removal of soil from the sides of the foundation, however, resulted in a change in the modes of vibration of the passive foundation, although the dynamic loads applied to the active foundation, were not altered.

The dynamic response of the passive foundation with the top 170mm of its sides free of soil adhesion (Fig.6.15) was similar to the foundation's response when it was fully embedded except that the vertical displacementsmonitored at the edge nearer to the active foundation were slightly -264 -
higher than the displacements recorded at the centre of the foundation. With further removal of soil from the sides of the passive foundation, rocking motions of the foundation became significant (Figure 6.16) and the difference between vertical motions measured at the near edge and vertical motion measured at the centre increased. When the sides of the foundations were completely free of soil adhesion, well defined rocking motion was developed as shown in Figure 6.18. The rocking motion expressed in terms of the angle of rotation of the top surface of the foundation at the instant of maximum motion(Figure 6.20) reached a peak at 32 Hz frequency and became negligible at frequencies exceeding 43 Hz. art. art in rates art bits art in the structure of the rates after the structure of the structure of the structure of

The displacements of the top surface of the passive foundation at the instant of maximum motion corresponding to three different frequencies of excitation are shown in Figure 6.19. As described in Section 6.4.4, vertical motions monitored at six positions on the surface of the foundation were used to obtain the displacement of the whole surface. Figure 6.19(a) shows that the projection of the axis of rotation on the top surface intersects the line joining the centres of the passive and the active foundations (i.e. XX axis in Fig.6.19) at a position about 150mm - 265 - from the centre of the passive foundation. The axis of rotation was not exactly normal to the vertical plane passing through the centres of the two foundations. The passive foundation was not perfectly symmetrical about the XX axis and this antisymmetry may have resulted in a skewed axis of rotation.. As stated in Section 6.4.4(b), horizontal radial motion monitored at three noncollinear positions on the passive foundation showed that the foundation did not develop any significant twisting motions. However, the amplitude of horizontal radial displacement at any particular frequency of excitation increased with decrease in depth of soil adhesion (Fig. 6.21), and the frequency of maximum horizontal amplitude measured when the foundation had no soil adhesion along its sides was less than the frequency of maximum amplitude for full embedment. However, the former coincides with the frequency of maximum angle of rotation shown in Figure 6.20. This suggests that the horizontal translation and the rocking motions are coupled. A similar conclusion was reached by Warburton (1973) and Richardson, et al(1972) who analysed a two-mass system attached to the surface of an elastic halfspace and found that vertical - 266 -

excitation of one of the masses resulted in vertical as well as coupled horizontal translation and rocking motions of the second mass. Considering the measured responses of passive foundation C (Fig. 6.25) and passive foundation A (Fig6.31). it appears that the effects of soil adhesion along the sides of these foundations on their responses are similar to those discussed earlier for the case of passive foundation B . For vertical excitation of the corresponding active foundation, passive foundation C, which was partially embedded, developed vertical translation, horizontal (radial) translation and rocking motions. as in the case of passive foundation B when it was partially embedded, whereas, passive foundation A, which was fully embedded, did not develop rocking motions. Thus, the test results indicate that soil adhesion along the sides of a passive foundation has a beneficial effect in that the relatively large peripheral displacements that may be developed during rocking of a surface foundation are effectively damped when the same foundation is fully embedded.

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#### 7.4.4. Interaction of Active and Passive Foundation.

Vertical displacements of active foundation A, monitored before and after placing passive foundation B in position are shown in Figure 6.24. The "primary response" (i.e. response measured before placing the passive foundation) is shown in full line. Any departure of the other response curves from this primary response can be attributed to the influence of the passive foundation on the response of the active foundation because the variation of force applied to the system with frequency of excitation was kept the same for all of the tests considered in Figure 6.24.

When the passive foundation was fully embedded the only significant deviation in the response of the active foundation from its primary response appeared at 40 Hz, the frequency of maximum amplitude of the vertical response of passive foundation B (Fig.6.11). Similarly, a comparison of each frequency response curve shown in Figure 6.24 with the corresponding frequency response curve for passive foundation B revealed that the frequency at which the perturbation appeared in the response of the active foundation was very close to the frequency of maximum amplitude of vertical displacement of passive

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foundation B. The perturbations in the frequency response curves are minor fluctuations compared to the primary response but the coincidence of the frequencies at which they occur with the corresponding frequencies of maximum amplitude of the passive foundation indicate that there was interaction between the two foundation, although the distance separating them was about 20 times the halfwidth of the active foundation.

The introduction of passive foundation C between active foundation A and passive foundation B (see Section 6.5.1) offered an opportunity to study the interaction of two passive foundations. The dimensions and locations of both foundations are shown in Figure 5.1. The displacement of passive foundation B measured by vertically oriented accelerometers on its surface, before and after placing passive foundation C in position, are shown in Figure 6.18 and Figure 6.28 respectively. The presence of passive foundation C resulted in slightly (about 10%) increased displacements of passive foundation B but the frequency of maximum amplitude remained unaltered. From Figure 6.29. which show the horizontal radial motions of passive foundation B, it appears that the horizontal radial motions of foundation B were affected, considerably, by the presence

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of foundation C. The frequency of maximum amplitude remained unaffected but the amplitude of displacements are heavily damped by the presence of foundation C. The damping effect created by the interaction of closely spaced foundations subjected to seismic forces has been recognised by Lee and Wesley (1973). Lee and Wesley's extensive theoretical study showed that the beneficial effect of interaction could reverse into an amplifying effect depending on the distance separating the structures. the natural frequencies of the structures and the velocity of shear wayes through the soil. Only one set of closely spaced passive foundations was considered in the current investigation, and further investigations with different sets of foundations and different spacings are necessary to define the effect of interaction on the dynamic response of closely spaced passive foundations.

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#### CHAPTER VIII

8.1.

# CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The following conclusions may be drawn from the experimental study on active foundation-soilpassive foundation systems described in the foregoing chapters.

(1) The dynamic behaviour of a passive foundation is greatly influenced by soil adhesion along its sides.

For vertical excitation of the corresponding active foundation, fully embedded passive found--ationsundergo vertical and horizontal radial motions.

Partial embedment or removal of soil from the vertical faces resulted in the development of substantial rocking motions of the passive foundation in addition to vertical and hori--zontal radial motions.

The rocking motions appear to be coupled to the horizontal radial motions.

(2) The total displacements of the passive found--ation in both vertical and horizontal direction increase with decrease in depth of soil adhesion.

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(3) The presence of the passive foundation affects the response of the active foundation and this effect is displayed as perturbations in the frequency response of the active foundation at frequencies closer to the frequency of maximum amplitude of the passive foundation. りまたたい あいとまであいたい あたいたいしょ 生たいたい 御たいようにたちがある

- (4) Large values of transmission ratio for an active foundation-layered soil-embedded passive foundation system occur at frequencies associated with the frequency of maximum amplitude of the passive foundation and the natural frequency of the layered soil medium. Transmission ratios are substantially independent of the mass ratio of the active foundation and the force applied to the active foundation.
- (5) The dynamic response of the foundation-soil systems considered in this investigation were non-linear and of the strain softening type. This non-linearity was not significant when the amplitudes of accelerations were less than 1.5ms<sup>-2</sup>.

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(6) The application of theories, developed for the oscillation of rigid circular disc on an elastic halfspace, to evaluate the dynamic response of active foundations considered in this investi--gation resulted in reasonable estimates of the frequency of maximum amplitude and the amplitudes of displacement at relatively low frequencies of excitation. However, maximum amplitudes predicted by the theory are 60% to 80% lower than the measured maximum amplitudes.

(7) The extension of elastic halfspace theory to evaluate the response of the passive foundation resulted in displacements 80% to 100% greater than the measured displacements mainly because, the ground motions at distances from the active foundations were much less than those predicted by the theory.

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## 8.2 SUGGESTIONS FOR FURTHER RESEARCH

The practical difficulties involved in conducting field tests imposed limitations on both the size and the number of foundations used in the present investigation. Furthermore, all of the tests were performed at one particular site. Thus, it is both desirable and necessary to conduct further experimentation involving different types of soils and different sizes of foundations, especially, large foundations, the dynamic response of which may be greatly influenced by the 'stratum effect' described in Section 6.1. いたち、「あいていていたい」「あいない、ないないない」」というないない、「あいないないないないないないないないないないないない、

The effect of passive foundation on the primary response of the corresponding active foundation and the significance of interaction between two passive foundations have been demonstrated but further experimentation with more closely spaced founda--tions is necessary to study these phenomena in detail.

Although valuable information on the beneficial effect of soil adhesion along the sides of a passive foundation was obtained from this investigation, the study of this important topic -274 -

is far from complete. Further investigation on this subject should consider partially embedded footings similar to passive foundation C, which represents a fully embedded foundation supporting a rigidly attached mass. The relationship between the mass and dimensions of the embedded part, the mass and dimensions of the exposed part, the shear resistance of soil and the rocking motions of the foundation will be useful in the design of foundations for vibration sensitive equipment.

From Figures6.14 and 6.37, it appears that the trnsmission ratios of an active foundation-soilpassive foundation system are independent of the mass ratio of the active foundation and the intensity of excitation. Confirmation of this finding by further experimental study involving wide range of force amplitudes and size of active foundations as variables will provide a practical method for estimating the response of existing foundations before installing a potential source of vibration in the vicinity.

The current investigation has not given any information on the response of the passive foundation when the corresponding active foundation is set into -275 - either rocking or horizontal translation. Further investigation involving horizontal or rotational excitation can be carried out with minor modifications to the vibrator. Horizontal excitation of the active foundation can be achieved by tilting the vibrator box through 90° and rotational motions can be developed either by introducing a known eccentricity to the applied vertical load with respect to the centre of gravity of the footing or by rearranging the fourshafts-layout inside the vibrator. It is recommended that any further experimental study with the vibrator should consider the use of vertical drive from a prime mover, preferably an electric motor, mounted on top of the vibrator.

Theoretical solutions available for the evaluation of the response of both active and passive foundations are not satisfactory. In the case of active foundations, further theoretical development should account for nonlinearity of the supporting soil and variation of soil conditions with depth. In the case of passive foundations, further analytical work should recognise and account for the differences between the dissipation of wave energy in real media and that in an ideal elastic halfspace.

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#### APPENDIX I

## FIELD MEASUREMENTS OF DYNAMIC SOIL PROPERTIES.

## (i) <u>Velocity of Shear Waves</u>.

The velocity of shear waves through the soil was measured by two seismic methods. The first method employed was the 'Surface-wave' method which was first introduced by Jones (1959), and later developed by Fry (1963). The basis of the method is to measure the length of surface waves (i.e. Raleigh waves), generated by steady state harmonic excitation of the soil surface, for different frequencies of excitation. The length of surface waves  $(L_R)$  is given by the distance between two successive points on the surface which vibrate in phase. The velocity of surface waves  $(V_R)$  at each frequency (f) is, calculated from the measured values of  $L_R$ , using the equation:

$$V_{\rm R} = f_{\bullet} L_{\rm R} \tag{a.1}$$

Since the difference between velocities of shear waves and surface waves in an elastic medium is negligible, the velocity of surface waves measured for a particular frequency is considered as equal to the velocity of shear waves  $(V_s)$  at that frequency. As the bulk of the surface waves in an halfspace

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travels through a zone of depth equal to the wave length and the average properties of this zone approximate to the properties of the zone at a depth of one-half the wave length,  $V_R$  computed from (a.1), and, hence,  $V_s$  is considered as the velocity of shear waves through the medium at a depthequal to half the measured wave length. The results are usually presented in the form of a shear wave velocity -depth profile. Further details of the method can be found in Richart, et al. (1970). For the current investigation, the vibrator described in Section 4.2 was used as the source of vibration. Two accelerometers, one stationary and the other moved along a radial line from the vibrator, were used to locate successive points vibrating in phase on the soil surface. The variation of length of surface waves with frequency of excitation is shown in Figure (a.1), and the shear wave velocity-depth profile is shown in Figure 6.3.

The second method used to measure velocity of shear waves was the 'Cross-hole' method developed by Stoke and Wood (1972). Two boreholes spaced 7.3 m were advanced into the ground to the same depth. An impulse rod with a small base plate carrying an AQ 40 accelerometer was placed in one of the boreholes and a 20 mm diameter pipe with a base plate carrying an 8306 accelerometer was

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placed in the other borehole. The outputs of the accelerometers were connected to a tape recorder through signal conditioning amplifiers. The tape recorder was switched on to record the outputs and the impulse rod was struck with a hammer blow. This procedure was repeated after advancing the boreholes in steps of 0.5m.

Accelerometer outputs, recorded at each depth, were fed into a.u.v. recorder to obtain a trace of the two wave forms on a common time base. From the trace, the time : taken by the shear waves (indicated by the second group of fluctuation in the wave form) to travel from the first borehole to the second borehole was evaluated. The velo--city of shear waves propagated at any particular depth was obtained by dividing the distance between the bore--holes by the time of arrival of shear waves measured on the u.v. trace corresponding to that depth. The accuracy of this method depended on identifying the second arrival in the trace.

### (ii)Poisson's Ratio.

The Poisson's ratio of the soil was computed from measured values of velocities of P-waves and shear waves The P-wave velocities were measured by the standard seismic refraction survey method using a geophone and a

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seismograph. Velocity of shear waves measured by the 'cross-hole' method were used in the computation of p Poisson's ratio. The following relationship between Poisson's ratio ( $\mu$ ), P-wave velocity ( $V_P$ ) and shear wave velocity ( $V_s$ ) was used to compute the Poisson's ratio of the soil:

$$\mu = \frac{v_{\rm p}^2 - v_{\rm s}^2}{v_{\rm p}^2 - v_{\rm s}^2}$$
(a.2)

The Poisson's ratio of the soil given by equation (a.2) varied between 0.34 and 0.4.

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## STATEMENT OF THE ADVANCED STUDIES UNDERTAKEN IN CONNECTION WITH THE PROGRAMME OF RESEARCH

The courses of study followed by the author to acquire additional knowledge related to the subject of the thesis are as follows:

> i). Special short courses for staff and post--graduates on statistical analysis of data given by Mathematics Department, Trent
>  Polytechnic in April 1977 and September 1978;

 ii). Short residential course on Vibration of Structures given by Computational Mechanics in conjunction with Durham University in March 1977

and

iii). Participation in the conference of The Society for Earthquake and Civil Engineering Dynamics on Instrumentation for Ground Vibration and Earthquakes, held in Keele University on 4 and 5 July 1977.

## DECLARATION

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The candidate declares that during the research programme whilst registered for the Council's degree of Doctor of Philosphy

· ...

- 1 he has not been a registered candidate for any other award of the CNAA or a University and
- 2 the material herein has not been used in any other submission.