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Strategic Market Games and Ricardo

Waseem Toraubally
University of Manchester

Abstract

We develop a Ricardian market game to show that in non-Walrasian economies, the Law of Comparative Advantage (LCA) à la Ricardo-Haberler (1817; 1936) can fail. Trade is driven, not by comparative advantages, but by strategic behaviour. This leads to a new and somewhat surprising result: it is shown in a Ricardian economy that at equilibrium, by both exporting and importing goods in which they have a comparative disadvantage, countries can Pareto improve on when they specialise as per the LCA, which in turn Pareto dominates autarky.

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Contact: Waseem Toraubally - waseem.toraubally@manchester.ac.uk.

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1. Introduction

The Law of Comparative Advantage (LCA) à la Ricardo–Haberler (1817; 1936) is a cornerstone of international trade theory. In its most basic two-country two-good format, it gives the following answer to the question of which goods a country will export, and which it will import. A country will produce more than it requires of the good in which it has a comparative advantage, and trade the surplus with the other country against the other good which it is less suited to produce. Ricardo postulated that free trade would *inevitably*, and even *unintentionally*, lead countries to specialise according to comparative advantages (see – e.g., Schumacher, 2013). Ricardo’s (1817) original model assumed perfectly competitive labour and goods markets. Yet, it is well known that in reality, countries, and their constituent workers, behave strategically (see – e.g., Taşbaşı, 2017; Siebert, 2007: p. 382; and Jacquemin, 1982). It is therefore important (and indeed a necessary step) to reanalyse, theoretically, the predictions of the traditional LCA when strategic interactions amongst agents are introduced.

In this paper, we construct a Ricardian model, and show, through a robust counterexample,¹ that the LCA fails. In the two-country two-good framework that we consider, contrary to what the LCA predicts, it is not comparative advantages, but strategic behaviour that drives trade. Intriguingly, at equilibrium in our example, countries can *Pareto* improve their situation by trading in both goods and not specialising at all, as opposed to when they specialise according to Ricardo’s principles.² The intuition for this apparent “anomaly” is that in non-Walrasian economies, market-clearing prices are sensitive to agents’ buy-and-sell decisions. Thus, any effort by a worker to benefit from shifting orders across markets (– e.g., buying less of the expensive commodity and/or more of the cheaper one), alters the corresponding market-clearing prices adversely. The resulting net effect affects the worker unfavourably, and s/he therefore chooses to stay put. Thus, this no-specialisation situation is sustainable as a market equilibrium. These claims are established in the context of *Strategic Market Games* (SMG) of the Shapley–Shubik (1977) tradition. An SMG is a non-cooperative game-theoretic model, which implements a general equilibrium outcome as a Nash equilibrium (N.E). Hence, SMGs constitute an elegant tool³ with which to model the influence of strategic interactions on trade in world markets.

Our results are obtained in a context where all of the original assumptions of Ricardo–

¹As in Koutsougeras (2003), since the solution to agents’ maximisation problem is intractable analytically, we use a robust numerical example to illustrate our main point.

²Note that both the market game in which agents specialise as per the LCA, and the one in which they do not specialise, allow for multiple equilibria. However, in this paper, we consider only subsets of equilibria of those two games. This has been done for the sole purpose of demonstrating that in imperfectly-competitive economies, one cannot rule out the possibility of there existing no-specialisation equilibria which Pareto dominate equilibria with specialisation. Whether or not *all* such no-specialisation equilibria are Pareto superior to equilibria with specialisation is an open question, which, though meaningful, is beyond the scope of this note.

³For excellent, rigorous treatments of the importance and usefulness of SMGs, see – e.g., Ziros (2011), Ziros (2015), and Koutsougeras and Ziros (2015).

Haberler are retained, except that of price-taking agents. Now, it is interesting to note, as Guren et al. (2015) argue, that most trade models assume either perfect factor mobility, or complete immobility, even though empirical evidence suggests that both assumptions are too extreme to appropriately reflect the impact of trade shocks on labour markets. In line with this observation, in this note, we treat labour as quasi-specific.⁴ By “quasi-specific,” we mean that there is a limit on the number of hours that workers can allocate towards the production of each good. This limit depends on the skill set of workers, which is exogenously determined.

In a previous paper, Cordella and Gabszewicz (1997) (henceforth, CG) use an SMG to show how the presence of oligopolistic agents affects the predictions of the original Ricardian model. Our model differs from theirs in many respects. In CG’s two-country, two-commodity framework, individuals derive utility from consuming only the good in which they have a comparative disadvantage. They show that with finitely many agents, for a large class of Ricardian economies, autarky is the *unique* equilibrium on the world market. In our model, however, workers derive utility from the consumption of *both* goods in the economy. Moreover, in our example, active trade takes place at equilibrium: both countries export *and* import from each other, *both* goods on the world market. This no-specialisation equilibrium *Pareto* dominates, both the equilibrium with specialisation, and autarky. Also, while strategic interactions by agents in CG do affect the equilibrium exchange rate of goods, individuals still take world prices of the two goods as given. Hence, contrary to our model – in which commodity prices are determined purely endogenously, by agents’ buy-and-sell decisions – CG’s provides no description of the commodity-price formation process.

Before closing this section, we remark that there are well-known special cases where two-way trade in the same good takes place. Such cases include the so-called “new trade theory:” increasing returns to scale and transport costs (Krugman, 1979; Brander, 1981), and; segmented markets and transport costs (Brander and Krugman, 1983). *None* of these effects are present in the Ricardian model that we consider in this paper. In the next section, we present our model, and we illustrate the failure of the LCA with an example in Section 3. Section 4 contains our conclusions.

2. The Market Game Γ

There are two countries F and H , and the set of agents is $N = F \cup H$, with $|F|, |H| \geq 2$. The set of commodities that can be produced by labour in both countries is denoted by $K = \{1, 2\}$. There is also a third good, m , which is not produced, and which, in addition to

⁴This assumption does in no way affect the predictions of the traditional LCA. In fact, as Ricardo (1817) and Haberler (1936) argue, even with quasi-specific (neither perfectly mobile nor completely immobile), or even in the extreme case of specific (completely immobile) factors of production, countries will still specialise as per comparative advantages. For an in-depth and intuitive analysis, we refer the interested reader to Haberler (1936): pp. 183-189.

yielding utility, also acts as money. The consumption set of each worker is thus identified with \mathbb{R}_+^3 . Technology is exogenously fixed and characterised by constant returns to scale, and is denoted by a_k^J , $J = F, H$. The technology in country J , a_k^J , determines the number of labour hours that worker $j \in J$ needs to produce one unit of commodity $k \in K$. As should be the case in a Ricardian world, we assume that $a_1^F/a_2^F \neq a_1^H/a_2^H$. Labour is homogeneous (as in CG) and quasi-specific (more explanation below) within a country, but heterogeneous and immobile across countries. Each $n \in N$ is described by a preference relation representable by a utility function $u_n : \mathbb{R}_+^3 \rightarrow \mathbb{R}$, and initial endowments, of money $e_{n,m} \in \mathbb{R}_{++}$, and of labour hours $Q_n \in \mathbb{R}_{++}$. To model the quasi-specificity of labour, we assume for each $j \in J$ that Q_j is ‘‘partitioned’’ into the production of the two goods based on the skill set of the worker – i.e., $Q_j = Q_{j,1} + Q_{j,2}$, where this division is determined by worker j ’s (exogenously-fixed) skill set. There are no transportation costs, and no government intervention.

There is full employment of labour, and workers can produce all commodities $k \in K$, as in CG. Viewed differently therefore, a worker j ’s, $j \in J$, $J = F, H$, endowment of labour hours for the production of a commodity $k \in K$ can in fact be interpreted as his/her endowment of k , $\frac{Q_{j,k}}{a_k^J}$.

Throughout this paper, we assume that the fixed exchange rate between the two countries is 1-1,⁵ and that preferences for each type of agents are convex, C^2 , and differentially strictly monotone.⁶

2.1. Interactions in the economy

Trade in the economy takes place via a system of trading posts (common to both countries), at which workers offer (q) goods 1 and 2 for sale, and place bids (b) for these commodities in terms of commodity m . We define the strategy set of each $j \in J$, $J = F, H$, as

$$\xi_j = \{ (b_j, q_j) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 : \sum_{k \in K} b_{j,k} \leq e_{j,m}; q_{j,k} \leq (Q_{j,k}/a_k^J), k \in K \}.$$

Given a strategy profile, we define $B_k = \sum_{n \in N} b_{n,k}$, $\phi_k = \sum_{n \in N} q_{n,k}$, $B_{-n,k} = \sum_{l \in N \setminus \{n\}} b_{l,k}$, and $\phi_{-n,k} = \sum_{l \in N \setminus \{n\}} q_{l,k}$. Transactions at each trading post clear through the price $p_k = B_k/\phi_k$. For a worker $j \in J$, $J = F, H$, we define the allocation of a commodity k , as

$$x_{j,k} = \begin{cases} (Q_{j,k}/a_k^J) + (b_{j,k}/p_k) - q_{j,k} & \text{if } k \in K; \text{ and } p_k \neq 0; \\ (Q_{j,k}/a_k^J) - q_{j,k} & \text{if } k \in K; \text{ and } p_k = 0; \\ e_{j,m} - \sum_{k \in K} b_{j,k} + \sum_{k \in K} q_{j,k} \cdot p_k & \text{if } k = m, \end{cases}$$

where, as is standard in the SMG literature, any division by zero, including 0/0, equals zero. According to the above mechanism, a worker $j \in J$, $J = F, H$, is allocated commodity

⁵This assumption affects neither the mechanism at work, nor our conclusions. This condition, which can most certainly be dropped, has been imposed so that unnecessary matters do not detract from the main message of this paper.

⁶– i.e., if u represents \succeq , then for all $\mathbf{x} \in \mathbb{R}_+^3$, $\partial u_n / \partial x_k > 0 \ \forall k = 1, 2, m$.

$k \in \{m\} \cup K$ in proportion to his bids and sales. Notice that the allocation rule above implies that a worker $j \in J$ does not *have to* offer for sale the commodities s/he produces. In fact, s/he can even consume everything s/he produces, *without these ever going onto the marketplace*.

Workers in country $J, J = F, H$, are viewed as solving the problem:

$$\max_{(b_j, q_j) \in \xi_j} \left\{ u_j \left((x_{j,k}(b_{j,k}, q_{j,k}, B_{-j,k}, \phi_{-j,k}))_{k \in K}, x_{j,m}((b_{j,k}, q_{j,k}, B_{-j,k}, \phi_{-j,k})_{k \in K}) \right) \right\}. \quad (1)$$

An equilibrium in our model is a profile constituting vectors of workers' bids for, and sale of commodities, $\{(b_j, q_j) \in \xi_j : j \in J, J = F, H\}$, which forms an N.E. So, given the strategies of workers other than some $j \in J$, an N.E with positive bids and offers is characterised by the following necessary and sufficient⁷ conditions for (1), for each $j \in J$:

$$\begin{aligned} \frac{\partial u_j}{\partial x_{j,k}} \cdot \left(\frac{B_{-j,k} \phi_k}{(B_k)^2} \right) &= \frac{\partial u_j}{\partial x_{j,m}} \cdot \left(\frac{\phi_{-j,k}}{\phi_k} \right) + \lambda_j^b, \quad k \in K; \\ \frac{\partial u_j}{\partial x_{j,k}} \cdot \left(\frac{B_{-j,k}}{B_k} \right) &= \frac{\partial u_j}{\partial x_{j,m}} \cdot \left(\frac{B_k \phi_{-j,k}}{(\phi_k)^2} \right) - \lambda_{j,k}^q, \quad k \in K; \\ \lambda_j^b \cdot (\sum_{k \in K} b_{j,k} - e_{j,m}) &= 0; \quad \sum_{k \in K} b_{j,k} \leq e_{j,m}; \quad \lambda_j^b \geq 0; \\ \lambda_{j,k}^q \cdot (q_{j,k} - (Q_{j,k}/a_k^J)) &= 0, \quad k \in K; \quad \lambda_{j,k}^q \geq 0, \quad k \in K; \quad q_{j,k} \leq (Q_{j,k}/a_k^J), \quad k \in K, \end{aligned} \quad (2)$$

where λ_j^b and $\lambda_{j,k}^q, k = 1, 2$, are the Lagrange multipliers associated with the constraints in (1).

Before proceeding, we present a (partially modified) result from Peck et al. (1992) which we will use in our example (see footnote 9 for more details):

Lemma 1. Let $\sigma = \{(b_n, q_n)\}_{n \in N}$ be an N.E of the offer-constrained market game $\Gamma(\bar{q})$. If at σ we have, $\forall n \in N$, that $b_n \in \mathbb{R}_{++}^2$ and $\sum_{k \in K} b_{n,k} < e_{n,m}$, then σ is also an N.E of the unconstrained market game Γ .

Specifically, in an offer-constrained market game, agents are constrained to send a fixed vector of commodities, \bar{q} , to the market. In this paper, the game $\Gamma(\bar{q})$ which will be analysed is as follows. If $\alpha, \alpha = 1, 2$, represents the good in which country $J, J = F, H$, has a comparative disadvantage, then in $\Gamma(\bar{q})$, every worker $j \in J$ is constrained to send $q_{j,\alpha} = 0$ to the market.

3. The failure of the LCA: an example

Consider an economy in which $N = F \cup H$, where $F = \{f_1, f_2\}$, and $H = \{h_1, h_2\}$. Both countries have the technology to produce goods $\{1, 2\}$, and the consumption set of each agent is a subset of \mathbb{R}_+^3 . Each $n \in N$ is endowed with 30 units of commodity m , and with labour hours, according to n 's skill set, as shown:

⁷Utility is a smooth concave function of x , and the consumption possibilities set (CPS) is *strictly* convex. Hence, by the Supporting Hyperplane Theorem, a unique optimum exists – i.e, there is a unique allocation vector that is a best response to the strategies played by other agents. For proof of convexity of the CPS, see – e.g, Dubey and Shubik (1978).

$$\begin{array}{r}
\phantom{Q_{f_1} = Q_{f_2} =} \phantom{Q_{h_1} = Q_{h_2} =} \phantom{(15, \frac{50}{3}),} \\
\phantom{Q_{f_1} = Q_{f_2} =} \phantom{Q_{h_1} = Q_{h_2} =} \phantom{(15, \frac{50}{3}),} \\
Q_{f_1} = Q_{f_2} = \phantom{(15, \frac{50}{3}),} \\
Q_{h_1} = Q_{h_2} = \phantom{(15, \frac{50}{3}),}
\end{array}$$

The number of hours required by workers in each country to produce each good, as determined by the technology, is as follows:⁸

$$\begin{array}{r}
(a_1^F, a_1^H) = (3, 4), \\
(a_2^F, a_2^H) = (\frac{10}{3}, 5).
\end{array}$$

The preferences of the agents are represented by the following utility functions:

$$\begin{array}{r}
u_{f_1} = u_{f_2} = 900x_{f,1} + 539x_{f,2} + 220x_{f,m}, \\
u_{h_1} = u_{h_2} = 153x_{h,1} + 343x_{h,2} + 68x_{h,m}.
\end{array}$$

From the above, and using (2), it can be shown that the following strategies form an N.E: $b_{f_1,1} = b_{f_2,1} = 7$; $q_{f_1,1} = q_{f_2,1} = 1$; $b_{h_1,1} = b_{h_2,1} = 2$; $q_{h_1,1} = q_{h_2,1} = 2$; $b_{f_1,2} = b_{f_2,2} = 3$; $q_{f_1,2} = q_{f_2,2} = 3$; $b_{h_1,2} = b_{h_2,2} = 11$; $q_{h_1,2} = q_{h_2,2} = 1$. The corresponding market-clearing prices to these strategies are $(p_1, p_2) = (3, \frac{7}{2})$, and each agent ends up with consumption:

$$\begin{array}{r}
\phantom{\mathbf{x}_{f_1} = \mathbf{x}_{f_2} =} \phantom{\mathbf{x}_{h_1} = \mathbf{x}_{h_2} =} \\
\mathbf{x}_{f_1} = \mathbf{x}_{f_2} = (\frac{19}{3}, \frac{20}{7}, \frac{67}{2}), \\
\mathbf{x}_{h_1} = \mathbf{x}_{h_2} = (\frac{29}{3}, \frac{71}{7}, \frac{53}{2}).
\end{array}$$

Suppose now that there is specialisation as per Ricardo–Haberler – i.e., according to comparative advantages. From (2),⁹ it may be verified that the following profile of strategies constitutes an N.E: $b_{f_1,1} = b_{f_2,1} = \frac{270}{49}$; $q_{f_1,1} = q_{f_2,1} = 0$; $b_{h_1,1} = b_{h_2,1} = \frac{360}{49}$; $q_{h_1,1} = q_{h_2,1} = 4$; $b_{f_1,2} = b_{f_2,2} = \frac{3773}{1058}$; $q_{f_1,2} = q_{f_2,2} = 2$; $b_{h_1,2} = b_{h_2,2} = \frac{2058}{529}$; $q_{h_1,2} = q_{h_2,2} = 0$. The corresponding market-clearing prices to the strategies above are $(p_1, p_2) = (\frac{45}{14}, \frac{343}{92})$, such that:

$$\begin{array}{r}
\phantom{\mathbf{x}_{f_1} = \mathbf{x}_{f_2} =} \phantom{\mathbf{x}_{h_1} = \mathbf{x}_{h_2} =} \\
\mathbf{x}_{f_1} = \mathbf{x}_{f_2} = (\frac{47}{7}, \frac{91}{23}, \frac{735642}{25921}), \\
\mathbf{x}_{h_1} = \mathbf{x}_{h_2} = (\frac{65}{7}, \frac{208}{23}, \frac{819618}{25921}).
\end{array}$$

⁸ F has an absolute advantage in the production of both goods. However, it has a comparative advantage in good 2, while H has a comparative advantage in good 1. We have chosen these figures precisely to show that while trade is not driven by comparative advantages, it is also not driven by absolute advantages.

⁹The game with specialisation is an offer-constrained one, and by Lemma 1, any N.E σ of $\Gamma(\bar{q})$ at which both $b_n \in \mathbb{R}_{++}^2$ and $\sum_{k \in K} b_{n,k} < e_{n,m}$ hold $\forall n \in N$, is also an N.E of Γ . Note, however, that at σ in $\Gamma(\bar{q})$, for agents in F (resp. H), only the N.E equations pertaining to bids in (2) are relevant when considering commodity 1 (commodity 2). Indeed, with (quasi-) specific factors of production, when countries specialise as per the LCA, they do not export the commodity in which they have a Comparative Disadvantage (C.D), since it costs them (relatively) more to produce this commodity, such that they receive (relatively) less per unit of this commodity that is exported. To illustrate this point in our example, let $\alpha, \alpha = 1, 2$, be the good in which country $J, J = F, H$, has a C.D. Now, notice that for any $j \in J$, $\partial u_j / \partial x_\alpha > p_\alpha \cdot (\partial u_j / \partial x_m) > p_\alpha \cdot (\partial u_j / \partial x_m) - \partial u_j / \partial x_\alpha$, such that workers are better off not exporting, and instead, simply consuming the good in which they have a C.D. Hence, countries specialising as per the LCA is akin to an offer-constrained game, in which, for good α , the only variable w.r.t. which workers $j \in J$ maximise their utilities is their purchases/bids.

A routine calculation shows that the utility that *each player* derives from not specialising is greater than the utility derived by specialising, which in turn, is greater than the utility derived under autarky.¹⁰ Hence, the no-specialisation outcome is a *Pareto improvement* upon the other two cases.

4. Discussion of example and conclusion

In the example above, it has been shown that both countries can improve upon the specialisation equilibrium, by both exporting and importing from each other the commodity in which they have a comparative disadvantage. The key idea behind this result is that in the no-specialisation game Γ , agents are allowed to make “wash-sales” – i.e, any agent $n \in N$, can enter the market for *any*¹¹ commodity $k \in K$ both as a buyer and a seller. This gives rise to a situation where for each $k \in K$, identical amounts of purchases and sales (equal to $\min\{b_{n,k}/p_k, q_{n,k}\}$) cancel each other out, *without influencing n 's net trade*. Yet, such strategies affect the thickness¹² of the market for k ; this in turn, reduces the other agents' ability to influence the market-clearing price. For clarity, consider an agent n , who, given the actions of others, plays a strategy $(\hat{b}_{n,k}, \hat{q}_{n,k})$ in the market for k , where $\hat{b}_{n,k} \cdot \hat{q}_{n,k} > 0$, and $\hat{b}_{n,k}/p_k - \hat{q}_{n,k} > 0$. It is trivial to verify that the (budget-feasible) strategy $(b_{n,k}^*, q_{n,k}^*) = (\hat{b}_{n,k} - \hat{q}_{n,k} \frac{B_{-n,k} + \hat{b}_{n,k}}{\phi_{-n,k} + \hat{q}_{n,k}}, 0)$ generates the same net trade for n , and hence, the same allocation. However, by playing $(\hat{b}_{n,k}, \hat{q}_{n,k})$, thereby increasing the thickness of the market for k , agent n is able to reduce the degree to which other agents are able to influence k 's price. An analogous argument holds for when $\hat{b}_{n,k} \cdot \hat{q}_{n,k} > 0$, and $\hat{b}_{n,k}/p_k - \hat{q}_{n,k} < 0$, in which case, $(b_{n,k}^*, q_{n,k}^*) = (0, \hat{q}_{n,k} - \hat{b}_{n,k} \frac{\phi_{-n,k} + \hat{q}_{n,k}}{B_{-n,k} + \hat{b}_{n,k}})$. Hence, in the non-Walrasian framework under consideration, agents take advantage of this feature by exercising their market power to manipulate prices in their favour, as a consequence of which, the LCA fails. Any endowments, and utility functions with the same marginal rate of substitution at the consumption allocation resulting from the above profile of strategies, would constitute an equilibrium with the same property, thereby attesting to the robustness of our example in endowment and utility spaces.

Another striking aspect of our example, which is of independent interest, is the following: in the (unconstrained) market game Γ , there exists no *interior*¹³ equilibrium where a worker $j \in J, J = F, H$, makes a negative net trade ($b_{j,\alpha}/p_\alpha - q_{j,\alpha} < 0$) in the commodity $\alpha, \alpha = 1, 2$, in which s/he has a comparative disadvantage (see the Appendix for proof of this claim). Hence, it is worthwhile to note that although agents do export the commodity in which they have a comparative disadvantage, they still buy more of it than they sell, at equilibrium.

¹⁰Autarky, in the model that we propose, is simply the trivial outcome in which no trade takes place at all, with $\mathbf{b} = \mathbf{q} = \mathbf{0}$. This profile of strategies clearly constitutes an N.E – see – e.g, Dubey and Shubik (1978).

¹¹Recall that this is not true of some commodities in the (offer-constrained) game with specialisation, $\Gamma(\bar{q})$.

¹²Following Peck et al. (1992), we say that the market for a commodity k is “thick” (thin) when ϕ_k is large (small) relative to $\sum_{f \in F} (Q_{f,k}/a_k^F) + \sum_{h \in H} (Q_{h,k}/a_k^H)$.

¹³An equilibrium is termed interior if every $j \in J, J = F, H$, solves (1) in the interior of ξ_j .

In this paper, we have shown that specialisation as per the LCA *may* improve welfare, but *need not always* do so, especially in the non-Walrasian framework that we adopt. Yet, the failure of the LCA does not have to destroy trade completely (as opposed to the very pessimistic result in CG). Rather, it implies a positive outcome: that two-way trade, even in goods in which countries have a comparative disadvantage, can still be mutually profitable for all countries engaging in it.

In future research, we will aim to extend the model considered in this work, to a two-stage game with perfectly mobile labour. In the first stage, workers would then choose their preferred “endowment” vector along the production possibility frontier, and subsequently trade strategically in the second stage.

Appendix

Claim 1. In the market game Γ of Section 3, there exists no *interior* equilibrium where a worker $j \in J, J = F, H$, makes a negative net trade ($z_{j,\alpha} \equiv b_{j,\alpha}/p_\alpha - q_{j,\alpha} < 0$) in the commodity $\alpha, \alpha = 1, 2$, in which s/he has a comparative disadvantage.

Proof: We first derive a crucial result, which stipulates that at *any*¹⁴ interior equilibrium (I.E) of Γ , all agents within the same country make net trades of the same sign in any commodity $k \in K$. The importance of this result will become evident in the sequel.

So, consider workers in country F first. For ease of exposition, throughout this proof, we will denote $\frac{\partial u_{f_1}}{\partial x_k} = \frac{\partial u_{f_2}}{\partial x_k}$ by $\delta_k, k = 1, 2$. Recall further, that for each $k \in K$, δ_k is constant. Next, pick any worker in F , say f_1 , and any commodity $k \in K$. Using (2), at an I.E, it has to be true that:

$$\delta_k \cdot \left(\frac{B_{-f_1,k} \phi_k}{(B_k)^2} \right) = 220 \cdot \left(\frac{\phi_{-f_1,k}}{\phi_k} \right) \Rightarrow (p_k)^2 = \frac{\delta_k}{220} \cdot \left(\frac{B_{-f_1,k}}{\phi_{-f_1,k}} \right).$$

W.l.o.g., consider the case where agent f_1 is a net seller of commodity k ($z_{f_1,k} < 0$). It can be verified that this implies that $\frac{B_{-f_1,k}}{\phi_{-f_1,k}} > p_k$, such that at any I.E in which agent $f_1 \in F$ makes a negative net trade in k , we must have:

$$(p_k)^2 = \frac{\delta_k}{220} \cdot \left(\frac{B_{-f_1,k}}{\phi_{-f_1,k}} \right) > \frac{\delta_k}{220} \cdot p_k \Rightarrow p_k > \frac{\delta_k}{220}.$$

Suppose now that at any such I.E, agent $f_2 \in F$ were a net buyer of ($z_{f_2,k} > 0$), or made a zero net trade ($z_{f_2,k} = 0$) in commodity k , such that $\frac{B_{-f_2,k}}{\phi_{-f_2,k}} \leq p_k$. Following the same procedure as above, it is easy to see that this implies that for f_2 , we must have $0 < p_k \leq \frac{\delta_k}{220}$, which is an impossibility, since $\nexists p_k$ such that $p_k > \frac{\delta_k}{220}$ AND $0 < p_k \leq \frac{\delta_k}{220}$. It can be easily proved that

¹⁴— i.e, even interior equilibria which are not “type-symmetric.”

an analogous conclusion is true for workers $h \in H$ as well. With this, we have established the following intermediate result:

Lemma 2. At any I.E of Γ , all workers within the same country make net trades of the same sign in any commodity $k \in K$ – i.e. $z_{j_1,k} \cdot z_{j_2,k} > 0$, while $z_{j_1,k} \cdot z_{j_2,k} = 0$ iff $z_{j_1,k} = z_{j_2,k} = 0$, $j_1, j_2 \in J, J = F, H$.

So, assume that at an I.E, an agent $f \in F$ were a net seller of commodity 1 (in which F has a comparative disadvantage), such that $p_1 > \frac{900}{220} = \frac{45}{11}$. Crucially, by Lemma 2, this means that *all* workers in F are net sellers of commodity 1. Therefore, this implies that there necessarily exists at least one agent $h \in H$ who is a net buyer of good 1 ($z_{h,1} > 0$). By Lemma 2, this means that all workers $h \in H$ are net buyers of commodity 1, such that $\frac{B_{-h,1}}{\phi_{-h,1}} < p_1 \forall h \in H$. Proceeding as above, we can finally show that at such an I.E, it must simultaneously be true $\forall h \in H$, that:

$$0 < p_1 < \frac{153}{68} = \frac{9}{4},$$

which contradicts our initial hypothesis that $p_1 > \frac{45}{11}$.

A similar approach can be used to show that no worker $h \in H$ can be a net seller of commodity 2 ($z_{h,2} < 0$) at *any* I.E of Γ . *Q.E.D.*

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