Journal Pre-proof

Comparative advantage with many goods: new treatment and results

Waseem A. Toraubally

PII: S0377-2217(23)00407-1
DOI: https://doi.org/10.1016/j.ejor.2023.05.027
Reference: EOR 18494

To appear in: *European Journal of Operational Research*

Received date: 3 April 2022
Accepted date: 22 May 2023

Please cite this article as: Waseem A. Toraubally, Comparative advantage with many goods: new treatment and results, *European Journal of Operational Research* (2023), doi: https://doi.org/10.1016/j.ejor.2023.05.027

Highlights

1. We construct a strategic market game with a continuum of commodities
2. We use a game-theoretic general equilibrium framework to show trade is driven by strategic behaviour
3. We prove that a Nash equilibrium exists
4. Both countries always engage in intra-industry trade in all sectors despite the existence of comparative advantages

Comparative advantage with many goods: new treatment and results*

Waseem A. Toraubally†

Abstract

This paper constitutes the very first treatment of the Shapley–Shubik (1977) market-game mechanism with a continuum of commodities. We develop an oligopolistic-competition model in which product prices are endogenously determined, via buyers' and sellers' strategic decisions, and we lay down and examine its mathematical structure. Taking agents' market power into account, we restudy the Ricardian Law of Comparative Advantage in a *many*-commodity framework, and obtain a (new) result that is in line with what is perceived in real-world markets: when agents act *strategically*, they do not specialise based on comparative advantages. For a large class of utility functions, we prove the existence of equilibria at all of which trade is driven neither by absolute nor comparative advantages, but exclusively by strategic decision-making.

Keywords: Game theory; Shapley–Shubik market games; Cournot oligopoly with a continuum of goods; Infinite-dimensional optimisation

1 Introduction

David Ricardo's (1817) ***Law of Comparative Advantage*** (RLCA) states that given two agents, each will specialise in the production of the commodities which they can manufacture at a relatively lower opportunity cost when trade is completely free. In this paper, we reanalyse the RLCA in a context with a continuum of goods à la Dornbusch, Fisher and Samuelson (1977) (henceforth, DFS), and show that it fails.[1] This suggests a rethinking of the traditional focus on comparative advantage—whereby, for example, the United States pursues the expansion of its financial markets but neglects its own manufacturing or energy production sectors—as it is demonstrably suboptimal. This is a crucial result, since the RLCA finds important applications in a myriad of economic, managerial and operations management frameworks (see the detailed discussion in §5).

*I thank Ken Baldwin, Huw Edwards, Randy Silvers, Anne Villamil, Gabriel Weintraub, participants of the BAFA NAG 2022 Conference and the NTU Internal Seminar series for helpful discussions. I am indebted to the late Peter Neary for his encouragement, and for many discussions on unconventional Ricardian models of trade. I extend my sincerest thanks and appreciation to Steve Spear, Kwok Tong Soo, Claudia Meo, and three anonymous referees of EJOR for extensive and very intelligent comments on earlier drafts, and to the Editor. Any errors are my responsibility.

†Nottingham Business School, Economics. Phone: +44 (0)115 941 8418. Email: waseem.toraubally@ntu.ac.uk.

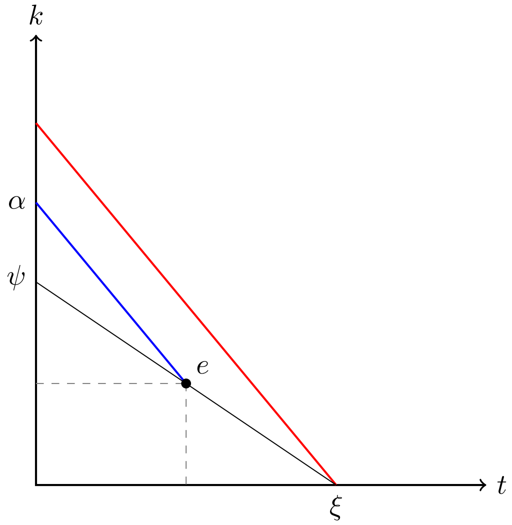
[1] Frameworks with a continuum of goods arise naturally in managerial economics, be it in models of continuous-time spatial competition (e.g., Ebina et al., 2022), dynamic oligopolistic pricing (e.g., Briceño-Arias et al., 2016; Colombo and Labrecciosa, 2021), or continuous trading of long-lived securities (Duffie and Huang, 1985).

We develop a simple game-theoretic model by fusing the approaches of Dubey and Shubik (1978), Spear (2003), Chen et al. (2018), and DFS. To be precise, we use a strategic market game which, aside from its obvious innovations, combines the Dubey–Shubik, Spear, and Chen–Korpeoglu–Spear mechanisms. This is then adapted to a comparative advantage framework in a DFS setup, with a *large number* of commodities. The outcome is an easily understandable construct that generates novel, powerful conclusions. Under the assumption of perfectly competitive labour and goods markets, DFS develop a model analysing the RLCA with a continuum of goods and they show that its predictions still hold in that setting. Our conclusion is markedly different to theirs: we assume that agents wield market power which they exercise to influence product prices (more on this in §§3 and 4), and we show that with a continuum of commodities, the predictions of the *RLCA never obtain*. This completely new result has serious repercussions, not least in international operations and supply chain management, patent pooling, and even in economic development, where important decisions are driven by the RLCA (alongside the dependencies it also creates). Our formulation, in line with all strategic market games,[2] is appealing because it features an explicit description of the price formation process, which is completely missing from perfectly-competitive models. Additionally, this work constitutes the first treatment of a strategic market game with a continuum of commodities. Since we moreover apply this new model to a Ricardian framework, for completeness, it is necessary that the proposed game be rigorously defined, and the conditions that characterise equilibrium in it be derived. These results, while interesting in their own right, also serve to enlighten the mechanism that drives trade in our model.

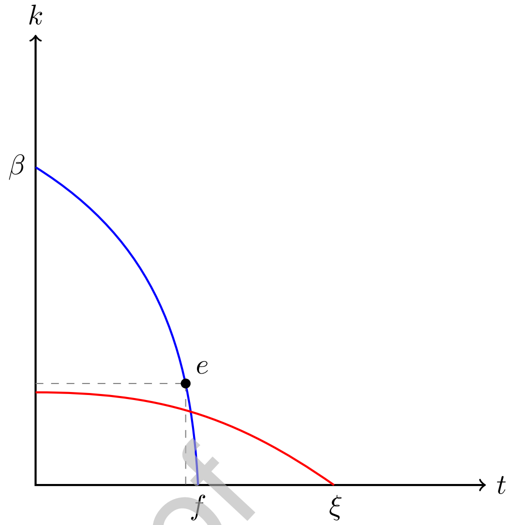
We expand on the continuum-of-good approaches developed by Neary (2002), Neary and Tharakan (2012), and Neary (2016) to develop a **fully** oligopolistic framework. Some context is important here. In each of the latter models, oligopolistic competition is modeled in the form of individual firms wielding market power only at a sectoral/local level. Since Neary (2002), Neary and Tharakan (2012) and Neary (2016) assume the existence of a continuum of sectors, with each sector comprising different firms, the upshot is individual firms being atomistic economy-wide. This is why Neary (2016) describes his approach as one which views firms as being "small in the large but large in the small" (p. 670). In contrast to the above and to monopolistically competitive models of intra-industry trade, in this work we analyse players that are purely oligopolistic from every angle: they have market power whenever and wherever they trade. That is to say, there is no assumed price-taking in any form. The very same agents interact in a continuum of markets, with each of them having market power in the market for each good. This is an important innovation and a timely addition to the trade literature, and to industrial organisation more generally. Indeed, in extant studies, the apparently simple notion of modeling players that are large both in the markets they trade, and in the economy as a whole, is complicated enough as to render intractable many attempts trying to formalise this notion. Reaction functions tend to be extremely ill-behaved, and equilibrium, even in the simplest models, may not exist (Neary, 2016). Our model remedies this deficiency. The trading mechanism we have constructed is tractable yet rich. Amongst others, the continuum of goods

[2] A strategic market game is a non-cooperative game-theoretic general equilibrium construct in which product prices are *explicitly* and *endogenously* determined by agents' buy-and-sell decisions. For examples of the many areas to which market games have been applied, see Spear (2003), Chen, Korpeoglu and Spear (2018), Goenka, Kelly and Spear (1998), Meo and Koutsougeras (2018), Peck, Shell and Spear (1992), Powers and Shubik (1998), Shubik (1987), and the references therein.

Figure 1: Illustration of the effects of strategic behaviour



(a) CPF with perfect competition



(b) CPF in a traditional market game

setup, while technically demanding, does not hinder our model's explanatory power when it comes to explicitly—and intuitively—showing how commodity prices are endogenously determined in the market for each good. Moreover, as we explain in §3.1, our model is well-behaved even out of equilibrium.

Our conclusion in this paper is unequivocal: even with a continuum of commodities, the RLCA *(always)* fails to obtain at equilibrium when agents are strategic. This is not to say that trade does not take place at all between players; rather, the latter do trade, just not as postulated by the RLCA. More precisely, *any* Nash equilibrium of the market game will have both sets of players producing, exporting and importing all goods—i.e., there is intra-industry trade. Because agents have non-negligible market power, any unilateral deviation from this equilibrium will unavoidably trigger price changes that make them worse off. Hence, even if a player has a comparative advantage in some goods, the process of complete specialisation forces prices and the corresponding terms of trade to change so drastically and unfavourably for any agent, that making any kind of gains from such a position becomes impossible.

A basic graphical analysis using an elemental two-good market game combining elements from Dubey and Shubik (1978), Spear (2003) and Toraubally (2022) will best help capture the *essence* of strategic behaviour in ours. The graphs in Figure 1 show a situation in which agents trade in *only* two goods, k and t. In Panel (a), the standard "classroom" example, markets are competitive, and point e represents the closed-economy equilibrium output on a particular player's production possibilities frontier, $\psi e\xi$. Free, perfectly-competitive trade then determines the relative prices at which agents can buy the two goods, yielding the blue *consumption possibilities frontier* (CPF), $\alpha e\xi$, before any specialisation takes place. The red CPF shows what occurs when agents specialise—in this case the player considered specialises in good t—under the assumption that the RLCA holds. Thus, competitive trade determines terms of trade that allow the agents considered to purchase more of both goods, making them strictly better off. Panel (b) replicates the situation in Panel (a), but under the *non-linear pricing* structure that arises in the elemental market game. The opening of free (oligopolistic) trade generates the blue, pre-specialisation CPF, βef. However, were the agent to specialise now, the red CPF would lie below the blue one, because as we explain in §4, after specialisation, the varying degrees of discounting required on *all* sales drastically change the terms of trade against the player that specialises. This terms of trade effect is directly analogous to what happens in the traditional Cournot game, where producing an extra unit of a good impacts the profitability of not just the marginal, but also of the inframarginal, units (see Tirole, 1988).

Our model makes an important contribution to a number of fields in which constant elasticity of substitution (CES) preferences are the main building block. In particular, our model provides a *tractable* formulation that circumvents the most important feature—and limitation—of the standard Dixit–Stiglitz model: that of equilibrium prices and markups being constant, irrespective of the number of goods (or varieties thereof) produced. Indeed, while constant markups are very convenient to use in economic geography, international trade, and macroeconomic theory amongst others, they do not fit the bill in situations where agents have market power. As is well known, most industrial organisation models are synonymous with markups which vary as market thickness (sales volume) varies. In our setup, we allow agents to have market power on both sides—demand and supply—of each market (of which there is a continuum), **and** in the economy as a whole. To be precise, agents are both **oligopsonistic** and oligopolistic, at both the sectoral and aggregate levels. This is all modeled in a framework using a class of very general additively separable utility functions which accommodate much more than just CES preferences (see also §5). In particular, there is no requirement for agents' (sub-) utility functions to be symmetric: both the symmetric and asymmetric cases can be accommodated just as easily. Our setup is general enough to also allow for both homothetic and non-homothetic utility functions. This flexibility makes it an important alternative, especially with regard to testability. As far as we know, this result is new.

In the next section, we summarise the main differences between our model and existing ones. In §3, we develop our framework, and explain how agents interact with each other. §4 contains a thorough equilibrium analysis of our model, alongside a detailed overview of how strategic behaviour drives our main results. We discuss testable implications, managerial applications, and conclude, in §5.

2 Literature review

Many trade models featuring a continuum of goods (or varieties of goods) have appeared over the last couple of decades, emphasising different reasons for trade. The overwhelming majority of these models are cast in perfectly-, or monopolistically-competitive frameworks, to make the study of these models analytically manageable. Some of the most prominent papers in that direction include Eaton and Kortum (2002) and Costinot (2009) in the Ricardian tradition,[3] and Krugman (1980, 1981) and Melitz (2003) in a strand of the literature that is now known as the New Trade Theory. Our method is completely different to existing ones in that it is, to the best of our knowledge, the first purely oligopolistic study of the RLCA with infinitely many goods.

[3] For more on such Ricardian models, see Eaton and Kortum (2012), and the references therein. Unsurprisingly, it is beyond the scope of our short literature review to mention many other influential papers that, alongside the ones we have discussed, now constitute a literature of mammoth proportions.

Costinot (2009), Costinot et al. (2012), and Eaton and Kortum (2002, 2012) use perfectly-competitive Ricardian setups, with *constant returns to scale* and a number of refinements, to provide *support for* the RLCA. Other models of monopolistic competition, initiated by Krugman (1979, 1980, 1981), Lancaster (1980), Helpman and Krugman (1985), Helpman (1987), use increasing returns to scale and product differentiation in general equilibrium as the main drivers of **intra-industry trade**. Melitz (2003) extends Krugman's (1980) setup to include heterogeneous firms in a monopolistically-competitive environment—in what is now the workhorse trade model with firm heterogeneity. Hsieh and Ossa (2011) develop a many-country, many-industry Ricardian model, but this features monopolistic competition of the Krugman (1980) variety, and Melitz-style firm heterogeneity. Nevertheless, Melitz (2003) notes that though monopolistic competition is commonplace in the literature, it comes with its well-known limitations (in the form of firms' markups being exogenously determined by the symmetric elasticity of substitution between varieties). Of note is the fact that in all of the above-mentioned lines of work, agents are taken to be atomistic (individually insignificant).[4] Thus, as aptly put by Melitz and Redding (2015: p. 4), "although this [monopolistic] framework provides a tractable platform for analyzing a host of firm decisions in general equilibrium, it neglects strategic interactions between [agents]." Ergo, monopolistic competition does not embody true/substantial progress over perfect competition in describing real-world *strategic* trade patterns today. Interestingly, these models are at odds with what the accumulating empirical data suggests: amongst others, Freund and Pierola (2015) show that *large individual* firms play a paramount role in comparative advantage, at least between developed countries, such that "models that treat individual firms as insignificant are not consistent with the evidence" (p. 1023)—see also Bernard et al. (2003), Bernard et al. (2007, 2018), and Chen et al. (2018). Our work addresses this hitherto largely overlooked part of the trade literature inasmuch as it provides a rigorous, tractable theoretical explanation of why agents fail to specialise based on the RLCA when they behave *strategically*.[5] This is especially relevant for managers and policymakers in the current international climate, with many situations, such as the prevailing trade tensions between the UK and the EU, between the US and its big oil-producing rivals, best represented by oligopolistic models. Importantly, the difference between Bernard et al. (2003), Freund and Pierola (2015), Bernard et al. (2007, 2018) and our work is that the former bring in oligopolistic market power for sellers (only), while in our case, we deploy a strategic market game in which each agent has market power both as a buyer and seller. Crucially, Bernard et al. (2003) and Bernard et al. (2007, 2018)

[4]Brander (1981) and Brander and Krugman (1983) use models in which the rivalry of oligopolistic firms leads to international two-way trade. Their results rely crucially on the domestic and foreign markets being segmented/separate. To keep their analyses tractable, they confine their treatments to ***partial equilibrium*** settings, and assume that agents across countries are identical. Our formulation is a general equilibrium one, and we do not require agents, even within the same country, to be identical in any way. As such, these models do not possess the same scope as ours, especially when it comes to intermarket linkages. Moreover, as we discuss in §4, though our construction features one common, fully integrated international market for each commodity, it does not preclude market segmentation per se. Thus, Brander's and Krugman's models may in fact be viewed as special cases of ours. Neary (2016) proposes a hybrid approach, where firms are viewed as being large within their sector, but insignificant in the economy as a whole, i.e., across sectors. However, as Freund and Pierola (2015) argue, such models overlook how large individual firms (superstars) account for much of the variation in trade ***across sectors***.

[5]All while retaining all the features—including constant returns to scale in production—of the traditional Ricardian model, except for price-taking agents. Recall that the defining characteristic of a Ricardian model of trade is neither perfect competition, nor perfect mobility and divisibility of labour, but differing constant-returns-to-scale technologies (see, e.g., Eaton and Kortum, 2002: p. 1745, and Bernard et al., 2003). For an interesting Ricardian model that relaxes the conventional perfect divisibility of labour (and also of goods) assumption, see Soo (2017).

allow firms to be large relative to sectors only, but assume that each firm has Lebesgue measure zero in the economy as a whole (see, e.g., Bernard et al., 2018: p. 571).

Our formulation also simultaneously extends existing ones in a number of important respects. Most of the works discussed in the two previous paragraphs rely on simplified modeling approaches involving the use of **symmetric** preferences, and agents which are either all homogeneous or all heterogeneous. As we shall show in §3, we do not impose any such restrictions. Agents within the same country can be taken to be heterogeneous â la Melitz (2003), totally homogeneous, or a combination of both. Our approach has some modelling advantages over that of DFS'. DFS, as we do here, consider a single (aggregate) factor of production: perfectly mobile labour. Now, in DFS, and models that build on it, comparative advantages are determined, not only based on technology, but also on the ratio of real wages across the two countries. In particular, wages, and by extension comparative advantages, are assumed to be determined perfectly competitively. In this light, another compelling aspect of our formulation is the fact that our setup allows us to recover factor prices from output prices—which cannot be done in DFS' model—thanks to the explicit price formation mechanism it features.

Additionally, moving from a finite-commodity setup to an uncountably-many-good one in a market game comprises many conceptual and technical difficulties. In existing trade models, using a continuum of goods is helpful because it makes the determination of the pattern of specialisation depend smoothly on the relative cost for each set of agents of producing each good. In a market game, however, a many-good system poses tremendous challenges, since the price of each and every good is derived endogenously, and explicitly. Once this problem has been overcome to create economically meaningful conclusions, however, the new method that we propose better captures real-world interactions between *strategic* agents than existing approaches do, as explained earlier (see also §5). Finally, we also prove that with oligopolistic agents, previously-thought-to-be inferior alternatives (no-specialisation equilibria) are actually optimal.

Last but not least, the astute reader may wonder whether the RLCA should apply in our context, since it is a result that essentially applies in first-best optimal equilibria, or when equilibrium allocations are isomorphic to the first-best optimum (such as in monopolistic competition with constant markups). For instance, in frameworks admitting variable markups, the market equilibrium deviates from the first-best optimum, irrespective of whether agents behave strategically or not—see, e.g., the seminal work by Melitz and Ottaviano (2008), and Mayer et al. (2021). In such cases, there is (typically) no reason to expect the RLCA to hold and indeed, intra-industry trade prevails. However, findings running counter to this argument do exist. Amongst others, Soo (2016) considers a model amalgamating Ricardian comparative advantages, monopolistic competition, and trade costs. Trade costs, in particular, do make the market equilibrium deviate from the first-best optimum. Yet, Soo (2016) shows in this setup (many countries and many goods) that if only one country has a comparative advantage in each sector, then complete specialisation as per the RLCA still takes place. Intra-industry trade only arises if more than one country has a comparative advantage in each sector. Ricci (1997) constructs a model integrating Ricardian comparative advantages, trade costs, increasing returns to scale, product differentiation, and monopolistic competition. While he finds that both inter- and intra-industry trade take place for some parameter values, he also finds that for some parametrisation of his model, regions specialise completely, engaging uniquely in inter-industry trade as the RLCA posits they should. Relatedly and conversely, Kikuchi et al. (2008) show, in a Ricardian model incorporating monopolistic competition, that when there are uncountably many sectors/goods, countries specialise completely as per the RLCA such that no intra-industry trade takes place. It is only when the number of goods is finite that intra-industry trade takes place. Fan et al. (2013) use a two-country, multisector version of Melitz (2003) and show that sectors in which one of the countries has a "strong" comparative advantage are characterised by inter-industry trade, while intra-industry trade takes place in those sectors in which neither country has a strong comparative advantage. Our work is related yet diametrically opposed to the above. Our model combines Ricardian forces, a continuum of goods, no market frictions, constant returns to scale, and endogenous markups; yet, intra-industry trade *always* takes place in all sectors/goods because every agent possesses market power which s/he uses strategically.

3 A market game with a continuum of goods

There are 2 countries, F and H. The set of all agents in the economy is $N = F \cup H$, with $|F| \geq 2$, $|H| \geq 2$, and $|N| < \infty$.

Consider next a measure space $(K, \mathscr{K}, \mu)$, where $K = [0, 1]$ denotes the set of consumption goods that can be produced in the world economy, the technologies for producing which are possessed by both F and H, $\mathscr{K}$ is the collection of all μ-measurable subsets of K, and μ is the Lebesgue measure when restricted to $\mathscr{K}$. $(K, \mathscr{K}, \mu)$ is thus complete, finite, and non-atomic. $L^\infty(K, \mathscr{K}, \mu)$ denotes the space which consists of (equivalence classes of) essentially bounded measurable functions $f : K \to \mathbb{R}$, where two measurable functions are equivalent if they are equal μ-$a.e.$ As well as the goods found in K, there is a special commodity, which we call m, that is not produced but which, in addition to its role in utility, also acts as money.[6] We will thus let our commodity space be identified with $\mathbb{R} \times L^\infty(K, \mathscr{K}, \mu)$. We remark that the L^∞***-norm*** of $f \in L^\infty(K, \mathscr{K}, \mu)$ is defined by

$$\|f\|_{L^\infty} = \operatorname*{ess\,sup}_K f = \inf \{t \in \mathbb{R} : \mu\{k \in K : f(k) > t\} = 0\}.$$

Agents $n \in N$ have no initial endowment of any good in $[0, 1]$, but are instead endowed with labour hours, with which to produce these commodities.[7]

Each country is characterised by its technology. The latter is exogenously fixed, exhibits constant returns to scale, and is denoted by $a^J(k)$, $J = F, H$. More precisely, the technology in country J, $a^J(k)$, determines how many labour hours it takes for an agent based in J to produce one unit of any commodity

[6] Our findings in this paper do not depend on the use of a numéraire. Fiat money would have worked just as well. However, using the latter would have made proving the existence of equilibria an even more technically demanding endeavour, especially when proving the (weak-star) compactness of agents' strategy sets. An additional cost would have been a cumbersome notation, with no additional gain in intuition. In any case, the use of a numéraire is standard, and widespread, in international trade models—see, e.g., Zhelobodko et al. (2012).

[7] Karatzas et al. (1994) consider an SMG with a continuum of "non-homogeneous" agents, and whose period-dependent endowments are i.i.d. random variables. While there may seem to be some parallels with our construction here, there is, amongst others, a crucial difference: in their model, production of the *single* commodity analysed is assumed to be determined exogenously, whereas here, the endowments are determined endogenously.

$k \in K$. Following DFS, we let $\zeta(k) := \frac{a^F(k)}{a^H(k)}$ be a continuous function, with $\frac{d\zeta(k)}{dk} > 0$. Additionally, we will normalise $a^F(k)$ to 1, μ-$a.e$ in K, and assume that $\zeta : K \to [\frac{1}{\Theta}, 1 + \frac{1}{\Theta}]$ is surjective, where $1 < \Theta$ is given. This assumption, which is without loss of generality, is made for the sake of tractability. Accordingly, country F (respectively, H) can then be interpreted as having a comparative advantage in the production of all goods for which $\zeta < 1$ ($\zeta > 1$), while neither F nor H has a comparative advantage in the production of the good for which $\zeta = 1$.[8] The above interpretation is very intuitive, as we next explain. With the goods having been arranged in the order of the comparative advantage of F over H, with $\frac{d\zeta(k)}{dk} > 0$, if we were to draw a line dividing the commodities in which F and H each have a comparative advantage, all the former will lie on one side of the dividing line, and all the latter on the other side, without the need to rearrange them (Haberler, 1936). For example, let $\zeta(a) < \zeta(b) < 1 < \zeta(c)$. It should then be clear that country F cannot enjoy a comparative advantage in goods a and c, while simultaneously having a comparative disadvantage in the production of good b. Hence, in our case, the above-mentioned dividing line logically occurs at the point where $\zeta = 1$.

Agents are taken to be perfectly mobile within a country, but immobile across countries. Each player $n \in N$ is described by a preference relation representable by a utility function[9] $U_n : \mathbb{R}_+ \times L^\infty_{\tau,+} \to \mathbb{R}$, and initial endowments, of money $e_{n,m} \in \mathbb{R}_{++}$, and of labour hours $Q_n \in \mathbb{R}_{++}$. The utility function of an agent $n \in N$, is assumed to be of the form[10]

$$U_n(x_n) = u_{n,m}(x_n(m)) + \int_{k \in K} u_{n,k}(x_n(k))d\mu(k), \tag{1}$$

where $x_n(\cdot)$ is n's consumption of good $k \in \{m\} \cup K$. For all $k \in \{m\} \cup K$: $u_{n,k}(\cdot)$ is twice-continuously differentiable in $x_n(k)$, $\frac{du_{n,k}(\cdot)}{dx_n(k)} > 0$, $\frac{d^2u_{n,k}(\cdot)}{d(x_n(k))^2} < 0$, and $\lim_{x_n(k)\to 0} \frac{du_{n,k}(\cdot)}{dx_n(k)} = +\infty$, with $u_{n,k}(0) = 0$. Now is a good time to highlight what our framework is *not*. In particular, our analysis is not merely a reformulation of a Walrasian trade model with two (representative) players in an imperfectly-competitive setup. On the contrary, it should be clear from our construction so far that agents within a country ***need not***, although they can, be homogeneous in any way. Furthermore, there is no requirement for agents' utility functions to be symmetric. Thus, what we propose in this work is a very general yet tractable model.

[8] In DFS and standard trade models, it is assumed that one country has a comparative advantage in all those commodities for which domestic unit labour costs are less than or equal to the other country's unit labour costs. If w_F and w_H denote wages in countries F and H, then F has a comparative advantage in all $k \in K$ for which $a^F(k) \cdot w_F < a^H(k) \cdot w_H \Leftrightarrow \frac{a^F(k)}{a^H(k)} < \frac{w_H}{w_F}$. But as we shall soon show, since there is a single market for each k, at which prices (and wages) are determined by players' buy-and-sell decisions, we have that $w_F = w_H$. Thus, our model has the nice property that even though *both* absolute and comparative advantages clearly exist, the goods in which F (H) has a comparative advantage, are also those in which it has an absolute advantage. As such, the theory that we propose in this paper shows that it is neither comparative nor absolute advantages that drive trade.

[9] V_+ stands for the positive cone of the vector space V. Analogous to Toraubally's (2019) approach, we define the partial order $\geq$ on V to be such that for any $f, g \in V$, $f \geq g$ iff $f(k) \geq g(k)$ for all $k \in K$. Thus, $V_+ := \{f \in V : f(k) \geq 0 \ a.e \text{ in } K\}$. L^∞_τ stands for $L^\infty(K, \mathscr{K}, \mu)$ endowed with the ***Mackey topology*** $\tau(L^\infty, L^1)$. The latter is the topology of uniform convergence on $\sigma(L^1, L^\infty)$-compact, convex, circled subsets of L^1—see the proof of Lemma 2 in the Appendix, or Bewley (1972), for further details. $\mathbb{R}$ will be assumed to be endowed with its usual Euclidean topology, ε, throughout this paper.

[10] This class of utility functions encapsulates a wide array of functional forms commonly used in economic theory, managerial economics and international trade, including constant absolute risk aversion (CARA), constant elasticity of substitution (CES), variable elasticity of substitution (VES) utilities, and many more. The interested reader is referred to Dixit and Stiglitz (1977) and Zhelobodko et al. (2012) for further details.

3.1 The trading mechanism

Any commodity $k \in K$, towards the production of which agents choose to allocate labour hours, is *manufactured* at the corresponding market/trading post, i.e., no good can be produced if not at a trading post.[11] There, agents may also place monetary bids (purchases) for every commodity $k \in K$. Every agent may thus, financial constraints permitting, consume vectors of each commodity that is produced.

Define $\mathbb{L} = L^\infty_\tau \times L^\infty_\tau$, where $\mathbb{L}$ is given the $\tau(L^\infty, L^1) \times \tau(L^\infty, L^1)$ product topology. Next, let $\mathbb{L} \supset \mathbb{L}_+ := L^\infty_{\tau,+} \times L^\infty_{\tau,+}$. The strategy set of any agent $n \in N$ is described by a measurable correspondence $S_n : K \to 2^{\mathbb{L}_+}$, such that

$$S_n = \left\{(b_n, l_n) \in L^\infty_{\tau,+} \times L^\infty_{\tau,+} : \mu\text{-}a.e \text{ in } K,\ b_n(k) \leq e_{n,m} \text{ and } l_n(k) \leq Q_n\right\},$$

where b_n and l_n stand for bids placed and labour hours offered, respectively. A strategy profile is a pair of measurable mappings $b := \bigtimes_{n\in N} b_n$ and $l := \bigtimes_{n\in N} l_n$ such that $(b_n, l_n) \in S_n\ \forall n \in N$. We may now introduce the following essential lemmata, which are part of our contribution.

Lemma 1. *The measurable maps b_n and l_n exist.*

Proof. See Appendix. □

Lemma 2. *The maps b_n and l_n are (Pettis-) integrable.*

Proof. See Appendix. □

With the above results in hand, for a given strategy profile $(b, l) \in \text{Gr}(S)$,[12] we can now define $B(k) = \sum_{n\in N} b_n(k)$, $\phi(k) = \sum_{f\in F} \frac{l_f(k)}{a^F(k)} + \sum_{h\in H} \frac{l_h(k)}{a^H(k)}$, $B_{-n}(k) = \sum_{i\in N\setminus\{n\}} b_i(k)$, and $\phi_{-n}(k) = \sum_{f\in F\setminus\{n\}} \frac{l_f(k)}{a^F(k)} + \sum_{h\in H\setminus\{n\}} \frac{l_h(k)}{a^H(k)}$. Observe that while $l_n(k)$ denotes the amount of labour hours which n allocates towards the production of k, given $a^F(k)$ and $a^H(k)$, it also corresponds to the total amount of *commodity* k that n sells onto the market. Consumption allocations of commodity $k \in \{m\} \cup K$, for any agent $n \in N$, may then be defined as follows:

$$x_n(k) = \begin{cases} \dfrac{b_n(k)}{B(k)}\phi(k) & \text{if } k \in K, \text{ and } B_k \cdot \phi_k \neq 0; \\ e_{n,m} - \displaystyle\int_{k\in K} b_n(k) d\mu(k) + \int_{k\in K} \frac{l_n(k)}{a^J(k)} \frac{B(k)}{\phi(k)} d\mu(k) & \text{if } k = m, \end{cases} \tag{2}$$

where $J = F, H$, and we adopt the market game convention that all divisions by 0, including $\frac{0}{0}$, equal zero whenever these appear in any of the expressions above—see Peck et al. (1992) or Shubik (1976) for an interpretation. Based on the above distribution rule, agent n is allocated commodity $k \in \{m\} \cup K$ in proportion to his/her bids and labour hours offered.

[11] This assumption has been made so degenerate/trivial outcomes are avoided. Though it is reminiscent of Shapley and Shubik's (1977) sell-all game—in that a trader can buy back some of, or everything, that s/he produces, but these must go through the market first—this assumption is fundamentally different, because here agents have no endowments of any goods per se. For more on this, please see §4.

[12] $\text{Gr}(S) := \{(k, \Lambda) \in K \times \prod_{n\in N} \mathbb{L}_+ : \Lambda \in S(k)\}$ stands for the graph of S.

A couple of points are in order. First, recalling that ϕ_k is the amount of k that is offered for sale, one sees that whenever $B(k) \cdot \phi(k) \neq 0$, the proportional allocation rule in (2) means that the fraction $\frac{B(k)}{\phi(k)} := p(k)$ has a natural interpretation as the ***price*** of commodity k. This simple but intuitive process provides a clear-cut description of how *market-clearing* prices—which vary purposively with agents' strategies, even out of equilibrium—are formed in a market game. Adopting a market game approach to analyse the important RLCA (be it in a marketing or any other economic framework) thus avoids the defect of perfectly-competitive models being ill-defined away from equilibrium (Shapley and Shubik, 1977). In particular, much like on real-world trading platforms, prices change in a reasonable manner: higher demand and supply drive prices up and down, respectively. From here onwards, we will use $p(k)$ and $\frac{B(k)}{\phi(k)}$ interchangeably, with each being understood to refer to the price of k.

Second, note that for $a.a\ k \in K$, and $\forall n \in N$, $0 \leq x_n(k) = \frac{b_n(k)}{B(k)}\phi(k) \leq \phi(k) \leq \frac{1+\Theta}{\Theta}\sum_{n\in N} Q_n$. Also, we have that $0 \leq x_n(m) \leq \sum_{n\in N} e_{n,m}$. Therefore, we can view $x_n(\cdot)$ as $x_n : \mathbb{L}^{|N|} \supset L^{\infty,|N|}_{\tau,+} \times L^{\infty,|N|}_{\tau,+} \to L^{\infty}_{\tau,+} \times \mathbb{R}_+$, where $\mathbb{L}^M$ and $L^{\infty,M}_{\tau,+}$ represent the M-fold products of $\mathbb{L}$ and $L^{\infty}_{\tau,+}$, respectively, such that $x_n(\cdot)$ is *well-defined* indeed.

Third, we expound a subtle characteristic of the price formation process lest the reader get the wrong impression about the latter. In our model, equilibrium prices are derived from the NE strategies of the underlying game where agents treat as given the bids and offers of others, and play their own (best-response) bids and offers. Consequently, what takes centre stage regarding how the economy arrives at an NE is the stability—or the absence thereof—of the mechanism which implements the NE. And indeed, in this respect, it was demonstrated by Kumar and Shubik (2004) that natural extensions of Walrasian tatônnement-like mechanisms to the strategic market game environment can fail to converge or fail to be locally stable. This observation does not affect the results in this paper, but is worthwhile remarking upon given that price formation in the market game is frequently held up as an unequivocal net positive.

Definition 1. A Nash equilibrium (NE) of our market game consists of agents' bids for commodities and labour hours offered such that

(i) Every agent's actions are best-responses given the expectations of other agents' actions;

(ii) The best-responses are consistent with all agents' expectations of other agents' actions.

At an NE, every agent $n \in N$ is viewed as solving the following problem:

$$\max_{(b_n,l_n)\in S_n} \left\{ U_n \left(\left(x_{n,k}(b_n(k), l_n(k), s_{-n}(k)) \right)_{k\in K}, x_{n,m}\left(\left(b_n(k), l_n(k), s_{-n}(k) \right)_{k\in K} \right) \right) \right\}. \tag{3}$$

A market for a good k is said to be ***active*** if $p(k) \gg 0$. Autarky—no trade at all, both within and across countries—in this framework is to be understood as a situation where (almost) no market is active, i.e., $\int_{k\in K} p(k)d\mu(k) = 0$. More precisely, with a continuum of commodities, one can always find a measure-zero (null) set of goods in which agents trade. This, in our present framework, still counts as autarky. As such, the statements "all", "almost all", "each" and "every" commodity are to be understood as referring to all $k \in K$, except possibly for a null set of commodities. Before closing this section, we present a few intermediate results which will be crucial for the presentation of our main findings.

Lemma 3. *The consumption possibilities set for each $n \in N$ is strictly convex.*

Proof. See Appendix. □

Lemma 3 is an extremely important result which cannot be dispensed with if we are to guarantee the existence of a supporting hyperplane in the allocation space under study. It allows us to characterise our programme as a convex one, such that any admissible strategy profile that is a *regular* point and constitutes a solution to (3) will dominate any other admissible strategy profile—i.e., it is a unique global optimum. Hence, we may in turn derive:

Lemma 4. *Any equilibrium of our game is characterised by the following necessary and sufficient condition for each $n \in N$:*

$$\begin{aligned}\delta\mathcal{L}_n(b,l;E) &= \int_{k\in K}\left(\frac{du_{n,k}(x_n(k))}{dx_n(k)}\frac{b_n(k)}{a^J(k)B(k)} + \frac{du_{n,m}(x_n(m))}{dx_n(m)}\frac{\phi_{-n}(k)B(k)}{a^J(k)(\phi(k))^2}\right)E_l(k)d\mu(k)\\ &+\int_{k\in K}\left(\frac{du_{n,k}(x_n(k))}{dx_n(k)}\frac{B_{-n}(k)\phi(k)}{\big(B(k)\big)^2} - \frac{du_{n,m}(x_n(m))}{dx_n(m)}\frac{\phi_{-n}(k)}{\phi(k)}\right)E_b(k)d\mu(k)\\ &+\int_{k\in K}E_b(k)d\lambda_n^b(k) + \int_{k\in K}E_l(k)d\lambda_n^l(k) - \int_{k\in K}E_b(k)d\psi_n^b(k) - \int_{k\in K}E_l(k)d\psi_n^l(k) = 0,\end{aligned} \tag{4}$$

where $\mathbf{0} \le (\lambda_n^b, \lambda_n^l, \psi_n^b, \psi_n^l)$—the set of Lagrange multipliers to the constraints in (3)—verifies $\langle G(b_n,l_n),(\lambda_n^b,\lambda_n^l,\psi_n^b,\psi_n^l)\rangle = 0$, with $G(\cdot)$ representing the set of constraints, and $E_b(\cdot)$ and $E_l(\cdot)$ representing the arbitrary increments associated with bids and offers, respectively. As before, $J = F, H$, depending on whether $n \in F$ or $n \in H$.

Proof. See Appendix. □

Even though (4) is a complex-looking equation, a number of important insights may be gleaned from it. We defer these to the next section.

4 Equilibrium analysis

In this section, we will analyse and discuss the main (endogenously-derived) features of our model at equilibrium, all of which will be needed to prove our main finding (Theorem 2). Along the way, we provide a thorough and intuitive discussion of the role that strategic behaviour plays in the failure of the RLCA. We dissect, formally yet very simply, why agents do not specialise at equilibrium.

Our first result below is of independent interest. Crucially, it also sheds light on the mechanism that drives trade in this paper, thereby serving to reinforce our conclusions.

Proposition 1. *At any NE, trade takes place in all commodities in K.*

Proof. From (1) and (4), we have for any $k \in K$ that $\lim_{b_n(k)\to 0}\frac{du_{n,k}(x_n(k))}{dx_n(k)} = +\infty$. Consequently, if $\exists\mathscr{A} \in \mathscr{K}$, with $\mu(\mathscr{A}) > 0$, such that $b_n(k) = 0$ for $a.a\; k \in \mathscr{A}$, then (4) yields a contradiction, as E is arbitrary and $\int_{k\in K}\frac{du_{n,k}(x_n(k))}{dx_n(k)}\cdot\chi_{\mathscr{A}}(k)d\mu(k) = +\infty$, while each of $\int_{k\in K}\frac{du_{n,m}(x_n(m))}{dx_n(m)}d\mu(k)$, $\int_{k\in K}E_b(k)d\psi_n^b(k)$, and $\int_{k\in K}E_l(k)d\psi_n^l(k) < \infty$. So, at any NE, we must have μ-$a.e$ in K, $b_n(k)\cdot p(k) > 0$ $\forall n \in N$—i.e., not only is the market for every commodity "active," but every agent also purchases $a.a$ commodities $k \in K$ from each other. Hence, at equilibrium, both countries trade in $a.a$ goods in K, such that there is international free trade. □

We spell out some interesting implications of this proposition. First, like in standard trade models, this result is tantamount to the concept of agents being variety lovers—i.e., agents wish to consume every good at equilibrium. Indeed, notice that all goods in K may be viewed, either as different varieties of a single good, or as ***truly different individual*** products altogether. However, the equivalence between Proposition 1 and variety-loving agents is not as straightforward as it may appear at first glance. This is because in existing market games, even if consumers' utility functions satisfy the boundary condition $\lim_{x_n(k)\to 0} \frac{du_{n,k}(x_n(k))}{dx_n(k)} = +\infty$, μ-$a.e$ in $\{m\} \cup K$, autarky—the trivial outcome of no bids placed and no offers made—still remains as a feasible and plausible equilibrium. Yet, this uninteresting outcome is circumvented in our proposed model, largely thanks to its simplicity: for every agent, their allocation of any $k \in K$ is increasing in both bids and labour hours offered, such that every agent wishes to be active on every market given that s/he is not endowed with any $k \in K$. In doing so, every agent trades with each other, thus leading to international trade.

Second, note that our construction itself is *not* incompatible with, nor does it preclude, outcomes such as production taking place within each country, with no international trade. Yet, this scenario does not materialise *at equilibrium* because even if there were more than one market for every commodity, due to there being no economic frictions of any kind, agents would still be freely able to move across trading posts to "shop around". And, as discussed in Toraubally (2018), the equilibrium market (trading-post) structure is determined by the distribution of prices across posts for each commodity. Therefore, so long as the price of a commodity is the same across any number of trading posts (for that same commodity), these can be consolidated w.l.o.g. into one common, fully integrated international market. That is to say, our modeling strategy in this paper remains valid irrespective of the number of trading posts per commodity, and regardless of the *location* of these posts. This is a strong result.

Third, this finding is important also from a methodological perspective. Indeed, for proving the existence of a nontrivial equilibrium at which each market is active, Dubey and Shubik (1978) consider a perturbed market game, in which it is assumed that an outside agency places a positive amount of bids and offers on each market for each good. In our case, using Proposition 1, we are able to proceed differently, without the need to consider any outside agency, such that all features of our model remain purely endogenously derived. We are now in a position to prove that:

Theorem 1. *There exists an equilibrium with active international trade in all commodities.*

Proof. See Appendix. □

Notice that without any further characterisation, Theorem 1 does not tell us anything about the patterns of specialisation that take place within the world economy. But as we prove in the Appendix, at any NE of our model, *every* agent uses up all his/her labour hours to produce—and by virtue of our trading mechanism, also **export to** and **import from** each other—*all* commodities in K. This conclusion leads us naturally onto the main contribution of this paper, which is that:

Theorem 2. *NE for our game exist, and at **every** possible NE, there is intra-industry trade (only) in every sector.*

Proof. See Appendix. □

Complete or even partial Ricardian specialisation never take place in our setup because the market-clearing price of each good is sensitive to every individual agent's purchases and sales. This price sensitivity persists no matter the number of agents, so long as $|N| < \infty$. This is a far cry from perfectly and monopolistically competitive models in which each lone actor is insignificant, and prices are given by Adam Smith's invisible hand. We will next decompose the role that strategic behaviour plays in driving the failure of the RLCA into its component parts.

The situation in Theorem 2 is sustainable as an equilibrium because strategic behaviour here implies that the marginal price of a commodity is not equal to its average price. To see this, let $\hat{s} = (\hat{b}, \hat{l})$ denote an equilibrium profile, with $\hat{b}_n(k) \cdot \hat{l}_n(k) > 0$ μ-$a.e$ in K, $\forall n \in N$, and let the strategies of all $i \in N \setminus \{n\}$ be given. Next, take any pair of goods $k, t \in K$, let $\frac{\hat{B}(k)}{\hat{\phi}(k)} := \hat{p}(k) \leq \hat{p}(t) := \frac{\hat{B}(t)}{\hat{\phi}(t)}$, and consider an agent $n \in N$ for whom $x_n(t) \geq x_n(k)$. Given agents' utility functions exhibit diminishing marginal utility, it is natural to wonder why agent n cannot do better by, e.g., reducing his purchases of t, and bidding more for commodity k, especially since the latter is at least as cheap. In fact, any such unilateral deviation makes n worse off, as can be inferred from the price formation mechanism. In particular, because all agents have market power, as soon as n reduces $\hat{b}_n(t)$, the price of t goes down. Crucially, it is not only the marginal unit that now sells at this lower price, but all units of t. Thus, not only does $x_n(t)$ go down, but the receipts from the sale of commodity t also do. At the same time, no sooner had n bidden more for k than its price went up. Again, it is not only the marginal unit, but all inframarginal units of k, that now sell at this *higher* price. Consequently, while $x_n(k)$ does increase, as do the proceeds from the sale of k, both effects are more than offset by the respective decreases in $x_n(t)$ and the receipts from the sale of t. Thus, by deviating, n can, *at best*, do no better than by playing $(\hat{b}_n, \hat{l}_n)$, such that $\hat{s}$ is indeed an NE.

Next, as discussed earlier, given the DFS-type technology, country $F(H)$ has a comparative advantage in the production of all goods for which $\zeta < 1$ ($\zeta > 1$). Yet, no specialisation takes place because each strategic agent simultaneously enters the market for every commodity both as buyer and seller—a strategy commonly referred to as making a ***wash sale***. To understand the wisdom behind making a wash sale in the present game, take $\hat{s} = (\hat{b}, \hat{l})$ to be as described above, pick any $n \in N$, and let $\hat{s}_{-n}$ be given. Consider *any* $k \in K$ for which $x_n(k) \ll \hat{\phi}_{-n}(k)$, and notice that the strategy $(\breve{b}_n(k), \breve{l}_n(k)) = \left(\frac{\hat{B}_{-n}(k) \cdot \hat{b}_n(k) \cdot (\hat{\phi}_{-n}(k) + \hat{l}_n(k))}{\hat{B}_{-n}(k) \cdot \hat{\phi}_{-n}(k) - \hat{b}_n(k) \cdot \hat{l}_n(k)}, 0\right)$ generates the same allocation of k to n. So why play $(\hat{b}_n, \hat{l}_n)$? This is because this strategy allows n to increase the *thickness* of the market, such that the impact of every other agent's bid is reduced. This way, n has a stronger hand in "nudging" the terms of trade to his advantage. But this is true of every $n \in N$, for $a.a$ $k \in K$. Hence, the strategic nature of agents causes them not to specialise. To make this concept crystal clear, a numerical example may prove instructive.

In Table 1, we sketch a simple scenario illustrating the effect of a wash sale. Going forward, it is helpful to recall each agent's optimisation programme in (3), which can be summarised simplistically as follows: she tries to maximise her consumption of $a.a$ $k \in K$ subject to spending as little as possible. Choose any $n \in N$, and pick the $k \in K$ for which $\zeta(k) = 1$—this is w.l.o.g. and has been done only to ease the

Table 1: Numerical illustration of wash sales

	No wash sales	With wash sales
$(B_{-n}(k), \phi_{-n}(k))$	$(13, 19)$	$(13, 19)$
$(b_n(k), l_n(k))$	$(\frac{130}{47}, 0)$	$(2, 6)$
$x_n(k)$	$\frac{10}{3}$	$\frac{10}{3}$

exposition. Given the strategies of all the other agents as in Table 1, the strategy $(b_n(k), l_n(k)) = (2, 6)$ yields an allocation $x_n(k) = \frac{10}{3}$. It can be easily verified that so does the strategy $(b_n(k), l_n(k)) = (\frac{130}{47}, 0)$. Nevertheless, by playing $(2, 6)$, n not only achieves $x_n(k) = \frac{10}{3}$, but does so spending less ($2 < \frac{130}{47}$), alongside obtaining revenue from the sale of k as well ($6 \cdot \frac{2+13}{6+19} > 0$). Thus, by behaving strategically and making a wash sale, n reduces the effectiveness (purchasing power) of every other agent's bid.

Finally, note that the market game delivers equilibrium outcomes that are individually rational, i.e., trade (both within and across countries) is mutually beneficial for each $n \in N$.[13] In particular, intra-industry trade in all goods benefits society because it brings about more strenuous competition in the market for each commodity, which leads to a reduction in sellers' monopoly power. In competitive economies, an individual agent entering both sides of the market for a commodity changes nothing at the aggregate level: none of the demand, supply, and therefore price, changes. But under oligopoly, even if only one agent were to engage in intra-industry trade, he would be able to influence the overall price and/or allocation in such a way as to make himself better off. And since our framework is characterised by complete information, every other agent does likewise. Indeed, Goenka, Kelly and Spear (1998) demonstrate that in strategic market games, there are always multiple, Pareto-rankable equilibria, and provided some regularity conditions are met, agents prefer thick- to thin-market equilibria. In our model, intra-industry trade is intimately related to wash sales, which involve agents entering both sides of the market for every commodity (see Table 1). This dramatically increases the thickness of markets. Hence, a similar conclusion to Goenka et al.'s (1998) is true in our setup as well: thick-market NE Pareto-dominate thin-market (complete specialisation) ones. Now, the strategic equilibrium arrived at might well be second best and be Pareto suboptimal. This is not to say that a benevolent social planner could or even should intervene. Without imposing additional structure on the system, the effectiveness of, and requirement for, government intervention are both ill-defined. Indeed, in the international sphere, mechanisms such as contracts and legal enforcement of reliance clauses, or direct government regulation, either do not exist, or operate with limited enforcement capabilities.

More can be said if one were to assume the existence of a social welfare function. NE of Γ are, in general, not Pareto optimal such that agents can be made better off by a reallocation of resources. Hence, inasmuch as the allocations from the NE are not Pareto optimal, a benevolent social planner would open the markets for successive rounds of retrading. However, a very careful distinction must be made: the planner will not necessarily strive towards achieving a Walrasian allocation, as it can very well be the case

[13] Recall that an allocation is said to be individually rational if it does not give any agent a payoff lower than his/her minmax value, $\underline{v}_n := \min_{s_{-n}} \max_{s_n} u_n(x_n(s_n, s_{-n}))$—see, e.g., Fudenberg and Tirole (1991: pp. 150-153).

that the latter yields a lower level of social welfare than a non-Walrasian NE would—recall that Pareto efficiency is not synonymous with equity.

4.1 A remark on trade costs

Our main aim in this paper was to present a bare-bones mathematical model which is tractable, simple to understand, and yet powerful in the conclusions it yields. For clarity of understanding, we therefore chose to do away with trade costs as we believe these would have detracted from the trading mechanism at play in our framework. This is not to say that our model cannot account for trade costs. It most certainly can. Trade costs can be modelled either in the form of iceberg costs, or in terms of net trade.

Perhaps the most natural way to model trade costs in our market game would be via iceberg costs: retain the assumption of one fully integrated international market, and impose transport costs that vary proportionally with distance à la Samuelson (1952). Whatever cost is incurred is then subtracted from the arriving volume such that only a fraction of what is produced is actually sold. As such, so long as the two countries are not equidistant from the international trading post, they will pay different effective prices for the same commodity. Such a specification would require a minor tweak to the allocation rule but leave agents' strategy sets untouched, which translates into very little to no changes to the primitives of our model. It is therefore not difficult to see that in such a scenario, our methods of proof and conclusions will remain unchanged, thereby reinforcing Krugman's (1980) point on how the resultant equilibrium "[...] depends on the assumed form of the [iceberg] transport costs, which shows at the same time how useful and how special the assumed form is (p. 954)." We formalise this argument below.

Denote the fully integrated international market by I. Then, as in Samuelson (1952), Krugman (1980, 1991), and Alvarez and Lucas (2007), assume that one unit of labour shipped from country $J, J = F, H$, to I results in τ_{JI} units of labour being offered at I. Akin to Feenstra (2015: p. 100), this formulation can be interpreted as the services of labour being used up in the transportation process. If, as we do in fact construe it, $(1 - \tau_{JI})$ represents a cost that varies proportionally with distance, then it is natural to assume that $0 < \tau_{JI} \leq 1$, with $\tau_{JI} = 1$ iff $J = I$. Hence, for any good $k \in K$, of the total labour that is transported to the trading post, only $\phi_k := \tau_{FI} \sum_{f \in F} \frac{l_f(k)}{a^F(k)} + \tau_{HI} \sum_{h \in H} \frac{l_h(k)}{a^H(k)}$ is actually traded and consumed. Correspondingly, for any $n \in N$, $x_n(m)$ becomes $e_{n,m} - \int_{k \in K} b_n(k) d\mu(k) + \tau_{JI} \int_{k \in K} \frac{l_n(k)}{a^J(k)} \frac{B(k)}{\phi(k)} d\mu(k)$, $J = F, H$, while (the formula for) x_k remains $\frac{b_n(k)}{B(k)} \phi(k)$. Applying these elementary changes to the allocation rule is sufficient to incorporate iceberg trade frictions into our model.

Nonetheless, this simple and seemingly elegant solution is unfortunately not without consequence.[14] We first provide some context to set the scene. Against the backdrop of perfect competition, Alvarez and Lucas (2007) demonstrate that proving the existence and uniqueness of (general) equilibrium in a trade environment featuring, amongst others, iceberg costs is not a trivial mathematical exercise. Indeed, the study of this class of models is so important that it has occupied the minds of trade theorists for a very long time. As Allen et al. (2020) argue, in such frameworks, the intricate general-equilibrium interactions and the very diverse set of assumptions on which different prototypes rest have significantly hampered

[14] I thank an anonymous referee for drawing this issue to my attention, for encouraging me to discuss the general-equilibrium implications of iceberg costs in the current setup, and for suggesting many of the references hereby cited.

our understanding of the inner workings of these models such that there is no straightforward answer to the questions of existence and uniqueness of equilibrium. Our model, though it has been cast in an oligopolistic setup and has a completely different allocation mechanism to competitive ones, is also not immune to similar technical concerns. Whilst the existence of equilibrium is unaffected once iceberg costs are accounted for, proving uniqueness (or otherwise) is a task which entails formidable difficulties, task which we do not pursue here—a legitimate limitation of our setup. Moreover, as with any other model deploying the iceberg-cost approach, the physical loss of goods associated with this specification causes serious distortions. In our case, it generates a deadweight loss given that agents' combined final allocation of every $k \in K$ falls. Another unaesthetic trait of iceberg trade costs is that our system does not remain closed anymore. By construction, some goods are lost in transit, and just disappear from the system.[15] This pulls in the opposite direction from our principal objective, which is to provide as explicit and as explanatory as possible a general-equilibrium model of price formation and resource allocation.

A perhaps more sophisticated (and satisfactory) way of tackling the trade cost problem for our purposes would be to impose a service charge, $c_J(k)$, $k \in K$, $J = F, H$, per unit of net trade as introduced in Toraubally (2018). This service charge is, in essence, very similar to iceberg trade costs, and would create a material separation between different markets for the same commodity. Importantly, it only distorts relative prices (both within and across markets), and entails no good being lost and the system remaining perfectly closed. Many recent papers study the implications of modeling trade costs that depart from the iceberg specification, e.g., Head and Ries (2001; ad-valorem tariff + non-tariff barriers), Hummels and Skiba (2004; ad-valorem + additive per-unit costs), Feenstra and Romalis (2014; ad-valorem + additive specific trade costs), Irarrazabal et al. (2015; ad-valorem + additive per-unit costs), and Perroni and Suverato (2023; per-unit export service). The specification we propose, while *very* close in spirit to the above-mentioned ones—especially Perroni and Suverato (2023)—is still different in one important respect (explained below) and should thus be viewed as being complementary to these approaches. We sketch how this can be achieved.

For each $k \in K$, let there be $T(k)$ trading posts—$T_F(k)$ in F and $T_H(k)$ in H, with $T_F(k)+T_H(k) = T(k)$. Agents in F would then own the trading posts in their country, and likewise for agents in H. For simplicity, let the service charge across all trading posts in F be the same, $c_F(k)$, and similarly for those in H, $c_H(k)$. In the spirit of iceberg trade costs, we can allow $c_F(k) = c_H(k) = c(k)$, although this is by no means necessary and $c_F(k) \neq c_H(k)$ would work just as well. This way, agents in F would have an additional cost when exporting their goods to country H, and vice versa. To put flesh on this concept, let $z_n^j(k) = \frac{l_n^j(k)}{a^J(k)} \frac{B^j(k)}{\phi^j(k)} - b_n^j(k)$ denote the net monetary trade of agent n in commodity k in market j. Then, the total commission/charge payable by any agent n is $\int_{k \in K} \left(\sum_{j \in T(k)} c(k) \beta_n^j(k) z_n^j(k) \right) d\mu(k)$, where $\beta_n^j(k) \in \{-1, 1\}$ depending on the direction of net trade n makes. And indeed, in such a scenario, one can get the so-called Law of One Price to fail at equilibrium. That is to say, trading posts for the same

[15] In benchmark monopolistically competitive models with a continuum of goods (or varieties thereof) and constant trade elasticity, bilateral trade flow variations occur either entirely on the intensive margin (Krugman, 1980) or exclusively on the extensive margin (Melitz–Pareto, 2003)—see Fernandez et al. (2022). In the setup we describe with iceberg costs, each large agent $n \in N$ still produces and exports every $k \in K$, albeit less of each. This means that it is the intensive, instead of the extensive, margin that is affected.

commodity would not all have the same price at equilibrium.[16] Interestingly, even in such a scenario, intra-industry trade will still take place. This is because the benefits of interacting in "thick" markets in strategic market games would counteract the disadvantageous terms of trade, i.e., agents still gain from making wash sales, the benefits of which are illustrated in the simple numerical example in Table 1. In particular, agents in F can import and export goods in such a way as to pay exactly the same amount that they receive in commission, and similarly for agents in H. Alternatively, consumers in F can both purchase from, and sell in, H in a way such that their net trades are just zero so they pay no service charge whatsoever, and conversely for agents in H. While seemingly pointless, the latter exercise serves a very important purpose: it increases the thickness of both markets. In both cases, agents benefit from what Goenka, Kelly and Spear (1998) prove: thick market equilibria Pareto dominate thin market equilibria. Hence, our analysis is robust to the introduction of trade frictions.

However, the inclusion of this apparently innocuous service charge would require changing the strategy sets and allocation rules, and the market game Γ would then become a generalised game à la Arrow–Debreu (1954). This in turn leads to a number of obstacles that need to be overcome. In the first instance, this specification would make for an extremely cumbersome notation.[17] Next, in light of the technical context analysed in our paper, especially proving the existence of equilibrium in an infinite-dimensional framework, there are other very lengthy mathematical details which would need taking care of, and these would be beyond the scope of this already technically heavy paper. Just as an example, proving the convexity of the Consumption Possibilities Set—which is the first step in the proof of Theorem 1—itself becomes a very complicated exercise. Furthermore, one cannot completely rule out the possibility of additional equilibria exhibiting different patterns of specialisation (more precisely, export and import) appearing, equilibria which would nonetheless co-exist with the type of equilibria described in Theorem 2. This is certainly not a problem, although the question then turns on whether these NE can be Pareto-ranked, which is a whole new ball game.

[16] It is now opportune to explain why our approach differs from the ones mentioned in the preceding paragraph. Pick any good k, and recall that agents are charged per unit of monetary net trade. Hence, it is not hard to see that even if the Law of One Price fails to obtain, an agent could still find himself paying the **same** relative price for k at equilibrium depending on his net trades across F and H, and the values of $c_F(k)$ and $c_H(k)$. This is not to say that this agent will actively and always strive to achieve such an outcome; it is merely a possibility. In a general-equilibrium setting, this happenstance is, as far as we aware, unique to market games, and is the very embodiment of individual agents possessing market power. Such an outcome can never occur in frictionless monopolistic or Walrasian settings. Now, to see how this situation is sustainable as a Nash equilibrium, suppose the *nominal* price of k is higher in F than it is in H. It would be tempting to think that an agent who made a net purchase in F and a net sale in H could do better by now, e.g., selling more in F and buying more in H. However, this is erroneous as in oligopolistic markets, this rejig involves a price effect. Indeed, by doing so, this *individually significant* instantly depresses the price of k (for every, and not only the marginal, unit) in F in view of the increased supply there. At the same time, the price (again, for every unit of k) in H rises due to increased demand for the good there. The net upshot is that the price, and relative-price, yield in such a way as to make any such unilateral deviation ineffective at best or detrimental at worst, such that he stays put—see the discussion in the Introduction, and Tirole (1988).

[17] See the previous paragraph, and note that $\int_{k\in K}\left(\sum_{j\in T(k)} c(k)\beta_n^j(k)z_n^j(k)\right)d\mu(k)$ only represents the commission payable, and does not include the net trade itself.

5 Testable implications, managerial applications, and conclusion

The last decade or so has seen the emergence of widespread availability of microeconomic data, which has led to heavier emphasis being laid on the study of firm behaviour at a microeconomic level. Amongst the most robust findings of this literature is that different firms charge variable markups, a fact which, together with other aspects of the data—such as incomplete cost pass-through—has proved hard to synthesise into a tractable theoretical model. In spite of this, the workhorse model in international trade still assumes either constant elasticity of substitution (CES) demand or monopolistic competition (MC) or both (CES $\vee$ MC), where "$\vee$" signifies inclusive disjunction. Yet, for many questions connected to pricing, or for analyses related to welfare, CES $\vee$ MC models sorely lack credibility (Arkolakis and Morlacco, 2017). To further complicate matters, for a large class of models which relax the CES $\vee$ MC assumption, predictions about markups and cost pass-through depend entirely on the demand side of the economy such that carefully selecting the structure of preferences is of utmost importance if the model is to be empirically useful. In this light, our general formulation is appealing for empirical purposes given it encompasses not only CES demand, but also other widely used substitutes to CES such as Pollak's Additive von Neumann-Morgenstern, Kimball, and Addilog utility functions, and quadratic mean of order r (QMOR) expenditure functions—see Arkolakis et al. (2019) for more details and references about the aforementioned specifications. This flexibility makes it an important alternative when it comes to analysing, and testing, various demand systems, as we next illustrate.

Since our model is one of intra-industry trade, it may be argued that it presents a strategic-behavioural take on the Country Similarity Theory/Linder hypothesis (LH). The LH (1961) posits that countries form comparative advantages in those goods for which domestic demand is high. It also postulates that rich (respectively, low-income) countries disburse a larger proportion of their income on high- (low-) quality goods. Accordingly, rich (low-income) countries invest in productive capacity to form comparative advantages in high- (low-) quality goods. Linder then argues that the more substantial is the overlap of production and consumption patterns between countries, the more intensively they will trade with one another (Hallak, 2010). Insofar as demands for *many* goods are non-homothetic, intensive trade between countries with similar demand structures implies intensive trade between similarly rich countries (Fajgelbaum et al., 2015). However, support for the LH in the literature has been mixed, with the empirical evidence leaning both sides, and strongly so—see, e.g., Fu et al. (2020), and Kitenge (2021). In this regard, thanks to the existence of comparative advantages, and by virtue of the demand specification we adopt, one can easily parametrise preferences to make countries very similar to each other, or, within the same framework, parametrise preferences such that they are vastly heterogeneous across countries. Our demand system is also flexible enough to capture cross-country differences in the demand for quality. This adaptability is especially useful for performing reduced-form analyses of the demand side of Linder's theory. Our work thus constitutes a very natural explanation and test of the LH.

Additionally, Porter (1980) notes that in most industries, competition is characterised by mutual dependence: agents perceive the effects of the other players' moves, and react to these. He argues that when buyers possess a nontrivial degree of bargaining power, they fight to bring prices down, they bargain for more goods, and they compete against each other. On the other hand, sellers have to select the right move, which entails finding one whose payoff is instantaneously determined, and also skewed as much as possible towards the agents' respective self interests. Our construction is rich enough to capture all these intricacies, and versatile enough to be used to test the robustness of the RLCA with respect to changes in market structure. And indeed, the majority of international trade today is carried out within global value chains (GVCs) which are dominated by large firms wielding substantial market power, and which individually affect specialisation patterns. A prominent feature of such trades is that of prices being formed through buyers and sellers negotiating *bilaterally* (Antras, 2015). Notwithstanding their importance from an empirical viewpoint, there is not much that is known regarding the pricing implications of agents possessing two-sided market power in trade, and much less from a general equilibrium perspective. Indeed, Alviarez et al. (2022) develop a theory accounting for large firms strategically using their (**two-sided**) market power to change the terms of trade in their favour, but for tractability reasons, this is done only at a partial equilibrium level and with finitely many goods. They construct an elegant flexible pricing framework which they use to measure the micro- and macroeconomic consequences of two-sided market power on US import prices, and they generate novel evidence. Nonetheless, their model does not feature intermarket linkages. Given that most of the economic decision-making that takes place even at aggregate level is essentially individualistic/strategic, a (game-theoretic) model that is able to capture such noncooperative interactions alongside the associated price formation dynamics seems most apt indeed. Our model bridges this gap. Its tractability and explicit price formation mechanism imply that it lends itself well, not only to sector level analyses of GVCs, but also to economy-wide level tests, in a continuum-of-good framework.

Further credence is given to this testable prediction by Plenert (2002) who explains that as far as interactions between large players are concerned, blindly following the RLCA creates dependencies, citing the example of the US shifting most of its oil production[18] to the Middle East, Latin America, and the North Sea between 1985 and 2008. Although its annual oil production subsequently increased nearly each year since 2009 until 2019 (US Energy Information Administration, 2020), this philosophy has left it vulnerable to, and heavily dependent on, cartel pricing. The significance of the latter point cannot be overemphasised today, with oil prices having plummeted to record new levels as a result of the glut caused by the Covid-19 outbreak. This has left a lot of cash-strapped US shale producers on the brink of bankruptcy, with casualties a certainty if "OPEC goes the other way and decides to fight another battle for market share by keeping the taps flowing" (Brower, 2020). Thus, what should be produced in an international corporate strategic plan of operation relies crucially on finding the optimal extent to which specialisation should be engaged into. Our model, and the empirical evidence, suggest that while the US should still import crude oil from other oil-rich countries, it should nonetheless retain its ability to produce oil as well, which it is now doing.

Equally importantly, our model, especially the no-specialisation outcome, finds very timely applications in the US technology industry, in the phenomenon of ***patent pooling***. Considering the ease with which

[18]Think of a perishable inventory system where the replenishment process is constant (see, e.g., Graves, 1982), such as a chemical plant or oil refinery, for which it is very expensive to shut down or discontinue the aforementioned process. Given the timing of demand requests, and assuming inventory is issued oldest first, a manager may view the "new" inventory that comes in at each time period—of which there are uncountably many—as a different commodity.

companies are able to acquire patents now, the big, well-established tech firms all routinely spend hefty sums to build patent portfolios on any new discoveries and inventions they produce. However, instead of each firm then engaging in costly multi-layer negotiations over licensing their new tech to other firms in the same sector, all the firms in the industry put their patents into a pool (administered by a contracted agent), and all pool participants are licensed to use all the patents in the pool at a reasonable royalty rate that is periodically renegotiated. As a result, any firm wishing to enter the tech industry and compete with an incumbent owning a wide array of patents[19] finds itself hampered due to intra-industry trade in an overlapping set of patent rights—a so-called patent thicket. As a general rule, royalties collected are then allocated to each pool member in proportion to their patent's value (see, e.g., Jeon and Nishihara, 2018).[20] What is interesting here is the fact that all of the firms hold fairly diverse portfolios of patents, rather than a small number of patents specific to their core businesses. The reason for this seems to be that ***diverse*** patent holdings enhance the individual strategic firm's bargaining power when it comes to negotiating the terms of licensing, with firms holding ***only specialised*** patents being forced to discount their use significantly, much like our construction stipulates. The primary value of patent portfolios to large firms like Microsoft is as bargaining chips in cross-license agreements. As Varian (2014) explains, the patent thickets constructed by each company function like the nuclear missiles which the US and USSR held during the Cold War. Both had enough missiles aimed at each other to ensure *mutual destruction* in case one party attacked, such that neither side dared to launch an aerial attack. The same concept is true of patent thickets, and of strategic behaviour in our model more generally (see the wash sales example in Table 1). It follows that despite the large numbers of patents and irrespective of where comparative advantages lie, intra-industry trade is optimal and always obtains at any equilibrium. Our model captures such interactions explicitly and in tractable fashion, in a many-good oligopolistic (and oligopsonistic) setup, and also puts forth predictions consistent with the goings-on in the US tech industry.

What then, can managers and policymakers juggling production, exports, and imports, infer from the simple model that we propose? That when faced with oligopolistic agents in a many-good scenario, free trade without resorting to protectionism is still beneficial to all parties involved. Moreover, our formulation yields a clear, unequivocal prediction: it pays for ***every*** agent involved to produce ***all*** the goods under consideration, both those in which they have comparative advantages, and disadvantages.

Obviously, the tech sector and international operations management are but a few of the areas in which the RLCA is extensively used to inform a host of multi-billion-dollar decisions. There are many other fields in which social scientists are heavily reliant on the potential gains from the RLCA, namely, in economic development (see, e.g., Lorenczik et al., 2017), in R&D and advertising (see, e.g., Erickson and Jacobson, 1992), in production and inventory management (see, e.g., Erickson and Jacobson, 1992), and even in academia (see, e.g., Shubik, 2002: p. 194)—hence further highlighting the far-reaching ramifications of our conclusions.

In this paper, we have provided a positive theory of intra-industry trade that is of relevance to managers,

[19]To illustrate, over 2003-04, Microsoft obtained over 1,000 patents (Varian, 2014). The same is also true of the cellphone market, where applicable standards which enable the production of interoperable end products use thousands of patented technologies as inputs (Sidak, 2009). Hence, for such markets, the use of a many-good framework is truly very fitting.

[20]Notice already how this bid-and-offer process and the allocation rule closely resemble the trading mechanism of our market game.

marketers, and policymakers. Our theory yields clear predictions of which and how many commodities each agent should produce and export. A word of warning is timely here. Our analysis has shown that when agents engage in strategic decision-making, the RLCA no longer holds. This in no way implies that DFS' model, and the subsequent papers that build on the latter, are wrong. Rather, what this work has shown is that extreme care needs to be taken when interpreting the predictions of Ricardian models, especially when these are framed in ***oligopolistic*** settings, when individual agents have market power on ***both*** sides of each market. Admittedly, our conclusions are inevitably driven by the adoption of norm-bounded subsets of L^{∞}, which we use due to the well-known measurability issues that arise when considering a measure space of commodities—issues which are inexistent when only finitely many goods are used (see, e.g., Toraubally, 2017). Notwithstanding this limitation, which does not violate Ricardo's premise (see, e.g., Haberler, 1936), we believe that our specification is justified as there is no *other obvious*, tractable way of modeling a continuum of goods in a market game framework.

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Appendix

Proof of Lemma 1. Following Bewley (1981), we will say that a sequence x_i in L^∞ converges to x in measure on sets of finite measure if for any E in K such that $\mu(E) < \infty$ and for any $\epsilon > 0$, $\mu\{k \in E : |x_i(k) - x(k)| \geq \epsilon\} \to 0$. Crucially, this sense of convergence defines a ***metrizable***, vector space topology on L^∞. Let $\mathscr{D}$ denote a metric for this topology. To define this metric, let $K_1 \subset K_2 \subset K_3 \subset \ldots \equiv \{K_i\}$ in K be of finite μ-measure, and such that $\bigcup_{i=1}^{\infty} K_i = K$. Then, if x and y are in L^∞, let

$$\mathscr{D}(x,y) = \sum_{i=1}^{\infty} \frac{1}{2^i \mu(K_i)} \left[\inf_{0<\alpha\leq 1} \left(\alpha + \mu\left\{k \in K_i : |x(k) - y(k)| \geq \alpha\right\}\right) \right].$$

The topology generated by $\mathscr{D}$ is the so-called ***topology of convergence in measure on sets of finite measure***. The measure space $(K, \mathscr{K}, \mu)$ is complete and finite. By the arguments in Toraubally (2018) and Appendix A of Toraubally (2019), we have that the graph of the correspondence S_n, $\mathrm{Gr}(S_n)$, is measurable. It is clear that S_n is non-empty valued. Now, on norm-bounded subsets of L^∞, the Mackey topology equals the topology of convergence in measure on sets of finite measure. As such, we have that these subsets are complete and separable metric spaces (when equipped with the metric $\mathscr{D}$ as defined above). Thus, the measurable selections b_n and l_n exist, by Aumann's Measurable Selection Theorem. □

Proof of Lemma 2. Consider the dual system $\langle L^1, L^\infty \rangle$, where $L^1(K, \mathscr{K}, \mu)$ is the space which consists of (equivalence classes of) integrable functions $f : K \to \mathbb{R}$. The Mackey topology $\tau(L^\infty, L^1)$ is the finest locally convex Hausdorff (l.c.h) topology on L^∞ such that the (topological) dual of L^∞ is L^1. The ***weak-star*** topology $\sigma(L^\infty, L^1)$ is the coarsest l.c.h topology on L^∞ such that the dual of L^∞ is L^1, while the ***weak*** topology $\sigma(L^1, L^\infty)$ is the coarsest l.c.h topology on L^1 such that the dual of L^1 is L^∞. For any locally convex topological vector space (l.c.t.v.s) Y, let its (topological) dual be denoted by Y^*. We may now proceed. L_τ^∞ is an l.c.t.v.s, and its dual separates points. By Lemma 1, b_n and l_n are measurable. As b_n and l_n take values in a metrizable space, they are also weakly measurable (see, e.g., Aliprantis and Border, 2006: p. 593). Given $\mu(K) < \infty$, it is trivial to see that the scalar function $\Lambda b_n : K \to \mathbb{R}$ defined by $(\Lambda b_n)(k) = \Lambda(b_n(k))$, is integrable with respect to μ for every $\Lambda \in (L_\tau^\infty)^*$. Similarly, $\Gamma l_n : K \to \mathbb{R}$ defined by $(\Gamma l_n)(k) = \Gamma(l_n(k))$, is integrable with respect to μ for every $\Gamma \in (L_\tau^\infty)^*$. Pettis-integrability of the weakly measurable maps b_n and l_n means that we now need to show that $\exists \mathscr{B}, \mathscr{L} \in L_\tau^\infty$ such that $\Lambda\mathscr{B} = \int_{k\in K} \Lambda(b_n(k))d\mu(k)$ for every $\Lambda \in (L_\tau^\infty)^*$, and $\Gamma\mathscr{L} = \int_{k\in K} \Gamma(l_n(k))d\mu(k)$ for every $\Gamma \in (L_\tau^\infty)^*$. So, let $\gamma_n = b_n, l_n$ and $\Psi = \Lambda, \Gamma$, where these are as described above, and put $M = \max\{e_{n,m}, Q_n\}$. First, note that we may view $\left(L^1, \sigma(L^1, L^\infty)\right)^* \equiv \left(L_w^1\right)^* = \left(L^\infty, \sigma(L^\infty, L^1)\right) \equiv L_{w^*}^\infty$, and accordingly, $(L_{w^*}^\infty)^* = L_w^1$. Take an *arbitrary* $\mathscr{A} \in \mathscr{K}$ and define a linear map $T_\mathscr{A} : L_w^1 \to L_w^1$ by $T_\mathscr{A}(\Psi) = \Psi(\gamma_n \chi_\mathscr{A})$, where $\chi_\mathscr{A}$ denotes the characteristic function of $\mathscr{A}$. Clearly, $\int_{k\in\mathscr{A}} \Psi(\gamma_n(k))d\mu(k) = \int_{k\in K} \Psi(\gamma_n(k)\chi_\mathscr{A}(k))d\mu(k)$. Next, note that for any l.c.t.v.s Y, the weak topology $\sigma(Y, Y^*)$ is generated by $\mathscr{S}$, the family of seminorms of the form $p(y) = p(y; y_1^*, y_2^*, \ldots, y_s^*) = \sup_{1\leq j\leq s} \left|\langle y, y_j^* \rangle\right|$, where $y_1^*, y_2^*, \ldots, y_s^*$ are an ***arbitrary*** finite system of elements of Y^* (see Yosida, 1952: p. 112). So, let us consider a finite system of elements $\Psi_1^*, \Psi_2^*, \ldots, \Psi_s^* \in L_{w^*}^\infty$, where $\Psi_j^* := \frac{M}{j}$ $a.e$ in K, and consider the seminorm $\hat{p}(\Psi) = \hat{p}(\Psi; M, \frac{M}{2}, \ldots, \frac{M}{s}) = \sup_{1\leq j\leq s} \left|\langle \Psi, \Psi_j^* \rangle\right|$. It can be straightforwardly verified that $\hat{p}$ is continuous, as clearly, there exist $\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_t \in \mathscr{S}$, $\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_t \neq \hat{p}$, and positive scalars $a_1, a_2, \ldots, a_t$, such that $\hat{p}(y) \leq \sum_{i=1}^{t} a_i \tilde{p}_i(y)$. But then, quite easily, we have, for any $\Psi \in L^1_w$, that

$$\int_{k\in\mathscr{A}} \Psi(\gamma_n(k))d\mu(k) = \int_{k\in K} \Psi(\gamma_n(k)\chi_{\mathscr{A}}(k))d\mu(k) \leq M \int_{k\in K} \Psi d\mu(k) = \sup_{1\leq j\leq s} \left|\left\langle \Psi, \Psi_j^* \right\rangle\right|.$$

That is, the linear functional defined by $\Psi \mapsto \int_{k\in\mathscr{A}} \Psi(\gamma_n(k))d\mu(k)$ is continuous on L^1_w as it is dominated by a continuous seminorm. Hence, the mapping $\Psi \mapsto \int_{k\in\mathscr{A}} \Psi(\gamma_n(k))d\mu(k)$ defines an element of $\left(L^1_w\right)^* = L^\infty_{w^*}$. But on norm-bounded subsets of L^∞, the weak-star topology $\sigma(L^\infty, L^1)$ coincides with the Mackey topology $\tau(L^\infty, L^1)$, which concludes our proof. □

Proof of Lemma 3. Our method closely follows Toraubally's (2022). Let $\mathbf{X}_n$ denote the set of all feasible allocations for any $n \in N$. Let $x_n^{'}$ and $x_n^{''} \in \mathbf{X}_n$, where $x_n^{'}$ and $x_n^{''}$ are obtained by the vectors of strategies $(b_n^{'}, l_n^{'})$ and $(b_n^{''}, l_n^{''})$, respectively. We need to show, given the strategies of all the other agents, that for all $\lambda \in [0,1]$, $\lambda x_n^{'} + (1-\lambda)\, x_n^{''} \in \mathbf{X}_n$, i.e., if $x_n^{*} \coloneqq \lambda x_n^{'} + (1-\lambda)\, x_n^{''}$, then we must show that $\exists (b_n^{*}, l_n^{*})$ feasible such that $x(b_n^{*}, l_n^{*}) = x_n^{*}$. So, **fix** any $\lambda \in [0,1]$, consider a commodity $k \in K$, and define,

$$\begin{aligned} x_n^{'}(k) &= x(b_n^{'}(k), l_n^{'}(k)); \\ x_n^{''}(k) &= x(b_n^{''}(k), l_n^{''}(k)). \end{aligned}$$

Since no confusion should arise, we drop the n subscript. W.l.o.g, let $l^{''}(k) \geq l^{'}(k)$. As $x(k)$ is increasing in $l(k)$ (holding $b(k)$ constant), it has to be true that

$$x(k) = x(b^{'}(k), l^{'}(k)) \;\leq\; x(b^{'}(k), l^{''}(k)) \equiv \bar{x}(k).$$

But $x(k)$ is strictly concave in $b(k)$ (holding $l(k)$ constant), such that we have

$$\begin{aligned} x^{*}(k) &= \lambda x^{'}(k) + (1-\lambda)\, x^{''}(k) \\ &\leq \lambda \bar{x}(k) + (1-\lambda)\, x^{''}(k) \\ &< x(\lambda b^{'}(k) + (1-\lambda)\, b^{''}(k), l^{''}(k)) \equiv x(b^{\lambda}(k), l^{''}(k)). \end{aligned}$$

$x(k)$ is continuous, and increasing in $b(k)$ (holding $l(k)$ constant). Since $0 = x(0, l^{''}(k)) < x^{*}(k) < x(b^{\lambda}(k), l^{''}(k))$, $\exists b^{*}(k) \in (0, b^{\lambda}(k))$ such that $x(b^{*}(k), l^{''}(k)) = x^{*}(k)$, by the *Intermediate Value Theorem*. Certainly, the same procedure can be followed for $a.a$ commodities in K. Thus, the set of feasible allocations of commodities produced is strictly convex. We next need to perform an appropriate analysis on $x(m)$, given that in our setup, $x(m)$ is not expressible as an explicit functional of $x(k), k \in K$. So, consider $x^{'}, x^{''} \in \mathbf{X}$, where the allocation $x^{'} = \left(x^{'}(m), (x^{'}(k))_{k\in K}\right)$ is achieved by playing $\left((b^{'}(k), l^{'}(k)\right)_{k\in K}$, and $x^{''} = \left(x^{''}(m), (x^{''}(k))_{k\in K}\right)$ is achieved by playing $\left((b^{''}(k), l^{''}(k)\right)_{k\in K}$, with $(x^{'}(k))_{k\in K}$, $(x^{''}(k))_{k\in K}$ being the same allocations derived in the first part of our proof. We must show, given $x^{'}(m)$ and $x^{''}(m)$, that $\lambda x^{'}(m) + (1-\lambda)x^{''}(m)$ is still attainable, where λ is as before. In particular, since for $a.a$ $k \in \{m\} \cup K$,

$x^{'}, x^{''} \in \mathbf{X}$ are *surjective* and continuous in b and l, if we prove that $\lambda x^{'}(m) + (1-\lambda)x^{''}(m) \leq x^{*}(m)$, where $x^{*}(m)$ corresponds to $(x^{*}(k))_{k\in K}$ induced by $\left((b^{*}(k), l^{*}(k)\right)_{k\in K}$ as previously, then we will be done. For what will follow, because of the nonatomicity of our measure space of commodities, it will be crucial to rewrite $x(m)$ as $x(m) = e_m + \int_{k\in K}\left(\frac{l(k)}{a^{J}(k)}\frac{B(k)}{\phi(k)} - b(k)\right) d\mu(k)$, where $J = F, H$ depending on whether $n \in F$ or $n \in H$. Furthermore, let $\frac{l(k)}{a^{J}(k)}\frac{B(k)}{\phi(k)} - b(k)$ be denoted by $z(b(k), l(k))$. What this simple manipulation allows us to do is work with each *individual* term (for *arbitrary* $b(k)$ and $l(k)$) under the integral, the reason for doing which will become apparent in the sequel. So, w.l.o.g., pick any $k \in K$. Consistent with our prior analysis, let $l^{''}(k) \geq l^{'}(k)$. As $z(b(k), l(k))$ is increasing in $l(k)$ (holding $b(k)$ constant), we have

$$z(b^{'}(k), l^{'}(k)) \;\leq\; z(b^{'}(k), l^{''}(k)).$$

Now, since $z_m(k)$ is linear in $b(k)$ (holding $l(k)$ constant), we have that

$$\begin{aligned} z_m^{\lambda}(k) &\equiv \lambda z(b^{'}(k), l^{'}(k)) + (1-\lambda)\, z(b^{''}(k), l^{''}(k)) \\ &\leq \lambda z(b^{'}(k), l^{''}(k)) + (1-\lambda)\, z(b^{''}(k), l^{''}(k)) \\ &= z(\lambda b^{'}(k) + (1-\lambda) b^{''}(k), l^{''}(k)) \equiv z(b^{\lambda}(k), l^{''}(k)). \end{aligned}$$

From the first part of our proof, we have that $x^{*}(k) = x(b^{*}(k), l^{''}(k))$, where $b^{*}(k) \in (0, b^{\lambda}(k))$. But since $z(b(k), l(k))$ is decreasing in $b(k)$ (holding $l(k)$ constant), we have that

$$z_m^{\lambda}(k) \leq z(b^{\lambda}(k), l^{''}(k)) < z(b^{*}(k), l^{''}(k)).$$

The above algorithm applied to $a.a$ goods in K yields

$$\begin{aligned} & z_m^{\lambda}(k) < z(b^{*}(k), l^{''}(k)), \quad \mu\text{-}a.e \text{ in } K \\ & \Longrightarrow \int_{k\in K} z_m^{\lambda}(k) d\mu(k) < \int_{k\in K} z(b^{*}(k), l^{''}(k)) d\mu(k) \\ & \Longleftrightarrow \lambda \int_{k\in K} z(b^{'}(k), l^{'}(k)) d\mu(k) + (1-\lambda)\int_{k\in K} z(b^{''}(k), l^{''}(k)) d\mu(k) < \int_{k\in K} z(b^{*}(k), l^{''}(k)) d\mu(k) \\ & \Longleftrightarrow \lambda\left(e_m + \int_{k\in K} z(b^{'}(k), l^{'}(k)) d\mu(k)\right) + (1-\lambda)\left(e_m + \int_{k\in K} z(b^{''}(k), l^{''}(k)) d\mu(k)\right) \\ & \qquad\qquad < e_m + \int_{k\in K} z(b^{*}(k), l^{''}(k)) d\mu(k) \\ & \Longleftrightarrow \lambda x^{'}(m) + (1-\lambda) x^{''}(m) < x^{*}(m), \end{aligned}$$

which completes our proof. □

Proof of Lemma 4. Let the positive bids and offers of all agents other than some $n \in N$ be given. By construction, $U_n(\cdot)$ is strictly concave in x. By Lemma 3, each agent's CPS, denoted by $\mathbf{X}_n$, is strictly convex. So, at an equilibrium with positive bids and offers, each agent $n \in N$ solves $\inf_{x_n \in \mathbf{X}_n}\{-U_n(\cdot)\}$. It is clear that if this problem has a solution, then it is unique. We may therefore move back to strategy space to derive the first-order necessary and sufficient conditions. This will be achieved by using a *Generalised Kuhn–Tucker theorem* (GKTT)—see Luenberger (1969: p. 249). To help the reader, we relate each symbol from the GKTT to those that we use in our model: $X \equiv \mathbb{L}$, $Z \equiv \left(L^{\infty,4}, \|\cdot\|_Z\right)$, $P \equiv L_+^{\infty,4}$, $f \equiv -\mathcal{U}$, $G \equiv G$, $x_0 \equiv \underline{s}$, $z_0^* \equiv \psi$, and $Z^* \equiv \left(L^{\infty,4}, \|\cdot\|_Z\right)^*$. The approach that we adopt in the analysis that follows has a "mix-and-match" flavour, in that it comprises a treatment of both non-normable topological vector spaces, and normed vector spaces. We proceed by noting that $\mathbb{L} \equiv L_\tau^\infty \times L_\tau^\infty := (L^\infty, \tau(L^\infty, L^1)) \times (L^\infty, \tau(L^\infty, L^1))$ is a topological vector space. L^∞, endowed with its essential supremum $\|\cdot\|_{L^\infty}$, is a normed vector space. Let $L^{\infty,M}$ denote the M-fold product of the normed space L^∞. We may then norm the product vector space $Z \equiv L^{\infty,4}$ by defining $\|z\|_Z = \sum_{i=1}^4 \|z_i\|_{L^\infty} < \infty, z_i \in L^\infty$. It is easy to see that the positive cone of Z, $L_+^{\infty,4}$, contains an interior point. By construction, $U_n(\cdot)$ is continuously Gâteaux-differentiable in x_n. $x_n(k)$ is trivially Gâteaux-differentiable in b_n and l_n μ-$a.e$ in K. Proceeding almost along the same lines as in Toraubally's (2018) Lemma 6, it can also be proved that $x_n(m)$ is (continuously) Gâteaux-differentiable in b_n and l_n, and linear in its (admissible) increments. So, the mapping $x_n := \left(x_n(m), (x_n(k))_{k\in K}\right)$ is continuously Gâteaux-differentiable in b_n and l_n. Thus we may, w.l.o.g., as in Toraubally (2018, 2019), view the composition $\mathcal{U} := U_n \circ x_n$ as $\mathcal{U} : \mathbb{L} \supset L_{\tau,+}^\infty \times L_{\tau,+}^\infty \ni (b_n, l_n) \to \mathbb{R}$,[21] such that $\mathcal{U}(\cdot)$, and hence $-\mathcal{U}(\cdot)$, are also continuously Gâteaux-differentiable functionals. Let $G(\cdot)$ be a mapping $G : \mathbb{L} \supset \mathbb{L}_+ \ni (b_n, l_n) \to \mathscr{C} \subseteq \left(L^{\infty,4}, \|\cdot\|_Z\right)$, such that

$$\begin{bmatrix} b_n(k) - e_{n,m} \\ l_n(k) - Q_n \\ -b_n(k) \\ -l_n(k) \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \mu\text{-}a.e \text{ in } K, \\ \mu\text{-}a.e \text{ in } K, \\ \mu\text{-}a.e \text{ in } K, \\ \mu\text{-}a.e \text{ in } K. \end{matrix}$$

$G(\cdot)$, as defined above, is a Gâteaux-differentiable mapping. Next, suppose that the strategy $(\underline{b}_n, \underline{l}_n) = (\underline{b}_n(k), \underline{l}_n(k))_{k\in K}$ minimises $-\mathcal{U}(\cdot)$ subject to $G(b_n, l_n) \leq \mathbf{0}$. We show that $(\underline{b}_n, \underline{l}_n)$ is a regular point (see Luenberger, 1969: p. 248). Choose an increment $E = E(\xi) \in \mathbb{L}$ such that for a given and sufficiently small $\xi \in \mathbb{R}_{++}$, we have

$$\begin{bmatrix} b_n(k) - e_{n,m} \\ l_n(k) - Q_n \\ -b_n(k) \\ -l_n(k) \end{bmatrix} + \begin{bmatrix} -\xi \\ -\xi \\ -\xi \\ -\xi \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \mu\text{-}a.e \text{ in } K, \\ \mu\text{-}a.e \text{ in } K, \\ \mu\text{-}a.e \text{ in } K, \\ \mu\text{-}a.e \text{ in } K, \end{matrix}$$

where the above matrices represent $G(\underline{b}_n, \underline{l}_n)$, $\delta G(\underline{b}_n, \underline{l}_n; E) := G'(\underline{b}_n, \underline{l}_n) \cdot E$, and $\mathbf{0}$, respectively, with $G'(\underline{b}_n, \underline{l}_n)$ denoting the Gâteaux derivative of $G(\cdot)$ at $(\underline{b}_n, \underline{l}_n)$. It is straightforwardly seen that each sum above is strictly negative, whether or not the constraints bind. Hence, $(\underline{b}_n, \underline{l}_n)$ is a regular point, i.e.,

[21] As opposed to $\mathcal{U} : \mathbb{L}^{|N|} \supset L_{\tau,+}^{\infty,|N|} \times L_{\tau,+}^{\infty,|N|} \ni (b, l) \to \mathbb{R}$—i.e., we have dropped the dependence on (b_{-n}, l_{-n}).

$G(\underline{b}_n, \underline{l}_n) \leq \mathbf{0}$, and $\exists E \in \mathbb{L}$, such that $G(\underline{b}_n, \underline{l}_n) + \delta G(\underline{b}_n, \underline{l}_n; E) < \mathbf{0}$. All the conditions of the GKTT are met. So, at an NE with positive bids and offers, $\exists(\psi_n^b, \psi_n^l, \lambda_b, \lambda_l) \geq \mathbf{0}$ in $\left(L^{\infty,4}, \|\cdot\|_Z\right)^*$, such that $\forall n \in N$ and $\forall E \in \mathbb{L}$, we must have

$$\delta\mathcal{L}_n(b,l;E) = \int_{k\in K}\left(\frac{du_{n,k}(x_n(k))}{dx_n(k)} \cdot \frac{b_n(k)}{a^J(k)B(k)}\bigg|_{(\underline{b},\underline{l})} + \frac{du_{n,m}(x_n(m))}{dx_n(m)} \cdot \frac{\phi_{-n}(k)B(k)}{a^J(k)(\phi(k))^2}\bigg|_{(\underline{b},\underline{l})}\right) E_l(k)d\mu(k)$$
$$+\int_{k\in K}\left(\frac{du_{n,k}(x_n(k))}{dx_n(k)} \cdot \frac{B_{-n}(k)\phi(k)}{\big(B(k)\big)^2}\bigg|_{(\underline{b},\underline{l})} - \frac{du_{n,m}(x_n(m))}{dx_n(m)} \cdot \frac{\phi_{-n}(k)}{\phi(k)}\bigg|_{(\underline{b},\underline{l})}\right) E_b(k)d\mu(k)$$
$$+\int_{k\in K} E_b(k)d\lambda_n^b(k) + \int_{k\in K} E_l(k)d\lambda_n^l(k) - \int_{k\in K} E_b(k)d\psi_n^b(k) - \int_{k\in K} E_l(k)d\psi_n^l(k) = 0,$$

where $(\lambda_n^b, \lambda_n^l, \psi_n^b, \psi_n^l) \geq \mathbf{0}$ verifies $\left\langle G(\underline{b}_n, \underline{l}_n), (\lambda_n^b, \lambda_n^l, \psi_n^b, \psi_n^l)\right\rangle = 0$; and as before, $J = F, H$, depending on which country n is a constituent of. □

Remark. The dual space of $\left(L^\infty(K, \mathscr{K}, \mu), \|\cdot\|_{L^\infty}\right)$ is identified with $ba(K, \mathscr{K}, \mu)$, the space of bounded additive functions on $\mathscr{K}$ which vanish on sets of μ-measure zero. An element of $ba(K, \mathscr{K}, \mu)$ is determined by the identity

$$\langle f, f^* \rangle = \int_{k\in K} f(k)d\lambda(k), \qquad f \in L^\infty(K, \mathscr{K}, \mu).$$

Proof. See Dunford and Schwartz (1957). □

Proof of Theorem 1. Recall that on norm-bounded subsets of L^∞, the Mackey topology $\tau(L^\infty, L^1)$ coincides with the weak-star topology $\sigma(L^\infty, L^1)$, a fact we will use throughout this proof. Note further that the weak-star topology $\sigma(L^\infty, L^1)$ is the topology of pointwise convergence on L^1—i.e., it is the topology on L^∞ induced by the product topology on $\mathbb{R}^{L^1}$ (where each $\gamma_n \in L^\infty$ is identified with a linear function on L^1). By Tychonoff's theorem, a subset of $\mathbb{R}^{L^1}$ endowed with the product topology, τ_P, is compact iff it is pointwise closed and pointwise bounded (see Aliprantis and Border, 2006: p. 218). As such, it is clear to see, $\forall n \in N$, that $[0, e_{n,m}]^{L^1}$ and $[0, Q_n]^{L^1}$, the spaces of all real functions on $L^1(K, \mathscr{K}, \mu)$ taking values in $[0, e_{n,m}]$ and $[0, Q_n]$, respectively, are compact in the pointwise topology. We next prove that the consumption possibilities set, which we denote by $\mathbf{X}_n$, for each $n \in N$ is compact in the $\varepsilon \times \sigma(L^\infty, L^1)$ topology. Pick any $n \in N$, and let n's strategies be denoted by $s_n \in S_n$. By the previous argument, we have that we may view $S_n \subset ([0, e_{n,m}]^{L^1}, \tau_P) \times ([0, Q_n]^{L^1}, \tau_P)$. Thus, by Alaoglu's Theorem (see, e.g., Berge, 1963: pp. 262-263), S_n is closed in $([0, e_{n,m}]^{L^1}, \tau_P) \times ([0, Q_n]^{L^1}, \tau_P)$, and is a $\sigma^2(L^\infty, L^1)$-compact topological space, where $\iota^M(\cdot,\cdot)$ denotes the M-fold product of $\iota(\cdot,\cdot)$. So consider any net of strategies $\{s_n^r\}_{r\in I}$ in S_n, and the corresponding allocations $\{x_n(s_n^r)\}_{r\in I}$, where I is an arbitrary index set. By passing to a subnet if necessary, we have that $s_n^r \xrightarrow{w^*} s_n^* \in S_n$. Since $x_n(\cdot)$ is $\tau^2(L^\infty, L^1)$-continuous in s_n—which follows from the observations we made after (2), and a straightforward application of Lebesgue's Dominated Convergence Theorem—we have that $x_n(s_n^r) \xrightarrow{w^*} x_n(s_n^*)$, which clearly lies in $\mathbf{X}_n$, which is thus closed. Our conclusion trivially follows, since $\mathbf{X}_n$ is a closed subset of $\left([0, \frac{1+\Theta}{\Theta}\sum_{n\in N} Q_n]^{L^1} \times [0, \sum_{n\in N} e_{n,m}], \tau_P\right)$, a compact set. We may now proceed. Consider any agent n's optimisation problem as in (3). $U_n(\cdot)$ is

$\varepsilon \times \tau(L^\infty, L^1)$-continuous (see Bewley, 1972) and strictly concave in x_n, and $x_n(\cdot)$ is an element of $\mathbf{X}_n$, which is $\varepsilon \times \sigma(L^\infty, L^1)$-compact and, by Lemma 3, strictly convex. Hence, there is a unique allocation, call it x_n^*, which by Weierstrass' Extreme Value Theorem exists indeed, that is a best-response to the strategies played by agents in $N \setminus \{n\}$. Reverting to strategy space, we thus have that there exists at least one combination of bids and offers that induces x_n^*. Let the set of such solutions be denoted by $\mathscr{X}_n(b_{-n}, l_{-n}) = \arg\max_{(b_n,l_n)\in S_n}\{U_n(x_n(b,l))\}$, where $(b,l) \in \bigtimes_{n\in N} S_n$. By the Maximum Theorem (see Berge, 1963: p. 116), $\mathscr{X}_n(\cdot,\cdot)$ is upper hemicontinuous. Moreover, on a compact Hausdorff range space, which $([0,e_{n,m}]^{L^1}, \tau_P) \times ([0,Q_n]^{L^1}, \tau_P) \supset S_n$ is, $\mathscr{X}_n(\cdot,\cdot)$ has closed graph iff it is upper hemicontinuous and closed-valued (see Aliprantis and Border, 2006: p. 561).[22] Define further the product correspondence $\mathscr{X} = \bigtimes_{n\in N} \mathscr{X}_n$. $\mathscr{X}(\cdot,\cdot)$ has closed graph and nonempty, convex values, as it is the product of upper hemicontinuous correspondences with nonempty, compact, convex, and closed values (see Aliprantis and Border, 2006: p. 568). $\left([0,e_{n,m}]^{L^{1,|N|}} \times [0,Q_n]^{L^{1,|N|}}, \tau_P\right) \supset \bigtimes_{n\in N} S_n$ is a nonempty compact convex subset of a l.c.t.v.s. Thus, by the Kakutani–Fan–Glicksberg fixed-point theorem (see Aliprantis and Border, 2006: p. 583), the set of fixed points of $\mathscr{X}(\cdot,\cdot)$ is compact and nonempty. These fixed points are NE of our market game. By Proposition 1, we know that at any NE, it is true that μ-$a.e$ in K, $\bigtimes_{n\in N} b_n(k)\cdot p(k) > 0$. □

Lemma 5. *At any equilibrium of our game, there is **full employment** of labour.*

Proof. Suppose not, i.e., let $\breve{s} = (\breve{b}_n, \breve{l}_n)_{n\in N}$ denote an equilibrium profile, where for some $n \in N$, we have that $\int_{k\in K} \breve{l}_n(k)d\mu(k) < Q_n$. Let the strategies of all agents other than $n \in N$ be given. Then, bearing in mind that μ-$a.e$ in K, $\frac{\partial x_n(k)}{\partial l_n(k)} > 0$ and $\frac{\partial x_n(m)}{\partial l_n(k)} > 0$, it is straightforward to see, by instead playing μ-$a.e$ in K, $(\hat{b}_n(k), \hat{l}_n(k)) = \left(\breve{b}_n(k), \breve{l}_n(k) + Q_n - \int_{k\in K}\breve{l}_n(k)d\mu(k)\right)$, that

$$U_n\left(\left(x_{n,k}(\hat{b}_n(k), \hat{l}_n(k), \breve{s}_{-n}(k))\right)_{k\in K}, x_{n,m}\left(\left(\hat{b}_n(k), \hat{l}_n(k), \breve{s}_{-n}(k)\right)_{k\in K}\right)\right) >$$

$$U_n\left(\left(x_{n,k}(\breve{s}(k))\right)_{k\in K}, x_{n,m}\left(\left(\breve{s}(k)\right)_{k\in K}\right)\right),$$

thus contradicting the claim that $\breve{s}$ was an NE to begin with. In particular, observe how $\int_{k\in K}\hat{l}_n(k)d\mu(k) = Q_n$. Hence, at any NE, there is full employment of labour. □

Proof of Theorem 2. Use Lemma 4, Proposition 1, Theorem 1, and Lemma 5. □

[22] By adopting the same approach as in Dubey and Shubik (1978), one may show that for every $k \in K$, there exists a constant $C(k)$, such that at any equilibrium, $C(k) < p(k) = \frac{B(k)}{\phi(k)}$, or $B(k) > C(k)\cdot\phi(k)$, which is bounded above zero. By Proposition 1, we know that $\bigtimes_{n\in N} b_n > 0$ at any NE. Now, suppose that there were no lower bounds on equilibrium bids. Then, for any $k \in K$, $\lim_{B_{-n}(k)\to 0} B(k) = b_n(k) > C(k)\cdot\phi(k)$. But it is clear to see that as $B_{-n}(k) \to 0$, the only best-response for n is to play $b_n(k) = \epsilon \to 0$, such that eventually, $b_n(k) < C(k)\cdot\phi(k)$, a contradiction. But n was arbitrarily chosen. So, for every $n \in N$, and every $k \in K$, $\exists \underline{b}_n(k) > 0$ such that at equilibrium, $b_n(k) \in [\underline{b}_n(k), e_{n,m}]$. Thus, as equilibrium bids and offers are bounded away from zero, $\mathscr{X}_n(\cdot,\cdot)$ has closed graph indeed.