## Abstract

This paper demonstrates a numerical pattern recognition method applied to curvilinear image structures. These structures are extracted from physical cross-sections of cast internal pistol barrel surfaces. Variations in structure arise from gun design and manufacturing method providing a basis for discrimination and identification.

Binarised curvilinear land transition images are processed with fast Fourier transform on which principal component analysis is performed. One-way analysis of variance (95 % confidence interval) concludes significant differentiation between 11 barrel manufacturers when calculating weighted Euclidean distance between any trio of land transitions and an average land transition for each barrel in the database. The proposed methodology is therefore a promising novel approach for the classification and identification of firearms.

**Key Words:** Forensic firearm identification; Barrel manufacture; Rifling; Principal component analysis; Weighted Euclidean distance; Euclidean distance.

## 1. Introduction

Analysis of curvilinear structures in the field of pattern recognition typically involves detection [1], matching [2] or tracking [3] of a known object or feature within an image. For example, unlike facial recognition where common salient features, such as the eyes and mouth [4] can be located, aligned and compared between individuals, the structures in this study do not exhibit these typical points of reference. This is due to the variable nature of gun barrel production, explained further in section 2. As a result, a robust holistic pattern recognition approach is required, which exploits the entire image and uses a statistical technique to extract the relevant features of barrel land transitions for identification. This study therefore combines spectral analysis and principal component analysis (PCA) to derive PCA signatures for each curvilinear transition image; further explained in sections 3 and 4. Statistical analysis of variance (ANOVA) of Euclidean distances (ED) computed between pairs of PCA signatures is also applied for quantitative identification and classification of barrels.

Due to technological advances in analytical surface profiling systems there has been a revived interest in the third dimension of measurement, depth [5]. This has influenced the direction of research in many disciplines, including the field of firearm identification where the surface topography of fired ballistic specimen is of primary interest. The class characteristics within surface features of rifled barrels have been used by forensic firearms examiners as a tool to eliminate suspect weapons from Police investigation for over a century. Such characteristics include the number and dimension of higher relief regions (lands) and lower relief areas (grooves) and the direction of the rifling twist. Fig. 1a illustrates the 9 mm diameter cross-section of a barrel cast, which has been cropped to the relevant area (600 x 581 pixels from 1024 x 768 pixels) and shows six land impressions (LI) and groove impressions (GI). The curvilinear region of interest is referred to as the transition, which spans from the edge of a land to the adjacent groove (see Fig. 1b); there are, therefore, 12 transition regions exhibited in the cross-section shown in Fig. 1a. As the image is from a cast, the barrel rifling profile is inverted.

The purpose of spiral rifling within a rifled gun barrel is to impart rotation onto a projectile resulting in gyroscopic spin stabilisation during flight, which is controlled by the barrel diameter and twist rate [6]. The ~ 0.1 mm deep rifling [7] engraves into the surface of the projectile thereby transferring to it the unique manufacturing tool marks and gross surface profile of the barrel lands and grooves [6]. It is these transferred surface features which are subsequently used for forensic firearm identification.

## 2. Gun Barrel Manufacture

In the manufacture of gun barrels, a number of methods can be used to generate the rifling. Historically, this was only achieved by cutting or scraping the barrel material away within the bore to create the grooves; however, more modern methods also include cold forging and electrochemical processes [6]. Currently there are three primary methods of rifling used by manufacturers; broach cutting [8,9], cold hammer forging [10-12] and button rifling [13,14]. Some manufacturers have also started to use electrochemical rifling (ECR) [6,15-19].

As the mechanical processes involved in the four methods are different, this may result in characteristic surface profiles (or form) exhibited by differently manufactured barrels. The most rapid changes in form are primarily observed in the transition region between individual adjoining lands and grooves within a barrel, as illustrated by Fig. 1b. The transition region also incorporates the form of the corresponding outermost edge of each land and groove, which are referred to as the land or groove edge radii.

Further to the rifling method, the depth of the grooves and the angle of the transition slope from land to groove can also be varied. Some organisations, such as the Sporting Arms & Ammunition Manufacturers' Institute (SAAMI) [20] and the Commission Internationale Permanente (C.I.P.) [21] have established guidelines for these measurements, although these geometric features are a result of weapon design and are typically determined by the manufacturer [22]. By segregating individual transitions from the overall rifling profile the influence of land width, a class characteristic used for barrel differentiation, is eliminated. The research hypothesis is that the transition profile is typical for a barrel manufacturer and potentially a specific firearm model, thereby characterised by a profile signature and utilised for manufacturer identification.

The aim of this preliminary study is to investigate the extent to which manufacturers of pistol barrels can be differentiated using these specific curvilinear regions of the barrel rifling surface topography. The principal goal will be to aid identification of the manufacturing source of barrels used illicitly within the criminal community; however, this principle could also be applied to cross-sectional images of other manufacturing components or curvilinear structures in the assessment of quality control, failure analysis and wear.

# 3. Compiling the Training Image Database

Due to the breadth of firearm types and calibres, this study focuses on the most popular calibre and firearm type used in United Kingdom (UK) gun crime; 9 mm x 19 mm (also known as 9 mm Parabellum) semi-automatic (or self-loading) pistols [23].

Evaluations of currently available optical three-dimensional imaging systems identified limitations for their application in analysing transition profiles due to the creation of artefacts within the recorded profile and/or missed regions of data caused by the steep vertical slopes observed between the lands and grooves [5] leading to an inaccurate representation of the transition geometry. As a result, visualisation of the transition features, such as angle of slope, radius of edges and surface roughness were achieved through barrel cast cross-sectioning and two-dimensional (2D) microscopy.

Within this section, the methodology for sample acquisition, cross-sectional imaging and post image processing are discussed.

## 3.1 Pistol Barrels

In the UK, barrels are classed as Section 5 firearms under the Firearms (Amendment) Act 1997. As a result, access to barrels has typically been limited to one used firearm per barrel manufacturer obtained from UK reference collections, although three brand new barrels have been included in the analysis.

Table 1 details the barrels that have been utilised in this investigation; all but one of the barrels (Glock 19) have a 'conventional' rifling profile, all but one (Smith & Wesson 6904) have 6 land transitions and all but one (Colt All American 2000) have right hand direction of twist. Each barrel has been designated a barrel code for easy referencing in the experimental results section. The manufacturers were selected based on their rifling method to ensure that all four modern rifling methods are represented. The 6 right rifling profile is the most common class characteristic exhibited by the 9 mm x 19 mm semi-automatic pistol [24] and hence pistol models of this type were primarily selected. Other class characteristics were included for comparative purposes.

## 3.2 Barrel Casting

The bore of each rifled barrel was replicated using Isomark Ltd two-part silicone casting materials; T-3 Grey (thixotropic, viscous compound) and F-1 Grey (fluid compound) have both

been utilised in the replication study (section 6). The resolution of these materials can be as high as 0.1  $\mu$ m [25], which is more than suitable for this investigation.

## 3.3 Cast Cross-Sectioning and Imaging

The barrel casts were cross-sectioned using a jig specifically designed to support the casting material during cutting to prevent distortion of the edge and promote formation of a planar and parallel imaging surface. Each nominal 9 mm diameter barrel cast was placed through two steel cylindrical support pieces with a 9 mm bore diameter. Slices of the casts were obtained using a double edged, non-serrated steel blade of a 52 ring gauge cigar cutter.

Three cross-sections were taken from each cast to generate a representative sample of transition profile images from different positions along the length of the barrel; referred to as  $C_1$ ,  $C_2$  and  $C_3$ .  $C_1$  was located 12 mm ± 1 mm from the muzzle end of the barrel,  $C_2$  in the centre of the barrel ± 1 mm and  $C_3$  located 20.2 mm + 0.2 mm from the chamber chamfer of the barrel, as shown in Fig. 2.

As there was insufficient resolution in Fig. 1a, each transition region was focussed on and imaged separately using the Meiji 7200 inverted microscope and 200x total magnification (20x objective lens and 10x eyepiece lens). For each land, there are two transitions; one on the left hand side and one on the right hand side as viewed from the chamber end of the barrel cast, let these be  $T_L$  and  $T_R$  respectively. Fig. 1b illustrates an example of a  $T_R$  taken from the Browning barrel cast. Due to the tool marks created on the cast surface during sectioning, a fully focussed image was typically compiled for analysis using images taken at multiple focal planes; this is discussed further in section 3.3.1. The image dimensions were 2048 x 1536 pixels and microscope accessories included a manual mechanical stage, Pixelink 3 megapixel CMOS camera and OmniMet<sup>®</sup> Capture and Measure Basic software.

LI 1 is denoted as the impression corresponding to the land located at the chamber end of the rifling at approximately the 12 o'clock firing position. Lands and corresponding LI are named

sequentially in the direction of the rifling twist; i.e. clockwise and counter-clockwise for right and left hand twist respectively.

# 3.3.1 Image Processing

All image processing was undertaken using macros and plug-ins devised for freeware ImageJ [26] v.1.44d.

Images acquired at multiple focal planes were compiled using a fusion algorithm ImageJ plugin, which extends the depth-of-focus for microscopy images using a complex wavelet approach [27,28] and the medium speed/quality trade-off plugin option.

An ImageJ macro was devised to automatically and consistently optimise the brightness and contrast for each image, apply the automatic minimum thresholding algorithm to binarise the image, fill in the majority of holes within the binarised regions using the *Binary* > *Fill Holes* option of the *Process* menu and scale the size of each image to 1024 x 786 pixels using the 0.5 scaling function in ImageJ.

An investigation was undertaken to determine the image input type that would yield the optimum results from PCA; the image types are shown in Fig. 1b-d and the results are detailed in section 5.

A selection of the binary transition images are given in Fig. 3 to highlight differences in transition form between the four manufacturing methods and two profile types (conventional and rounded polygonal). Here, two corresponding transition sides ( $T_L$  and  $T_R$ ) of two curvilinear land-groove impression transitions (typically LI 2 and 5) have been illustrated for eight different barrel manufacturers (barrel codes AA, BA, EA, FA, HB, HC, JA and KB).

# 3.4 Fast Fourier Transform

Each binary image was then processed using 2D fast Fourier transform (FFT) generating a spectral image that defines the frequencies within the binary image. To optimise further the analysis, the following modifications were made to the 2D FFT image:

- Only the modulus of FFT is analysed to render the training database immune to translational shifts, resulting in symmetrical spectral images;
- FFT space is halved in both dimensions to remove the highest spatial frequencies from the analysis as we have checked that no important frequency components are located in the discarded region (see Fig. 4);
- 3. The intensity of the central pixel of the cropped, modulus FFT is forced to zero to remove the effect of the binary image's mean pixel intensity. The purpose and effect of this modification is further explained in section 5.1.

Within the modulus FFT there are two high pixel intensity "jets" that radiate from the central pixel (*x* axis zero frequency); one at the normal to the *x* axis and the other ~ 25 ° to the normal. From investigation using artificial images these features represent the transition sides (labelled as LI and GI in Fig. 1b) and the transition slope respectively and are annotated in Fig. 4b.

A clockwise rotation of the binary input image results in a clockwise rotation of the primary jets. Thus, rotations of the transition greater than 1 or 2 ° within the analytical field of view will significantly affect the comparability of the FFT in the analysis and therefore such rotations should be minimised during data acquisition.

# 4. Principal Component Analysis

PCA is a numerical search algorithm based on the Karhunen-Loeve expansion, which determines the vectors that best describe the distribution of images within an entire database of images; it has been used extensively, for example, in the field of pattern recognition [29], biometrics [30] and facial recognition [31,32,33].

Briefly, a database of 2D images is supplied as the input, also often called "the training database". For this we used cropped, modulus 2D FFT images of our raw images. The PCA algorithm analyses the set of modified spectral images to identify which features of the image are statistically significant in differentiating between them. These significant features are known as principal components or eigenimages and characterise the statistically independent variations between the images ranked in order of increasing importance. Their relevant importance, for any given binary image, is given by the value of the corresponding "PCA score".

In this application, let each 2D FFT image in the database be characterised by T(x,y); an array of 514 (*K*) by 193 (*N*) pixels each having an 8-bit greyscale intensity value. This can be thought of as the vector, *KN*, comprising of 99 202 points and the set of images will occupy a low-dimensional subspace [34].

The  $T_R$  database of transition profiles in this work was comprised of 267 images. Let each set of m spectral images be called  $T_1$ ,  $T_2$ ,  $T_3$ ,...  $T_n$ . The average spectral image is computed using the definition  $M = \frac{1}{n} \sum_{i=1}^{n} T_i$  and each image differs from the average image within the database by the quantity  $D_i = T_i - M$ . PCA generates a new set of m orthogonal images of the statistically significant features within the set of original images, called eigenimages (E), by diagonalising the covariance matrix, which describes the distribution of  $D_i$  with respect to itself.

Typically there is a cut-off point (see section 5.5) which can be determined to minimise the number of eigenvectors required to yield an effective identification or recognition for an image within the database.

An original image can be reconstructed using  $T_i = M + \sum_{j=1}^m \lambda_i^j E_j$ , where  $\lambda_i^j$  is the PCA score

of order *j* for the *i*<sup>th</sup> spectral image of the training dataset. However, as a result of undertaking PCA on modulus spectral images, the binary transition images cannot be reconstructed from

calculating the inverse FFT and therefore, the eigenimages identified by PCA cannot directly be interpreted as features belonging to the transitions.

In the application of PCA to match a transition profile to a particular barrel manufacturer, the mean PCA score ( $\overline{\lambda}_k^j$ ) is calculated from the set of scores belonging to a specific barrel where  ${}_m \lambda_k^j$  is the PCA score of order, *j*, *k* is the barrel code in the database (refer to Table 1) and m = 1 to *f* is the transition index number. The number, *f*, of transition images depends on the barrel under consideration i.e. the number of lands in the rifling profile. For a particular barrel which exhibits 6 grooves and is cross-sectioned three times, *f* will be 6 x 3 = 18. Equation 4.1 provides a 'typical' signature for the *k*<sup>th</sup> barrel within the training database, on the basis of which any set of PCA scores can be compared to, in order to establish a "match" or a "mismatch".

$$\overline{\lambda}_{k}^{j} = \frac{1}{f} \sum_{m=1}^{f} {}_{m} \lambda_{k}^{j}$$

$$(4.1)$$

# 4.1 Quantitative Comparison using Euclidean Distance

There are a number of distance measurements suggested to quantitatively compare a pair of vectorial quantities, however, the ED measurement has been commonly used and has shown to be successful in the application of facial recognition [35].

For the image database the ED can be defined for matching transition images and mismatching transition images. Matching EDs ( $ED_m$ ) result from computing the ED between the PCA scores of any transition image that originated from a barrel and its corresponding mean PCA score, whilst mismatching EDs ( $ED_{mm}$ ) are calculated between any transition image from that barrel and the mean PCA score ( $\overline{\lambda}_k^j$ ) from a different barrel within the database.

The equations 4.2 and 4.3 explicitly show the calculation of  $ED_m$  and  $ED_{mm}$ . The value of ED may be calculated with fewer than the *m* available PCA scores. If there are only three most significant PCA scores, this would define a three-dimensional space for a particular image; if seven significant scores for each transition image are used then there are seven dimensions

within the Euclidean space,  $\lambda^1$ ,  $\lambda^2 \dots \lambda^7$ .  $\Delta$  represents the optimum number of PCA scores utilised in the signature, which is discussed further in section 5.5. The ED defines the scalar distance between the two PCA score signatures; the smaller the value of ED the more similar the two input images are.

$$ED_m^i(m,\overline{m}) = \sqrt{\sum_{j=1}^{\Delta} ({}_m\lambda_i^j - \overline{\lambda}_i^j)^2}$$
(4.2)

$$ED_{mm}^{ik}(m,\overline{m}) = \sqrt{\sum_{j=1}^{\Delta} ({}_{m}\lambda_{i}^{j} - \overline{\lambda}_{k}^{j})^{2}}$$
(4.3)

The degree of success for identifying the barrel from which any transition image originates from can be quantified by calculating the percentage overlap between the values of  $ED_m$  and  $ED_{mm}$ , after all possible matches (267) and mismatches (3 738) allowed within our dataset have been considered.

# 4.1.1 Weighted Euclidean Distance

The significance of principal components can be weighted to bias the ED towards a particular goal of special relevance to the investigator. The level of weighting is known as a weighting factor ( $\omega$ ) and individual weighting factors are applied to each PCA score, i.e.  $\omega_1, \omega_2, \omega_3, \dots \omega_j$ . This therefore modifies the equations for  $ED_m$  and  $ED_{mm}$  to those given in Equation 4.4 and 4.5 respectively, where WED abbreviates weighted Euclidean distance.

$$WED_{m}^{i}(m,\overline{m}) = \sqrt{\sum_{j=1}^{\Delta} \omega_{j} (_{m} \lambda_{i}^{j} - \overline{\lambda}_{i}^{j})^{2}}$$
(4.4)

$$WED_{mm}^{ik}(m,\overline{m}) = \sqrt{\sum_{j=1}^{\Delta} \omega_j ({}_m \lambda_i^j - \overline{\lambda}_k^j)^2}$$
(4.5)

The weights used to compute the ED can provide enhanced flexibility to the method, by being updated so as to optimise the sensitivity in identifying, for example, one particular manufacturer or one particular feature of the transition. For one specific application, if the database size is made larger, the weighting factors do not need to be updated, unless the newly included

transitions exhibit a combination of features that are not already present in the training database.

In this study, WED is optimised by computing the scalar percentage overlap of matching EDs and mismatching EDs as described in section 4.1 by the multivariate PCA scores. Weighting factors are then optimised for each PCA score, with the ultimate aim of reducing the percentage overlap using a purpose-written Matlab code. Section 5.5 discusses the results of this optimisation.

## 5. Experimental Results and Discussions

This section of the paper discusses the outcome of using our bespoke algorithm required for quantitative image analysis using binarisation, FFT, PCA and WED. It also details the results of our ability to identify individual barrels and barrel manufacturers.

# 5.1 Selection of Training Database Image Input Type

Using an initial, un-optimised code to run PCA, three image input types (shown in Fig. 1b-d) were used to investigate whether any of these resulted in enhanced clustering of the PCA score values for transition images belonging to specific barrels. Binary threshold images (Fig. 1c) appeared to show optimal grouping for the 7 most significant PCA scores, suggesting that PCA exhibits potential use for this quantitative application and FFT spectra from these images should be selected for further development. Binarisation also served to remove the striations visible on the face of the cross-section, which are insignificant toolmarks and artefacts produced from the cross-sectioning tool surface.

# 5.2 Minimisation of Translational Shifts within Analysis

Although the modulus 2D FFT has been computed, the intensity of the central pixel within the FFT image equates to the average pixel intensity of the original image. The ratio of black and white pixels in the binary image will strongly bias the central pixel of the modulus FFT image, particularly so for vertical shifts. As a result, the central pixel of the spectral images used in the

training database was forced to zero to eliminate vertical translations from the binary images. This processing method was abbreviated CPZ (central pixel zero).

When transition profiles exhibit horizontal shifts, new information will necessarily be included in the analytical field of view, which results in small changes in the corresponding modulus spectral image that cannot easily be eradicated from the analysis.

5.3 Effect of Incorporating both  $T_L$  and  $T_R$  Images within the ED Identification Both training image databases,  $T_L$  and  $T_R$ , were independently processed using the CPZ PCA algorithm and a sample of the eight most significant scores were utilised to determine the degree of overlap between  $ED_m$  and  $ED_{mm}$ . A third dataset was also formulated to determine whether combining both transitions of each land improved differentiation between manufacturers. The third study was compiled by concatenating the eight most significant PCA scores for each image within the  $T_R$  database with those eight from the corresponding  $T_L$ database, yielding a signature of 16 PCA scores from which the ED can be calculated. Table 2 details the results of the investigation.

The significant result of this investigation is that utilising both transition profiles for each land to calculate the ED from concatenated PCA scores of the two databases, reduces the percentage overlap between the histogram by a factor of 1.5. This demonstrates that the two transitions within a land profile carry statistically independent information and their combined use thus increases the potential for differentiation between barrel manufacturers.

Ideally, the histogram for all possible  $ED_m$  and  $ED_{mm}$  should not overlap at all, but by chance, two different manufacturers may produce very similar transition profiles resulting in significant overlap in the EDs calculated. Overlap may also result from the natural deviation of transition profiles brought about through barrel manufacturing, subsequent use of the firearm, as well as systematic error within the imaging process. However, the presented methodology can be further optimised to minimise the overlap (see section 5.6).

The final aspect that required consideration was the effect of rotation of the sample within the field of view during imaging. The modulus FFT is sensitive to rotations of the sample; however, the effect of these cannot be eliminated from, or accounted for, in the algorithm at this stage due to errors potentially introduced by the manual registration of the samples within the field of view of the camera. Such errors were kept to a minimum during image registration prior to image acquisition by careful reference to the grid setup within the OmniMet<sup>®</sup> imaging software. The use of the grid was shown to minimise the effect of horizontal and rotational shifts by controlling the position of the transition within the imaging field as much as possible. We estimate that slight deviations in imaging brought about through rotational shifts will thereby account for a small proportion of the deviations within the PCA score values and therefore the ED calculations.

## 5.4 Differentiation of Barrels using Raw Euclidean Distance

Fig. 5 illustrates the ability of our method to differentiate between barrels made by different manufacturers based on quantitative comparison using ED. The greatest separation achieved by PCA is between Glock (barrel code FA) and the other barrels. This was expected as the profile of the Glock rifling is a rounded polygonal shape, compared to the other 14 barrels, which have a conventional rifling profile.

There was also 0 % overlap between a SIG Sauer P226 (barrel code HB) and the other barrels, as well as between a Walther P990 DAO barrel (barrel code KC) and the other barrels used in this study. However, it is interesting to note that these two pistol barrels have sister barrels within the training database, barrel codes HA and KB respectively, which are already statistically dissimilar. I.e. barrel codes HA and HB are barrels manufactured by SIG Sauer for the P226 model pistol using hammer forged rifling and KB and KC are manufactured by Walther for the P990 DAO pistol using button rifling. The difference between the two corresponding sets of barrels is that one was well used and one was relatively new; barrel HA was from an older, well used P226 pistol whereas HB barrel was from a demonstration P226 pistol that had less than 100 rounds fired through it, KB was from a new P990 DAO pistol that had less than 20 rounds fired through its barrel and KC was from a pistol that was extensively used (~ 15 000

fired rounds). This suggests that it may be possible to also use this analysis method to differentiate between well used (potentially worn) barrels and the transition profiles of a new, less-worn barrel. However, as the exact histories of these pistols and the consistency of transition profiles produced by manufacturing tolerances are not known at this time, further interpretation cannot be made and will be undertaken as part of future work.

Each *ED<sub>mm</sub>* population can now be classified into *ED<sub>mm</sub>* for individual mismatching barrels, a one-way ANOVA is undertaken for each barrel and followed by either the Tukey Honestly Significant Difference (HSD) [36] or Dunnett T3 [37, 38] post-hoc method of pairwise comparison. Dunnett T3 was utilised rather than Tukey HSD in comparisons where the variances of the *WED<sub>m</sub>* and *WED<sub>mm</sub>* datasets are heterogeneous i.e. for all barrel codes except DA, FA, HA, KB and KC, which are commonly manufactured using forged methods of rifling. This results in 6 uniquely identifiable barrels (barrel codes BA, FA, HA, JA, HB and KC) where the mean *ED<sub>m</sub>* is statistically different to all 14 *ED<sub>mm</sub>* (for barrel code JA, F(14, 255) = 196, *p* = < 0.001, MS<sub>error</sub> = 5.00 x 10<sup>29</sup>,  $\alpha$  = 0.05 and Dunnett T3, *p* = < 0.03). Therefore, a further development to the algorithm needs to be investigated in order to further separate *ED<sub>m</sub>* and *ED<sub>mm</sub>* populations and enhance differentiation between barrels. This can be achieved by weighting the PCA scores as discussed in section 4.1.1.

# 5.5 Optimisation of PCA Scores used in Calculation of Weighted Euclidean Distance The percentage overlap between transition images $WED_m$ and $WED_{mm}$ for the 14 other barrels was monitored to determine the number of significant PCA scores from each database of images, $T_L$ and $T_R$ , that should be concatenated for any calculation of WED values.

The ultimate goal for the assignment of weighting factors was to reduce the percentage overlap between the population of matching and mismatching WEDs to a value as close to zero as possible. This involved limiting the number of PCA scores for each eigenimage within the  $T_L$  and  $T_R$  databases to  $\Delta = 3$ , concatenating them, calculating  $WED_m$  and  $WED_{mm}$  (as previously explained in section 4.1.1) and finally computing the percentage overlap. The weighting factor search was then repeated for increasing values of  $\Delta$  until the percentage overlap reached a

plateau, such that increasing  $\Delta$  did not yield a significant reduction in percentage overlap. The results are depicted in Fig. 6. The optimum concatenated PCA score signature was produced when  $\Delta = 14$  (i.e. 7 most significant scores from the two databases), which further reduced the overlap to 4.5 %.

Fig. 7 shows the optimum weighting factors for each PCA score within the optimised concatenated signature where the first 7 scores of the signature originate from the  $T_L$  database and the last 7 scores originate from the  $T_R$  database. The greater the weighting factor value, the higher the importance of that principal component in the calculation of WED.

Within the 14 concatenated PCA score signature, score 1 from  $T_L$  and  $T_R$  databases have both been reduced to nearly zero, indicating that these are the least important principal components when differentiating between barrel manufacturers. Further to this, the 4<sup>th</sup> significant PCA score in the  $T_L$  database also has very low importance in the WED calculation for minimising percentage overlap in order to maximise barrel differentiation.

To further interpret these weighting factors, the corresponding eigenimages (mean centred modulus FFT images) can be displayed. The 7 most significant eigenimages are shown in Fig. 8 for both  $T_L$  and  $T_R$  databases. From artificially generated land transitions it was determined that the modulus FFT comprised of two primary "jets" of high pixel intensity originating from the *x* axis zero frequency of the modulus FFT. All the real transitions within the two training databases produce comparable modulus FFT exhibiting similar primary jets. Three aspects of the two jets in the modulus FFT can easily be quantitated to discriminate between barrels:

- 1. The relative angle of the jets;
- 2. The relative broadness or sharpness of the jets;
- 3. The relative amplitude of the jets.

By conserving the polarity (positive and negative pixel intensities) of the eigenimages, it is possible to determine which eigenimage is responsible for modulating each of the three parameters. Table 3 outlines the interpretation of these parameters with respect to the relevant eigenimages and the level of importance for barrel identification determined by the value of the

weighting factors. In summary, the relative angle of the transition sides and transition slope is more important in the discrimination of barrels compared to the depth of the transition.

The most important components within the concatenated PCA score signature are from the 5<sup>th</sup> and 4<sup>th</sup> significant PCA scores from  $T_L$  and  $T_R$  databases respectively. However, from investigations so far, we cannot identify which morphological image features correlate to these specific eigenimages. The importance of scores within the two halves of the concatenated signature are not symmetrical; two separate databases, and therefore two PCA searches, were run, which resulted in slightly different features of the images within the two databases being statistically more important. From Fig. 8 you can see that  $E_1$  to  $E_3$  are mirror images of each other in the two training databases inferring that the top three features are ranked similarly, however, from  $E_4$  onwards, different features of the databases become important. Fig. 9 illustrates this further by comparing  $E_1$  and  $E_4$  for  $T_L$  and  $T_R$  databases.

# 5.6 Differentiation of Barrels using Weighted Euclidean Distance

Using the optimised weighting factors for the 14 scores within the concatenated signature,  $WED_m$  and  $WED_{mm}$  can now be displayed and analysed for each of the 15 barrels in the training database.

If Fig. 5 is compared to Fig. 10, it clearly shows that separation between the mean  $WED_m$  and  $WED_{mm}$  for each barrel has been significantly increased by weighting the PCA scores within the concatenated signature. The percentage overlap between  $WED_m$  and  $WED_{mm}$  has thus been significantly reduced from a maximum of 38 % to a maximum of only 9 %. There is 0 % overlap for 5 of the barrels within the database; John Slough of London (barrel code EA) and new Walther (barrel code KB) barrels can now also be uniquely identified.

One-way ANOVA followed by Tukey HSD (applicable only for barrel codes EA, FA and LA) or Dunnett T3 is applied to the individual barrel data resulting in unique differentiation of 12 of the 15 barrels in the training database. This is because the mean  $WED_m$  of 12 barrels are statistically different to the mean  $WED_{mm}$  of the other 14 barrels in the database (for barrel code

DA, F(14, 255) = 74, p = < 0.001,  $MS_{error} = 3.21 \times 10^{29}$ ,  $\alpha = 0.05$  and Dunnett T3, p = < 0.01). Using this statistical analysis method, there is a 5 % chance that the conclusion reached is not correct based on the 95 % confidence interval.

Of the barrels that can be statistically differentiated, it is noteworthy that the land profiles of new SIG Sauer P226 (barrel code HB) and P250 (barrel code HC) barrels are statistically different to each other. Thus, the two rifling methods, hammer forged and ECR respectively, may also be able to be identified if a SIG Sauer barrel without a serial number was submitted as forensic evidence.

The three barrels that cannot be differentiated between using this methodology are the Springfield (barrel code IA), Colt (barrel code CA) and Smith & Wesson (barrel code LA) barrels.  $WED_m^{IA}$  statistically overlaps with both  $WED_{mm}^{CA}$  and  $WED_{mm}^{IA}$ , which means that this barrel can be differentiated from 12 other barrels in the training database; all but the Colt and Smith & Wesson barrels.  $WED_m^{CA}$  and  $WED_{mm}^{LA}$  overlap statistically, as do  $WED_m^{LA}$  and  $WED_{mm}^{CA}$ , however these can be differentiated from the Springfield barrel and therefore  $WED_m^k$ , when k = CA and LA, are statistically different to the mean PCA score signatures for the 13 other mismatching barrels. As not all of the barrels can be differentiated and uniquely identified, an investigation was undertaken to increase the number of land transitions used in the identification to establish whether this improves differentiation between the barrels.

# 5.6.1 Using any Trio of Lands to Calculate WED

In order to use more than one land in the calculation of WED, a mean concatenated PCA score signature was calculated between any trio of land transition profiles within one particular cross-section, for example LI 1, LI 2 and LI 3. If a barrel cast has 6 LI, then there are 20 possible WED calculations for one cross-section. The mean concatenated PCA score signature for a barrel manufacturer ( $\overline{\lambda}_k^j$ ) was then subtracted from the mean trio of land transition profiles.

This resulted in a reduction in percentage overlap of the  $WED_m$  and  $WED_{mm}$  population histogram from 4.5 % to 3.3 % and Fig. 11 shows the individual barrel analysis for trio  $WED_m$ and  $WED_{mm}$ . This results in an optimum false acceptance rate (FAR) for the technique of 1.7 % and a false rejection rate (FRR) of 1.7 %.

Calculation of WED using any trio of lands further increases the differentiation between barrels used in this training database, such that 8 barrels now have 0 % overlap between *WED<sub>m</sub>* and *WED<sub>mm</sub>*; Browning (barrel code BA), the well used SIG Sauer P226 barrel (barrel code HA) and the new electrochemical SIG Sauer P250DC (barrel code HC) being the latest inclusions within this group. The FB Radom (barrel code DA) and Ruger (barrel code GA) also have virtually 0 % overlap. The Colt (barrel code CA) now yields the highest percentage overlap, which has slightly increased from 6.3 % to 6.6 % through averaging of any three concatenated LI PCA scores and there has been a slight increase for Beretta (barrel code AA) barrel from 3.7 % to 3.8 %. However, overlaps have reduced or remained at 0 % for all other barrels in the database. Even barrels containing visually highly similar transition features, such as those present within the Colt and S&W barrels (see Fig. 12), can now be differentiated using our bespoke quantitative approach.

One-way ANOVA between the mean  $WED_m$  and mean  $WED_{mm}$  for each barrel in the database now yields a statistically significant difference for all 15 barrels (for barrel code IA, F(14, 885) = 998, p = < 0.001,  $MS_{error} = 1.09 \times 10^{29}$ ,  $\alpha = 0.05$  and Dunnett T3, p = < 0.001). Therefore, all barrels could be uniquely identified when any trio of lands from a particular barrel cast crosssection is used in the calculation.

This supports the suggestion that CPZ PCA and calculation of WED can be used as a means to differentiate between barrel manufacturers using 3 land transition profiles from any barrel cast cross-section. This may also be the case for identifying between worn and new barrels or the method of rifling used within a manufacturer.

As transition profiles were isolated during our analysis, the proposed method of identification is independent of using other class characteristics, such as width of lands and grooves and direction of twist, commonly used by forensic firearm examiners in the elimination of a barrel manufacturer. As a result, in instances where barrels have remarkably similar class characteristics this novel method may be applied to further eliminate or offer verification of the barrel manufacturer.

Table 4 quantitatively compares the three ED measurement algorithms to illustrate that using weighted PCA scores and multiple land impressions in the calculation of WED significantly reduces the FAR/ FRR for most of the barrels in the training database. As a result, this algorithm should be preferentially implemented in future work.

# 6. Repeatability

Finally, the methodology of barrel casting and imaging of a selection of barrels needed to be investigated to ensure that the method of data collection was repeatable.

Four of the barrels featured in the training database were selected for re-casting based on the presence of irregular features in the observed transition profile and the spread of the corresponding *WED<sub>m</sub>*. The barrels selected comprised of:

- Browning Hi-Power (barrel code BA);
- Ruger KP89 (barrel code GA);
- Springfield P9C (barrel code IA);
- Star Firestar (barrel code JA).

The first repeat cast was obtained using Isomark F-1 Grey casting material and three further repeat casts were obtained using Isomark T-3 Grey casting material for each of the four barrels. Both products were tested to establish whether the viscosity and formulation of Isomark material used in the analysis produced significantly different results following PCA.

The algorithm for analysing repeat casting uploads binary transition images from repeat barrels, undertakes CPZ PCA and projects the new (difference) images onto the eigenimages obtained from the training database to determine PCA score values for the repeat sets. If a new PCA search had been undertaken then a new set of eigenimages would have been produced, making conclusions on the repeatability very difficult to draw. The set of scores were limited to the 7 most significant scores for images in corresponding  $T_L$  and  $T_R$  repeat databases, which were concatenated to obtain a signature of 14 scores. The same optimised weighting factors (detailed in Fig. 7) are applied to calculate  $WED_m$  between all combinations of lands for each of the four repeat casts and for the four separate barrels tested. For this investigation, WEDr are calculated between any signature belonging to one repeat cast of the same barrel and any other signature belonging to one of the three repeat casts from the same barrel. For example, with the Browning Hi-Power barrel F-1 repeat cast,  $WED_m$  are calculated from intra-comparisons of the lands within this cast and three individual sets of WED<sub>r</sub> are calculated from the inter-comparison of eigenimages from F-1 cast with those from each T-3 barrel repeat cast. This methodology ensured that the repeatability test was fair as the mean PCA score signature for each barrel manufacturer was not used and thereby eliminated any potential for normalisation of the signatures against a mean value. As a result, the full spread of WEDs could be observed and utilised for each repeat cast.

Only the  $C_3$  cross-section location is imaged for this investigation as the cross-sectional position is more precisely located due to the use of the chamber edge of the cast as a positioning device. This therefore minimised the natural variation in transition profile as much as possible for the repeatability study.

Fig. 13 represents the results of the repeatability experiment. The spread of  $WED_m$  and corresponding  $WED_r$  are homogeneous for each set of 4 repeat casts for each of the four barrels and this incorporates both systematic error (discussed in section 6.1) as well as the true deviation in transition profiles between the 6 lands in each barrel. However, the following general conclusions can be drawn from this analysis:

- The spread of WEDs within the 6 Springfield P9C land transitions and 6 Star Firestar land transitions are greater than the Ruger KP89 and Browning Hi-Power barrels. This is a result of the greatest variation in the transition profiles between the 6 lands of this barrel and this can be confirmed by visual observation;
- The second smallest spread of *WED<sub>m</sub>* is shown to be with the Ruger KP89;
- The Browning Hi-Power land transitions typically exhibit least variation in transition profile. This appears to be because there is visibly less variation in the transition images for the 6 lands in the barrel compared to those from other three barrel manufacturers. However, there appears to be greater variation in *WED<sub>m</sub>* for the fourth T-3 repeat barrel cast than the other three, possibly due to slight differences in the location of the cross-sectional point along the barrel or more likely through systematic errors incorporated during imaging. As a result, the effect of systematic error requires further investigation.

Undertaking one-way ANOVA results in the acceptance of the null hypothesis; there is no significant difference in the mean  $WED_m$  of the four repeat casts and the  $WED_r$  taken from the same barrel, however, there is a 1 % chance that this conclusion is incorrect. This is the case for all four of the barrels analysed (for barrel code BA second repeat cast, F(3, 101) = 3.13, p = 0.029, MS<sub>error</sub> = 2.05 x 10<sup>29</sup>,  $\alpha = 0.01$ ), which shows that the methodology of casting and imaging is repeatable and reliable within the spread of WED data acquired.

# 6.1 Systematic Error

Systematic errors can arise from a number of factors within the imaging methodology, although due to the manual nature of the imaging process this is typically a result of deviations in the orientation of the transition in the field of view of the microscope objective lens. The transitions employed in this research do not have a readily identifiable datum. Relative alignment of transitions is therefore accomplished by aligning the land profiles to appear approximately horizontal in the raw image field. Our approach is informed by the ideal case of the land profiles forming arcs at a fixed radius. The relatively small arc lengths enable them to be approximated to a straight line. In practice, perturbations about the ideal arc are accommodated by manual means. Ideally such errors will be negligible or a very small component of the spread of WEDs.

However, investigation suggests that the degree or proportion of systematic error varies according to the similarity of the transition profiles within a specific barrel. For example, the Glock (barrel code FA) has a very small spread of *WED<sub>m</sub>* as the variation in transition profile is very small resulting in the systematic error being of similar magnitude. The Star (barrel code JA) barrel on the other hand exhibits vastly different transition profiles, thereby generating a greater spread of *WED<sub>m</sub>* and a level of systematic error that is comparatively small.

Investigation has also shown that the geometric shape of the transition may influence the degree of systematic error. Transition profiles that exhibit a small (sharp) land edge radii, such as those in the Colt barrel (barrel code CA), produce a smaller spread of systematic error in comparison to profiles which have larger (rounded) land edge radii, such as the Browning barrel (barrel code BA). This is due to the investigator's ability to subjectively determine the apex of the land edge during image registration, which may introduce the potential for variability between operators. However, our investigations determined such systematic errors did not significantly affect the analysis as operators were trained in data acquisition protocols.

Due to the impact of operator error on calculation of WEDs, if this application was to be taken forward for further development, there would be significant potential for enhancement of the methodology. The cast cross-section location would be cut more precisely and the whole cross-sectional surface of the cast would be imaged simultaneously at a high resolution (true optical resolution > 6400 dpi), such that the regions of interest (~ 100 µm deep transition regions) could be selected and enhanced for automatic analysis as opposed to imaging these features individually. This development would enable a robust investigation of the automated alignment of transitions extracted from different barrels to be studied, together with the evaluation of the overall relative alignment of transitions within a given barrel, and in comparison to other barrels. Theoretically, the software could incorporate algorithms to account for or eliminate rotations between images, such as using automatic feature centring and image registration, which would significantly reduce the time associated with data acquisition and eliminate any influence of between-operator variation. However, these methods typically require reference images or images that exhibit common features that can be automatically identified and aligned. For

example, in facial recognition, the eyes, nose and/or mouth can be located and utilised for alignment [39]. However, this is not the case with land transition images as the land edge radii of the transition may be small or large, there may be several peaks that could be mistaken for transition edges and the overall transition profile has been shown to vary from one land to the next even within the same barrel. Therefore it is likely that some subjective human input would still be required for rotation minimisation and image alignment, such as manual identification of the features to be aligned.

## 7. Conclusions and Further Work

The numerical pattern recognition method presented in this paper supports the hypothesis that curvilinear image structures extracted from land transition profiles, inherent in rifled barrels, can provide the basis for a barrel signature. The curvilinear structures arise from a combination of barrel design and manufacturing method. The resultant discrimination and identification of a barrel and its manufacturer is achieved with high reliability. The implications for forensic firearms practitioners are significant as the curvilinear structures examined in our work are not currently recognised as a class characteristic. By hypothesis, the introduction of an additional class characteristic into the standard examination protocol will enhance the capability of forensic firearms identification.

The proposed methodology involved binarisation, 2D FFT analysis and CPZ PCA of microscope images along with the calculation and statistical hypothesis testing between sets of known matching and mismatching WEDs. Optimal differentiation between manufacturers occurs when a concatenated average PCA score signature is created from any three lands within a barrel; all 15 barrels can be uniquely identified using one-way ANOVA ( $\alpha = 0.05$ ). This means that all 11 barrel manufacturers can be identified using this analytical method. The percentage overlap of the histogram between *WED<sub>m</sub>* and *WED<sub>mm</sub>* for the training database was 3.3 %, resulting in a FAR/ FRR of 1.7 %.

This quantitative approach is potentially free from the operator's influence and has the ability to search quickly in very large databases. Therefore, this technique could be used as a 'filter' prior

to subjective human analysis, by allowing the user to focus their attention on a smaller, more relevant sample.

Our findings are a significant advancement in establishing a new pattern recognition tool designed to advance the scientific principles underpinning forensic firearms identification. Further research will be undertaken to examine the consistency of transition profiles in multiple newly manufactured and used barrels for particular pistol barrel manufacturers. Expansion of the dataset would also help to determine the scope of the validity of the method and assess the use of a 'mean' transition profile as a comparator within the CPZ PCA algorithm. We also aim to expand our approach to encompass tribological studies involving barrel manufacturing tolerances and barrel wear rates.

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