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Bio-inspired design and 4D Printing of Multi-stiffness Wavy Metamaterial Energy Absorbers/Dissipators with Shape Recovery Features

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A R T I C L E I N F O

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A B S T R A C T

This study proposes a four-dimensional (4D) printing design of novel wavy metamaterials for energy absorption/dissipation applications. When designing energy absorbers (EAs), ensuring stability, high energy absorption capacity, and shape memory behavior becomes a pivotal consideration. The conventional re-entrant metamaterial is reformulated by replacing the oblique supports of the unit cells with wavy struts, and various curvatures and arrangements of these supports are analyzed for their impact on the mechanical behavior of the metamaterials. The straight horizontal beams of the lattice metamaterials are replaced with wavy connections with inspiration from the cactus fiber to further enhance the material ductility and strength before and after inner contacts, respectively. A finite element analysis (FEA) simulates the deformation patterns and shows mechanical stress distributions within the structures under quasi-static compression. Following this, the lattice structures are additively manufactured from polylactic acid (PLA) silk ultra to validate the FEA results. A good correlation is observed between the FEA and experiments. From both experiments and simulations, a direct relation between the curvature of the wavy ligaments and the stress values is noticed showing the lower the stiffness and stress concentration with the higher curvature. Due to the dependency of structural stiffness on the curvature of the wavy ligaments, multi-stiffness wavy unit cells containing different curvatures can be combined to introduce a hybrid design containing low and high-stiffness unit cells. Under quasi-static compression, the hybrid design leads to a layer-by-layer yield and corresponding multi-plateau region related to each yield on force-displacement relation. In addition, during lattice deformation, the wavy design facilitates earlier contact points and frictional interactions between the walls. These mechanisms enable stress redistribution across broader surfaces, significantly enhancing energy absorption and dissipation through contact-based mechanisms. Due to these newly introduced energy absorption and dissipation capabilities of the proposed metamaterials, multiple applications of the proposed metamaterials can be considered such as in airplane wings, crash boxes, and bio-protection devices.

1. Introduction

The energy absorption capacity of a structure refers to its ability to convert kinetic energy into internal energy through various mechanical phenomena such as plastic deformation, shear, torsion, bending, and even fracture [1]. In today's transportation, the importance of energy absorbers within the industry cannot be underestimated. Ensuring the safety of individuals becomes paramount when it comes to using cars, bicycles, and motorcycles. In Canada alone, nearly 2000 lives are tragically lost annually due to car accidents on roads [2]. Designers of energy absorbers must exert their utmost efforts to provide enhanced safety measures.

In the realm of energy absorption/dissipation, mechanical metamaterials demand serious consideration as a highly promising candidate [3]. They are intricately designed structures deriving their mechanical properties primarily from the unique configurations of their unit cells, making them apart from the parent material [4]. Owing to their exceptional mechanical performance, they have found widespread application across various industries. For instance, in the sports industry, they are integrated into helmets as absorbers, enhancing safety measures [5]. In the aerospace sector, they are extensively employed to maximize sound transmission loss within aircraft cabins, optimizing

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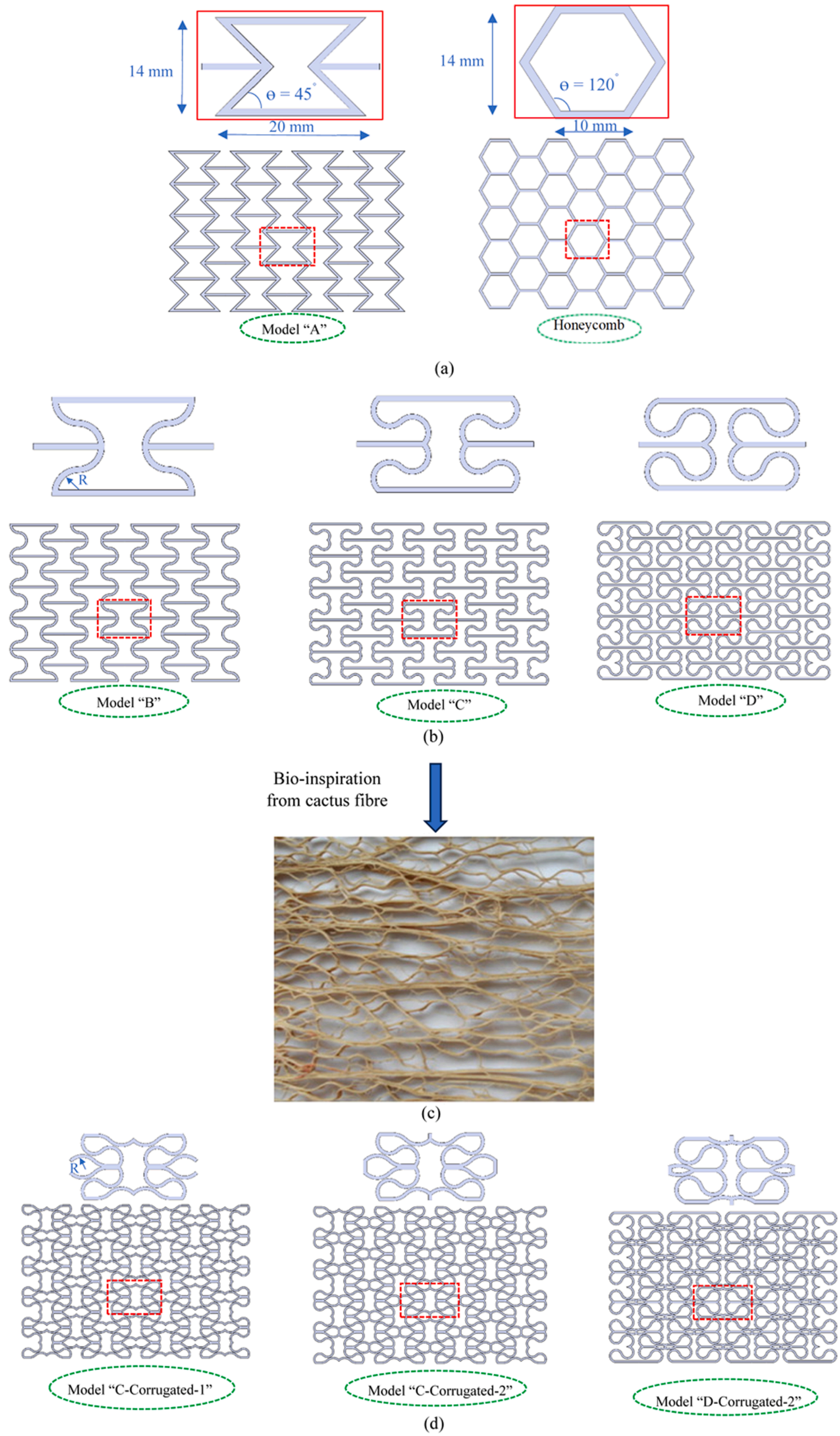

Fig. 1. (a) common re-entrant and honeycomb metamaterial, ***(b)*** wavy metamaterials, ***(c)*** cactus fibre, reprinted from [53], and ***(d)*** corrugated-based metamaterials.

Table 1
Geometrical information of the metamaterials*.

Parameter (mm)/ Structure	Wall thickness (mm)	Radius (mm)	Radius of the corrugated design (mm)
Model A	1	–	–
Model B	0.9	3.4	–
Model C	0.75	2.4	–
Model D	0.5	2.7	–
C-Corrugated−1	0.6	2.4	1
C-Corrugated−2	0.5	2.4	1
D-Corrugated−1	0.4	2.7	1
Honeycomb	1.2	–	–
Wavy tips	0.3	3.4** 2.4**	–

*The dimensions of all samples are considered constant at 100 × 70 × 15 mm.
* The mass of all samples is considered constant at 35 g.
** The radius related to the wavy design, a combination of model "B" and "C", is 3.4 mm and 2.4 mm respectively.

Table 2
Printing parameters.

Printing parameters	Value
Nozzle diameter	0.4 mm
Nozzle temperature	220C
Bed temperature	35C
Printing speed	$200\ \frac{mm}{s}$
Infill direction	± 45
Infill percentage	100 %

passenger comfort [6]. Due to the excellent dynamic behavior of metamaterials [7–9], the automobile industry capitalizes on the incorporation of metamaterials in car bumpers, elevating energy absorption capacity and consequently safeguarding passengers in the event of a collision [10]. These examples underscore the significance of mechanical metamaterials as transformative solutions with diverse applications across multiple industries.

From the mechanics point of view, one crucial mechanical parameter influencing the behavior of structures is Poisson's ratio, quantifying the relationship between transverse and axial strain [11]. Based on this parameter, mechanical metamaterials can be classified into different groups, including positive Poisson's ratio (PPR) metamaterials [12–19], zero Poisson's ratio (ZPR) metamaterials [20–25], and negative Poisson's ratio (NPR) lattices [26–34]. When designing reliable energy absorbers, mechanical stability must be prioritized, as it is closely linked to the energy absorption capacity of the structure [20]. The more stable the deformation patterns, the higher the energy absorption capacity. Extensive research has been conducted in recent years to explore the energy absorption capabilities of metamaterials, yielding significant insights in this field.

In the pursuit of advancing energy absorber/dissipator designs, notable contributions have been made by scholars. Garland et al. [35] introduced sliding Coulombic friction mechanism to absorb energy via sliding motion. Bacigalupo et al. [36] introduced energy absorption in metamaterials through self-recovery behavior, which occurs via frictional sliding between adjacent disks, dissipating energy during deformation. Ingrole et al. [37] introduced hybrid-designed auxetic honeycomb energy absorbers, showing remarkable improvements in compressive strengths and energy absorption capacity. However, their work also revealed noticeable instabilities within their designs. Similarly, Gunaydin et al. [38] demonstrated that anti-tetra chiral designs exhibit higher specific energy absorption (SEA) when compared to modified re-entrant hexagonal lattices. Nevertheless, certain instabilities were observed in their compression tests. In a different approach, Alomarah et al. [39] proposed re-entrant chiral-based auxetic structures, which exhibited considerable enhancements in energy absorption capacity compared to honeycomb structures. However, it is important to note that the occurrence of instabilities remains an inevitable challenge within their designs. These studies shed light on the continuous efforts in developing energy absorbers, highlighting both advancements and areas of improvement in achieving optimal stability and enhanced energy absorption capabilities.

In contrast, extensive research has been dedicated to investigating the stability of energy absorber metamaterials under compression, yielding promising outcomes. Notably, Hamzehei et al. [20] presented a groundbreaking approach by introducing novel DNA-based re-entrant metamaterials exhibiting zero Poisson's ratio (ZPR) behavior with diverse initial reaction forces. These pioneering ZPR metamaterials demonstrated remarkable characteristics, combining astronomical energy absorption capacity with exceptional stability. Additionally, Hamzehei et al. [10] proposed an innovative method of incorporating horizontal bars into conventional octagonal unit cells. By integrating horizontal bars, they achieved multi-stiffness three-dimensional (3D) ZPR blocks, offering a harmonious blend of stability and high energy absorption capacity on a 3D scale. These notable contributions emphasize the significance of exploring advanced strategies in energy absorber design, focusing on achieving both stability and exceptional energy absorption capacity. Through such advancements, the potential for groundbreaking solutions in the field of energy absorber metamaterials is unveiled, inspiring further research and development in this promising domain.

Even though there is a firm relationship between stability and energy absorption capacity in an energy absorber, one significant feature that can make the metamaterials special for energy absorption application is shape memory (recoverable) behavior. This behavior refers to a unique property exhibited by certain materials, allowing them to regain their original shape after an external stimulus like a heating-cooling process, so-called 4D printing [40]. This characteristic presents a significant opportunity in order to design and develop an absorber with reusable capabilities, thereby optimizing resource utilization and cost-effectiveness [41–43].

When it comes to tensile mechanical loads, maximizing tensile strength and the structure's ability to undergo significant deformation before failure, so-called ductility, are two important mechanical aspects that wavy metamaterials can cover them [44–46]. Apart from tensile loads, this paper aims to reveal the potential of wavy metamaterials under quasi-static compression. On this matter, several studies on curve/arc designs of lattice structures have been carried out [47–52]. Meena et al. [47] introduced an S-shaped auxetic unit cell, showing lower stress concentration and a more negative value of Poisson's ratio than the common auxetic unit cell. Hao et al. [48] designed arc re-entrant unit cells inspired by arch bridges. Their designs provided lower values of stress and stable deformation patterns attributed to the plastic hinges under quasi-static compression and impact loads. Liu et al. [49] proposed a novel accordion cellular honeycomb core exhibiting close to zero Poisson's ratio behavior containing in-plane corrugated U-type (curve) beams. Their results showed better in-plane morphing and out-of-plane load-bearing capabilities. Inspired by cuttlebone, Chen et al. [50] designed buckling resistance of carbon fiber curved-wall honeycomb which can efficiently delay the early onset of buckling failure mode under quasi-static compression. Wang et al. [51] investigated the elastic properties of the spline-based curvy honeycomb under quasi-static compression, showing that the higher curvature of the spline cell wall leads to a smaller equivalent elastic modulus and an increase in structural resistance against out-of-plane buckling. Feng et al.[52] introduced newly designed curved-based honeycombs, showing higher energy absorption capacity compared to the conventional rhombus honeycombs. Each of the above studies focuses on specific applications such as stress redistribution, enhanced out-of-plane stability, and improved energy absorption. The wavy design, characterized by flexibility and early contact during deformation, offers potential for advanced engineering applications with both high ductility and

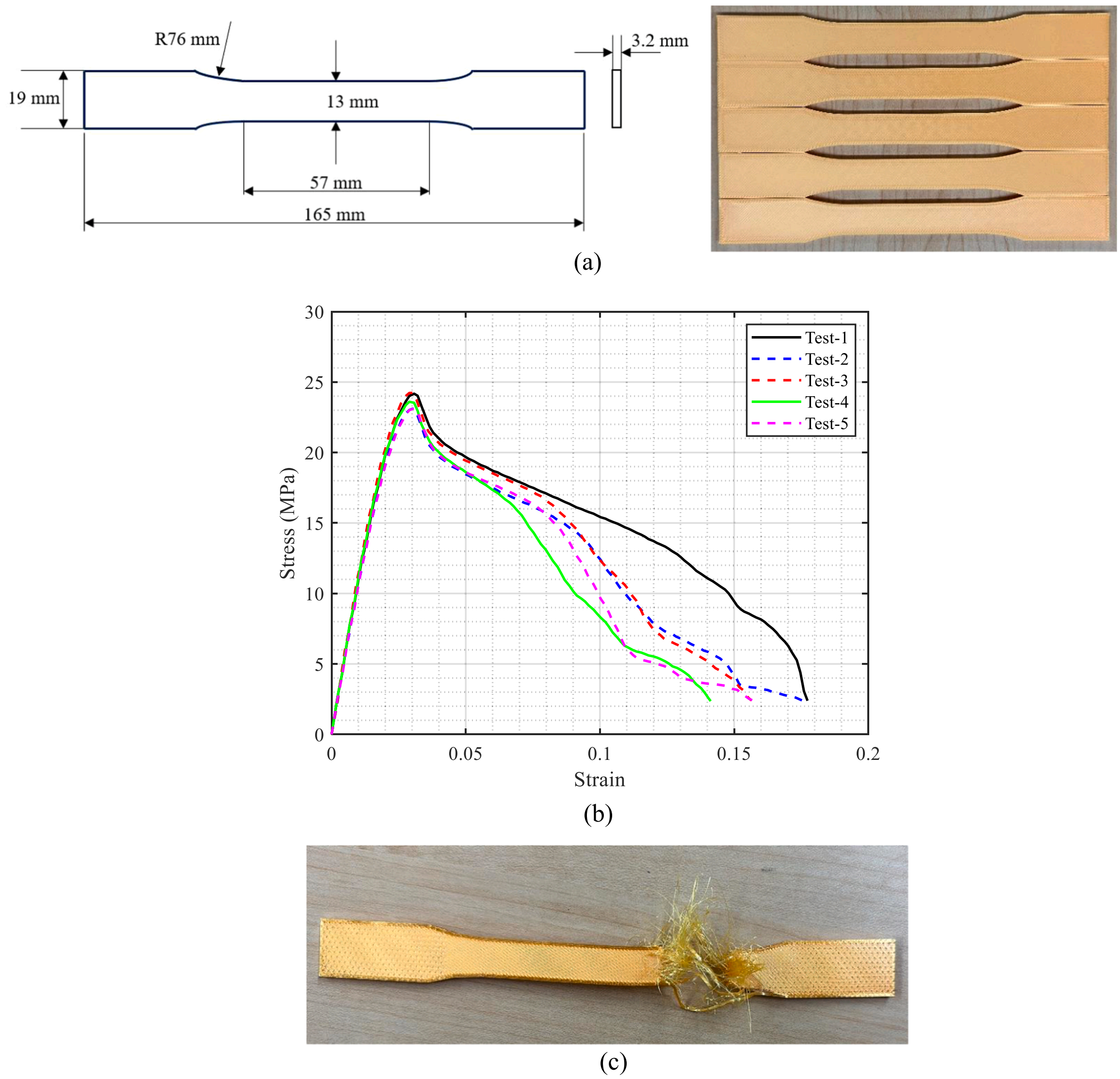

Fig. 2. (a) Schematic view with the corresponding 3D-printed dog-bone samples, (b) stress-strain relations related to the PLA silk ultra, and (c) fibrous behavior under tensile loadings.

strength, though these aspects remain underexplored. Additionally, the flexibility of the nonlinear shape arrangement in wavy designs allows for straightforward adjustments to achieve multiple unique properties, including energy dissipation alongside energy absorption. Key mechanisms underlying the mechanical behavior of lattice structures with curved elements such as the effects of curvature, element arrangement, and contact interactions during deformation require further study and comprehensive analysis.

This research introduces new design methods and mechanisms to enhance the functionality of wavy metamaterial structures, focusing on achieving unique mechanical properties, particularly in energy absorption and dissipation. By replacing the struts of conventional re-entrant auxetic unit cells with wavy counterparts, smart wavy metamaterials have been developed to address common limitations in re-entrant designs such as instability, limited energy absorption/dissipation capacity, and susceptibility to fracture under compression load. The study further explores the effects of curvature on the structural behavior of wavy designs, revealing that increased curvature in wavy ligaments reduces initial reaction forces, stiffness, and enables earlier densification, resulting in more stable deformation under quasi-static conditions. Bio-inspired load-bearing elements are proposed to provide balanced ductility, high strength, and enhanced energy absorption/dissipation properties. Various strategies for arranging and orienting the wavy elements are also discussed, offering stability control, quasi-zero-stiffness (QZS) feature, and improved energy dissipation through contact-based wall interactions. This research guides energy absorber and dissipator designers in reformulating re-entrant metamaterial designs and adjusting geometric parameters to achieve a stable, effective, and reusable energy absorber with superior energy absorption and dissipation capabilities.

2. Conceptual design

This study takes the conventional re-entrant unit cell into account, see Fig. 1a (model “A”). Then, the unit cell ligaments are replaced with wavy ligaments with different curvatures, see Fig. 1b (models “B”, “C” and “D”). Following this, the wavy metamaterials are considered, and the horizontal ligaments of the unit cells are replaced with corrugated designs inspired by cactus fibers, see Fig. 1c. Referring to the cactus fiber, the curved design of structural links and supports in lattices can induce multi-contacts between wavy fibers, providing better support to constant loads. In addition, during the deformation of such wavy fibers, friction between contacts can cause better energy dissipation. Referring to these behaviors and properties of cactus fiber, we replaced all straight

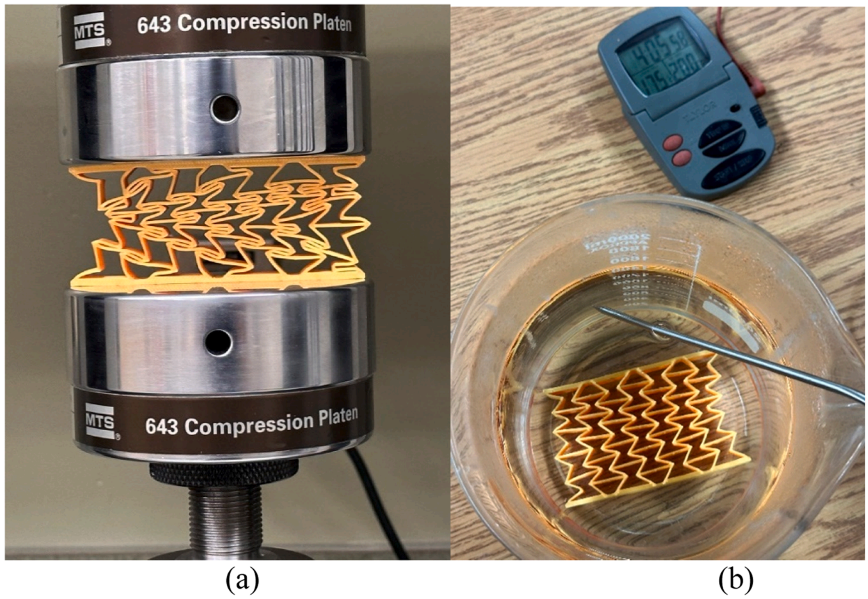

Fig. 3.. The experiments related to ***(a)*** quasi-static compression, and ***(b)*** shape recovery behavior.

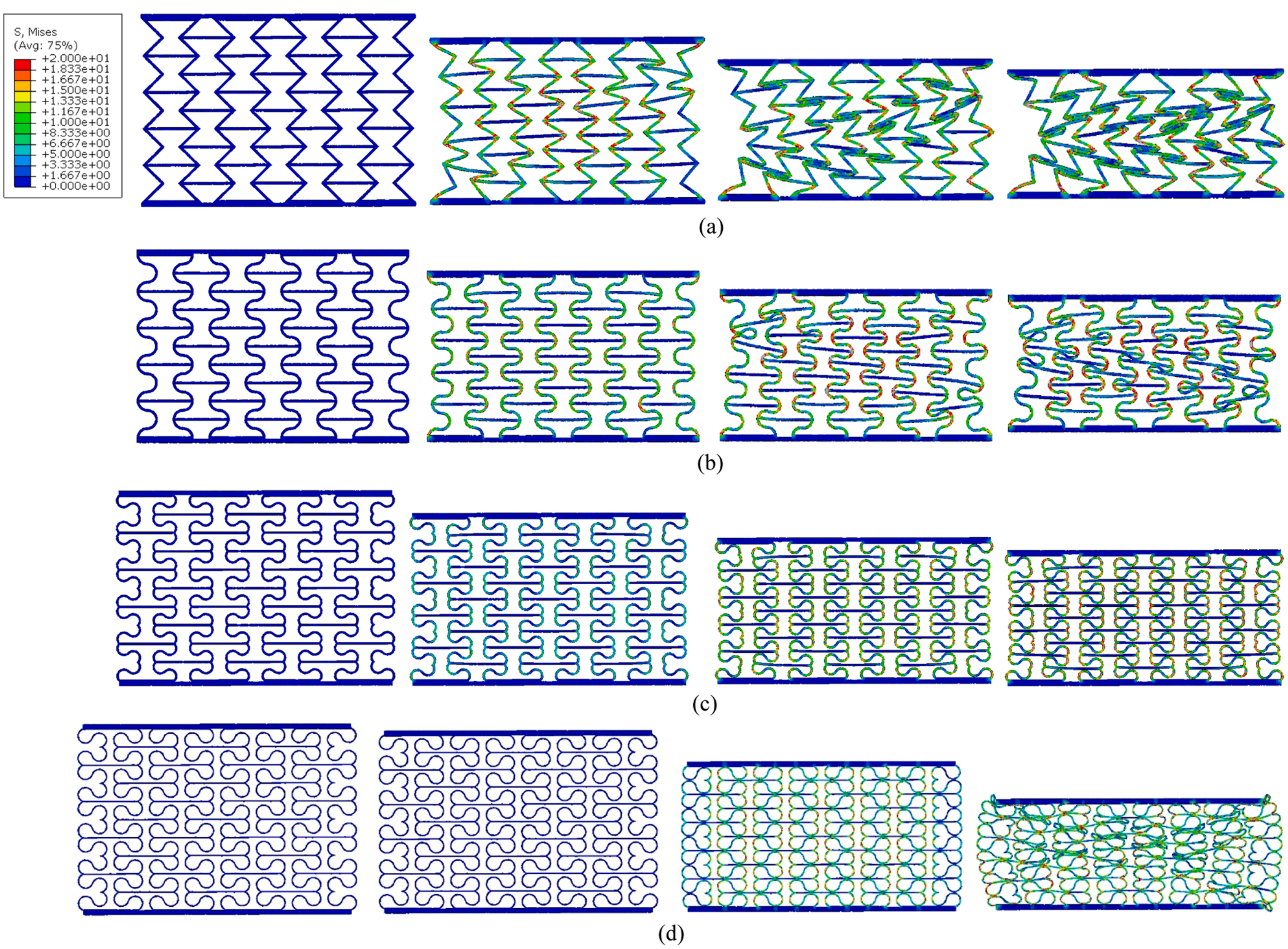

Fig. 4.. Deformation patterns of metamaterials ***(a)*** “Model-A”, ***(b)*** “Model-B”, ***(c)*** “Model-C” and ***(d)*** “Model-D” under quasi-static compression.

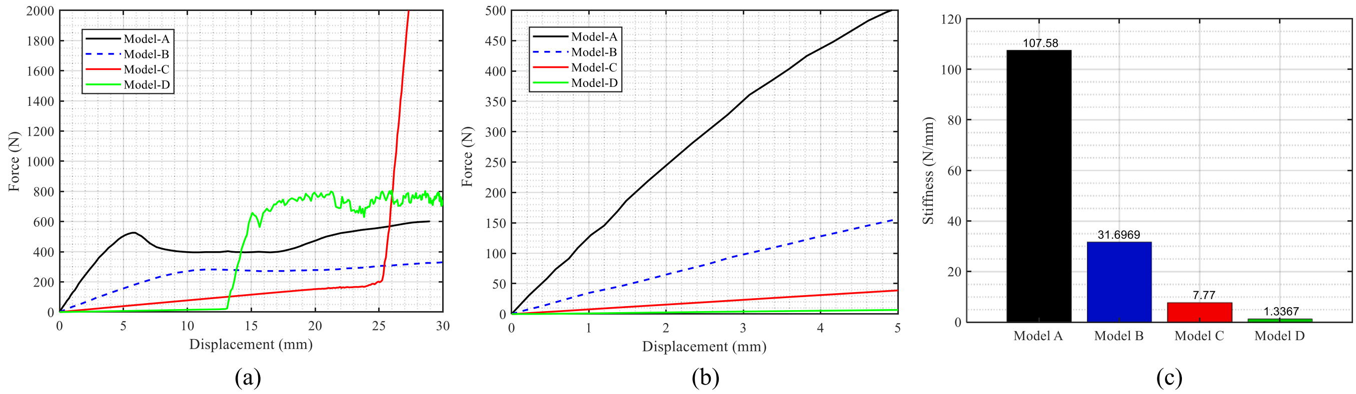

Fig. 5.. Force-displacement relations of metamaterials obtained from the FEA ***(a)*** elastic-plastic region, ***(b)*** elastic region, ***(c)*** stiffnesses.

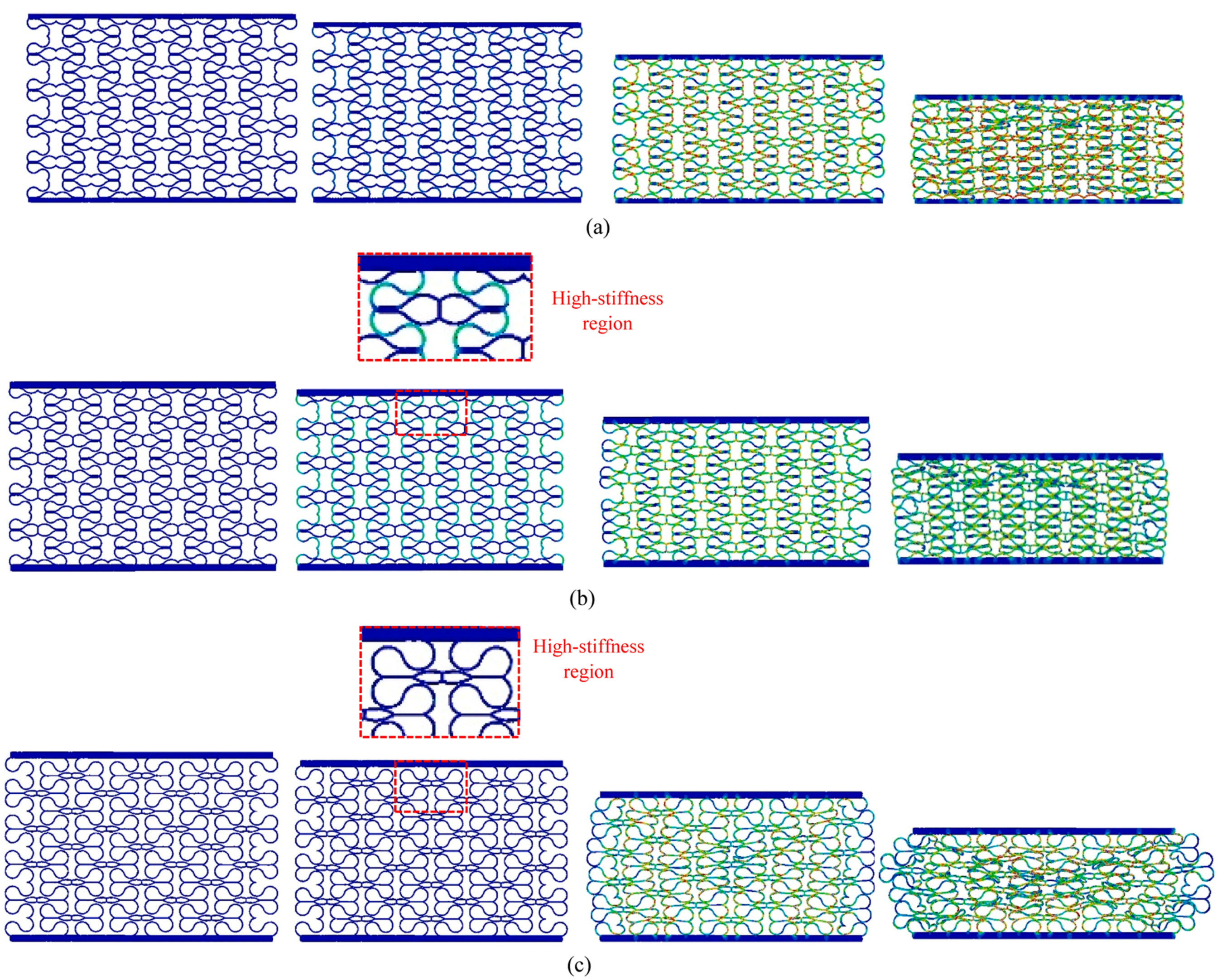

Fig. 6.. The effect of horizontal corrugated beams on deformation patterns of metamaterials ***(a)*** C-corrugated-1, ***(b)*** C-corrugated-2, and ***(c)*** D-corrugated-2.

connections in models 'B', 'C,' and 'D' with corrugated ones, leading to the corresponding corrugated designs, as shown in Fig. 1d. It is also worth noting that to compare the mechanical performances of the newly designed wavy metamaterials with their potential competitors like honeycombs, traditional honeycomb design is also considered, see Fig. 1a. The geometrical details of the metamaterials are provided in Table 1. It is worth mentioning that to make the results comparable, the mass of all samples is considered constant at 35 g.

3. Material & methods

3.1. Fabrication

For the fabrication of the samples in this study, the Bambu Lab X1

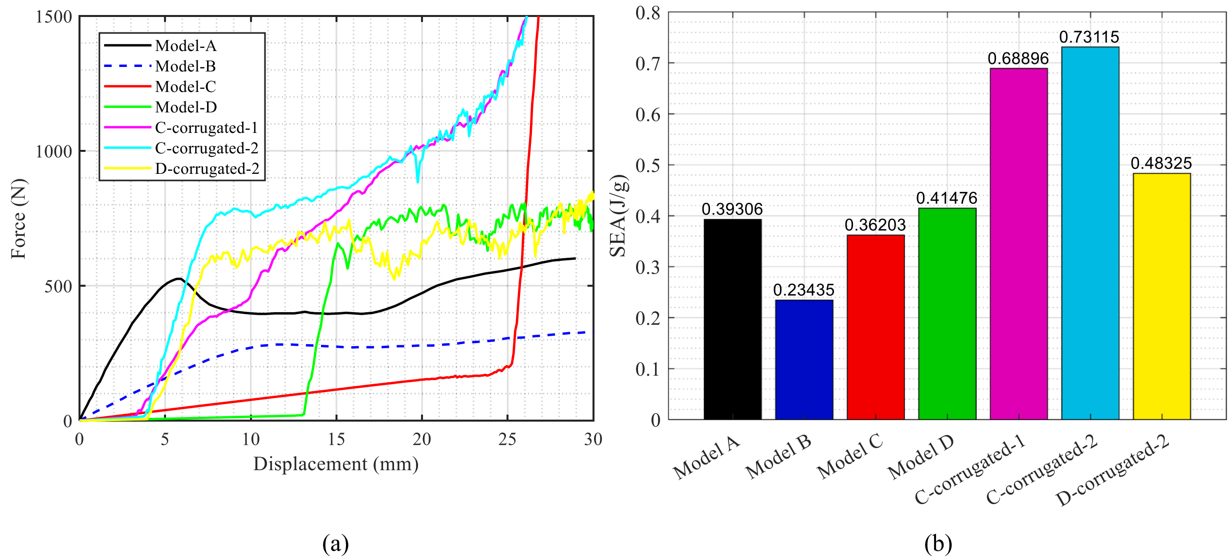

Fig. 7.. The ***(a)*** force-displacement relation obtained from the FEA, and ***(b)*** SEA of the proposed metamaterials.

carbon machine was employed with the following setup, see Table 2.

3.2. *Material & finite element analysis (FEA)*

This study considers Polylactic Acid (PLA) silk ultra providing formability and shape memory behavior via a heating-cooling process. To achieve the mechanical properties of the PLA silk ultra, five dog-bone samples were additively manufactured, see Fig. 2a. Tensile tests were performed according to the standard ASTM-D-638–14 [54] at a speed of 2 $\frac{in}{min}$. The corresponding stress-strain relations can be seen from Fig. 2b. This presents the engineering stress-strain curves for five PLA silk-ultra dog-bone specimens subjected to tensile testing, demonstrating the material's elastic-plastic behavior. Since PLA silk ultra is a composite material containing polylactic acid (PLA) with silk fibres [55], fibrous behavior was observed during testing, see Fig. 2c. This can lead to increased resistance to deformation and superior load-bearing capabilities, making it suitable for applications requiring high mechanical performance such as energy absorption applications. The deformation patterns of metamaterials under quasi-static compression are simulated using the ABAQUS explicit package. Detailed information regarding the FEA and its validation through experimental studies are provided in the supplementary information file.

3.3. *Experiments*

To evaluate the mechanical properties of the proposed metamaterials, a quasi-static compression test was carried out at a speed of 5 $\frac{mm}{min}$ and the maximum strain of 40 % via the MTS universal testing machine. To evaluate the recoverable behavior of metamaterials, a heating-cooling process is carried out via a beaker, and a digital thermometer for sensing water temperature, see Fig. 3.

4. Results & discussion

Different wavy grades, unit cell layouts, and curve arrangements can lead to several interesting and superior mechanical responses and behaviors. Section 4.1 shows how the wavy unit cells with different curvatures affect the deformation patterns and mechanical properties of the metamaterials, including energy absorption/dissipation capacities. As aluminum honeycombs are widely used for energy absorption applications [56], Section 4.2 compares the mechanical performance of the wavy metamaterial design showing the highest energy absorption capacity with the aluminum honeycomb. Section 4.3 shows how to provide stability under quasi-static compression by a simple replacement of the common unit cells “A” with the wavy ones. In the following, Section 4.4 shows how to consider hybrid design, a metamaterial design containing soft and stiff unit cells, to achieve multi-plateau regions (quasi-zero stiffness) on force-displacement relations. In the following, Section 4.5 shows how different wavy ligaments with different curvatures can be considered in one unit cell to provide relative motion between the walls of the unit cells for energy dissipation applications. Section 4.6 shows the shape memory feature (fully recoverable behavior) via a heating-cooling process. In the end, Section 4.7 proposes the potential applications of the wavy metamaterials in different areas such as aerospace, automobile and sports industries. It is important to highlight that, while this research presents a range of energy absorbers and dissipators, they are not asserted to be the highest-capacity options available. Other energy absorbers or dissipators with greater capacities may exist.

4.1. *“The effects of wavy unit cells on mechanical properties”*

4.1.1. *Straight horizontal links between unit cells*

This section reveals the effect of wavy ligaments on the deformation patterns obtained from the FEA. Fig. 4a shows the common re-entrant metamaterial deformation pattern. Within the common re-entrant metamaterial, model-A, certain regions experience higher stress concentrations than others. These localized stress concentrations can trigger instabilities by initiating buckling or bending within the unit cells where the stress is concentrated, resulting in instabilities. On the other hand, the wavy unit cells lead to lower stress concentrations, providing a more uniform distribution of stress throughout the material, see Fig. 4b, c, and d. Indeed, when subjected to external loads, wavy metamaterials with lower stress concentrations can better transfer the loads across the structure. This prevents stress concentration in particular regions, reducing the risk of localized deformations. This means that wavy metamaterials are superior to conventional metamaterial in terms of energy absorption, as there is a direct relation between energy absorption capacity and stability.

The curve designs also lead to the introduction of multi-stiffness unit cells. From Fig. 5, it can be seen that the more curvature, the lower the stiffness. This means that the wavy unit cells are capable of introducing different energy absorbers with different initial reaction forces. For example, model “A” shows the highest initial reaction forces, while models “C” and “D” show low initial reaction forces. There is a reduction of 98 % in the stiffness of the structure “D” compared to “A” (a decrease

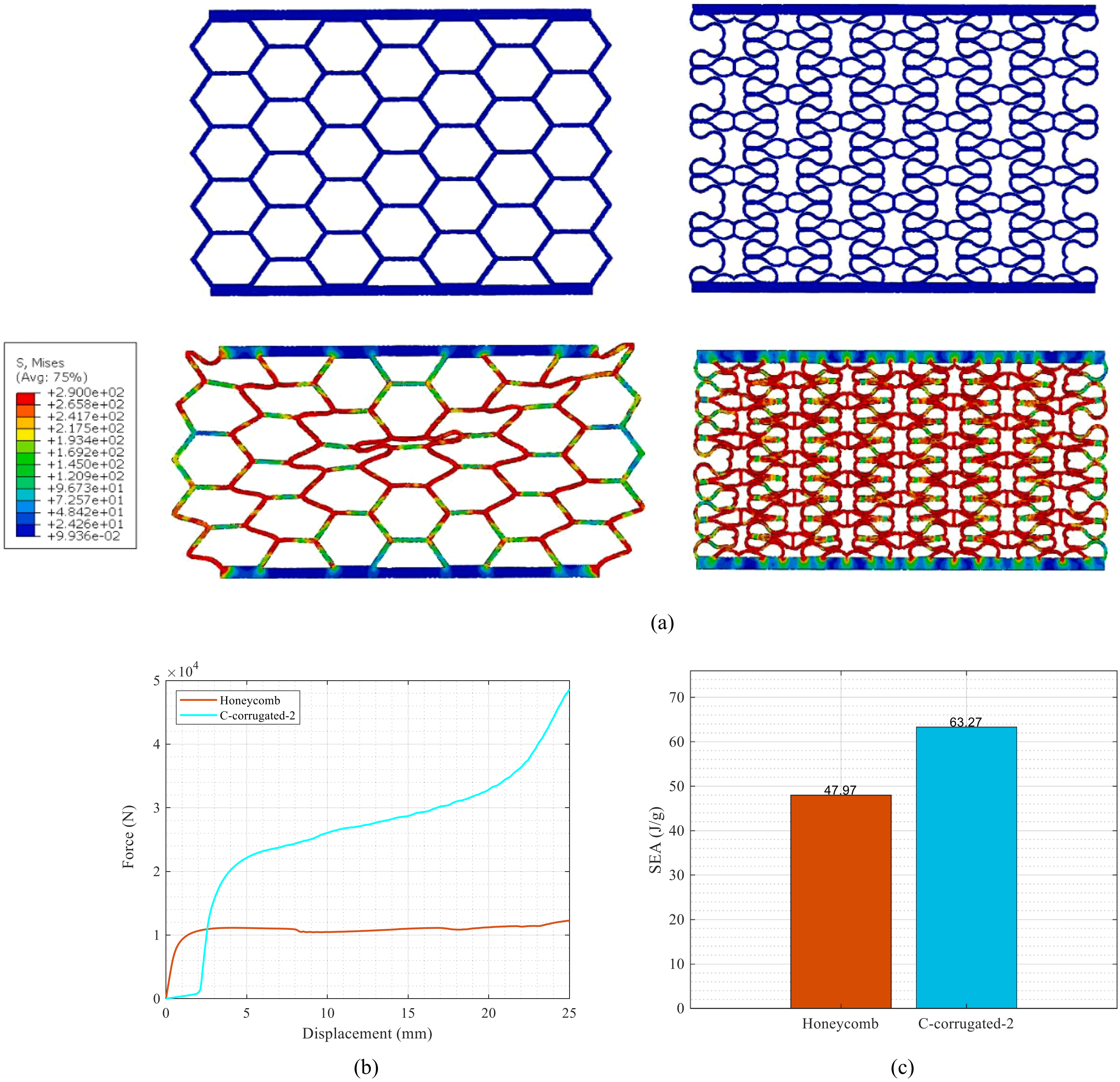

Fig. 8. The (a) deformation patterns of aluminum honeycomb and "C-corrugated-2" model under quasi-static compression, (b) force-displacement relation obtained from the FEA, and (c) SEA with assigned aluminum properties.

in structural stiffness from 107.58 $\frac{N}{mm}$ to 1.3367 $\frac{N}{mm}$). This makes the wavy metamaterials suitable for applications where occupant safety is a priority [57].

Indeed, models "C" and "D" behave similarly to crushable foams, where materials initially deform under low loads with minimal resistance, and then gradually increase their resistance as they compress further [58].

4.1.2. Corrugated horizontal links between unit cells

In this section, the effect of corrugated horizontal beams is investigated. The results are provided by the FEA. The straight horizontal beams are replaced with the corrugated ones, see Fig. 6. This leads to an earlier densification under compression due to cell wall contacts. It is worth mentioning that some vertical struts are considered in models "C-corrugated-2" and "D-corrugated-2". After cell wall contacts, these vertical struts provide high-stiffness regions within the lattice structures with severe resistance under compression, see the regions shown in red-dashed rectangles. This leads to a higher energy absorption capacity. Indeed, these high-stiffness regions lead to the bending of the walls in their vicinities, resulting in more plastic deformation and higher energy absorption capacities, see Fig. 7. For example, the SEA related to models "C-corrugated-1", and "C-corrugated-2" increases by up to 90 % (changes from 0.36203 J/g to 0.68896 J/g) and 101.95 % (changes from 0.36203 J/g to 0.73115 J/g) compared to model "C". This denotes the efficiency of the corrugated horizontal beams in enhancing energy absorption capacity through wall contact during compression. It is worth mentioning that although there is an increase in the energy absorption capacity of the model "D-corrugated-2" compared to the model "D" by up to 16.51 % (increases from 0.41476 J/g to 0.48325 J/g), that increase is not as high as the energy absorption capacity of the models

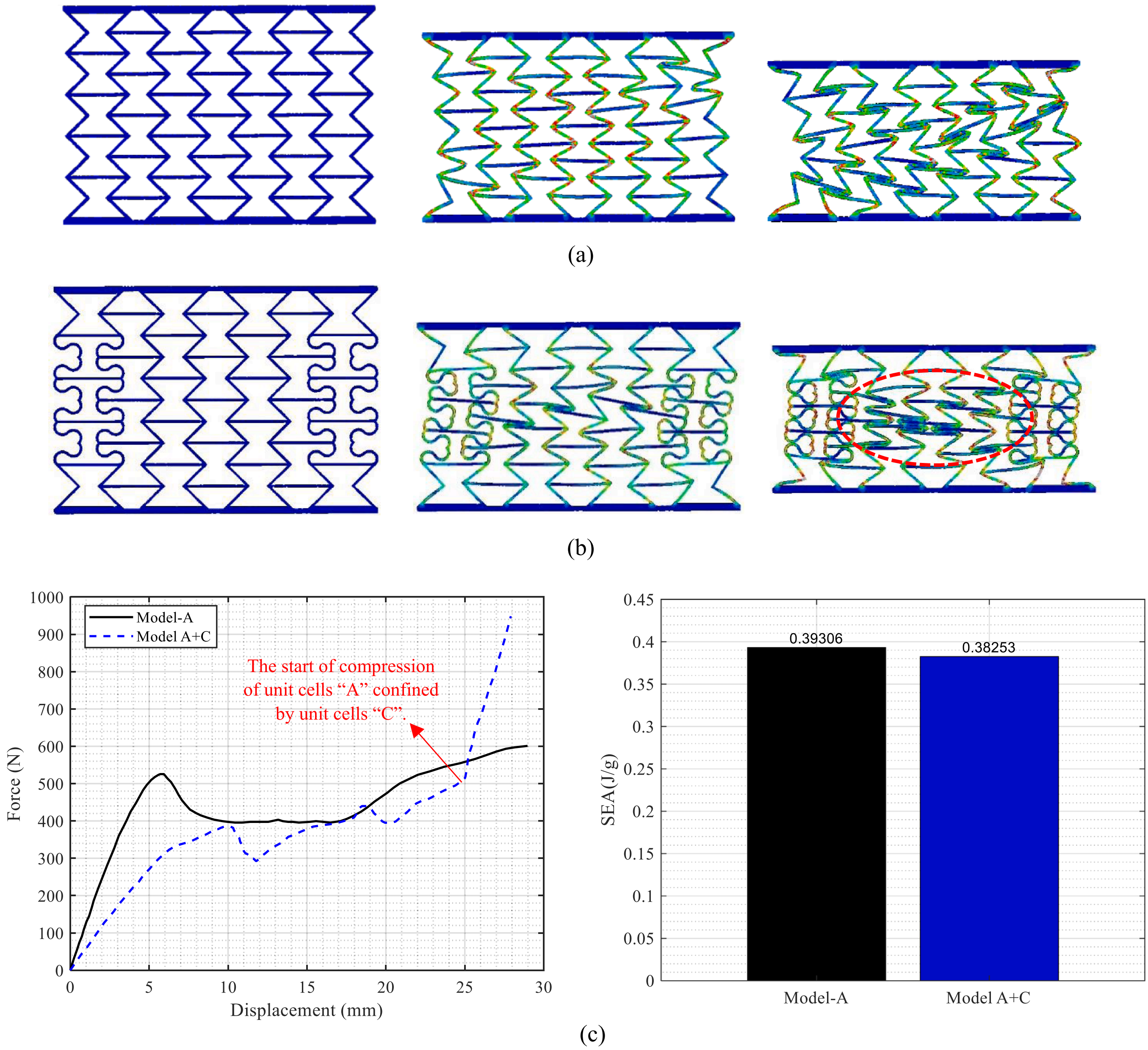

Fig. 9. The deformation patterns of (a) common model "A", (b) model "A+C" on maintaining structural stability, and (c) corresponding force-displacement relation obtained from the FEA.

"C-corrugated-1" and "C-corrugated-2". The main reason mainly stems from the lower stiffness of the unit cell "D" compared to "C". Due to the lower stiffness of unit cell "D", this unit cell is less capable of withstanding significant plastic deformation compared to unit cell "C", resulting in an earlier failure under compression when it comes to high-stiffness regions.

4.2. *"Comparison of "C-corrugated-2" model with aluminum honeycomb"*

Aluminum honeycombs are widely used as energy absorbers due to their lightweight structure [56, 59, 60], and high specific strength. Section 4.1.2 and Fig. 7 show that the "C-corrugated-2" structure exhibits the highest energy absorption capacity. FEA is carried out by assigning aluminum properties to that structure to compare the mechanical properties of that structure with those of common competitors like aluminum honeycombs at the same mass, 90 g. The mechanical properties of aluminum are provided in the supplementary information, see Table S2. Fig. 8a shows the deformation patterns of the aluminum honeycomb and the "C-corrugated-2" model under quasi-static compression. The boundary conditions are the same as fixing the bottom surfaces and applying compression from the top. The results demonstrate a higher energy absorption capacity of the corrugated wavy design than the aluminum honeycomb, due to the contact resulting from the early densification of the wavy lattice structure in contrast to the traditional honeycomb structure (Honeycomb SEA (J/g) = 47.97, C-corrugated-2 SEA (J/g) = 63.27).

4.3. *"Stability control"*

This section sheds light on the capability of hybrid designs, a combination of high and low-stiffness unit cells, to provide stability under mechanical loads. The results rely on the FEA. In comparison with the deformation patterns of model "A" showing instability, see Fig. 9a, embedding low-stiffness unit cells on the border of a re-entrant metamaterial structure is indeed a promising approach to enhancing its

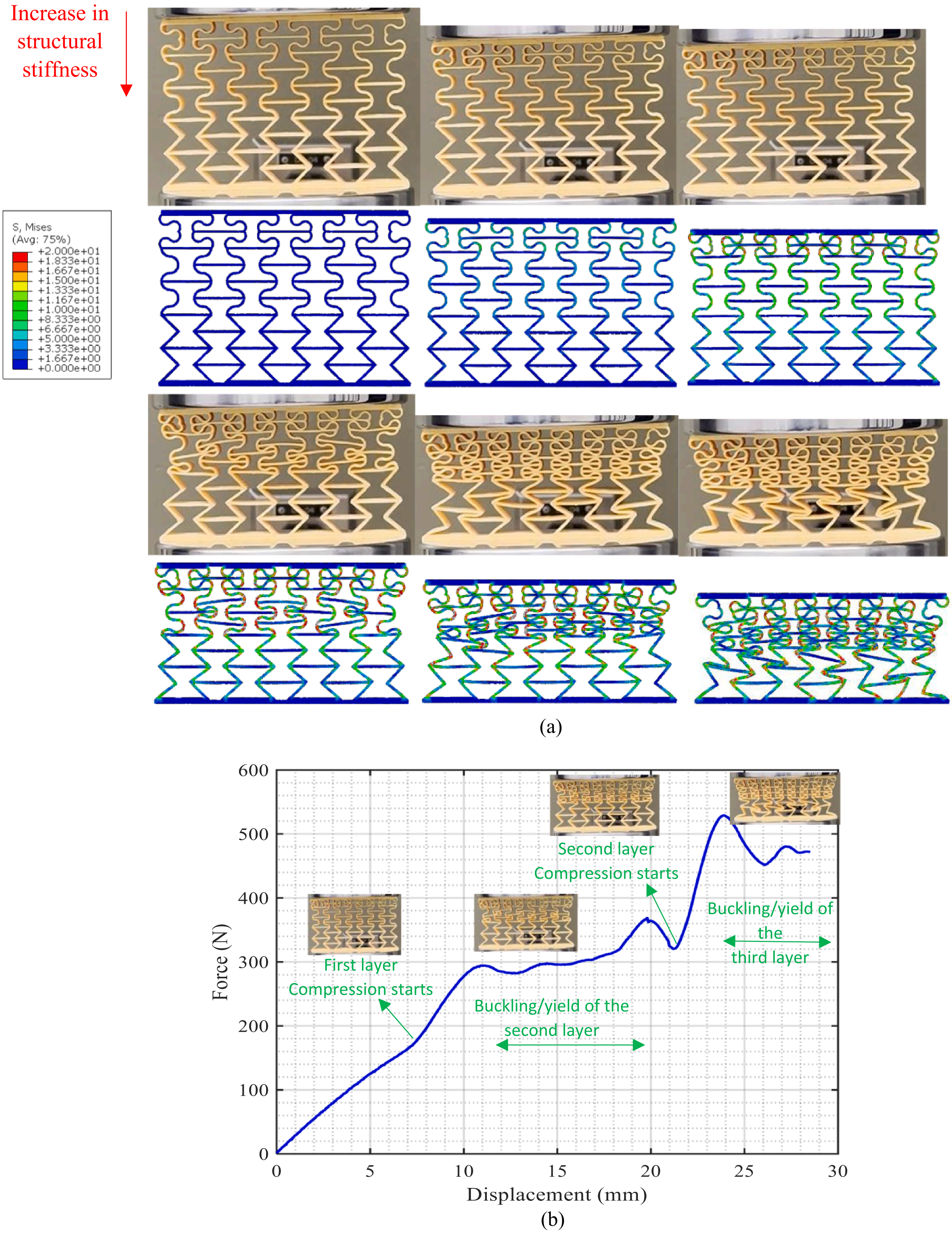

Fig. 10.. The ***(a)*** deformation patterns of multi-stiffness metamaterial, and ***(b)*** the corresponding force-displacement relation obtained from the experiments.

stability under compression, see Fig. 9b. By strategically placing these low-stiffness unit cells on the border of the metamaterials, stress concentrations can be mitigated, reducing the likelihood of localized failure/buckling. Consequently, one effective way to provide stability in common metamaterial model "A" is embedding the low-stiffness unit cells on the border of the structure.

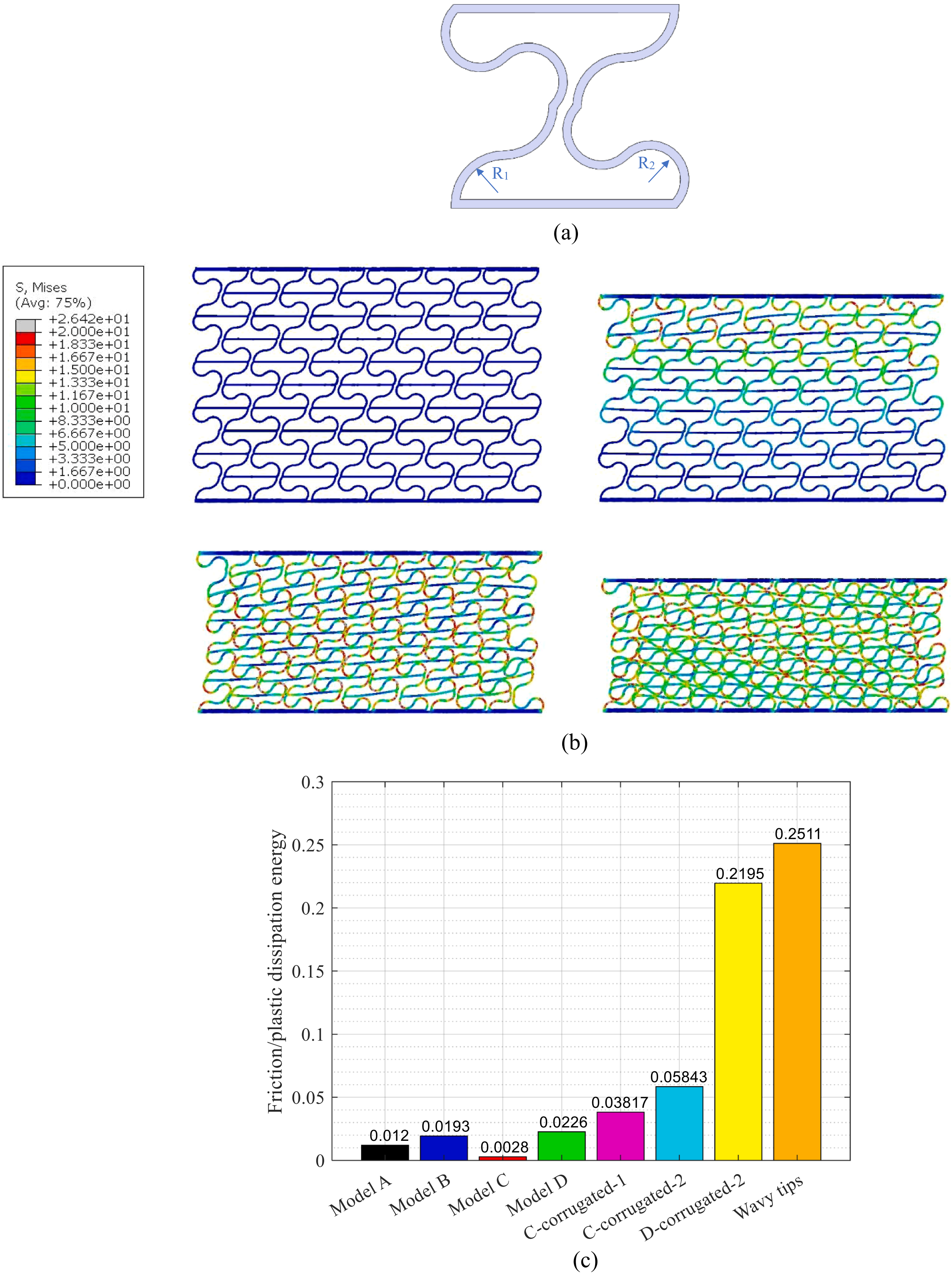

Fig. 11.. The ***(a)*** “wavy tips” unit cell with R_1= 3.4 mm and R_2= 2.4 mm, ***(b)*** the corresponding deformation patterns, and ***(c)*** the ratio of friction to plastic dissipation energy.

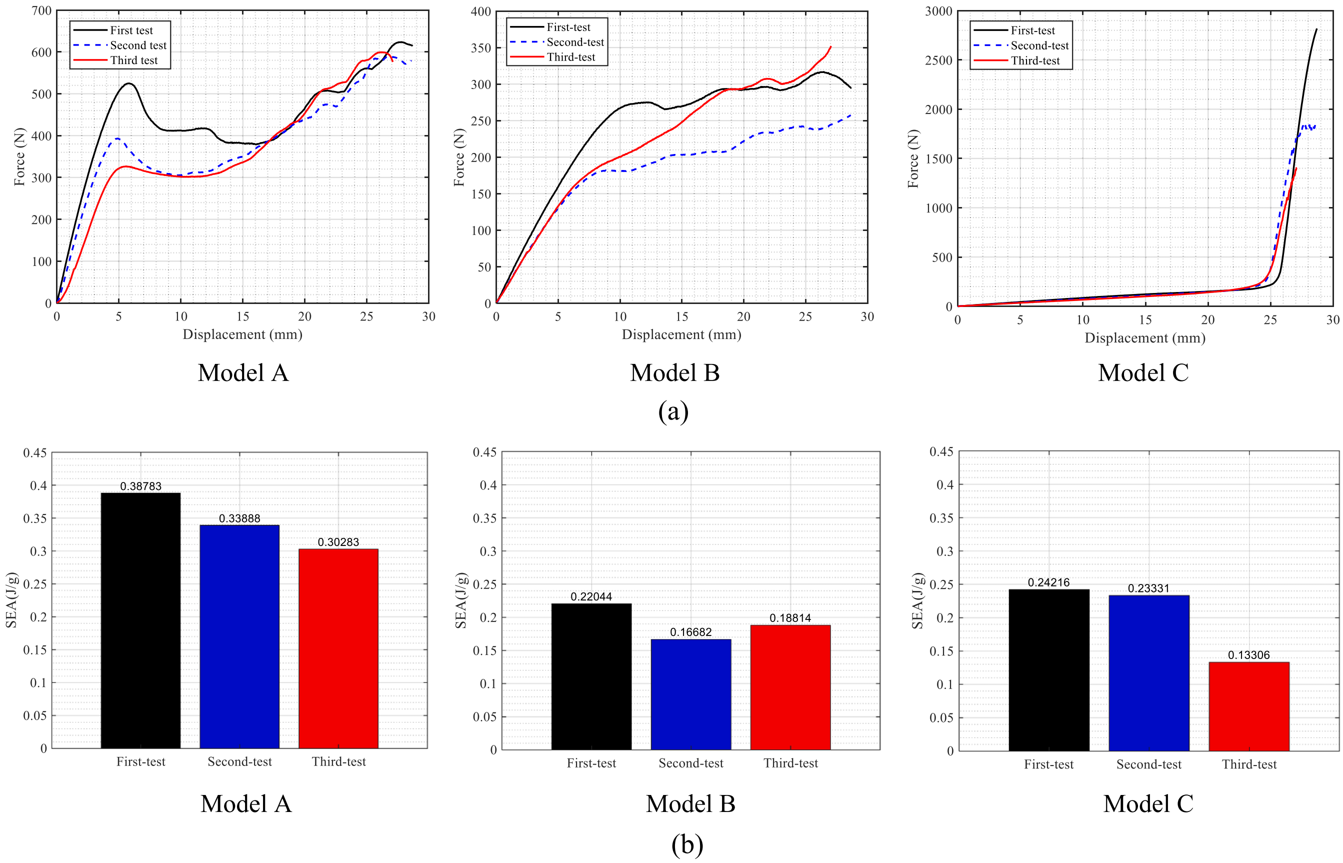

Fig. 12.. The ***(a)*** heating-cooling effects on force-displacement relations, and ***(b)*** the SEA corresponding to models “A”, “B”, and “C”.

4.4. *“QZS feature of hybrid metamaterials”*

This section elucidates how the gradient in stiffness within metamaterials, ranging from soft to stiff, can give rise to a QZS characteristic, obtained from the FEA and experiment. When a hybrid metamaterial is subjected to compression, the applied load causes stress distribution throughout the soft layer. The soft unit cells undergo significant deformation without transmitting stresses to the other layers. As the compression continues, the soft row containing the soft unit cells yields, see Fig. 10 a. This yielding behavior of the soft layer results in a plateau region on the force-displacement relation. Meanwhile, the stiff unit cells within the hybrid metamaterial resist deformation and provide structural stability under compression. It is worth mentioning that each yield within the soft layers contributes to forming a plateau on the force-displacement relation, see Fig. 10 b. The stable deformation patterns and plateau regions of the proposed metamaterial make it well-suited for vibration isolation [21, 61] applications or used as shock absorbers [62] that effectively mitigate vibrations while maintaining stability under mechanical loads.

4.5. *“Relative motion and energy dissipation”*

This section shows how the internal design of the unit cell containing different curvatures in the unit cell, so-called “Wavy tips”, can lead to a relative motion between the cell walls. The results corresponding to plastic and frictional dissipation are obtained from the FEA. It considers the first law of thermodynamics, saying that the change in a material’s kinetic and internal energy over time is equal to the total work done by surface forces and body forces on the material. Plastic Dissipation is the energy absorbed by a material due to irreversible plastic deformation. The plastic dissipation energy is computed by integrating the product of the stress, $\boldsymbol{\sigma}$, and the plastic strain rate, $\dot{\boldsymbol{\varepsilon}}_{pl}$, over the volume of the material, **v**, and over time, **t**, see Eq. 1 [63].

$$\iint_0^t \boldsymbol{\sigma} \quad : \dot{\boldsymbol{\varepsilon}}_{pl} \boldsymbol{dv} \ \boldsymbol{dt} \tag{1}$$

Frictional Dissipation is the energy lost due to frictional forces during the contact interaction between surfaces. It is calculated by integrating the product of the velocity vector, **v**, and traction force, **τ**, over the contact surface area, **s**, as provided in Eq. 2 [63].

$$\int_S \mathbf{v}.\boldsymbol{\tau} \mathbf{dS} \tag{2}$$

As can be seen from Fig. 11 a, different curvatures of the beams in one unit cell lead to relative motion and corresponding frictions. This leads to the simultaneous absorbing and dissipating of energy via frictional motions once mechanical loads are applied, see Fig. 11 b. This makes the proposed “wavy tips” unit cell suitable for energy absorption and dissipation applications. From Fig. 11 c, it can be inferred that the ratio of friction to plastic dissipation energy of the “wavy tips” metamaterial is almost twenty times greater than that of the common model “A”, changing from 0.012 to 0.2511, confirming the practicality of “wavy tips” model in energy dissipation applications.

4.6. *“Shape recovery feature”*

This section explores how metamaterials can return to their original shape after being subjected to heating and cooling through experiments. The usability of metamaterials extends beyond simple shape recovery behavior. As discussed in Section 3.2, the PLA silk ultra, as parent material, behaves like fibres, allowing metamaterials to be used multiple times while keeping their properties through repeated loading and unloading processes. Indeed, the robustness of PLA silk ultra ensures that the metamaterials can be used multiple times while retaining their

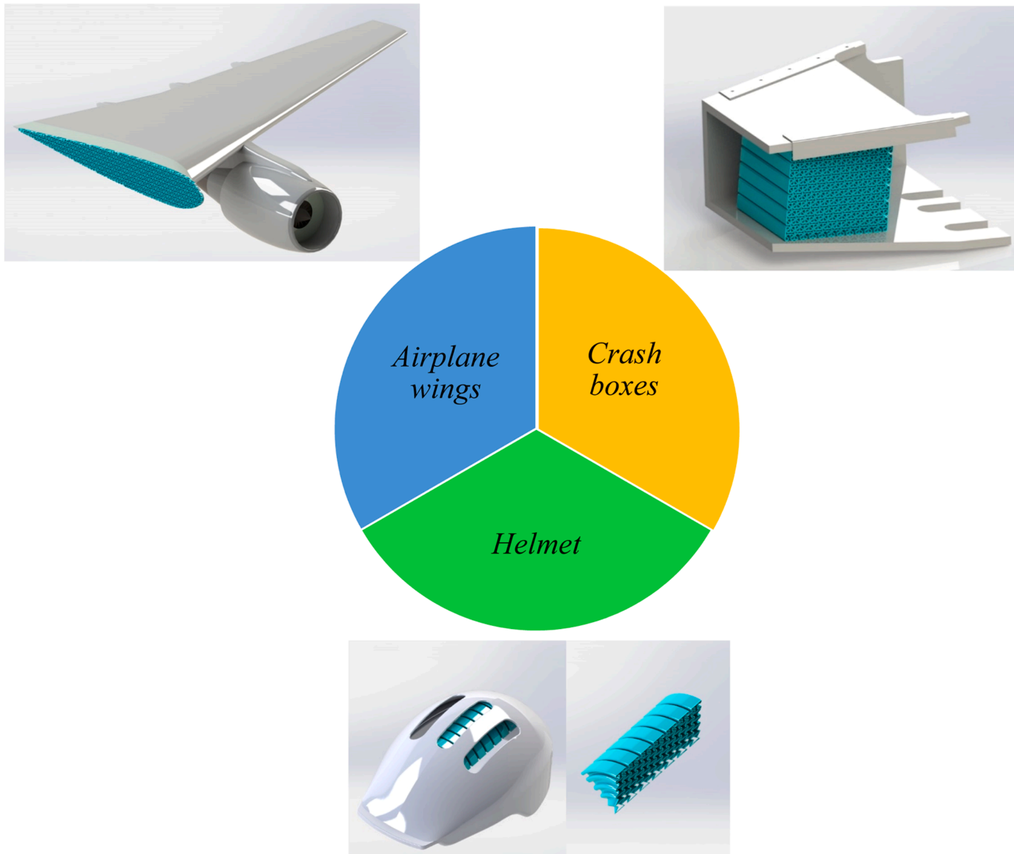

Fig. 13.. The potential applications of the proposed wavy metamaterials.

functional integrity and providing a cost-effective and reliable solution. It is also worth noting that one interesting aspect of metamaterials' behavior is the slight decrease in structural stiffness and energy absorption capacity after repeated cycles of loading and unloading, see Fig. 12. This reduction is typically minimal and does not significantly affect the material's overall performance. On the flip side, take model "B" into account. While there is a slight drop in energy absorption capacity in the second test, an increase in energy absorption capacity can be seen in the third test, see Fig. 12. This increase and recovered hardening behavior could be related to the re-alignment of broken polymer chains during the shape recovery process through heating. With greater deformation, the mechanical properties of PLA silk materials may not fully recover if a fracture occurs. Although fracture is not addressed in this study, the testing was carried out at 40 % of strain, which sufficiently demonstrates the superior properties of PLA Silk Ultra compared to traditional PLA.

4.7. Potential applications

This section introduces possible applications of the multi-stiffness wavy metamaterials introduced in this study, see Fig. 13. The first potential applications could be in airplane wings. Airplane wings experience different stress levels during various flight phases such as takeoff, cruising, and landing [64]. Multi-stiffness wavy metamaterials can adapt their stiffness in response to these changing stresses. In addition, the ability to change stiffness can help in optimizing the aerodynamic shape of wings, reducing drag, and improving fuel efficiency. They can also enhance the wing's ability to absorb impacts such as bird strikes without catastrophic failure by locally increasing stiffness. When it comes to the automobile industry, the role of multi-stiffness metamaterials is hard to ignore. Take crash boxes, as a potential application, into account. Crash boxes are designed to absorb impact energy during a collision [65]. The variable stiffness property of the metamaterials allows for a more progressive and controlled collapse, enhancing the protection of occupants by ensuring that the crash energy is absorbed efficiently. Another potential application could be in the sports industry like helmet design [66]. Helmets need to protect the head from varying levels of impact. Multi-stiffness wavy metamaterials can adapt their stiffness to absorb both low and high-energy impacts effectively. Indeed, helmets incorporating wavy metamaterials can better distribute the stresses arising from impact, reducing strain on the neck and making cycling safer.

5. Conclusion

To address common deficiencies occurring in common metamaterials under quasi-static compression, including instability, fracture, and low energy absorption/dissipation capacities, this study proposed novel wavy metamaterials with the inspiration from cactus fibre. Experiment and numerical studies show significant effects of the wavy grade and wave cell layout and arrangement on mechanical properties, stability, and enhanced energy absorption/dissipation capacities. Superior material characteristics, like programmable stiffness, stability enhancement, and obvious inner friction, can be realized by adjusting the design parameters and tuning the pre-load of the wavy metamaterials. The key findings are summarized as follows.

- Stability enhancement in common metamaterials can be achieved by embedding wavy unit cells in regions prone to instability.
- Introducing different curvatures leads to multi-stiffness unit cells, where the more curvature implies the lower the local stiffness. There is a reduction of 98 % in the stiffness of the structure "D" (wavy struts

with large curvature) compared to "A" (straight struts) (a decrease in structural stiffness from 107.58 $\frac{N}{mm}$ to 1.3367 $\frac{N}{mm}$).

- The more curvature results in earlier densification and higher energy absorption capacity, making wavy metamaterials promising for energy absorption applications. The energy absorption of contact-based wavy metamaterial, model "C-corrugated-2", increases by up to 86 % compared to "A", conventional re-entrant structure, (The SEA increases from 0.39306 J/g to 0.73115 J/g), and increases by up to 31.89 % compared to the conventional aluminum honeycomb (The SEA changes from 47.97 J/g to 63.27 J/g).
- Simultaneous inclusion of low and high-stiffness unit cells generates multi-plateau regions in force-displacement relation, offering quasi-zero stiffness characteristics.
- To dissipate energy through the frictional motion of cell contact walls, introducing neighbor curves with different curvatures is necessary, highlighting the importance of variation in curvature within one unit cell. This leads to an increase in the ratio of friction/plastic dissipation energy from 0.012 J/g (common auxetic design) to 0.2511 J/g (wavy tips design).
- Wavy metamaterials exhibit shape memory features (fully recoverable behavior) through heating-cooling processes, indicating their reusability without damage or fracture, ensuring their practicality and durability under cyclic loading-unloading conditions.

CRediT authorship contribution statement

Ramin Hamzehei: Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Nan Wu:** Writing – review & editing, Supervision, Methodology, Conceptualization. **Mahdi Bodaghi:** Writing – review & editing, Supervision, Software, Methodology, Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.engstruct.2024.119538.

Data availability

Data will be made available on request.

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