Computational design of a large dataset of viscoelastic meta-structures for inverse design in low-frequency vibration attenuation

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ABSTRACT

In the context of acoustic stealth, the attenuation of low-frequency vibrations generated by submarine rotating machinery remains a major challenge. Existing passive and active solutions still have limitations to effectively mitigate vibrations across varying operating conditions. The research work introduces a computational design approach that integrates generative design and finite element analysis to build a comprehensive dataset of viscoelastic meta-structures, characterized by their transmissibility. This dataset will serve as a foundation for future machine learning-enabled inverse design, paving the way for optimized vibration attenuation strategies.

Keywords: Vibration attenuation, Viscoelasticity, Generative Design, Meta-Structure, Finite element analysis

1. INTRODUCTION

Radiated noise poses a significant threat to submarine stealth, as it can be detected by passive sonars, potentially compromising the detection, localization, and identification of the submarine. Despite continuous advancements, mitigating this noise remains a major challenges for engineers. Previous research has primarily focused on attenuating noise from external sources, such as the propeller[1] and the hydro/aerodynamic interactions between the hull and surrounding fluid.[2] However, at low speeds, it has been shown that the main radiated noise source is the primary source of radiated noise originates internally, specifically from submarine machinery.[3] Several noise absorption technologies have been developed for marine power systems and comprehensively reviewed by Qiu et al.[4] These include locally resonant structures (LRS),[5] distributed dynamic vibration absorbers (DDVA),[6] particle damping (PD),[7] quasi-zero stiffness (QZS) meta-structures,[8] and active vibration control (AVC) systems.[9]

In the context of submarines, engineers are particularly interested in lightweight vibration-absorbing technologies that can serve as suspension systems, positioned between the rotating machinery and its cradle. This leads to the exploration of mechanical metamaterials, which have demonstrated exceptional potential over the past decade due to their lightweight architecture, high energy absorption and vibration isolation performance (VIP). As an example, a phononic crystal with the same surface area but featuring a graded internal structure exhibits varying VIP.[10] This highlights the importance of metamaterial design. Similarly, gradient effects in layered structures have demonstrated significant VIP at low frequencies.[11] Sandwich structures also represent promising candidates for VIP, as shown by Zhang et al.,[12] where they function as LRSs, as demonstrated by Li et al.[13] Nature has also inspired the development of original structures with exceptional VIP at low frequencies.[14] Many of these bio-inspired designs draw inspiration from biological structures, such as animal paws or human legs, while others mimic the intricate geometry of spider webs, as explored by Ruan and Li.[15] Additionally,

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metamaterials exhibiting negative Poisson's ratios, have been investigated for VIP potential. For example, Gao et al.[16] developed a PD metamaterial coupled with a re-entrant auxetic structure, achieving enhanced VIP at low frequencies.

However, optimizing metamaterial design requires more than geometric manipulation alone. To fully explore new possibilities, generative design and machine learning becomes essential for leveraging the solution space and solving inverse design problems. This research strategy necessitates the construction of a dataset containing geometries and their corresponding mechanical performances. In this context, geometric parameters serve as the primary input variables.[17] Built on this, this work presents a computational design approach that integrates generative design for creating meta-structures based on geometric parameters and finite element simulation for obtaining their transmissibility. The proposed computation is then verified with experimental tests.

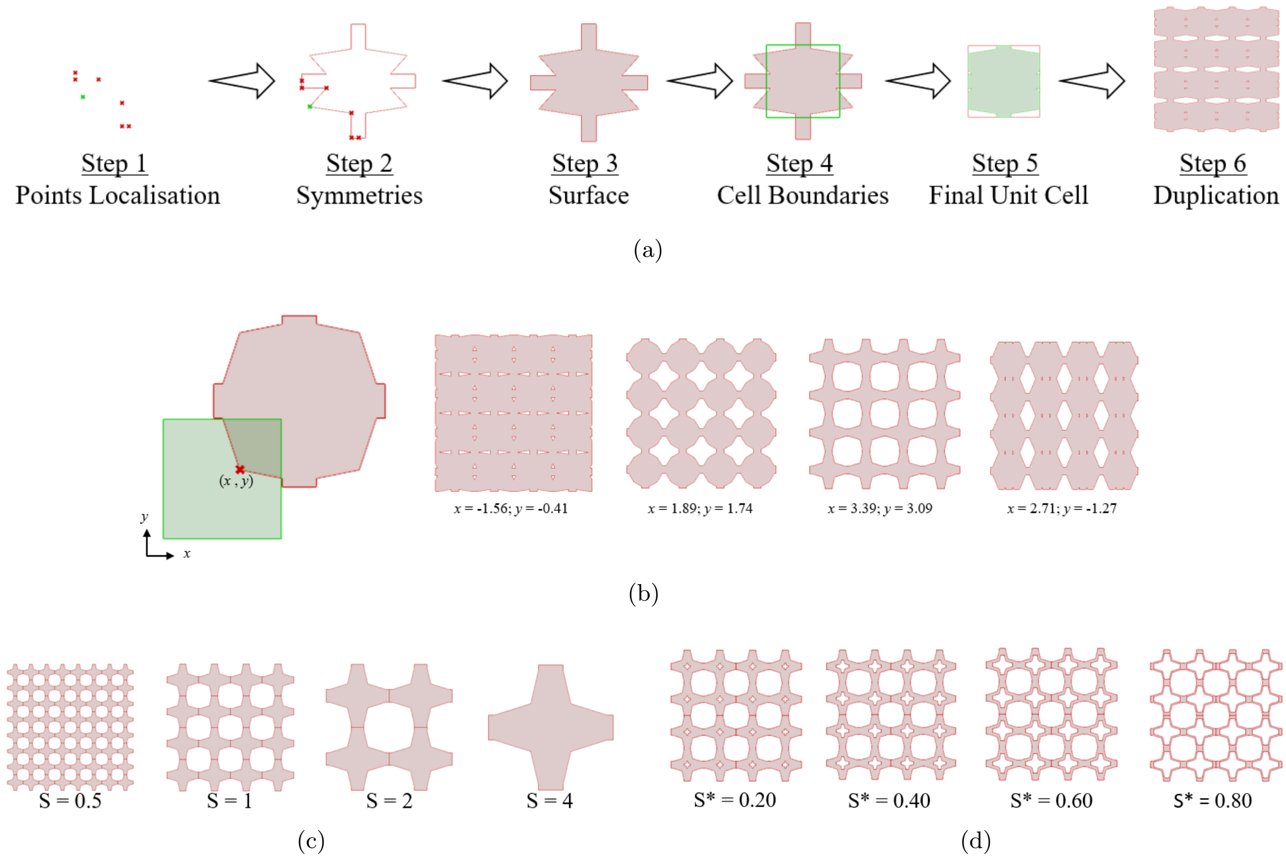


Figure 1: Generative design: (a) meta-structure creation steps; (b) various configurations of meta-structures for different geometric parameters; (c) number of unit cells; and (d) lightness.

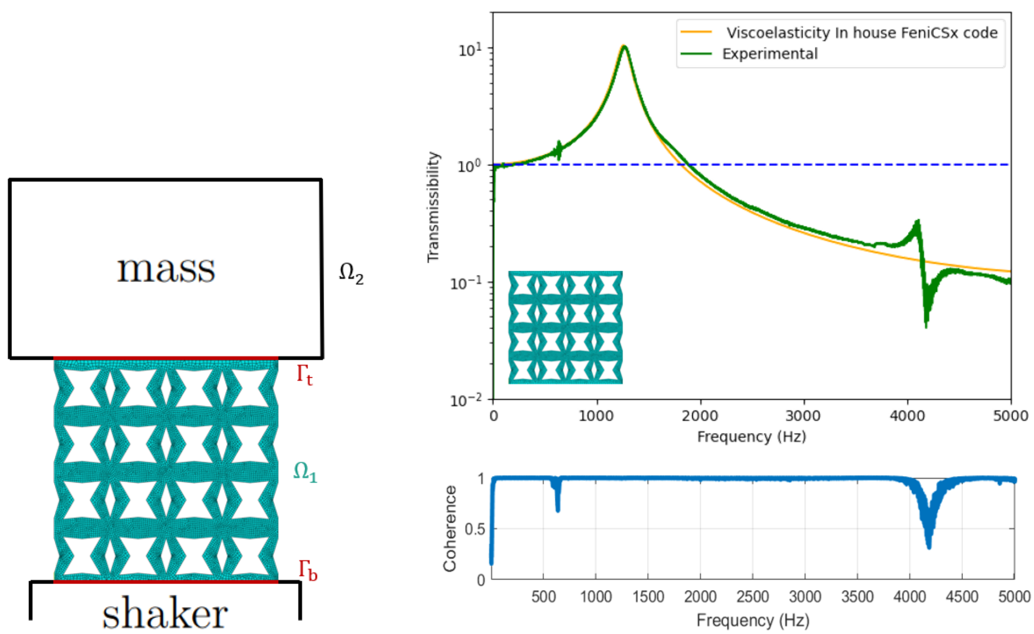
2. GENERATIVE DESIGN OF META-STRUCTURES

To develop meta-structures for inverse design problems, generative design approach was employed and implemented within the commercial software Rhinoceros 7 and its Grasshopper plugin. To generate the designs and their corresponding geometric variables, we used the following geometric parametrization:

- Point coordinates: The meta-structure is designed as periodic 2D lattice. As depicted in Fig. 1a, due to the periodicity of the structure, only a single unit cell needs be defined, which is then duplicated to form the complete lattice. Each unit cell is initially contained within a 1 cm^2 square, but its actual dimensions can exceed this boundary, allowing for a broader range of design options. Fig. 1b show various lattice structures generated. Some configurations appear fully dense, while other contain significant voids. By using an incremental steps of 0.1 for each geometric parameter, we were able to generate over 3,000 unique lattice structures.

- Number of cells: The meta-structure was designed within a 16 cm^2 square domain, initially composed of 16 unit cells. To introduce further variability, a scaling parameter S was defined, allowing for different cell arrangements. As shown in Fig. 1c, the meta-structure can contain 64, 16, 4, or a single unit cell, significantly altering its mechanical properties.
- Lightness: To control the structure's mass distribution, a scaling transformation was applied to the unit cell. This transformation, denoted S^*, consists of scaling the unit cell and then performing a subtraction operation between the original and scaled versions. This parameter can take 61 discrete values, and its effect on the overall structure is illustrated in Fig. 1d.

With just the point coordinates variations, the design space exceeds 3,000 possibilities. Incorporating the number of unit cells increases this over 10,000 configurations. Finally, introducing the lightness variation parameter expands the design space to approximately 850,000 unique structures. While this extensive set of meta-structures provides a promising foundation for constructing the envisioned dataset, performance metrics must also be associated with each geometry to enable meaningful analysis.



(a) (b)

Figure 2: (a) Study configuration and (b) simulation-experiments comparison.

3. MODELING AND NUMERICAL SIMULATION

To investigate the mechanical performances of a designed meta-structure acting as a suspension system, we consider the simple mounting system illustrated in Fig. 2a. An item of mass is supported by the meta-structure, and a sinusoidally varying ground displacement excites the system. The transmissibility $T(\omega)$ across the system is then defined, at a given angular frequency ω, as the magnitude of the displacement ratio (mounting mass over the ground displacements). Several regimes are generally observed:

- $T(\omega) \approx 1$: the mounting item oscillates similarly to the ground. There is no amplification or attenuation. This typically happens at low frequencies.

- $T(\omega) > 1$: the oscillations of the mounting item is larger (in amplitude) than the ones of the ground. There is amplification, which might be important (a phenomenon known as resonance).
- $T(\omega) < 1$: there is vibration attenuation. This is a desirable case, and we are looking for meta-structures that maximize this configuration over the frequency range of interest.

Modeling. Regarding the modeling of such a mounting system, we consider the mounting item as a rigid body of mass m and moment of inertia tensor $\mathbf{I}$. The meta-structure is considered as a deformable body made of a viscoelastic material defined by its density ρ, its storage modulus E, its Poisson's ratio ν, and its loss factor η. We consider a perfect coupling between the mounting mass and the meta-structure. Finally, we denote by $\boldsymbol{u}_1 : (\Omega_1, \mathbb{R}_+) \to \mathbb{C}^d$ the displacement field within the meta-structure and, respectively, by $\boldsymbol{u}_2 : \mathbb{R}_+ \to \mathbb{C}^d$, and $\boldsymbol{\theta}_2 : \mathbb{R}_+ \to \mathbb{C}^{d'}$ the displacements and rotations of the mounting item (expressed at the center of mass). These fields are solutions of the following dynamic problem:

$$\text{[Linear momentum balance]} \quad \rho \ddot{\boldsymbol{u}}_1(\mathrm{x}, t) = \mathrm{div}(\boldsymbol{\sigma}_1(\mathrm{x}, t)), \quad \text{in} \quad \Omega_1, \tag{1a}$$

$$\text{[Constitutive law]} \quad \boldsymbol{\sigma}_1(\mathrm{x}, t) = (1 + i\eta)\mathbf{C} : \boldsymbol{\varepsilon}(\boldsymbol{u}_1(\mathrm{x}, t)), \quad \text{in} \quad \Omega_1, \tag{1b}$$

$$\text{[Euler's first law]} \quad m \ddot{\boldsymbol{u}}_2(t) = -\int_{\Gamma_t} \boldsymbol{\sigma}_1 \cdot \boldsymbol{n} \, dl, \tag{1c}$$

$$\text{[Euler's second law]} \quad \mathbf{I} \ddot{\boldsymbol{\theta}}_2(t) = -\int_{\Gamma_t} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{n}) \times \boldsymbol{r} \, dl, \tag{1d}$$

$$\text{[Continuity of the disp.]} \quad \boldsymbol{u}_1(\mathrm{x}, t) = \boldsymbol{u}_2(t) + \boldsymbol{\theta}_2(t) \times \boldsymbol{r}(\mathrm{x}), \quad \text{over} \quad \Gamma_t, \tag{1e}$$

$$\text{[Imposed harmonic disp.]} \quad \boldsymbol{u}_1(\mathrm{x}, t) = u_{\mathrm{in}} \exp(i\omega t)\mathbf{e}_y, \quad \text{over} \quad \Gamma_b, \tag{1f}$$

where $\boldsymbol{\varepsilon}$ denotes the linearized Green-Lagrange strain tensor, $\boldsymbol{\sigma}$ the linearized Cauchy stress tensor, $\mathbf{C}$ the Hooke material tensor, and $\boldsymbol{n}$ the outward unit normal to Ω_1.

Numerical resolution. The problem given by Eq. (1) can be solved using the finite element (FE) method. More specifically, we employ the popular open-source computing platform FEniCS[18] together with the open-source FE mesh generator GMSH[19] to build an in-house python package. It accepts STEP files as inputs, and computes the transmissibility (among others) automatically.

Results and validation. In order to validate the numerical model prior to generating a large dataset, we conduct experiments and compare the experimental results with the numerical ones, as depicted in Fig. 2b. More specifically, we printed several meta-structures using PolyJet technique with the Stratasys Objet260 Connex3 printer. VeroClear™ (Stratasys) polymer material has been used, whose properties have been identified as $E = 1.2\,\mathrm{GPa}$, $\nu = 0.312$, $\rho = 1.18\,\mathrm{g\,cm^{-3}}$, and $\eta = 0.1$. The meta-structure has side lengths of 4 cm, while the aluminum mass has a length and width of 5.4 cm and a height of 3.6 cm. We observe good agreements between the experimental and numerical results as shown in Fig. 2b. Note that the pics around 4 kHz for the experimental results are biased as the coherence is low in this frequency range (*i.e.* data cannot be trusted there).

4. CONCLUSION

The proposed computational design approach, implemented using Grasshopper and FEniCSx, has enabled the generation of over 800,000 unique meta-structures. Incorporating additional parameters, such as rotation, would further expand the design space beyond a billion possibilities. Given this scale, traditional analysis methods become impractical, necessitating the use of machine learning to efficiently explore and optimize the design. Moreover, by automatizing and validating the numerical simulation framework, a comprehensive dataset can now be generated, therefore paving the way for addressing inverse design problems in a scalable manner.

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REFERENCES

[1] Özden, M., Gürkan, Y., Özden, A., Canyourt, T., and Korkut, E., "Underwater radiated noise prediction for a submarine propeller in different flow conditions," *Ocean Engineering* (2016).

[2] Huo, L. and Fei, S., "Design on the fairwater shape and his influence on the radiation noise of submarines," *Journal of Vibroengineering* (2016).

[3] Ginoux, J.-M. and Bertinier, M., "La maîtrise de la discrétion acoustique dans un programme de navire," *Acoustique & techniques* (2011).

[4] Qiu, Y., Xu, W., Hu, Z., Fu, J., and He, M., "Review of lightweight vibration isolation technologies for marine power devices.," *Shock and Vibrations* (2024).

[5] Ruan, Y., Liang, X., Hua, X., Zhang, C., Xia, H., and Li, C., "Isolating low-frequency vibration from power systems on a ship using spiral phononic crystals," *Ocean Engineering* (2024).

[6] Li, L., Li, B., and Xu, Z., "Design of distributed dynamic absorbers for vibration suppression of panel structures," *Acta Mechanica Sinica* (2024).

[7] Wang, C., Xiao, W., Wu, D., Wang, S., Lin, C., Luo, Y., Xiao, J., Yao, K., and Xu, Z., "Study on bandgap characteristics of particle damping phononic crystal," *Applied Acoustics* (2020).

[8] Yang, Z., Wang, Y., Huang, Z., and Rao, Z., "Characteristic analysis of a new high static low dynamic stiffness vibration isolator based on the buckling circular plate," *Journal of Low Frequency Noise, Vibration and Active Control* (2021).

[9] Yang, T., Wu, L., Li, X., Zhu, M., Brennan, M., and Liu, Z., "Active vibration isolation of a diesel generator in a small marine vessel: An experimental study," *Applied Sciences* (2020).

[10] Li, X., Cheng, S., Yang, H., Yan, Q., Wang, B., Xin, Y., Sun, Y., Ding, Q., Yan, H., and Li, Y., "Analysis of low frequency vibration attenuation and wave propagation mechanism of graded maze structure," *Physica B: Condensed Matter* (2023).

[11] Li, S., Yang, J.-S., Yang, F., Schmidt, R., Fan, Y.-L., Wu, L.-Z., and Schröder, "Fabrication and vibration isolation capacity of multilayer gradient metallic lattice sandwich panels," *Mechanical Systems and Signal Processing* (2022).

[12] Zhang, K., Ding, B., Hong, F., Liu, H., Yu, J., and Deng, Z., "A compressible square-frustum sandwich structure for low-frequency vibration suppression: Numerical and experimental analyses," *Thin-Walled Structures* (2023).

[13] Li, H., Hu, Y., Huang, H., Chen, J., Zhao, M., and Li, B., "Broadband low-frequency vibration attenuation in 3d printed composite meta-lattice sandwich structures," *Composites Part B* (2021).

[14] Yan, G., Zou, H.-X., Wang, S., Zhao, L.-C., Wu, Z.-Y., and Zhang, W.-M., "Bio-inspired vibration isolation: Methodology and design," *Applied Mechanics Reviews* (2021).

[15] Ruan, H. and Li, D., "Band gap characteristics of bionic acoustic metamaterials based on spider web," *Engineering Structures* (2024).

[16] Gao, X., Wei, J., Huo, J., Wan, Z., and Li, Y., "The vibration isolation design of a re-entrant negative poisson's ratio metamaterial," *Applied Sciences* (2023).

[17] Deng, B., Zareei, A., Ding, X., Weaver, J., Rycroft, C., and Bertoldi, K., "Inverse design of mechanical metamaterials with target nonlinear response via a neural accelerated evolution strategy," *Advanced Materials* (2022).

[18] Baratta, I. A., Dean, J. P., Dokken, J. S., Habera, M., Hale, J. S., Richardson, C. N., Rognes, M. E., Scroggs, M. W., Sime, N., and Wells, G. N., "DOLFINx: The next generation FEniCS problem solving environment," (Dec. 2023).

[19] Geuzaine, C. and Remacle, J., "Gmsh: A 3-D finite element mesh generator with built-in pre- and post-processing facilities," **79**(11), 1309–1331.