

**Exploring a Dialectic Professional Development programme through the
experiences of secondary mathematics teachers: A Reflexive Thematic Analysis**

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Data access statement

The data supporting the findings reported in this paper are openly available from the Zenodo repository at [10.5281/zenodo.14982761](https://doi.org/10.5281/zenodo.14982761).

Abstract

Teacher professional development contributes to shaping an education system and to a nation's economic competitiveness. However, the desire for conformity in professional development risks leaving some teachers feeling pedagogically confused, disappointed in the compromises they make to their beliefs, and frustrated at the practices they are asked to adopt.

In this study I suggest an emerging specification for mathematics Dialectic Professional Development (DPD) which is characterised by the philosophical concept of contradiction. I design an example mathematics DPD programme and interpret it through the lens of contradiction using a qualitative case study approach. In seeking to gain greater insight into mathematics DPD, I draw on insights from three secondary school mathematics teachers in the East Midlands of England who participated in my programme over one academic year. These insights, in the form of completed programme documents and semi-structured interviews are then interpreted through Reflexive Thematic Analysis.

As a result of my study, I suggest contradiction can be used to interpret beliefs about mathematics education as interdependent, moments of variation as acts of self-sabotage, and belief trajectories as showing journeys not destinations. These themes suggest beliefs about mathematics education are contradictory, professional development programmes should include moments of variation, and that teachers determine the direction of their professional learning. I conclude that mathematics DPD offers a structured approach for mathematics teachers to not only develop new practices but to also understand their response to these new practices.

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Chapter 1.

Introduction

1.1 Synopsis of Chapter 1

In this thesis I provide a new application of theory to make an interpretation of an original mathematics professional development programme I have designed. In this chapter, I provide a discussion of the educational context that has led me to design the professional development programme, the theory I use, and the research question I seek to answer. I begin by setting my personal experiences within the wider educational contexts of mathematics teaching and mathematics professional development. Furthermore, I discuss an influential professional development experience which ultimately contributed to the development of my work. I then move on to explain the philosophical perspective of Hegelian contradiction which I adopt throughout this thesis. Following a brief discussion of the scope of education, I provide my interpretation of teacher beliefs and professional development. These two concepts are further explored in Chapters 2 and 3 respectively, but an introduction here helps provide an understanding of the direction of my work. Therefore, drawing on my professional experiences and my philosophical perspective, I introduce my research question and explain how I seek to answer it in the organisation of this thesis.

1.2 The context of mathematics education in England

I summarise my perspective on mathematics education in England by looking at the overlapping contexts of schools and teacher professional development. These

reflections are intended to provide insight into the evolution of my thinking and offer an understanding of my position as a researcher. Furthermore, in my discussion of professional development, I place an emphasis on my involvement with the Maths-for-Life programme (Heal 2021, Woodford and Wake 2023) which has influenced my thinking and ultimately led to my desire to carry out this study.

1.2.1. The mathematics education context of schools in England

Wright (2012) suggests some of the tensions in mathematics education in England can be understood by referencing the Math Wars in the United States. Adopting this approach, I therefore broadly recognise the existence of the competing factions of the traditionalists advocating a conservative teaching style, and the reformers advocating a more progressive style (Schoenfeld 2004). For this initial discussion, I summarise a conservative approach as ‘characterised by an authoritarian, transmission model of teaching’ (Wright 2012, p. 7), and a progressive approach as ‘typified by the promotion of practical activities and personal exploration’ (Wright 2012, p. 8).

From progressive toward conservative

When I first became a mathematics teacher in 2002, I was highly influenced by the Cognitive Acceleration in Mathematics Education (CAME) Thinking Maths resources (Shayer and Adhami 2007). At the time it felt like a natural way to teach, and I can now see the focus on a realistic problem, a period of small group collaborative learning, and a period of whole class discussion (Shayer and Adhami 2007) would have positioned me toward the progressive camp. However, over the intervening years both my approach, and the prevailing approach in classrooms in England, gradually began to move toward a more

conservative application. The ‘resurgence of teacher-led pedagogies’ (Wright, Fejzo and Carvalho 2022, p. 28) could be seen in classrooms with a turn toward ‘bite-sized learning objectives and a tendency for mathematics teachers to path-smooth’ (Foster 2013, p. 563).

The reasons for movement toward conservative approaches are complex, but it is suggested influential factors included the development of an assessment focused curriculum, the de-professionalisation of teachers (Foster 2013), and teachers acquiescing toward policies that saw them be successful in school (Wright 2012). Moreover, recent government education policy in England has been focused on the creation of a mathematics mastery market (Boylan and Adams 2024). This mastery market includes organisations such as the Ark multi-academy trust and their Mathematics Mastery programme, along with the National Centre for Excellence in the Teaching of Mathematics (NCETM) and their Teaching for Mastery programme (Boylan and Adams 2024). Echoing a more conservative approach, the Mathematics Mastery programme is said to rely on:

the premise that learning is most effective when concepts are presented by teachers in a highly structured and unambiguous way. Examples are carefully chosen and sequenced, with minimal conceptual steps, to enable learners to draw correct inferences and to avoid cognitive overload (Wright, Fejzo and Carvalho 2022, p. 28).

In 2014, I became a regional lead of the National Centre for Excellence in the Teaching of Mathematics (NCETM) Mathematics Hubs programme (Lewis and Maisuria 2023). In this role I had the opportunity to influence classroom approaches through the principles of Teaching for Mastery and the ‘five big ideas’ (NCETM 2023). Attempting to classify Teaching for Mastery as either a conservative or progressive approach toward mathematics education is not simple. For example, one of the NCETM’s five big ideas

reflects a more conservative stance in encouraging curriculum coherence through ‘small, connected steps’ (NCETM 2024). However, I also recognise the big idea of mathematical thinking could be understood as a more progressive approach in encouraging the development of reasoning in students.

From conservative back toward progressive

Despite being unable to classify the Mathematics Hubs programme easily, it did lead me to gradually question the big ideas of Teaching for Mastery, and to question their use in classrooms. Though I had unconsciously moved toward conservative approaches over my career, Teaching for Mastery led me back to a more conscious support for progressive approaches. I think my personal experience of movement between progressive and conservative is typical of many other mathematics teachers. In my experience, mathematics teachers would rarely describe themselves as being either progressive or conservative, and their classroom practice tends to be a mix of many beliefs. Understanding progressive and conservative as two ends of a spectrum feels more appropriate, and I suggest many teachers continuously move between these ends over time as they engage with professional development. However, I recognise that alongside a growth in the mathematics mastery market in England there has been a resurgence of more conservative leaning approaches. Nevertheless, I would describe very few teachers as being purely conservative and instead see many as blending different beliefs and approaches.

1.2.2 The mathematics educational context of professional development in England

Whilst I suggest both Mathematics Mastery and Teaching for Mastery demonstrate a growth in conservative leaning professional development programmes, they exist alongside many progressive professional development programmes. In 2018, I left my Mathematics Hub role for an opportunity to work on developing a more progressive approach to mathematics education through the Maths-for-Life programme (Heal 2021, Woodford and Wake 2023). This programme was funded by the Education Endowment Foundation to support students in England resitting their national mathematics exam. It built on work by Swain and Swan (2007) and involved the development of new lesson resources and a professional development programme to be evaluated through a large scale randomised controlled trial. The thinking of Malcolm Swan has been highly influential to me and will be seen throughout this thesis in my initial views of teacher beliefs about mathematics education, and my initial views of teacher professional development.

Development of the Maths-for-Life programme

The Maths-for-Life programme included a modified form of lesson study intended to examine five pedagogic principles of collaborative learning, formative assessment, cognitive conflict, closure, and models of structure (Woodford and Wake 2023). For instance, we created a new lesson to facilitate teacher learning, and encourage student thinking, using matching cards (Woodford and Wake 2023). Within a realistic context, groups of students would be asked to place cards to describe scenarios using both ratio notation and fraction notation (the principle of collaborative learning). By presenting the ratios in the form part to part, and the fractions in the form part to whole, it was anticipated

opportunities for discussion would be afforded and misconceptions revealed (the principle of cognitive conflict). Once students had placed their cards, teachers were asked to introduce a further set of cards to each group displaying a bar model diagram (the principle of representations). Finally, teachers were asked to avoid intervening in student discussions too early and encouraged to facilitate student-to-student discussion (the principle of formative assessment) before clarifying learning at the end of the lesson (the principle of closure).

A critical incident in the Maths-for-Life programme

As part of the modified lesson study approach that was adopted, I had the privilege of watching the ratio-fraction lesson being taught with a class of post-16 students. I watched three students working together to place the cards and saw many of the anticipated discussions and anticipated misconceptions take place. At the point when it would have been helpful to introduce the set of bar model diagram cards, the class teacher came over to look at the students' progress. At this moment the teacher told the students their work was completely wrong and wiped out their card placements with a wave of the arm. The teacher followed this by explaining a stepwise procedure detailing exactly how the students should work out which ratio cards matched with which fraction cards. The class teacher acted in a way which felt like the opposite of what the whole professional development programme had been encouraging. In the post-lesson discussion, the class teacher reflected on this moment by explaining they felt it was simply the correct thing to do at the time. Exhibiting what I have already characterised as a more conservative

approach the teacher suggested the students had reached cognitive overload and would benefit from a small-step explanation.

As a result of this incident I wondered whether there was a need to create even clearer lesson plans and to dictate teacher actions even more precisely. The irony here was that I became tempted to use more conservative approaches to encourage teachers to adopt progressive approaches. Instead, I started to realise the importance of giving teachers ‘time to reflect on their new experiences’ (Swan and Swain 2010a, p. 168) since change is complicated. I now see the teacher’s intervention with the students as neither a success nor a failure, but as part of their journey. My involvement in the Maths-for-Life programme focused my attention on two elements of the teacher’s journey. I felt I had an insufficient understanding of teachers’ reactions to the programme, and an insufficient understanding of how teachers held a blend of beliefs about mathematics education. My challenge has been to recognise that if I believe in a more progressive approach to education, then I have a responsibility to improve my understanding of a teacher’s journey in a professional development experience.

1.3 Exploring my philosophical perspective

My approach to gaining deeper insight into the journey of a teacher has been influenced by the philosophical perspective I adopt in this study. I have been drawn to an interpretation of Hegel offered by McGowan (2019) and his claim that contradiction is Hegel’s ‘great ontological insight’ (McGowan 2019, p. 93). The precise meaning of contradiction, and how it is an ontological feature, requires further explanation. However, at this point, I highlight the importance of this philosophical perspective since it underpins

my entire thesis. It has shaped my conception of the pursuit of knowledge, led to a reinterpretation of the concepts I use, shaped the design of the professional development materials I created, and influenced my interpretation of collected data. Therefore, in this section I seek to summarise McGowan's (2019) thinking and introduce some of the applications to my work which are further explored in future chapters.

1.3.1 Hegelian dialectics as an epistemology

I begin with an overview of McGowan's (2019) explanation of the epistemology of Hegelian dialectics since this later underpins the theory of learning I utilise in my professional development programme. McGowan (2019) provides an explanation of Hegelian dialectics through a contrast with a more traditional interpretation of it as process of determinate negation. Singer (2001) summarises the traditional interpretation of determinate negation as the identification of an initial position (the thesis), an opposite position (the antithesis), and the subsequent creation of a new position (the synthesis). I note this traditional interpretation of Hegel has had some influence on education, such as in Stojanov's (2018) educational philosophy, Williams and Ryan's (2014) design of professional development, and MacCarthy's (2021) approach to thematic analysis. I return to consider each of these areas within this thesis by relating them to the explanation of Hegelian dialectics offered by McGowan (2019). Whilst the simplicity of determinate negation is alluring, I see it as an inaccurate rendering of Hegel which more usefully serves as a frame to understand the true trajectory of his work (McGowan 2019). In line with McGowan (2019), I see the traditional formulation as providing an opportunity to understand the true nature of a thesis, as revealing the redundancy of an external antithesis, and as highlighting reconciliation rather than synthesis.

The true nature of a thesis

The traditional formulation of determinate negation relies on the view that a contradiction is revealed through the contrast between a thesis and an antithesis. Initially, a contradiction can be understood as the contravention of the Aristotelian law of noncontradiction (Aristotle 2001) in that something cannot be both true and not true at the same time. In contrast, McGowan (2019) argues the contradiction is not external to the thesis but is inherent to its conception. Accordingly, seeking to apply the law of noncontradiction to any ‘position’ (McGowan 2019, p. 12) leads to the revelation of its own contradiction. McGowan (2019) illustrates this with the position of rejecting the certainty of all knowledge. In such a case, a contradiction is revealed in the claim of ‘universal certainty about the lack of certainty’ (McGowan 2019, p. 19). The position contravenes the law of noncontradiction in that for it to hold it must simultaneously be both true and not-true. It is McGowan’s (2019) contention that *The Phenomenology of Spirit* by Hegel (2018) consists of the examination of a sequence of positions with the inherent contradiction revealed in each one. Therefore, the word contradiction moves from simply signifying the contravention of a law, to describing a thesis as a position at ‘odds with itself’ (McGowan 2019, p. 12), ‘simultaneously negating itself’ (McGowan 2019, p. 12), and containing a ‘self-betrayal’ (McGowan 2019, p. 28).

The redundancy of an external antithesis

If a contradiction can be revealed through the self-betrayal of the original thesis, then an antithetical position becomes redundant. Indeed, as McGowan (2019) argues, creating an antithesis risks allowing contradiction to be hidden through the creation of

opposition. This is an important distinction since the issue is not denying the existence of seemingly antithetical positions, but the risk they pose in facilitating the avoidance of internal contradiction. Woodford, Clapham and Serret (2023) provide the example of a mathematics teacher who created an opponent out of the school leadership team for championing antithetical pedagogical approaches. In doing this the teacher was able to ignore any contradictions in their own pedagogical beliefs and instead directed their frustrations into anger with the school leadership team (Woodford, Clapham and Serret 2023). I suggest the antithetical pedagogy is not the problem, but that the response to it enabled the teacher to avoid considering their own pedagogy. Creating an antithetical opponent is unnecessary and risks disguising the existence of contradiction in favour of difference (McGowan 2019).

Reconciliation rather than synthesis

In McGowan's (2019) construction of the dialectic process, the antithesis is considered redundant and therefore it is no longer possible to create a new position through synthesis. Instead, McGowan (2019, p. 14) suggests what has become understood as synthesis is better interpreted as reconciliation to contradiction. Reconciliation takes place when the existence of contradiction is accepted, it no longer poses 'enough of a problem to catch thought's interest' (McGowan 2019, p. 18), and one moves on to explore a new position. The previous example of the existence of contradiction when rejecting the certainty of any knowledge cannot be resolved. An individual would simply have to accept the contradiction exists and move on to examine a new 'more appealing' (McGowan 2019, p. 18) position. Eventually this new position would also reveal a contradiction, and so the

dialectic process continues. McGowan (2019) argues movement of thought does not follow from resolving a contradiction, nor from creating a new idea through synthesis. Instead, movement of thought occurs when an individual becomes reconciled to the un-resolvability of contradiction.

Summarising dialectics as an epistemology

I summarise McGowan's (2019) construction of Hegelian epistemological dialectics, not as a three-part formula, but as an experience which reveals contradiction. Moreover, contradiction is understood as 'the inability of anything to be identical with itself' (McGowan 2019, p. 16). A similar sense can be seen in Engley's (2023, p. 747) definition of dialectic thinking as seeing 'the interdependence of things on their own internal opposition'. For McGowan (2019, p. 14) this epistemological process becomes the 'driving force of all movement' as an individual moves from one contradictory position to explore a new contradictory position.¹ Whilst some have suggested thinking advances when a contradiction is solved (Pouwels and Biesta 2017), McGowan (2019) argues advancement occurs when an individual becomes reconciled to the impossibility of solving the contradiction. I return to this understanding of the epistemology of Hegelian dialectics and contradiction in my discussion of a learning theory underpinning a new form of professional development in Chapter 3.

¹ McGowan (2019, p. 18) recognises Hegel 'did not directly articulate this as his project', but argues it is an inescapable truth illustrated throughout the trajectory of his work.

1.3.2 The ontological insight of Hegelian dialectics

In addition to contributing a better understanding of Hegelian dialectics, McGowan (2019, p. 93) also claims contradiction is Hegel's 'great ontological insight'. Some have argued ontology should supersede epistemology (Guba and Lincoln 1994), but this position is not universal. For instance, Žižek (2012, p. 17) reverses the order when he states, 'what appears to us our inability to know the thing indicates a crack in the thing itself'. Similarly, McGowan (2019, p. 89) suggests our ability to think contradiction 'must have its condition of possibility in what is'. Repeated reconciliation *to* contradiction eventually leads to reconciliation *with* contradiction. As an individual realises 'contradiction is unsurpassable' (McGowan 2019, p. 21), they are left with the ontological insight that 'everything is also what it is not and has its identity in what negates it' (McGowan 2019, p. 85). For McGowan (2019) this feature of contradiction can be seen in everything from the natural world to human nature², and from logic to claims about God. I believe I have a responsibility to take this ontological claim just as seriously and so seek to understand how it can contribute to my understanding of the experience of a mathematics teacher professional development programme.

1.4 The implications of my philosophical perspective

The implication of seeing contradiction as an ontological feature has affected my initial understanding of concepts central to this work. I begin with a brief explanation of a contradiction informed perspective of education. From this, I provide my interpretation of the concepts of teacher beliefs and professional development through a contradiction

² Hegel (2018, p. 12) articulates this as 'everything hangs on grasping and expressing the true not just as *substance* but just as much as *subject*'.

informed lens. These elements are discussed in greater detail in subsequent chapters but are mentioned here to provide an initial understanding of my research question.

Furthermore, an explanation of how I apply contradiction to the analysis of my research data takes place in Chapter 5.

1.4.1 A Hegelian perspective on education

In considering the scope of education, Biesta (2020) notes the German language draws on the two words of *Bildung* and *Erziehung* to describe education. Whilst recognising there is debate over the meaning of the German words, one approach is to suggest they indicate education involves the ‘work of the self’ (Biesta 2020, p. 1019), and support for this work respectively. Accordingly, Hegelian educational philosophers such as Stojanov (2018) have sought to develop an interpretation of the interplay between *Bildung* and *Erziehung*. I appreciate Stojanov’s (2018) suggestion that students need to learn to apply, and articulate, their beliefs in relation to taught knowledge. This aligns well with Biesta’s (2020, p. 1015) suggestion that education should be concerned with ‘the question of how the ‘I’ exists as ‘I’’. However, I depart from Stojanov (2018, p. 14) when he suggests this development should be supported through the taught process of a thesis-antithesis-synthesis interpretation of determinate negation. I align more closely with Biesta (2020, p. 1018) in suggesting ‘the “I” is not the outcome of a process of cultivation’. Instead, Biesta (2020, p. 1020) recognises *Erziehung* is more about arousing the desire of others to exist in the world by balancing ‘one’s own desires’ and those of others. Drawing on these two uses of the word education, Biesta (2020) broadens his claims to suggest the scope of education includes both a cultivation and an experience explanation. This provides a recognition of the need to cultivate ‘capacities and capabilities’ (Biesta 2020, p.

1015), supplemented by a consideration of how the self should respond to these new skills. I anticipate an understanding of Hegelian dialectics, founded in contradiction (McGowan 2019), may provide an approach to support the work of *Erziehung* in the positioning of the self.

Whilst I recognise I have provided a very brief interpretation of a Hegelian view of education, it highlights the opportunity for a different way of thinking. Indeed, I further apply this perspective to teacher education in the form of professional development throughout this work. However, I also see this interpretation of education as applying to the specific domain of mathematics education. For instance. I maintain teachers of mathematics have a responsibility to encourage students to apply mathematical knowledge to their values, and to reveal the inherent contradictions. Ernest (2021, p. 40) suggests the teaching of mathematics is problematic when it is presented as ‘neutral, value-free and bears no responsibility for any of its applications’. Instead, I believe students should be encouraged to consider society’s over-valuation of mathematics and to confront any ‘dark side’ (Ernest 2021, p. 29) of potential contradictions that exist.

1.4.2 A Hegelian perspective on two key concepts

In addition to affecting my underlying perspective around education, contradiction also informs my understanding of concepts used in this study. Here I introduce my initial contradiction informed interpretation of the concept of mathematics teacher beliefs, which I further discuss in Chapter 2. Similarly, I also introduce an initial contradiction informed interpretation of the concept of teacher professional development which is further explored in Chapter 3. Together these two concepts lead me to define my approach to professional

development in Chapter 3, and to exemplify this approach in Chapter 4 in my design of the Rethinking Approaches in Mathematics (RAM) programme.

Teacher's beliefs about mathematics education from a Hegelian perspective

Earlier in this chapter, I summarised conservative beliefs as focusing on a transmission model of teaching, and progressive beliefs as focusing on personal exploration. Swan (2006b) extended this categorisation of teachers' beliefs about mathematics education by using three mutually exclusive categories of Transmission, Discovery and Connectionist. This is explained further in Chapter 2, but a teacher might be placed in the Transmission category if they believed mathematics should be conveyed to students using a 'chalk and talk' (Swan 2006b, p. 59) approach. The three belief categories satisfy Aristotle's previously mentioned law of noncontradiction in that they cannot be both true and not-true at the same time. In traditional logic, a teacher categorised in the Transmission category could not be considered as not-Transmission at the same time. Whilst rarely being explicit, I suggest this logic of mutually exclusive classification underpins many approaches to categorising teachers' beliefs about mathematics education.

However, in this work I intend to apply McGowan's (2019, p 16) contradiction informed perspective of the 'inability of anything to be identical with itself', to beliefs about mathematics education. For me, a traditional belief statement only provides a partial revelation of the teacher's beliefs around mathematics education. Applying a contradiction informed perspective means I anticipate beliefs about mathematics education may be

understood in terms of identity and non-identity³, and as depending ‘on what negates it’ (McGowan 2019, p 7). Therefore, as an application of McGowan (2019), I anticipate a mathematics teacher categorised as Transmission may also reveal ways in which they could simultaneously be understood as not-Transmission. How these ideas of identity and non-identity, along with ideas of negation, can be interpreted in practice are developed in this thesis.

Mathematics teacher professional development from a Hegelian perspective

A traditional conception of teacher professional development is that it should bring about changes to teacher beliefs, changes to teacher practices, and changes to student outcomes (Guskey 2002, Boylan, Coldwell *et al.* 2018). Debate exists around the order in which these changes should take place, with Swan (2011) supporting the view that changes to teacher practice give rise to changes in student outcomes, leading to changes in teacher beliefs. Irrespective of the order, recent teacher professional development in England has been marked by a demand for fidelity (Strom and Viesca 2021) to facilitate the provision of causal explanations of impact on student outcomes (Gorard, See and Siddiqui 2017). From a McGowan (2019) informed perspective, presenting an approach to teaching, without providing opportunities to examine the inherent contradiction, has potential negative consequences. Without opportunities for reflection on contradiction, there is a risk contradiction is mistaken for difference and opponents are created on which to direct anger. Indeed, it has been argued teacher professional development without reflection on contradiction has contributed to some teachers feeling pedagogically confused,

³ Hegel (2010, p. 51) articulates this thought when he states an identity is really ‘the identity of the identity and non-identity’.

disappointed in the compromises they make to their beliefs, and angry at the practices they have been encouraged to adopt (Woodford, Clapham and Serret 2023).

According to a contradiction informed perspective, I believe teacher professional development should take care not to claim a noncontradictory approach to teaching. In fact, more than that, I suggest teacher professional development should provide opportunities to examine the contradictions inherent within the examined approach. It is through this process of revealing contradiction that I see the work of the self taking place and through which teacher beliefs about mathematics education can be formed. Furthermore, I suggest it is not conformity that should be the focus in professional development, but the noticing of departure from the anticipated. In line with McGowan (2019), I suggest a teacher's unconscious actions within a professional development programme may help to reveal their true desires since 'what one thinks or claims about oneself falls aside in the face of what one does' (McGowan 2019, p. 43).⁴ Therefore, I anticipate actions, where teachers react to the programme or make changes, as being noteworthy features of teacher professional development.

1.5 Answering my research question

Having introduced a contradiction-influenced perspective of the concepts of teacher beliefs and professional development I now map out the remainder of this thesis. I

⁴ Hegel (1892, p. 256) articulates this thought when stating 'it is true that a man is nothing but the series of his actions'.

do this by explaining my research question, and then by explaining how each chapter contributes to answering the question.

1.5.1 Articulating my research question

I am interested in understanding how a contradiction informed philosophical perspective can be used to provide a theoretical explanation of a mathematics Dialectic Professional Development (DPD) programme. Mathematics DPD is founded on McGowan's (2019) interpretation of Hegelian contradiction and is further explained in Chapter 3. In addition, I provide details of my design of a specific case of mathematics DPD called the Rethinking Approaches in Mathematics (RAM) programme in Chapter 4. The theoretical explanation of RAM will be developed based on my interpretation of insights provided by participating teachers. Hence, I am not seeking to study participants, but to understand how contradiction provides insight into the design and experience of the programme. I then intend to relate these interpretations and explanations of RAM back to my understanding of mathematics DPD. Therefore, understood through McGowan's (2019) interpretation of Hegelian contradiction, I aim to answer the research question:

how can mathematics teachers' experiences of a Dialectic Professional Development programme be interpreted through the lens of contradiction?

1.5.2 Organisation of this thesis to answer my research question

To answer my research question in this thesis, I review key concepts in greater detail, explain my design of the RAM programme, explain the methodology I adopt, and

discuss my approach to analysis and interpretation of data. A summary of each chapter, and how they contribute to my thinking, is provided below.

Chapter 2: Teachers' beliefs about mathematics education

In the next chapter I review the conceptualisation and positioning of educational beliefs in relation to other linked constructs. From this I introduce the specific sub-category of teacher beliefs about mathematics education and examine a model developed by Swan (2006b, 2006a, 2014). Swan's model is utilised, and eventually re-interpreted through a lens of contradiction, in my analysis of teacher beliefs about mathematics education in Chapters 6 and 8.

Chapter 3: Professional development in mathematics education

Swan's work has not only influenced my thinking around beliefs about mathematics education, but also my initial approach to mathematics teacher professional development. In Chapter 3, I utilise a professional development framework to analyse a particular programme developed by Swan which I see as representative of his thinking. Using this as a starting point, and by drawing in other literature, I create an emerging definition of mathematics DPD. This emerging definition provides thinking in each of the six suggested elements of the professional development framework. Importantly, this includes a contradiction informed discussion of reconciliation to contradiction as a theory of learning, and of agents of change being understood in terms of unconscious actions in moments of variation. These ideas subsequently influence the design of the RAM

programme in Chapter 4 and as explained in Chapter 5, informs some of the data I subsequently prioritise for analysis.

Chapter 4: The Rethinking Approaches in Mathematics programme

In Chapter 4, I exemplify the emerging definition of mathematics DPD from the previous chapter in the form of the RAM programme. This one-year programme has been designed for this study to take place with a department of school mathematics teachers for students aged 11 to 16 with the intention of helping to answer my research question. Within the chapter I draw attention to the careful development of a mathematical problem and the subsequent modified form of lesson study I utilise within the programme. This chapter contributes an understanding of the anticipated experience of participants, and signposts potential moments of variation I take note of in my analysis of data.

Chapter 5: Methodology and research design

In the fifth chapter I explain the methodological approach of case study which I adopt to answer my research question. The case is defined as the one-year RAM programme as described in Chapter 4, and as experienced by three teachers in an 11 to 16 school mathematics department in the East Midlands of England. To facilitate an understanding of my interpretation, I provide contextual information about both the school and the three participating teachers. Furthermore, within this chapter I account for the data I use in this study, and my use of Reflexive Thematic Analysis (Braun and Clarke 2021b). Together, my data and analytic framework, contribute to the themes and interpretations described in Chapters 6, 7 and 8.

Chapters 6, 7 and 8: Analysis of themes

In Chapters 6, 7 and 8 I provide the results and discussion of my three themes which were developed from the analytic framework introduced in Chapter 5. Each chapter provides both the results and discussion of a different theme with the intention of explaining how contradiction can provide an understanding of teachers' experiences of the RAM programme. In Chapter 6, I consider the theme that teachers' beliefs about mathematics education can be conceived of as interdependent. In Chapter 7, I consider the theme of how moments of variation are acts of self-sabotage. Then, in Chapter 8, I consider the theme of belief trajectories show journeys not destinations.

Chapter 9: Conclusion

Finally, in Chapter 9, I summarise my themes, make recommendations, and present the implications of my research. I argue a contradiction informed perspective provides an original interpretation of teacher's experiences in a professional development programme. I identify the value of a contradiction informed interpretation of beliefs about mathematics education as foundational for mathematics DPD. I argue mathematics DPD provides a model of teacher professional development which satisfies the scope of education in line with Biesta's (2020) explanation of learning as cultivation and existence. Finally, I suggest my work has implications for clarifying government education policy such as that in England around the mathematics mastery market (Boylan and Adams 2024).

1.6 Summary of Chapter 1

In this introductory chapter I have provided the educational experiences and philosophical underpinning that have led me to this work. I acknowledge my beliefs and approaches to mathematics education have varied over my career but identify myself toward the progressive end of the spectrum. This perspective means I am committed to designing teacher professional development that contributes to the development of progressive pedagogies but recognises the contradictions within. Furthermore, I am interested in moments of programmes where participants react to the intended design, and where they display blended beliefs about mathematics education. To help understand these issues, I adopt a philosophical perspective based on McGowan's (2019) interpretation of Hegelian contradiction. I view contradiction as an ontological insight whereby 'everything is also what it is not and has its identity in what negates it' (McGowan 2019, p. 85). Whilst I will link both teacher beliefs and professional development to the work of Swan, I have also signalled my intention to interpret the concepts of teacher beliefs and professional development in terms of contradiction. With these re-interpretations in mind, I introduced my research question and desire to understand how McGowan's (2019) theory around contradiction can provide deeper insight into a mathematics DPD programme.

Chapter 2.

Teachers' beliefs about mathematics education

2.1 Synopsis of Chapter 2

In the first chapter I introduced the professional experiences and philosophical perspective which have led to the development of this thesis. I provided my research question and thereby signalled my intention to interpret a teacher professional development programme through the lens of contradiction. In this chapter, I review the conception of teacher beliefs by locating them in relation to other constructs, defining them according to three important features, and then examining one specific model of teachers' beliefs about mathematics education. The model I focus on was designed by Swan (2006a, 2006b), and I therefore refer to his thinking throughout this chapter when locating and defining beliefs. I am drawn to Swan's (2006a, 2006b) model of teacher beliefs as he developed it for use alongside the professional development programmes he created. A more detailed discussion of his approach to professional development takes place in Chapter 3. An understanding of Swan's (2006a, 2006b) framework is important since I use it in the mathematics Dialectic Professional Development (DPD) programme I design for this study and use it to interpret the data I collect. Ultimately, I suggest a re-interpretation of Swan's model, according to my contradiction informed perspective, to support mathematics DPD.

2.2 Locating beliefs

Before examining a definition of teacher beliefs, I first locate them in relation to other similar constructs. Therefore, I briefly explain the relationship between beliefs and

identity, between beliefs and knowledge, and between beliefs and other constructs in the affective domain. Within each of these sub-sections I relate wider education literature to the positions I infer were held by Swan, and to the positions I take.

2.2.1 Beliefs and identity

Research on teacher identity has been a growing area of interest and significance in mathematics education research (Skott 2022). However, debate persists around a precise definition of teacher identity (Lutovac and Kaasila 2018, Graven and Heyd-Metzuyanim 2019, Hong, Cross Francis and Schutz 2024). Views range from seeing beliefs as synonymous with identity (Ingram *et al.* 2020), to seeing them as one of many ‘multiple intersecting’ factors affecting identity (Hong, Cross Francis and Schutz 2024, p. 162). Whilst Swan (2006b, p. 59) made no direct reference to the term teacher identity in his work, he does suggest beliefs ‘help people to understand themselves, to understand their environment and to form social groupings around shared values’. This social aspect of Swan’s explanation resembles the explanation of identity provided by Wenger (1998, p. i) who suggests it is the emergence of ‘who we are’ in a specific social context. Willis *et al.* (2023) have built on this idea to suggest mathematics teacher identity can be understood as a function of beliefs about teaching, a sense of belonging to a mathematics community, and an enthusiasm for mathematics. Therefore, I hold the position that mathematical beliefs emerge in the specific social practices of mathematics education and contribute to the shaping of an individual’s identity (Boaler and Sarah 2017).

2.2.2 Beliefs and knowledge

The second way in which I locate beliefs is through a brief discussion of its relation to knowledge. I suggest beliefs are operationally viewed as a form of knowledge in terms of cognition, but as distinct from knowledge in terms of practice.

Beliefs as a form of knowledge

Firstly, in terms of cognition, Chick and Beswick (2018) claim the only difference between beliefs and knowledge is the level of consensus each attracts. Furinghetti and Pehkonen (2002) expand the basis of the distinction to suggest a statement of knowledge can be viewed as one hundred percent true. Therefore, beliefs are conceived of as a form of knowledge, with the difference being that knowledge has ‘been affirmed as true on the basis of objective proof or consensus of opinion’ (Kagan 1992, p. 73). Whilst I accept beliefs operate in a cognitively similar fashion to knowledge, a slight distinction now follows in terms of philosophy. In Kantian terms, a statement of knowledge would be considered analytic based on being true by definition, whilst a statement of beliefs would be considered synthetic based on requiring empirical verification (Kant 2009). I return to consider this differential later in my work when applying a more Hegelian perspective to the interpretation of statements of belief.

Despite beliefs and knowledge being ‘inextricably intertwined’ (Pajares 1992, p. 325), they are further differentiated by how required teacher knowledge has been identified in mathematics education. Skott (2022, p. 3) comments knowledge is specifically used to refer to the ‘knowledge of subject matter’, and not around beliefs, for mathematics teachers. Therefore, numerous models have developed around what mathematics teachers

should know, with the most ‘dominant frameworks’ (Skott 2022, p. 3) drawing on the work of Shulman (1986). For instance, Shulman’s (1986) influence is seen in the teacher model of Ball, Thames and Phelps (2008) who define a series of knowledge domains within the general categories of Subject Matter Knowledge and Pedagogical Content Knowledge. Similarly, the Mathematics Teachers’ Specialized Knowledge model (Carrillo-Yañez *et al.* 2018, Carrillo 2020) suggests sub-categories of knowledge under the headings of mathematical knowledge and pedagogical content knowledge.

Beliefs as distinct from knowledge

Some researchers suggest knowledge alone cannot account for the differences between the practices of mathematics teachers (Ernest 1989a, Ernest 1989b). Instead, given similar levels of teacher knowledge, Ernest (1989a, 1989b) claims underpinning teacher beliefs account for any variation that exists. This approach can be seen in many models, including Schoenfeld’s (2015) goal-oriented model of teaching where he identifies beliefs as one of the three factors affecting classroom decisions alongside goals and resources. Likewise, in the Mathematics Teachers’ Specialized Knowledge model (Carrillo-Yañez *et al.* 2018, Carrillo 2020), underpinning beliefs are identified as an important factor, though receive little discussion. Swan (2014) also recognises this distinction and identifies the importance of both subject knowledge and beliefs about mathematics education on teachers’ practice. Indeed, in each of these examples, beliefs about mathematics education are understood as being shaped by beliefs in the domains of mathematics, teaching mathematics, and learning mathematics. I return to consider each of these domains in greater detail later in this chapter. Therefore, to summarise, I align with

Beswick (2011), who suggests atomisation of knowledge is insufficient, and an additional consideration of teachers' beliefs provides a better insight into teacher development.

2.2.3 Beliefs and the affective domain

Thirdly, whilst beliefs are conceived of as residing in the cognitive domain as a form of knowledge (Pajares 1992, Furinghetti 1996), they are also acknowledged as having consequences in the affective domain (McLeod 1992, Schuck and Grootenboer 2000). Within the affective domain, beliefs are considered alongside the related concepts of values (DeBellis and Goldin 1999), attitudes, and emotions (McLeod 1992) as influencing practice. More widely, Schuck and Grootenboer (2000) suggest beliefs, values, attitudes, and emotions can be understood as laying on a continuum of increasing affective response and decreasing cognitive response. In practical terms, this translates to an increasing intensity of feeling alongside decreasing levels of stability across the continuum (McLeod 1992). Therefore, from this perspective, beliefs are seen as the least intense in feeling but the most stable and resistant to change of the four concepts. The idea of a continuum is further highlighted when emotions are understood as arising in response to a situation, attitudes as learnt in response to a situation, values as the criteria shaping decisions, and beliefs as positions held (Lomas, Grootenboer and Attard 2012). This perspective aligns well with Swan (2014, p. 623) who described how attitudes become values 'as they are thoughtfully chosen, prized, cherished, affirmed and acted on repeatedly'.

Whilst understanding the spectrum of beliefs-values-attitudes-emotions as helpful, it is not intended as an exhaustive list of concepts that reside in the affective domain.

Ingram *et al.* (2020, p. 148) suggest it is an area of ‘muddiness’ and question the value of the distinctions. Indeed, within mathematics education, Schoenfeld (2019, p. 2) suggests the distinctions ‘serve little purpose when one is trying to explain human behavior’. This leads Schoenfeld (2010) to introduce an umbrella word of orientations to cover a range of affective domain concepts including beliefs. He argues distinctions are unnecessary and suggests orientations capture a ‘sense of what matters’ (Schoenfeld 2020, p. 359) to a teacher. Such an approach appears reasonable if, as in the case of Schoenfeld, the intention is to create a post hoc model that accounts for classroom decisions. However, in this thesis I value the distinctions and terminological clarity since I seek to understand an interpretation of beliefs, rather than simply explain classroom decisions.

2.3 Conceptualising beliefs

Having positioned beliefs in relation to other constructs, I now explain teacher beliefs through discussing their conceptualisation by Swan and linking to wider literature. Debate exists around a precise definition of beliefs within education (Swan 2006b, Hannula *et al.* 2016, Martínez-Sierra *et al.* 2020), and so some researchers rely on listing features. For example, across various texts, Swan describes teacher beliefs as mental constructs (Swan 2007), as being resistant to change (Swan 2006b, Swan 2007), and as influencing the way teachers ‘perceive and deploy’ tasks (Swan 2007, p. 226). These features accord with what Skott (2015) suggests are an agreed set of features, leading to his definition of beliefs as:

individual, subjectively true, value-laden mental constructs that are relatively stable results of substantial social experiences and that have

significant impact on one's interpretation of and contribution to classroom practice (Skott 2015, p. 19).

Both Skott and Swan highlight the main features of beliefs as mental constructs, as being resistant to change, and as affecting practice. Therefore, in the following sub-sections I examine these three features and consider their implications for my work.

2.3.1 Beliefs as mental constructs

As previously discussed, beliefs can be considered as a form of knowledge that are either not proven or not universally agreed upon. Hence, stating beliefs are mental constructs means they are viewed as being subjective and open to change (Skott 2015). Swan (2007, p. 226) similarly identifies these two elements when describing his view that beliefs 'evolve with experience' and are 'permeable mental structures'.

The formation of teacher beliefs through experience

The formation of beliefs through experience aligns with my view that teacher beliefs emerge through engagement in the social practice of mathematics education (Boaler and Sarah 2017). These social practices include experiences at both the national and local level. For example, at a national level, Stigler and Hiebert (2009, p. 87) suggest teaching within different countries has evolved over a long period of time based on a 'stable web of beliefs and assumptions that are part of the [national] culture'. They claim that in mathematics education this stable web of beliefs centres on beliefs in the domains of mathematics, teaching mathematics, and learning mathematics (Stigler and Hiebert 2009). In the national context of England, Dickinson *et al.* (2020, p. 343) suggest the dominant classroom culture is one where students 'see mathematics as a question of

learning rules which lead to answers based on received wisdom'. Such a perspective is further supported when Boylan (2021) suggests, in comparison to Shanghai, teaching in England is currently dominated by teacher led explanations and individual student practice.

However, I also acknowledge the influence of local variations on mathematics teachers' beliefs. Swan and Swain (2010b, p. 175) draw attention to the importance of local context by suggesting changes in teachers' beliefs are more likely' following the trial of new practices. Swan and Swain (2010b) applied this principle in local professional development interventions, but a similar outcome can be seen in the Shanghai Mathematics Teacher Exchange (Boylan *et al.* 2019). Here, teachers participating in the exchange reported changes in their 'beliefs about how pupils learn' (Boylan *et al.* 2019, p. 96) in contradistinction to the national web of beliefs. Furthermore, I also note there are suggestions teachers' mathematics beliefs are influenced by the existing local culture. For example, Francome and Hewitt (2020, p. 490) record the case of a secondary school in England where students were taught in mixed attainment groups and question whether this 'encourages alignment' of teachers to more student-centred beliefs. Thus, I would argue national and local experiences contribute to the formation of mathematics teachers' beliefs.

The changing of teacher beliefs

Once teacher beliefs are recognised as being formed by experience, it is important to recognise ways in which these experiences are conceived. Indeed, some of the earliest work on teacher beliefs refers to looking to 'correct' (Green 1998, p. 50) the beliefs of others and to turn subjective beliefs into 'objectively reasonable beliefs' (Fenstermacher

1978, p. 170). Green (1998) helpfully categorises experiences intended to contribute to belief change into two broad strategies of indoctrination and instruction.

Green (1998) distinguished between beliefs influenced through instruction and indoctrination based on whether evidence is used. Indoctrination is a word with negative connotations and so caution needs to be exercised in labelling an experience in such a way. However, a possible example is given by Ellis, Getti and Mansell (2024, p. 133), who cite the Teaching and Leading Innovation Fund in England as leading teacher professional development in low social mobility areas but providing no appeal to evidence or ‘overarching rationale’. I suggest professional development through indoctrination could also include the growing trend of school level directives around techniques teachers must adopt (Ellis, Gatti and Mansell 2024). For example, Rubel and Stachelek (2018, p. 18) cite the example of a school which ‘advocated a constellation of pedagogical techniques’ taken from the book *Teach Like a Champion* (Lemov 2021). Though the appeal to evidence is not clear, I note Rubel and Stachelek (2018, p. 18) quote a mathematics teacher who explained ‘we have to do that with Teach like a Champion’, despite the practice conflicting with the teacher’s personal beliefs about mathematics education.

Green’s (1998) second identified way to influence beliefs is through instruction that appealed to evidence. The proliferation of randomized controlled trials in England (Sims *et al.* 2023) provides one example of the trend toward seeking change through interventions based on evidence. However, due to low powered trials, there are suggestions that claims of impact are over-exaggerated (Sims *et al.* 2023). There are examples of mathematics education programmes which I interpret as seeking to influence teacher

beliefs through instruction. For instance, I suggest the Improving Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) programme (Hodgen *et al.* 2014) provides an example of influencing beliefs and practices based on evidence claims. As well as being founded in research-based principles of formative assessment, the ICCAMS programme claims to contribute to students making the equivalent of two years of progress in one year (Hodgen *et al.* 2014).

My interpretation of Swan's view on experiences influencing teacher beliefs is that he worked on the instruction, rather than indoctrination, end of the spectrum. Swan did not seek to enforce changes and encouraged change through the evidence of experimentation and reflection. Indeed, for me it is this element of inviting teacher reflection on an experience that distinguishes instruction from indoctrination. For instance, Swan stated his approach to experiences was to 'invite teachers to take risks and adopt new practices so that they have cause to reflect' (Swan *et al.* 2013, p. 947) and 'changes in beliefs are more likely to follow changes in practice' (Swan and Swain 2010b, p. 175). The conception of beliefs as subjective, personal mental constructs that are open to change means they are an important element of teacher professional development programmes. For me, this provides an important reminder to avoid seeking experiences which influence beliefs through indoctrination, and to focus on evidence and reflection as part of a teacher's journey.

2.3.2 Belief as resistant to change

The second feature of beliefs mentioned by Swan and Skott (2015) is the idea of stability and resistance to change. Schoenfeld (2020, p. 374) argues beliefs 'are borne of experience over time, and they change slowly'. Similarly, Swan claims beliefs are 'often

resistant to change’ (Swan 2006b, p. 59), and that teachers cling to their beliefs ‘tenaciously’ (Swan 2014, p. 947). One reason provided by Swan (2006b, p. 59) for the resistance of beliefs to change is through conceiving of them as operating in a ‘system’.

Belief systems

As with some other mathematics education researchers (Furinghetti and Pehkonen 2002, Thompson 1992, Eichler and Erens 2015, Skott 2022), Swan’s understanding of teacher beliefs is linked to Green’s (1998) claim that beliefs cluster together in a system rather than exist in isolation. These clusters of related beliefs may include ones which are resistant to change, and others which are more open to examination and alteration (Green 1998). Though individual beliefs may move from being resistant to open, the clustering affords some level of protection to change at the system level (Green 1998). This clustering conceptualisation appears relatively common across mathematics education research with beliefs in the domains of mathematics, teaching mathematics, and learning mathematics often viewed as operating in a system (Aguirre and Speer 1999, Swan 2006b, Schoenfeld 2010, Carrillo-Yañez *et al.* 2018, Skott 2022). Thus, the notion of belief clustering means the belief in one domain is seen as more resistant to change through its association with beliefs in other domains (Green 1998).

In addition, Cross-Francis (2015) comments beliefs are not necessarily clustered together based on logical relationships. Instead, contrasting beliefs may be grouped on a ‘quasi-logical’ (Green 1998, p. 47) basis based on teachers’ perceptions of compatibility. For instance, a teacher could quasi-logically perceive mathematics is discovered (from the domain of beliefs about mathematics), and that mathematics should be transmitted to

students in the form of procedures (from the domain of beliefs about teaching mathematics). One implication of this is beliefs across domains may appear conflicting to an outsider but not necessarily to an individual (Op't Eynde, de Corte and Verschaffel 2002).

2.3.3 Beliefs as affecting interpretation and engagement

The third widely agreed feature of beliefs is they affect interpretation and engagement (Skott 2015) in activities. As Swan states, a mathematics teacher's beliefs influence the way they 'perceive and deploy' (Swan 2007, p. 226) a task and explain why teachers 'engage in certain practices and not others' (Swan 2006b, p. 59). However, Swan also acknowledged the influence of other factors, and suggested classroom management, student expectations, and available resources can compromise a teacher's beliefs (Swan *et al.* 2013). More generally, Ernest (1989a, 1989b) argued differences in beliefs about mathematics education can account for differences in practice of two teachers with similar levels of knowledge. Some researchers therefore suggest the existence a causal link connecting teachers' beliefs, teacher's practices, and student learning (Eichler, Erens and Törner 2023). However, seeking to track changes to teachers' beliefs is complicated by a difference in teachers' self-reported and actual beliefs (Swan *et al.* 2013). Both Swan (2013) and Schoenfeld (2020) cite the case of Mrs Oublier (Cohen 1990) to illustrate how a teacher may think their beliefs have changed, but that observers saw no difference in practice. This suggests caution should be taken when attempts are made to measure changes to teacher beliefs following the experience of a mathematics programme.

2.4 Swan's model of teachers' beliefs about mathematics education

Having located and defined teacher beliefs, I now provide a detailed insight into the model of teachers' beliefs about mathematics education that Swan (2006b) developed. Swan's model focuses on a system of beliefs about mathematics education which is a development of an approach suggested by Askew *et al.* (1997), who in turn built on the work of Ernest (1989a, 1989b). This is the model I utilise in this study as both part of the professional development activities for teachers, and to provide data for interpretation in my study. In addition, I apply my contradiction informed philosophical interpretation in Chapters 6 and 8 to suggest a reinterpretation of the model.

The system of beliefs about mathematics education

Op't Eynde, de Corte and Verschaffel (2002) include beliefs about mathematics education in their synthesised framework of systems of mathematics related beliefs alongside beliefs about the social context, and beliefs about the self. Op't Eynde, de Corte and Verschaffel (2002, p. 29) go on to suggest the system of beliefs about mathematics education consists of three clustered and 'interrelated subsets' of beliefs about mathematics, beliefs about teaching mathematics and beliefs about learning mathematics. These are the same three belief domains suggested by Ernest (1989a, 1989b), are widely used across mathematics education literature (Aguirre and Speer 1999, Schoenfeld 2010, Carrillo-Yañez *et al.* 2018, Skott 2022, Vesga-Bravo, Angel-Cuervo and Chacón-Guerrero 2022), and are adopted by Swan (2006b) in his model.

It has been suggested other belief domains could be added to the cluster of beliefs about mathematics education to help provide precise explanatory beliefs for teacher

actions (Cross Francis 2015). For instance, Mosvold and Fauskanger (2013) support the inclusion of a fourth domain of beliefs about the knowledge needed for teaching mathematics. Similarly, Xie and Cai (2021) supplement the three domains with an additional two of beliefs about students and beliefs about teachers. However, in this thesis I will remain consistent with Swan (2006b) and focus on the categories he identified in the three main domains.

2.4.1 The domain of beliefs about mathematics

Beliefs about mathematics are established as a key determinant toward the teaching of mathematics (Thompson 1984, Hersh 1998). Furthermore, Ernest (1989b) suggested beliefs in this domain are foundational since they influence beliefs in the other two domains. Despite recognising beliefs about mathematics are often implicit and may be difficult to articulate for a teacher, Ernest (1989b) names a hierarchy ranging from Instrumentalist, to Platonist, to Problem-Solving. Such an approach is acknowledged as being ‘widely used’ (Mosvold and Fauskanger 2013, p. 45) across mathematics education literature. Therefore, in the left-hand side of Table 2.1, I provide the category names and descriptions taken from Ernest (1989a), with the equivalent names and descriptions taken from Swan’s (2006b) teacher validated model on the right-hand side.

Category name (Ernest 1989a)	Description of mathematics (Ernest 1989a)	Category name (Swan 2006b)	Description of mathematics (Swan 2006b)
Instrumentalist	A set of unrelated, utilitarian facts, rules and skills which are accumulated for an external purpose.	Transmission	A given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to learners.
Platonist	A static, but unified, body of knowledge which is discovered rather than created.	Discovery	A creative subject in which the teacher should take a facilitating role, allowing learners to create their own concepts and methods.
Problem Solving	A dynamic, continually expanding creation of the human mind which is open to revision through enquiry.	Connectionist	An interconnected body of ideas which the teacher and learner create together through discussion.

Table 2.1 Beliefs about mathematics

Adapted from Ernest (1989a) and Swan (2006b).

Whilst there are clear links between the descriptors in Table 2.1, there is also a difference in emphasis. Ernest (1989a) draws from a philosophy of mathematics perspective, whilst Swan (2006b) placed additional emphasis on the application of the philosophical perspective to the ‘sub-discipline’ (Golding 2017, p. 461) of school mathematics. For instance, the three categories presented by Ernest (2006b) can be summarised as mathematics is an instrument (Instrumentalist), is discovered (Platonist), or is created (Problem Solving). Hence, for Ernest (2006b), mathematics is seen as either a set of rules, a fixed body of knowledge, or an expanding body of knowledge respectively. However, in Swan’s (2006b) comparable descriptors he emphasised the practical implications of the view in a school setting. The Transmission category retains the Instrumentalist emphasis on rules but additionally identifies the teacher as conveying the rules to the learners. The Discovery category retains the Platonist sense of discovering mathematics but adds the teacher creates opportunities for the learner. The Connectionist category retains the Problem-Solving idea of creating mathematics but also identifies the teacher as connecting learning through discussion.

Conceivably, Swan’s descriptors may have been influenced by the numeracy descriptors provided by Askew *et al.* (1997) to describe beliefs about what it is to be a numerate pupil. By exemplifying what these beliefs look like in a classroom, I suggest Swan (2006b) overlaps into the domains of teaching mathematics and learning mathematics. For example, looking at the Transmission statement, I would argue Swan’s (2006b) descriptors identify the teacher’s role is to convey information and the learners’ role is to listen. This moves away from the original philosophical categorisation that mathematics is seen as a ‘set of tools’ (Xie and Cai 2021, p. 749). Swan’s (2006b) adjusted

descriptors still capture a sense of teacher's beliefs about mathematics, but an acknowledgement should be made that the original philosophical perspectives are supplemented with practical implications.

2.4.2 The domain of beliefs about teaching mathematics

Thompson (1992, p. 135) recognised beliefs about teaching mathematics are 'eclectic', unlikely to precisely match theoretical categories, and built over many years of experience. Rolka and Roesken-Winter (2015) additionally suggest teachers' own experiences of learning mathematics are particularly influential towards their beliefs around its teaching. Kuhs and Ball (1986) identify four distinct views around the teaching of mathematics which they name content focused emphasising performance, content focused emphasising conceptual understanding, learner-focused, and classroom focused. Thompson (1984, p. 137) notes the fourth view, classroom focused, concerns 'the structure of lessons and general pedagogical skills' rather than being focused on mathematics pedagogy. This appears a reasonable explanation for why Ernest (1989a) reduces the categories to three views which he names Instructor, Explainer and Facilitator. In Table 2.2, I summarise Ernest's (1989a) category names and descriptors and provide the equivalent categories and descriptions from Swan's (2006b) model. It is helpful to note Swan retained the category names of Transmission, Discovery and Connectionist in this domain that were first introduced in the domain of beliefs about mathematics.

Category name (Ernest 1989a)	Description of teaching mathematics (Ernest 1989a)	Category name (Swan 2006b)	Description of teaching mathematics (Swan 2006b)
Instructor	Mastery of skills alongside correct performance.	Transmission	Structuring a linear curriculum for the learners; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when learners ‘fail’ to grasp what is taught.
Explainer	Developing conceptual understanding with unified knowledge.	Discovery	Assessing when a learner is ready to learn; providing a stimulating environment to facilitate exploration and avoiding misunderstanding by a careful sequencing of experiences.
Facilitator	Problem posing and assisting in problem solving.	Connectionist	A non-linear dialogue between teacher and learners in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked upon.

Table 2.2 Beliefs about teaching mathematics

Adapted from Ernest (1989a) and Swan (2006b).

Whilst Ernest (1989a) provided brief descriptions, he additionally noted beliefs around teaching mathematics are linked to a teacher's view of curricular materials. He identified teachers may have a strict adherence to a textbook, may seek to modify and enrich a textbook, or may seek to construct their own learning sequence. However, Ernest did not go so far as to say there is a one-to-one relationship between the three teaching categories and the three approaches to curricular materials. In contrast, Swan (2006b) does make a much more explicit link by attaching references to a linear curriculum, to careful sequencing, and to non-linear dialogue in his three descriptors.

Askew *et al.* (1997) suggest the three categories in the domain of beliefs about teaching mathematics reflect different points on the teaching-learning spectrum. They view teaching as taking priority in a Transmission perspective, learning as taking priority in a Discovery perspective, and teaching and learning as being complementary in a Connectionist perspective. Swan (2006b) implied the same in his descriptions with the additional exemplification of how the belief might appear in a classroom. For example, in the Transmission category Swan (2006b) added that a teacher structures the curriculum, gives verbal explanations, provides questions, and corrects misunderstandings (Swan 2006b).

I also argue Swan (2006b) created a slight change in emphasis with the Discovery and Connectionist categories compared to Ernest's (1989a) original intentions. Swan (2006b) created a Discovery category focused on providing experiences, whereas Ernest's (1989a) Explainer category placed an emphasis on a teacher's role in bringing conceptual understanding. Similarly, Swan's (2006b) Connectionist category focuses on the teacher

contributing to discussion rather than on Ernest's (1989a) original emphasis on seeing teaching as posing problems. Considering this, I do not dismiss Swan's (2006b) categories as wrong, but recognise he adjusted the descriptors to capture features he saw as important.

2.4.3 The domain of beliefs about learning mathematics

In the domain of beliefs about learning mathematics, Ernest (1989b) suggested views exist around two key constructs. Firstly, he identified mathematics learning may be viewed on a scale from passive reception to active construction of knowledge. Secondly, he suggested learning may be viewed as existing between requiring compliance and providing autonomy. The recognition of this complexity may help to account for why he suggested both four (Ernest 1989a) and six (Ernest 1989b) potential categories within this domain. Beswick (2012) reduces these perspectives to three which, for convenience, I adopt in Table 2.3 though retain the sense of Ernest's (1989a) original wording. In addition, since Ernest (1989a) did not provide category names in this domain, the first column in Table 2.3 are my suggestions, whilst Swan continues to use the same three category names of Transmission, Discovery and Connectionist.

Category name	Description of learning mathematics (Ernest 1989a)	Category name (Swan 2006b)	Description of learning mathematics (Swan 2006b)
Compliant	Compliant learner behaviour focusing on the mastery of skills, through the reception of knowledge.	Transmission	An individual activity based on watching, listening and imitating until fluency is attained.
Active	Learners actively construct their own understanding.	Discovery	An individual activity based on practical exploration and reflection.
Autonomous	Learners explore and autonomously pursue based on their own interests.	Connectionist	An interpersonal activity in which learners are challenged and arrive at understanding through discussion.

Table 2.3 Beliefs about learning mathematics

Adapted from Beswick (2012), Ernest (1989a) and Swan (2006b).

I summarise Ernest's (1989a) three views about the nature of learning mathematics as compliant obedience, active pursuit of understanding, and autonomous pursuit of own interests. This same sense is seen in the descriptors of Askew *at al.* (1997), who suggest the three perspectives see learning as following instructions, as a product of interacting with problems, and as following from interactions with others respectively. Swan (2006b) once again retained the essence of the descriptions but added additional description based on what might be seen in a mathematics classroom. So, the Compliant category is reflected in Swan's (2006b) Transmission category as an individual activity, where learners watch and imitate. I also note Swan (2006b) decided to identify Transmission and Discovery approaches to mathematics learning as individual activities, whilst the Connectionist perspective is identified as taking place in groups. For Ernest (1989a), active learning is understood as taking place individually or in groups, and the focus is on making sense of provided mathematical opportunities. There is a potential risk that a teacher does not identify with Swan's (2006b) Discovery category if they attend to the emphasis on individual work, rather than on the practical exploration element.

2.4.4 Summary of Swan's category descriptors

To summarise, Swan (2006b) created a set of nine descriptors that closely resemble the thinking of Askew *at al.* (1997), and have clear links to the philosophical underpinnings suggested by Ernest (1989a). He retained the essence of Ernest's (1989a) explanations but ended up with a simplified model by limiting himself to three categories within each domain. Generally, since Swan (2006b) based his descriptions on observed teacher actions, he included additional practical details compared to Ernest's (1989a) original descriptions. As noted above, there are times when Swan's (2006b)

exemplifications slightly modify the intentions of Ernest's (1989a) more philosophical descriptors. I think it is helpful to recognise Swan's descriptors in the domain of beliefs about mathematics moved away from philosophical beliefs and toward exemplification of practice that infer beliefs. In noting this emphasis, I am seeking to highlight this may affect teachers' interpretations of the descriptors. However, I also note Swan (2006b) validated his belief statements against teacher's own qualitative descriptions of their practice, student's descriptions of their teacher's practices, and classroom observations. He comments that independent researchers recognised the 'remarkable consistency' (Swan 2007, p. 226) that this cross-validation gave.

2.4.5 The cluster of beliefs about mathematics education

Swan (2006a, 2006b) used his descriptors and domains in the form of a self-reported belief questionnaire intended to help quantify beliefs of mathematics teachers. The questionnaire consisted of the 9 statements shown in Tables 2.1, 2.2 and 2.3 grouped by belief domain. Within each belief domain, teachers were asked to show their preferences by weighting the three belief descriptors to sum to 100. Using these domain weightings, Swan calculated a mean average Transmission, Discovery and Connectionist weighting. I illustrate this process in Table 2.4 with notional belief weightings, though for simplicity replace the category descriptions with the category names.

	Beliefs about mathematics	Beliefs about teaching mathematics	Beliefs about learning mathematics	Mean average
Transmission	60%	50%	55%	55%
Discovery	25%	25%	25%	25%
Connectionist	15%	25%	20%	20%
Total	100%	100%	100%	100%

Table 2.4 Illustration of belief weightings about mathematics education

By calculating the mean average for each belief category, Swan (2006a) effectively created a measure for the system of beliefs about mathematics education. In addition, Swan (2007) summarised the categories of Transmission, Discovery and Connectionist when applied to the system of beliefs about mathematics education. Inevitably, the belief descriptions draw on wording from the individual domains and are summarised in Table 2.5.

Category from Swan (2007)	Description of beliefs about mathematics education from Swan (2007)
Transmission	Mathematics seen as a series of ‘rules and truths’ that must be conveyed to students, teaching is ‘chalk and talk’, and learning is through individual practice until fluency is attained.
Discovery	Mathematics seen as a human creation, teaching is facilitating and reactive, learning is encouraged through individual exploration and reflection.
Connectionist	Mathematics seen as a network of ideas, teaching takes place through proactive challenging of student thinking, learning takes place through collaborative discussion.

Table 2.5 Beliefs about mathematics education

Adapted from Swan (2007)

To summarise, the weightings that teachers ascribe in Swan's (2006b) belief questionnaire are used to provide a mean average teacher's beliefs regarding mathematics education. Swan (2006b) then described the teacher's beliefs based on their highest mean average weighting for the cluster of beliefs about mathematics education. For example, in the illustration of Table 2.4, the highest mean average is 55% and so the teacher would be described as having Transmission beliefs. I discuss my use of Swan's belief questionnaire in more detail in Chapter 5 and explain my contradiction informed re-interpretations in Chapters 6 and 8.

2.4.6 The existence of conflicting beliefs

The use of domains and categories to understand teachers' beliefs about mathematics education leads to the potential for inconsistent conflicts. There may be inconsistencies between stated beliefs and enacted beliefs (Skott 2015), inconsistencies between domains (Swan 2006b), and inconsistencies within domains (Askew *et al.* 1997, Swan 2006b). I examine each of these inconsistencies in the context of Swan's (2006b) model, but suggest it reveals a deeper gap in research around the philosophical underpinning of frameworks.

Inconsistencies between stated and enacted beliefs

Skott (2015) recognises there may be a difference between the stated beliefs of teachers and those that are enacted in the classroom. He suggests this can be caused by teachers perceiving self-reported questionnaires as being for a researcher, and that there is a lack of shared understanding in terms being used. In addition, Cross Francis (2015) suggests

the differences may exist when a researcher fails to include the necessary explanatory beliefs in their model. Therefore, it is recommended researchers draw on a variety of data sources including short-answer questionnaires, lesson observations, stimulated recall, interviews (Skott 2015), and lesson plans (Vesga-Bravo, Angel-Cuervo and Chacón-Guerrero 2022). Indeed, Swan (2006b) adopted this triangulation approach in his model and utilised pen-portraits of the teachers, lesson observations, and questionnaires to support inferences. However, it must still be recognised that such models rely on a researcher seeking to fit a teacher into their model and imposes a set of belief descriptors on a teacher rather than allow them to ‘elicit their own’ (Skott 2015, p. 20). In addition, I note Swan (2006b, p. 59) argued the use of weightings within a domain meant teachers could avoid having to ‘choose between what they perceive to be false dichotomies’.

Inconsistencies between domains

The second potential area in which inconsistencies may exist is between the domains being studied. For instance, in Swan’s (2006b) model it is possible that a teacher has inconsistencies between the domains of beliefs about mathematics and beliefs about teaching mathematics. It is conceivable a teacher identifies most strongly with a Discovery belief about mathematics but also believes teaching mathematics requires a Transmission approach. Swan (2006a, p. 194) accepted the creation of a single dominant belief about mathematics education is a ‘crude’ process. Furthermore, as discussed earlier, these beliefs may appear inconsistent to the researcher, but not to the teacher (Op’t Eynde, de Corte and Verschaffel 2002).

Inconsistencies within domains

Thirdly, and as a consequence of Swan asking teachers to weight all three categories, inconsistencies may be seen within domains. For example, using Tables 2.4 and 2.5, a teacher is understood as believing a teacher must convey mathematics to students (Transmission), and facilitate their exploration (Discovery). Swan (2007, p. 226) suggested the categorisations show ideal types and in practice teachers combine elements of each of the categories ‘even where these appear to conflict’. Similarly, in the model Swan drew on from Askew *et al.* (1997, p. 28), they note many teachers in their sample ‘combined several characteristics of two or more’ of their belief categories.

To explain the existence of conflicting beliefs within a domain, Swan (2011) therefore suggested teachers call on different beliefs depending on the context of the lesson. For instance, a belief that the learning of mathematics takes place individually may be called upon to develop fluency, but the belief that the learning of mathematics takes place collaboratively may be called upon to develop reasoning. Alternatively, a teacher may adopt a Transmission approach to teaching mathematics with a low attaining class and a Discovery approach with a higher attaining class. The mutual exclusivity of the categories is maintained, and teachers are understood as switching depending on need. Irrespective, of the suggestion of switching Swan still labelled a teacher according to a single dominant category at the level of the system of beliefs about mathematics education.

2.5 Uses of Swan's model of teacher beliefs about mathematics education

In this section I briefly review how Swan's belief questionnaire has been used by other researchers and relate it to my work in this thesis. I structure this by considering how it has been adopted to identify teachers' current beliefs, and how it has been used to measure changes to teachers' beliefs following an intervention.

2.5.1 Identifying teachers' current beliefs

Swan (2006a) and other researchers (Beeli-Zimmermann 2019, Francome and Hewitt 2020, Calleja 2022) have used the questionnaire to describe teachers' initial beliefs regarding mathematics education. The mean averages are calculated across the three belief domains and teachers classified according to their dominant belief about mathematics education. However, adopting this approach meant Swan (2006a) excluded some teachers from his studies who showed no dominant belief. For instance, one participant had dominant belief weightings of 34% in both the Transmission and Discovery categories and so was not included in the analysis (Swan 2006a). Whilst I see the creation of simplified statements of teachers' beliefs as helpful, the process also reveals the danger of educational dualism. Seeking to classify teachers into a category based on their beliefs about mathematics education weightings can lead to the exclusion of some teachers.

Once teachers' dominant beliefs are ascribed, researchers have used this to make inferences. For example, Beeli-Zimmermann (2019) shows teachers with high Connectionist beliefs noted certain elements of a mathematics intervention. She therefore suggests

professional development experiences should be tailored for the different categories of teachers. Similarly, Francome and Hewitt (2020) utilise Swan's (2006a) questionnaire to consider the beliefs and practices of two groups of teachers from secondary schools in England. They note teachers who must account for a wide range of student attainment in their class tend to show practices more in line with Connectionist beliefs. So, whilst I see mathematics teachers' beliefs as complex, I agree that understanding current beliefs can provide insight into teacher experiences.

2.5.2 Measuring change to teacher beliefs

In addition to establishing teachers' initial beliefs, Swan's (2006a) belief questionnaire has also been used to identify changes in beliefs about mathematics education. For example, Swan (2006a) and Calleja (2022) have collected pre- and post- intervention questionnaire data to claim an intervention leads to changes in participants' beliefs regarding mathematics education. I treat this with some caution since, as already discussed, beliefs should also be understood as resistant to change. Additionally, the interventions teachers participated in (Swan and Swain 2010b, Swan *et al.* 2013, Calleja 2022) were designed to encourage the consideration of beliefs and practices that I see as aligning most closely with the Connectionist descriptors. Following these interventions, three belief trajectories for teachers were identified, with teachers moving from Transmission to Discovery, from Transmission to Connectionist, and from Discovery to Connectionist (Swan and Swain 2010b, Calleja 2022). However, it seems little surprise that interventions designed to support the development of Connectionist practices were reported as leading to increased Connectionist beliefs.

2.6 Summary of Chapter 2

I began this chapter by locating beliefs in terms of identity, knowledge, and other related constructs in the affective domain. I recognise there is significant overlap with each of these areas, but in this study I focus on beliefs about mathematics education held by teachers. I suggested mathematics teachers' beliefs are a component of identity, that they are cognitively similar to knowledge, and in the affective domain are seen as the least intense in feeling but the most resistant to change. In addition, I noted that whilst beliefs are cognitively similar to knowledge, they can also be considered distinct since knowledge alone cannot account for differences between teachers' practices.

After locating teacher beliefs, I moved on to consider how they are conceptualised by referring to three widely agreed upon features. Firstly, I suggested teacher beliefs are seen as personal mental constructs which are based on experience and therefore seen as being open to change. Secondly, I explained how teacher beliefs are seen as resistant to change, and that this resistance is partially explained through the ideas of a system of beliefs. Thirdly, I noted how teacher beliefs are understood to affect interpretation and engagement and therefore a causal link is identified to student outcomes.

Having located and defined teacher beliefs I then moved on to provide a detailed account of the model of mathematics teachers' beliefs proposed by Swan (2006b). Swan created a model that provides a numerical measure for the system of a teacher's beliefs about mathematics education. Using these numerical measures, teachers' beliefs can be classified

according to the highest, or dominant, value. Therefore, teachers are considered to have either Transmission, Discovery or Connectionist beliefs about mathematics education.

Finally, in this chapter I noted how Swan's (2006b) teacher belief questionnaire has been used in two main ways. Firstly, it has been used to create an interpretation of teachers' initial beliefs, and secondly it has been used to measure change in teacher beliefs following an intervention. Whilst I will go on to suggest an alternative way to understand the measure of current beliefs, I do not intend to use the model to measure change. Since beliefs are considered resistant to change, in Chapter 5 I explain my alternative use of the model to describe teachers' views of the Rethinking Approaches in Mathematics programme.

Chapter 3.

Professional development in mathematics education

3.1 Synopsis of Chapter 3

In the two previous chapters I aligned my philosophical perspective to McGowan (2019), and examined a model of beliefs about mathematics education provided by Swan (2006b). Swan's model of beliefs is closely related to his work on professional development programmes for mathematics teachers and I again base discussion in this chapter on his thinking. In my research question, stated in Chapter 1, I identified a desire to gain insight into a mathematics Dialectic Professional Development (DPD) programme. Therefore, my purpose in this chapter is to review Swan's thinking around professional development and use it to describe my approach of mathematics DPD. Using a framework proposed by Boylan, Coldwell *et al.* (2018), I explore Swan's approach, draw on other literature, and apply a contradiction informed perspective to provide an emerging definition of mathematics DPD. In the next chapter, I exemplify this emerging specification with the design of the Rethinking Approaches in Mathematics (RAM) programme which I utilise in this study.

3.2 Introduction to professional development

In this sub-section I define teacher professional development, highlight the reason it is considered important, and introduce a framework by which models of professional development can be analysed and developed. I then employ this framework in the remainder

of the chapter to allow me to consider Swan's approach to professional development, and from this develop an emerging specification of mathematics DPD.

3.2.1 Defining professional development

Professional development is a 'contested and fuzzy' (Czerniawski *et al.* 2021, p. 39) term which overlaps with other disputed terms including professional learning (Boylan and Demack 2018), professional learning experience (Heck *et al.* 2019), and career professional development (O'Brien and Jones 2014). As O'Brien and Jones (2014, p. 684) state, 'the question of whether the terms are used, understood or differentiated in practice is a long way from being answered'. Some researchers such as MacPhail *et al.* (2019) claim a distinction can be made based on intention. They identify professional development as the result of intentional and structured events, and professional learning as the result of informal and unstructured events (MacPhail *et al.* 2019). A similar perspective is taken by Sims *et al.* (2021, p. 7), who create a narrow definition by restricting it to only include 'structured, facilitated activity'. The alternative approach, which I adopt in this study, is to refer to professional development as the experience a teacher engages in, and professional learning as the resulting outcome for a teacher (Boylan and Demack 2018).

3.2.2 The importance of professional development

Governments across the world view professional development as offering a route to improve teaching and the educational outcomes of students (Cordingley *et al.* 2020, Sancar, Atal and Deryakulu 2021, Sims *et al.* 2021, Trivedi 2022). Indeed, Schleicher (2018, p. 63)

claims a key component of any high performing school system is the education of teachers and helping them ‘grow in their careers’. Furthermore, improved student outcomes are viewed as essential to increasing ‘nation-states’ economic competitiveness’ (Kennedy 2014, p. 691) and creating a ‘vibrant society’ (Shah and Campbell 2023, p. 1). Aside from economic gains, it is suggested professional development ensures teachers stay up to date with recent practices (Gardner, Glassmeyer and Worthy 2019); improves teacher confidence and self-efficacy (Fletcher-Wood and Zuccollo 2020); and increases the likelihood of teachers staying in the profession (Walker, Worth and Van den Brande 2019).

Considering the associated benefits, governments must make decisions around the allocation of finance and time to teacher professional development (McChesney and Aldridge 2021). Inevitably, decisions across countries vary with teachers in Singapore being allocated 100 hours of professional development per year (Schleicher 2019), compared to an estimated 43 hours for teachers in English secondary schools (Van den Brande and Zuccollo 2021). These 43 hours in England come at a cost of £3000 per teacher per year (Van den Brande and Zuccollo 2021) with teacher time being the largest contributing factor. In their review of the mathematics system in England, Noyes et al. (2023) highlight the range of professional development interventions that exist and the diversity of stakeholders. Without specifying a format, they call for an increased focus on supporting non-specialist mathematics teachers with subject knowledge, and inspiring experienced specialists by ‘re-engaging them with mathematics’ (Noyes *et al.* 2023, p. 41). These challenges, time costs, and monetary costs mean models of professional development require careful consideration and specification.

3.2.3 Understanding models of teacher professional development

In the rest of this chapter, I analyse the approach to mathematics teacher professional development taken by Swan and use this to draw out my explanation of mathematics DPD. I do this with the intention of helping to explain the design approach I took for the RAM programme I created for this study. Swan revealed a broad approach to the design of mathematics professional development as inviting teachers to adopt new practices followed by providing opportunities to reflect and then seeing if teachers ‘perhaps modify their beliefs’ (Swan *et al.* 2013, p. 947). Such an approach aligns closely with the professional development design model suggested by Guskey (2002) of using professional development to change teachers’ practices, leading to a change in students’ outcomes, and a change in teachers’ beliefs. Commenting on Guskey’s model, Boylan, Coldwell *et al.* (2018, p. 122) suggest it can support designers of professional development ‘to understand how changes in teacher attitudes and beliefs occur’. This aligns well with my intentions in this study, and I therefore include a specific model by Swan, and the wider thinking of Guskey, in my analysis of professional development.

To support the analysis of professional development, Boylan, Coldwell *et al.* (2018) suggest a framework consisting of six elements which I summarise, and re-order, in Table 3.1.

Framework element	Summary of the framework elements
Philosophical foundations	The philosophical underpinning of the professional development programme.
Purpose	The intended purpose of the professional development programme.
Components	The relationship between the elements of the model.
Scope	The nature of the activities within the professional development programme.
Theory of learning	The theory of change and/or learning underpinning the professional development programme.
Agents of change	The influences on a professional development programme that affect professional learning.

Table 3.1 A framework for professional development

Adapted from Boylan, Coldwell et al. (2018)

In the remainder of this chapter, I analyse Swan's thinking around professional development according to the framework elements shown in Table 3.1. Though Swan created several professional development programmes, I will initially draw on the Promoting Inquiry based learning in Mathematics And Science education across Europe (PRIMAS) project (Swan *et al.* 2013), which I see as representative of his thinking. Some elements of the Boylan, Coldwell *et al.* (2018) framework are not explicitly clear in the article by Swan *et al.* (2013) and I therefore make inferences where necessary. In addition, and where appropriate, I

draw in wider education literature to develop my thinking and specify implications for the contradiction informed approach of mathematics DPD.

3.3 The philosophical foundations of professional development

Boylan, Coldwell *et al.* (2018, p. 131) infer Guskey's model of professional development as being founded in a social realist approach which recognises the existence of 'generative mechanisms which produce observable regularities across the social world'. In a similar fashion, I infer Swan as holding a social realist perspective. Swan sought to identify patterns in changes to teachers' beliefs about mathematics education, which were dependent on their initial beliefs, following the experience of a professional development programme. When seeking to understand teachers' experiences, this seems a helpful perspective to inform the design of mathematics DPD. Strom and Viesca (2021) take a slightly different approach to classifying philosophical perspectives with respect to professional development and suggest the categories of dualism and multiplicity. I make a brief consideration of these two approaches below and add a third suggestion of dialectics in line with the thinking of McGowan (2019).

3.3.1 Dualism informed professional development

As per Strom and Viesca (2021, p. 210) I suggest a common philosophical perspective in work on professional development is that of dualism which 'conceives of reality in binaries or separations'. A perspective founded in dualism advances the idea there is a single universal reality, reduces complexity to simplicity, and assumes agency lays with human actors (Strom

and Viesca 2021). I concur with Strom and Viesca (2021, p. 221) that this approach has been highly influential in the development of professional development and has led to a focus on conformity and fidelity. Furthermore, it has been argued this traditional dualist conception of teacher professional development may contribute to some teachers feeling pedagogically confused, disappointed in the compromises they make to their beliefs, and angry at the practices they have been encouraged to adopt (Woodford, Clapham and Serret 2023). I suggest professional development founded in dualism places an ‘emphasis on sameness’ (Strom and Viesca 2021, p. 221), and would risk allowing the contradiction to be avoided in favour of creating opponents (McGowan 2019).

3.3.2 Multiplicity informed professional development

In rejecting dualism as an underpinning philosophy for teacher professional development, Strom and Viesca (2021) call for an alternative approach of multiplicity. They suggest this move to multiplicity would shift thinking from seeing ‘the world in individual, discrete units to an ‘and, and, and’ worldview of entangled, co-constructed multiplicities’ (Strom and Viesca 2021, p. 210). Strom and Viesca (2021) further suggest professional development founded in multiplicity is characterised by difference rather than sameness. I infer this perspective as underpinning the work of Swan *et al.* (2013) where the aim was not to force teachers to change their beliefs and practices, but to consider different approaches. Swan provided resources and opportunities to help teachers develop practices that would align to what he saw as the Connectionist perspective (Swan *et al.* 2013). In contrast to a dualist underpinning, I do not believe Swan viewed a single belief category as correct but sought to

support teachers in an under-resourced and under-considered belief category. The perspective favoured by Strom and Viesca (2021) is appealing but is problematic from a contradiction perspective since it retains the categorisation it claims to reject. In contrast, I claim a dialectic approach to professional development renders the assumptions of dualism obsolete through questioning the premise of noncontradiction.

3.3.3 Dialectic informed professional development

As discussed in chapter 1, I see McGowan's (2019, p. 85) claim that 'everything is also what it is not and has its identity in what negates it' as an ontological insight. So, whereas Strom and Viesca (2021) suggest the defining characteristic of teacher professional development should be difference, I suggest it is contradiction (Woodford, Clapham and Serret 2023). As McGowan (2019) argues, space and time allow contradiction to be viewed as external rather than internal and can therefore be mistaken for difference. There is a risk that a professional development programme is only seen as promoting a different approach, rather than highlighting inherent contradiction. Mathematics DPD should not suggest to participants there are different noncontradictory approaches to mathematics education, but one experience of contradiction. Indeed, I argue a mathematics DPD programme has a responsibility to surface the contradictions that exist as part of the process of reflection.

I additionally suggest a mathematics DPD programme, must show a contradiction informed understanding of teacher beliefs. As discussed in Chapter 2, teacher beliefs about mathematics education can be defined according to statements as seen in Swan's (2006b) use

of the Transmission, Discovery and Connectionist categories. Following a professional development programme, a participant might claim they have moved from a certain set of beliefs to a different set. However, McGowan (2019, p. 118) would describe this perspective as ‘utterly false’ since what we are not, is always integral to what we are. From a contradiction informed perspective, I suggest teacher belief categories may be better understood by the ‘failure to be self-identical’ (McGowan 2019, p. 214), and that ‘at all moments, what I am not remains integral to what I am’ (McGowan 2019, p. 118). Therefore, I anticipate the categories of Transmission, Discovery and Connectionist only provide a partial understanding of teachers’ beliefs about mathematics education. In this thesis, I will seek to explore how this understanding of teacher beliefs can be seen in a mathematics DPD programme.

3.4 The purpose of professional development

The second element of the Boylan, Coldwell *et al.* (2018) framework I consider is around the purpose of professional development. The wider debate around the purpose of professional development includes a discussion of teacher identity (Boylan, Coldwell *et al.* 2018) which I understand as ‘who teachers are, believe they are, and want to be’ (Garner and Kaplan 2019, p. 8). However, in discussing linear path models of professional development such as that of Guskey and Swan, Boylan, Coldwell *et al.* (2018) suggest the debate around purpose centres on supporting teachers versus changing student outcomes. As an additional consideration, it should be recognised both views may utilise student outcomes, in the form of test scores, as one measure of the programme. However, the distinction helps to focus attention on whether the purpose of mathematics DPD should be teachers or students.

3.4.1 Influencing teacher beliefs

Boylan, Coldwell *et al.* (2018, p. 134) identify ‘professional experimentation’ as a way in which teacher beliefs can be influenced. This description aligns well with the approach described by Swan *et al.* (2013, p. 945) of providing opportunities in professional development to examine Connectionist orientations rather than ‘traditional transmission-based approaches’. Swan *et al.* (2013, p. 947) express their desire to support teachers to ‘modify their beliefs’ through experimentation in new practices followed by opportunities for reflection. These opportunities for reflection are described as including the sharing of teaching experiences, discussing ‘pedagogical implications’ (Swan *et al.* 2013, p. 946) and reviewing the growth of new beliefs. I see this as a supportive and thoughtful approach, though from a contradiction perspective I would add the importance of providing opportunities for teachers to consider the contradictions revealed within the process of reflection. I suggest a professional development programme, with a purpose of contributing to the formation of teacher beliefs, will be characterised by isolated lessons and opportunities for reflection, rather than a sequence of lessons designed to improve student attainment.

3.4.2 Changing student outcomes

An alternative purpose for professional development is suggested by Desimone (2009, p. 183) as ‘student achievement’. It is interesting to note Guskey (2003, p. 750) also suggested a modified perspective of the ultimate purpose of professional development as being ‘improvement in student learning outcomes’. Whilst acknowledging the importance of teacher change, Filges *et al.* (2019, p.6) argue this is only done to ‘serve the ultimate beneficiaries of

the intervention, that is, the children’. Similarly, Sims *et al.* (2021, p. 5) prioritise impact on student attainment when they state a purpose of helping all teachers be ‘as effective as the best teachers’. In contrast to the purpose of influencing teacher beliefs, I suggest professional development programmes in this category are characterised by sequences of lessons designed to develop student understanding.

Examples within mathematics education of focusing on student outcomes can be inferred, although there is always an element of supporting teacher change within the programme. For example, I see the Improving Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) programme as providing sequences of lessons designed for students, based on developing classroom dialogue and teacher’s use of formative assessment (Pampaka *et al.* 2021). The impact on student outcomes is the primary purpose, along with a recognition that participants will also develop their pedagogy. Similarly, I infer the designers of the Realistic Mathematics Education (RME) programme as seeing their purpose to be changing student outcomes through developing alternative approaches to the “‘transmissionist’ classroom cultures’ (Dickinson *et al.* 2020, p. 342) in England. Indeed, Dickinson *et al.* (2020, p. 342) note that without providing more RME resources, some teachers reverted to more traditional approaches following the professional development programme. This issue of resources appears important, with Boyd and Ash (2018, p. 222) suggesting a well-designed mathematics textbook scheme, with an associated pedagogy, has the potential to improve student outcomes and ‘provoke change in embedded teacher beliefs that have proved resilient in previous reform efforts’. In addition, Sims *et al.* (2021, p. 53)

suggest lasting change in teachers is more likely when there is an emphasis on developing behaviour that aligns to ‘the school’s current practice, or aspirations’.

3.4.3 The purpose of mathematics DPD

I would not exclude the possibility of designing mathematics DPD with the purpose of changing student outcomes. However, I concur with Evans (2014) in being cautious about attempts to identify direct causal impact on students through professional development programmes. The reality of change is complex since:

‘new’ ideas or ways of thinking that have been planted within teachers’ consciousness may take time to blossom and to become gradually assimilated into their practice – and in the interim such ideas or perspectives may have been augmented (or diluted) through interaction with a myriad of other (often unrecognizable or unidentifiable) influences (Evans 2014, p. 188).

Therefore, for mathematics DPD, I suggest student outcomes are not ‘considered integral’ (Evans 2014, p. 188) to the purpose. In line with Swan *et al.* (2013), I argue the purpose of mathematics DPD is supporting the formation of teacher beliefs about mathematics education. Therefore, rather than being characterised by sequences of lessons, mathematics DPD should be characterised by experimentation and opportunities for reflection. This statement should also be understood in the context of the philosophical perspective of dialectics shared earlier in this chapter. Unlike Swan, my perspective is not to only offer support to develop other approaches and beliefs, but to also support teacher’s responses through an understanding of contradiction. To summarise, I suggest the purpose of

mathematics DPD is the formation of teachers' beliefs through an understanding of contradiction.

3.5 The components of professional development

Boylan, Coldwell *et al.* (2018, p. 123) describe the components of professional development as 'the essential elements of the model and the relationships between them'. Since Swan *et al.* (2013) utilised a linear approach to connecting the components, I focus my discussions on two alternative path models. In addition, I refer to Opfer and Pedder's (2011) complexity model but see this as a more useful way to understand professional learning rather than a support for the design of professional development (Boylan, Coldwell *et al.* 2018).

3.5.1 A linear model for the design of professional development

In their model, Swan *et al.* (2013) identify the components of professional development, teacher experimentation, student outcomes, teacher reflection, and teacher beliefs. Reflecting the purpose of influencing teacher beliefs, these components are considered to linearly relate to one another with an initial input of professional development leading to teacher experimentation, changes in student outcomes, teacher reflection (as part of the professional development programme), and a potential change in teacher beliefs. Significantly, it is reflection on the change in student outcomes, in terms of motivation and achievement, that Swan *et al.* (2013, p. 947) claim leads to 'changes in attitude'. This chain of logic reflects Guskey's (2002) general model of professional development and can be seen implicitly in other mathematics professional development initiatives. For example, a very

similar approach can be seen in the Maths-for-Life professional development trial in further education colleges which sought to support teachers to develop practices around dialogic learning (Wake 2022, Woodford and Wake 2023). The professional development programme included centrally designed lesson resources and employed an adjusted form of lesson study to look at student outcomes and support the reflections of teachers (Wake 2022, Woodford and Wake 2023).

3.5.2 An alternative linear model for the design of professional development

Reflecting the purpose of changing student outcomes, an alternative linear model suggested by Desimone (2009) adjusts the sequence of change. This model posits the component of professional development changes teacher beliefs, which consequently changes teacher practice, and leads to change in student outcomes (Desimone 2009). I tentatively suggest this linear process can be seen in the design of the Shanghai Teacher Exchange programme (Boylan and Adams 2024). Recognising I write in broad terms, government ministers in England encouraged the adoption of East Asian practices with the aim of improving student learning outcomes. Therefore, the input of the Shanghai Teacher Exchange programme in terms of observing Chinese teachers was intended to change teachers' beliefs, which would in turn change teacher practice, leading to changes in student results. As with any linear path model, the process appears to end, but it is reasonable to suggest any changes in student outcomes leads to subsequent changes in teachers. Indeed, changes to student's mathematical vocabulary and 'increased confidence, resilience and problem-solving abilities' (Boylan *et al.* 2019, p. 76) led to changes in participants' 'beliefs about mathematics teaching'

(Boylan *et al.* 2019, p. 20). In contrast to the previous linear path model, opportunities for reflection (as part of the professional development process) are more difficult to identify.

3.5.3 A complex model for professional learning

As with several other researchers (Opfer and Pedder 2011, McChesney and Aldridge 2021, Strom and Viesca 2021), I question the value of the dominant, linear, conceptualisation of professional development. Instead, drawing on complexity theory, Opfer and Pedder (2011) suggest the relationship between components of professional learning can be understood as three interacting subsystems of the school, the teacher, and the professional development activity. To this I would add a fourth influence of the larger educational system, as identified by Strom, Martin and Villegas (2018). A diagrammatic representation of these systems of professional learning is offered in Figure 3.1.

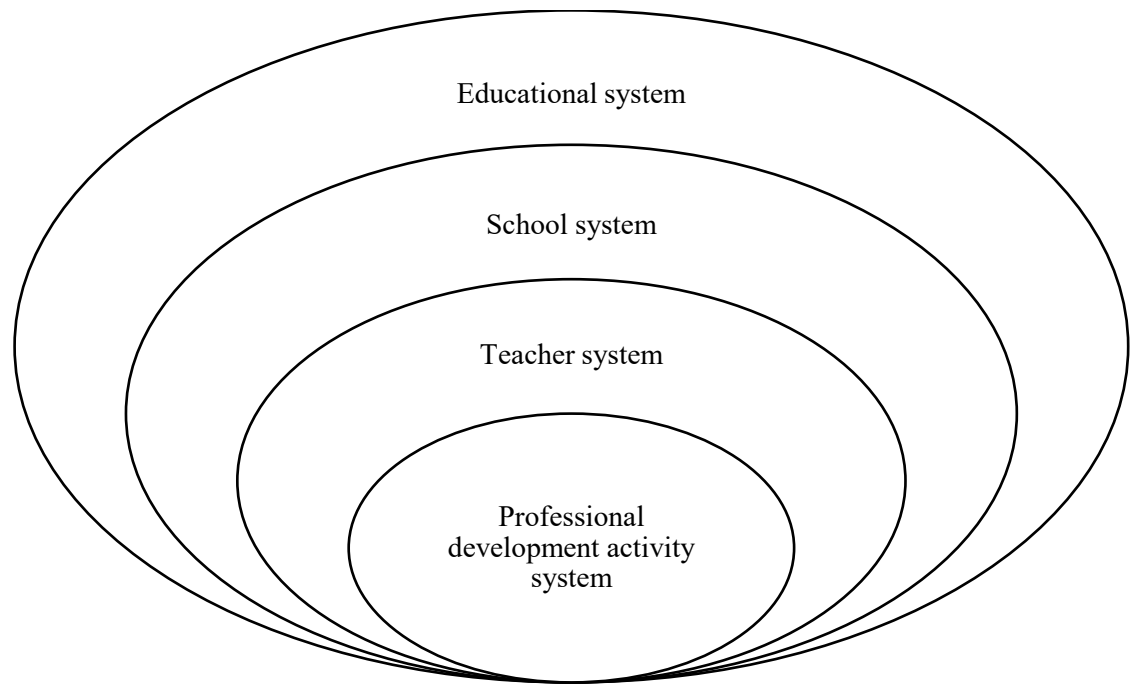


Figure 3.1 The systems of professional learning

At the educational system level, I recognise the dominance of neo-liberal discourses of performativity in England (Boylan, Coldwell *et al.* 2018). Moreover, after government intervention in education following the COVID-19 pandemic, Jones and Ball (2022, p. 1) suggest we are at ‘the start of another regeneration’ of neo-liberalism. Similarly, influences at the school system level in England can be seen in local adaptations of the borrowed teaching for mastery policy (Clapham 2024). In addition, at the local school system level, there is a ‘resurgence of teacher-led pedagogies’ (Wright, Fejzo and Carvalho 2022, p. 28) in England. At the level of the teacher system, as described in Chapter 2, mathematics teacher’s existing beliefs are recognised as affecting their trajectories of change following a professional development experience. Together, these influences highlight the risk of linear conceptions of

professional development, and of believing programmes can lead to consistent outcomes across all settings. As Strom and Viesca (2021, p. 221) note, recognising complexity explains why ‘it is simply not possible to ‘train’ teachers to implement pedagogies’.

3.5.4 The components of mathematics DPD

Despite recognising the value of complexity, I see Opfer and Pedder’s (2011) model as more helpful for understanding the professional learning a teacher experiences. For the design of mathematics DPD, I return to the linear path model components utilised by Swan. However, I suggest Swan recognised wider influences on professional learning but saw them as influencing a teacher’s experimentation and reflection. For instance, amongst reasons for teachers being resistant to change, Swan identified factors at each of the system levels in Figure 3.1. He noted how teachers referred to national examination pressures, student attitudes, and personal beliefs about mathematics education as reasons for not adopting new practices. Therefore, for the design of mathematics DPD, I adopt the linear path model of seeing an initial professional development input encouraging teacher experimentation, leading to reflection on changes to student outcomes, supporting the formation of teacher beliefs about mathematics education.

3.6 The scope of professional development

Drawing on Boylan, Coldwell *et al.* (2018), one way to understand scope is to consider the elements that contribute to the professional development programme. Therefore, in this sub-section I review the design suggested by Swan *et al.* (2013), before examining a more

general list of features of effective professional development and the notion of causal mechanisms. This leads to the identification of elements I suggest are important in the design of mathematics DPD.

3.6.1 The scope of the PRIMAS professional development programme

Swan *et al.* (2013) conceived the scope of PRIMAS at the meso-level (Boylan, Coldwell *et al.* 2018) in relating their designed activities to professional learning. They suggested a five-part structure to the programme of identifying current beliefs and practices; considering contrasting practices; examining a provided lesson plan; teaching a lesson and analysing the lesson (Swan *et al.* 2013). They acknowledge this model ‘has resonance with Japanese Lesson Study’ (Swan *et al.* 2013, p. 946), though additionally recognised the need for adaptations in a European context. For instance, on a practical basis they suggested there would be no requirement for teachers to watch other teachers’ lessons nor for an outside expert to support lesson analysis.

There has been a growing interest in lesson study, and Japanese lesson study specifically, as a form of professional development in both England and the rest of the world (Seleznyov, Goei and Ehren 2024, Wake 2024). One distinguishing feature of Japanese lesson study is the prioritisation of developing mathematical thinking in students rather than demonstrating a model lesson (Woodford 2024). Seleznyov (2019) identifies the critical components of Japanese lesson study as a school-wide research focus, collaborative creation of a lesson plan, the teaching and observation of a research lesson, a formal post-lesson

discussion, and the involvement of an outside expert. Shimizu and Kang (2022) describe the importance of the post-lesson discussion to the model of Japanese lesson study in supporting teacher reflections. They note how the formal process encourages participating teachers to focus their reflections on the development of pedagogies to support student learning and problem solving. Shimizu and Kang (2022) further suggest it is the focus on student learning that facilitates the professional growth of teachers. This Japanese influenced approach can be seen in the work of Swan *et al.* (2013) and has been adopted, and adapted, by Wake (2018, 2022, 2024). Wake (2024) reports that following the use of an adjusted form of lesson study, teachers had a better understanding of applying the principles of Teaching for Mastery (in a post-16 context), changed their immediate practice, changed their intended practice for future years, and saw improved student understanding and engagement. Once again, the value of an emphasis on reflection around student outcomes can be seen.

3.6.2 General components of effective professional development

Before considering the applicability of features of the design of Swan *et al.* (2013) to mathematics DPD, I first note the wider debate around features of effective programmes. For example, Desimone (2009) argues effective professional development is subject focused, involves active learning, is spread over time, is a collective endeavour, and coheres with existing curricula. However, this list is not intended as a definitive list with, for example, Darling-Hammond, Hyler and Gardner (2017) concurring with the first four elements but also suggesting that coaching, mentoring, and feedback should be included.

Within mathematics education, Joubert and Sutherland (2009) suggest a similar list of effective elements, stating professional development must be sustained over time; include elements of mathematics, learning mathematics and teaching mathematics; should be grounded in classroom practice; and should provide opportunities for reflection and enquiry. I note this study is often quoted in mathematics research (Foster 2022, Treacy and Leavy 2023, Wake *et al.* 2023), which may create a compounding effect around the essential nature of these elements. In contrast, in the evaluation of the Shanghai Mathematics Teacher Exchange (Boylan, Maxwell *et al.* 2018), it is suggested effective professional development includes the features of being sustained over time; adapted to local requirements; tailored to teacher needs; focused on adaptation rather than reproduction; include opportunities for enquiry; and provide opportunities for collaboration. This lack of agreement around essential elements of effective professional development suggests caution needs to be adopted. As Asterhan and Lefstein (2024, p. 11) advise, it is the desire to find ‘simple answers to complex questions’ that can lead to attempts to produce a list of effective features of professional development.

Rather than seek to synthesise the already synthesised lists of effective features, I concur with Sims and Fletcher-Wood (2021) and note the flaws in logic. They argue the creation of such lists may be based on inappropriate inclusion criteria and on incoherent inference processes. With respect to inference, Sims and Fletcher-Wood (2021) suggest the existence of a feature within an intervention should not be used to conclude the feature is necessary. So, the presence of opportunities for reflection in a successful intervention is not sufficient to imply it is essential. Asterhan and Lefstein (2024) extend the criticism by

identifying that the professional development intervention itself, and not its features, is usually the object of study in research. They therefore suggest any claims about effective features should be based on studies that trial the intervention in different ways. To some extent, this approach can be seen in the Centres for Excellence in Maths Teaching for Mastery trial (Wake *et al.* 2023) which included a comparison of an intervention with lesson study to the same intervention without lesson study. Indeed, Wake *et al.* (2023) suggest the trial with the lesson study element led to additional student progress compared to that without.

A causal mechanism approach to professional development

In addition to effective features of professional development, there is also support for finer design granularity in terms of causal mechanisms (Sims *et al.* 2021). These mechanisms are seen as ways to change behaviour and appear to be grounded in cognitive and behavioural science. For example, Sims *et al.* (2021) group fourteen causal mechanisms into four categories of instilling insight, motivating goal-directed behaviour, teaching new techniques, and embedding practice. I suggest reducing design to causal mechanisms is what Wenger (1998, p. 265) warned against when he stated the mechanics of learning should never ‘become the primary focus of educational design’. Since the causal mechanism approach places little value on teacher reflection it is not a perspective I will base decisions on when designing mathematics DPD.

3.6.3 The scope of mathematics DPD

For mathematics DPD, I recognise the five elements suggested by Swan *et al.* (2013) help to place a focus on teacher beliefs and reflection. In Table 3.2, I suggest adjustments to the five elements (both in order and emphasis) that ensures consistency with my contradiction informed perspective. I highlight the importance of opportunities for reflection around contradiction in three of the stages. This is intended to support my view that the purpose of mathematics DPD is the formation of beliefs through an understanding of contradiction.

Element	Name	Description
1	Examination of existing beliefs	Teachers should have a structured opportunity to identify their current beliefs about mathematics, teaching mathematics, and learning mathematics.
2	Examination of mathematics	Teachers should have opportunities to examine different mathematical approaches to solving a problem, and to reflect on the beliefs about mathematics they view as underpinning the mathematics problem.
3	Examination of teaching mathematics	Teachers should have opportunities to examine a pre-designed research lesson plan, and to either commit to certain key actions, or justify an alternative approach they will adopt. Teachers should reflect on the beliefs about teaching mathematics they believe underpin the design of the lesson plan.
4	Teaching of a lesson	Teachers should have opportunities to teach the research lesson individually, according to their adapted version of the research lesson plan.
5	Examination of learning mathematics	Teachers should have the opportunity to discuss the learning of the students in their research lesson. Teachers should reflect on the beliefs about learning mathematics they saw in their research lesson.

Table 3.2 Suggested elements of mathematics DPD

Together the five stages in Table 3.2 create an adjusted form of lesson study which I see as suitable for mathematics DPD. At the end of each of elements 2, 3 and 5, teachers are asked to reflect on beliefs about mathematics, teaching mathematics, and learning mathematics respectively. In doing this, there is an anticipation that the contradictory nature of the underpinning beliefs behind the approach being examined will be revealed.

Whilst I do not suggest an adjusted form of lesson study is the only design that is appropriate for mathematics DPD, I do suggest it aligns with the stated purpose. Indeed, Stigler and Hiebert (2016) suggest modifications of Japanese Lesson study may be useful in providing opportunities for teachers to reflect on their cultural beliefs. Swan *et al.* (2013) recognised the resemblance of their model to Japanese lesson study, with the practical adaptations suggested of not requiring teachers to watch the research lessons of others and not utilising an outside expert to analyse lessons. I will adopt the same practical adaptations for the model of lesson study I utilise in mathematics DPD. In addition, Nguyen and Tran (2023) note how a cross-cultural lesson study process led to changes in mathematical knowledge and beliefs about learning mathematics for participants. They suggest the use of resources ‘adopted from different cultures’ (Nguyen and Tran 2023, p. 831) is pivotal in encouraging teacher reflections. Thus, I anticipate an adjusted form of lesson study including a research lesson plan drawing on approaches from different countries, is a useful feature to encourage teachers to reflect on the contradictions inherent within cultural beliefs about mathematics education.

3.7 The theory of learning in professional development

The fifth element of the Boylan, Coldwell *et al.* (2018) framework I consider is around the theory of learning. Once again, I begin by considering the theory of learning adopted by Swan *et al.* (2013) before applying my McGowan (2019) informed perspective.

3.7.1 A social theory of learning for professional development

I infer the work of Swan *et al.* (2013) as falling under the broad category of a sociological perspective with a clear statement around the influence of ‘social constructivist and socio-cultural theories of learning’ (Swan *et al.* 2013, p. 946). Similarly, the cause-and-effect approach of social realism can be seen in the linear conception of professional development eventually leading to changes in teacher beliefs of Swan *et al.* (2013). Other models of mathematics professional development in England have also drawn on theories within the broad umbrella of a sociological perspective. For example, Wake (2022) draws on activity theory and communities of practice to explain learning at both the collective and individual level. Similarly, the ICCAMS intervention utilised ‘peer support networks’ (Pampaka *et al.* 2021, p. 16) to aid in the development of approaches.

3.7.2 A dialectic theory of learning for professional development

Within mathematics education, Williams and Ryan (2014, 2020) have provided an example of socio-cultural, dialectic thinking in mathematics professional development. They claim a well-designed lesson study process can challenge teachers’ current practices, and lead to a new set of beliefs about mathematics education. They adopt an epistemological

conception of dialectics as determinate negation through the thesis-antithesis-synthesis model (Williams and Ryan 2020, Williams and Ryan 2014) discussed in Chapter 1. For instance, they might see a teacher's beliefs as the thesis, observed student outcomes as the antithesis, and a new approach adopted as the synthesis. I see this approach as representative of professional development that employs dialectics as a theory of learning, but view it as an inaccurate, and insufficient, interpretation of Hegelian dialectics.

3.7.3 A theory of learning for mathematics DPD

For me, mathematics DPD should not be about moving through a process which is designed to 'progress from the old to the new level of thought' (Williams and Ryan 2014, p. 379) through the synthesis of two positions. Instead, I suggest professional learning takes place in mathematics DPD through the revelation of contradiction. I believe teachers should be encouraged to examine a new approach, reflect on the beliefs about mathematics education underpinning the approach, and be encouraged to notice the contradictions that exist. Rather than seeking to resolve the contradiction I take the perspective that the contradiction cannot be resolved (McGowan 2019). Ultimately, mathematics DPD should include cycles of examining different approaches and revealing the contradictions within each. I suggest professional learning takes place in the social context of teachers forming their beliefs about mathematics education through their desire to trial new approaches, examine the new approaches, identify the contradictions, and move on to trial another approach. Dialectics as a learning theory becomes a source of animation, not by seeking to eliminate contradiction, but by finding 'a path to sustain it' (McGowan 2019, p. 16).

It is possible to interpret the experiences of some of the teachers in the PRIMAS project according to my dialectic theory of learning. Swan *et al.* (2013) reported 31 teachers (out of 71) began their professional development programme with what is described as a Transmission set of beliefs about mathematics education. These teachers experienced the same programme as the other 40 teachers and identified ‘the shortcomings of transmission methods’ (Swan *et al.* 2013, p. 951). Whereas Swan *et al.* (2013) interpret this as a reason for the teachers to move to a different set of beliefs, I re-interpret the teachers as identifying contradiction in the Transmission approach. In addition, this example provides an indication that teachers with different beliefs about mathematics education may have different views of the methods being supported by the professional development programme. Whilst Swan *et al.* (2013) saw the teachers’ subsequent movement to a new belief position as following from the resolution of contradiction, I suggest they may have moved from one contradictory approach to another.

3.8 Agents of change in professional development

In this section I review the theorisation of the agents of change (Boylan, Coldwell *et al.* 2018) conceptualised by Swan *et al.* (2013) in their model of professional development. Following links to wider education literature, I apply my contradiction informed perspective to suggest an understanding of teacher agency in mathematics DPD.

3.8.1 Agents of change in the PRIMAS model

Swan *et al.* (2013) conceived of change as a linear process which involves the input of the professional development programme, trialling new practices, reflecting on the new practices, and the modification beliefs. Therefore, I suggest the agents of change are understood as the professional development programme, the students who experience the new practices, and the teacher involved.

The influence of professional development programmes

Drawing on activity theory, Swan *et al.* (2013) theorise the influence of the professional development programme resources they design. They suggest the resources they create (tools) are mediated by the curriculum goals (rules), the school (communities) and the roles of participants (division of labour). A similar approach is seen when Wake (2022, p. 17) affirms the importance of careful design so that ‘each of the elements we produce should not be left to chance’. Therefore, I suggest the importance of careful design in all elements of a lesson study process.

The influence of students

Swan *et al.* (2013) acknowledge the role of students in bringing about change to teacher’s beliefs. They suggest beliefs about mathematics education change once teachers see the new practice ‘improves student motivation and achievement’ (Swan *et al.* 2013, p. 947). There is some indication of this occurring in practice in the paper by Swan *et al.* (2013),

however they also note factors that countered the improvements in student motivation and achievement. For instance, Swan *et al.* (2013) identify how some teachers felt students could not cope with the changed practices, and others worried about a negative influence on national exams.

The influence of teachers

Following the trialling of new resources, Swan *et al.* (2013) suggest the teacher is then responsible for change through reflection. Here Swan *et al.* (2013) utilised their lesson study model so that teachers ‘reflect on the growth of new practices and beliefs’ (2013, p. 946). The importance of teacher agency in change can also be seen in the identification of trajectories for teachers based on the teacher’s initial beliefs about mathematics education. For instance, most teachers initially classed as having Discovery beliefs prior to the professional development programme moved to Connectionist following the intervention (Swan *et al.* 2013).

3.8.2 Agents of change in mathematics DPD

For mathematics DPD I suggest changes in teacher beliefs are influenced by the design of the research lesson for professional experimentation, the learning that is seen, and the teachers’ current beliefs. I value the opportunity to ‘create constructive tensions’ (Garner and Kaplan 2019, p. 28) where teachers can look to ‘handle, and not eliminate, the resistance, ambiguities and tensions’ (Hordvik, MacPhail and Ronglan 2020, p. 10). In this work I will refer to these constructive tensions as occurring in *moments of variation* which I explain by drawing on McGowan’s (2019) description of the divided self.

The divided self

Following through on his claim that contradiction is an ontological feature, McGowan (2019) extends his thinking to provide an explanation of some human actions. In short, McGowan (2019) suggests the contradiction of human nature can be seen in the conscious and the unconscious of the divided self. I will explain this with an example before drawing out important aspects of the claim and applying it to teacher professional development. McGowan (2019) illustrates his thinking with a person consciously seeking to overcome the contradiction between wanting to live in a nice house and their lack of money, through working long hours. The individual may indicate a conscious desire to overcome the contradiction, but their unconscious actions reveal their true intentions. For example, the individual may unconsciously oversleep and be late for work, act inappropriately at a party, or have an affair with the manager's spouse. In each of these cases, McGowan (2019) claims the individual creates the conditions to sustain the contradiction. McGowan (2019) argues the individual's actions reveal it is working long hours, not the nice house, that have become the real source of satisfaction. Despite conscious claims of what is desired, McGowan (2019) argues the unconscious seeks to self-sabotage and sustain the conditions for the real source of satisfaction.

In terms of teacher agency, the important features to note in this example are the role of conscious desires and unconscious actions. For McGowan (2019, p. 42), the existence of our unconscious fits with a contradiction informed ontology and reveals we are 'inextricably out of joint' with ourselves. Our conscious desire may be to overcome contradiction and

achieve harmony, but this is thwarted by our unconscious desires. Drawing on Freud's work in *Beyond the Pleasure Principle*, McGowan (2019) argues it is our unconscious that drives our actions so that we subvert our conscious desires and sustain engagement with the contradiction. We unconsciously find satisfaction 'through the failure to attain' (McGowan 2019, p. 50) that which we consciously think will resolve the contradiction. Hence, I suggest in professional development, teachers should be understood as agentic though with a recognition they may subvert their conscious desires through unconscious actions. To generalise this argument, I suggest McGowan (2019) identifies an individual seeking to overcome the contradiction between A and B through C. Actions are then interpreted as a desire to maintain the contradiction between A and B, with the inference that satisfaction is found in C.

To provide an example in an education context, I suggest a teacher might consciously seek to overcome the contradiction between improving student outcomes (A) and a lack of student engagement (B) by trialling a new approach suggested in a professional development programme (C). However, their unconscious might find ways to subvert the new approach so the contradiction would be maintained. For example, the teacher might change the suggested approach or make a subversive adjustment during the lesson. The teacher unconsciously maintains the contradiction between student outcomes (A) and engagement (B). Hence, it could be argued the teacher finds unconscious satisfaction in repeating experiences of professional development (C) rather than in achieving better student outcomes (A). Whilst I

acknowledge better student outcomes may provide temporary satisfaction, I also suggest this would soon be replaced by the desire for even better examination outcomes in the next year.

In mathematics DPD, I intend to take notice of the contrast between conscious claims and contrasting actions. The importance of actions is highlighted when McGowan (2019, p. 43) suggests what we claim about ourselves ‘falls aside’ in comparison to what we do. Therefore, in mathematics DPD I will design for, and take note of, moments of variation where there is an opportunity for conflict between what a teacher claims they want and an interpretation of what is seen. I further develop this idea in Chapter 5 when applying my analytic framework to my collected data.

3.9 Summary of Chapter 3

In this chapter I have used a professional development framework suggested by Boylan, Coldwell *et al.* (2018) to interrogate the approach taken by Swan. Through considering Swan’s approach to my philosophical perspective, I have created an emerging specification for the design of mathematics DPD. This specification is summarised in Table 3.3 and forms the foundation of the design work discussed in the next chapter.

Framework element	Summary for mathematics DPD
Philosophical foundations	Hegelian contradiction as an ontological feature in line with the thinking of McGowan (2019).
Purpose	The formation of teacher beliefs about mathematics education through an understanding of contradiction.
Components	A linear path of professional development, to teacher experimentation, to reflection on changes to student outcomes, to potential changes in teacher beliefs.
Scope	Activities designed around an adjusted form of lesson study which facilitate an examination of beliefs about mathematics, teaching mathematics and learning mathematics.
Theory of learning	Professional learning understood as reconciliation to the existence of contradiction in beliefs about mathematics education. The process of reconciliation requires repeated study and reflection on alternative approaches and beliefs about mathematic education.
Agents of change	The teacher is agentic, though as a divided self has both conscious and unconscious desires. Moments of variation may reveal where the unconscious seeks to remain engaged in contradiction through subverting the conscious desires.

Table 3.3 A framework for mathematics DPD

Chapter 4.

The Rethinking Approaches in Mathematics programme

4.1 Synopsis of Chapter 4

Having conceptualised mathematics Dialectic Professional Development (DPD) in the previous chapter, I now introduce the resources designed specifically for this study. I created the Rethinking Approaches in Mathematics (RAM) programme, for mathematics teachers of students aged 11-16, according to the framework elements of mathematics DPD summarised in Table 3.3. Underpinned by my philosophical perspective and purpose, I discuss the design of the programme shaped around the components of a linear path model. Initially I provide an in-depth discussion of the problem that formed the basis of the research lesson which was studied as part of the adjusted lesson study process. Following this discussion, I focus on the interaction between the final three framework elements of mathematics DPD in terms of scope, theory of learning, and agents of change. Therefore, I firstly explain the adjusted form of lesson study I designed to satisfy the scope element. Secondly, I explain how my theory of learning led to design decisions around the foregrounding of beliefs about mathematics education. Thirdly, I explain how an understanding of the conscious and the unconscious, drawn from the agents of change framework element, influenced my design decisions around moments of variation.

4.2 Introduction

In developing this study, I considered whether I should utilise an existing professional development programme or design my own. Using an existing programme may have been possible and may have placed an emphasis on interpreting unanticipated moments of variation. However, having specified mathematics DPD in the previous chapter the approach felt sufficiently distinct to Swan's work to warrant the creation of a new set of resources. Whilst I share the same desire to support teachers through reflection, I think my emphasis on contradiction as a theory of learning requires new design. Similarly, I suggest my understanding of McGowan's (2019) divided self means I felt there was a need to design opportunities for moments of variation into the professional development programme. Therefore, in this chapter, I explain my design of the RAM programme intended to exemplify the framework for mathematics DPD. Before explaining the structure of the RAM programme, I first explain the design of the research lesson that became the focus of the adjusted form of lesson study. Although presented sequentially here, the true nature of the development was more cyclic with the design of the research lesson affecting the design of the RAM programme, which in turn led to adjustments in the design of the research lesson and so on.

4.3 The design of the RAM research lesson

As discussed in Chapter 3, I intended to utilise an adjusted form of lesson study withing the RAM programme. Therefore, I sought to design a research lesson that could be studied by the participants, and which would lead them to consider the contradictory nature of

beliefs about mathematics education. The intention of the research lesson was not to claim it offered the ‘correct’ way to teach, nor to try to tell teachers they should adopt this approach for all their future lessons. Instead, I wished to use the research lesson to prompt reflection around beliefs about mathematics education based on an approach taken in another country. Mellone *et al.* (2019, p. 199) refer to this as ‘cultural transposition’ where cultural practices in one country can be challenged through a consideration of pedagogy taken from another. Furthermore, Nguyen and Tran (2023) note such cultural transposition can impact both teacher knowledge and teacher beliefs.

4.3.1 Drawing on Japanese lesson design

Whilst resources for cultural transposition may come from many countries, I chose to design a lesson based on thinking and research from Japan. I grounded this decision on my experiences of Japanese education, and through identifying clear cultural education differences between England and Japan. Japanese approaches to mathematics education stem from a different set of underlying beliefs about mathematics, teaching mathematics, and learning mathematics compared to those typically held by Western teachers (Stigler and Hiebert 2009). Stigler and Hiebert (2009) contrast the predominant Western belief that mathematics is a set of procedures with the dominant Japanese view that mathematics is about relationships. In a similar fashion, Mullis *et al.* (2020) suggest the style of lessons Japanese students experience leads to them outperforming students in England in terms of problem solving.

Furthermore, in 2015 I witnessed several Japanese mathematics lessons in person across a range of age groups, whilst participating in the International Math-teacher Professionalization Using Lesson Study (IMPULS) programme. As part of the IMPULS programme, I witnessed lessons that were designed, and taught, based around the Teaching Through Problem-solving (TTP) structure (Takahashi 2021). Takahashi (2021) states a TTP lesson includes the phases of reviewing previous learning, posing a new problem, individual student problem solving, teacher led comparison and discussion, and highlighting key concepts. For me, this style of lesson offered a contrast to the structure I typically saw in English schools and prompted deep personal reflection. Notably, I was challenged to consider how learning was drawn from the experience of problem solving, the use of a single mathematics problem throughout the lesson, an extended period of individual student problem solving, the ability of teachers to compare alternative methods, and the careful use of the blackboard during the lesson. These are all features which I believed might become a source of reflection for teachers in an English education context.

4.3.2 The Japanese mathematics lesson

The Japanese mathematics lesson I decided to base my research lesson on has been used in Japan for many years (Baldry *et al.* 2023). It featured in the 1999 Trends in International Mathematics and Science Study (National Center for Education Statistics 1999), and I witnessed a version of the lesson being taught in Japan as part of the IMPULS programme in 2015. Using a TTP style lesson structure, students are asked to individually solve the problem shown in Figure 4.1

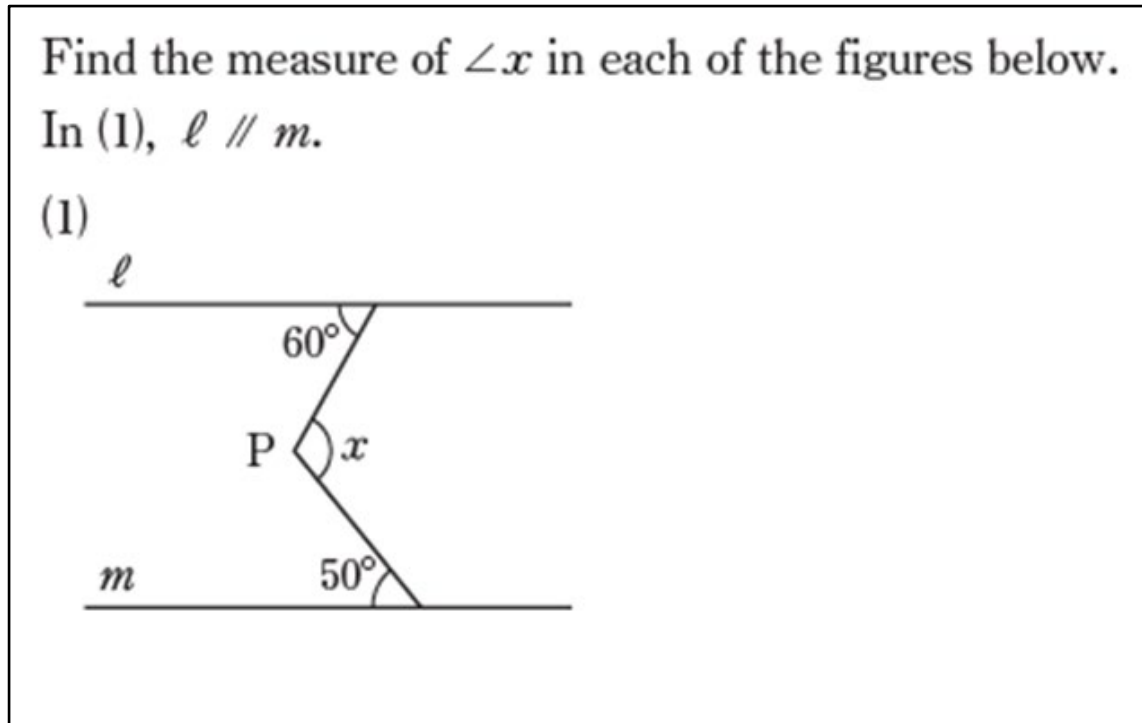


Figure 4.1 The missing angle problem from *Fujii et al.* (2012)

The topic of the question (angles between parallel lines) is familiar to teachers in England, but from my experience the specific problem is less familiar. In contrast to Figure 4.1, questions on this topic in England tend to focus on a simpler variant of a pair of parallel lines with a single transversal line passing through them. The problem is particularly suitable for a TTP style lesson since it fulfils the requirement of Takahashi (2021) of potentially being solved in a variety of ways. However, I required the problem to be suitable not only for the classroom, but also for a professional development programme for teachers. Therefore, I sought to ascertain whether the problem would be challenging, and of interest, to teachers in England.

4.3.3 Examining the mathematics problem

Ascertaining whether the problem shown in Figure 4.1 would be appropriate for teachers in England took place in two stages. Firstly, to provide an indication of the methods that could be reasonably anticipated I asked a group of trainee mathematics teachers to attempt the problem. Secondly, I decided to look at the effects of varying the size of the angles through discussion with an experienced mathematics colleague. Ethical permission and informed consent for this were gained from participants in line with the procedures described in more detail in Chapter 5.

Trialling with trainee mathematics teachers

To elicit whether the problem would be likely to lead to multiple methods in an English context, I asked fifteen trainee mathematics teachers to attempt the problem shown in Figure 4.2. These trainee teachers were part of a University programme I taught on, and all fifteen trainees provided informed consent for me to use their thinking. The trainee teachers provided their approaches to the problem in an online environment and were unable to see each other's work until they were all revealed together at the end.

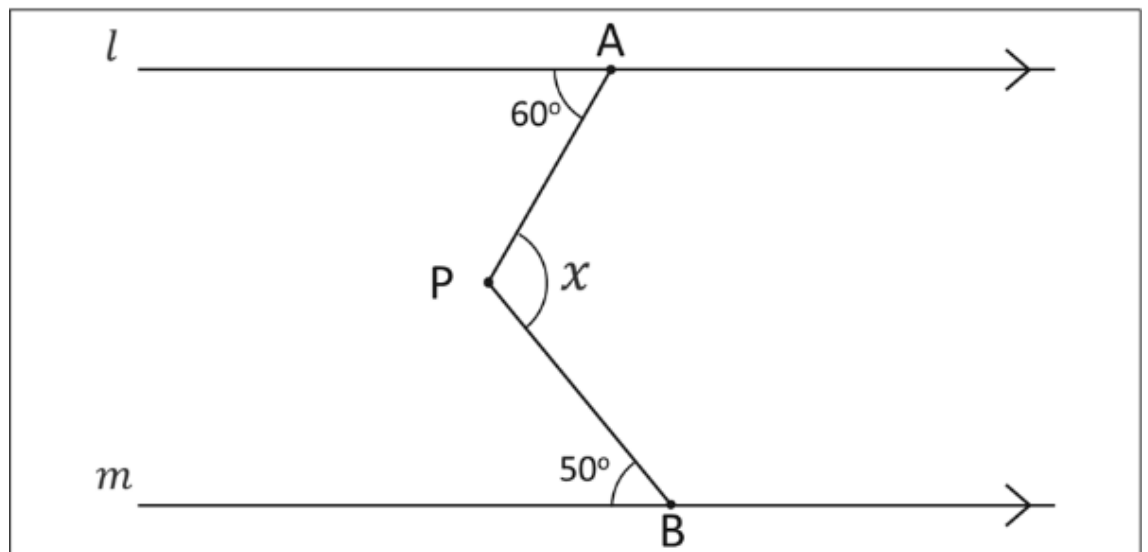


Figure 4.2 My copy of the trialled problem

In Table 4.1, I display the methods of the trainee teachers, grouped according to the nature of the auxiliary line, alongside a fifth (unsuccessful) method. This is the classification approach utilised in the Japanese lesson I witnessed and is mentioned briefly by Foster (2023) as providing a domain specific problem-solving strategy. In summary, the trainee mathematics teachers attempted to add an auxiliary line either parallel to l and m , perpendicular to l and m , connecting A and B , or as an extension of AP/BP . The fifth method involved students not adding an auxiliary line and merely working out angles they could see.

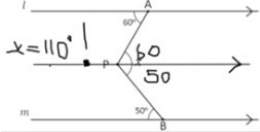
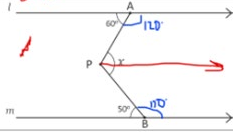
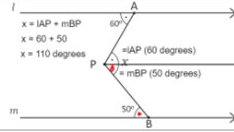
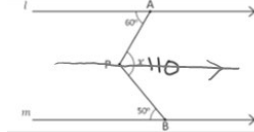
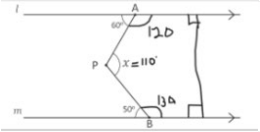
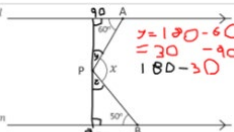
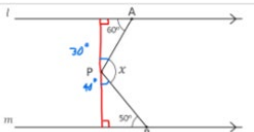
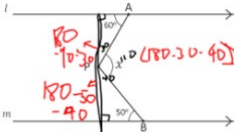
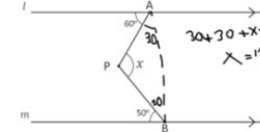
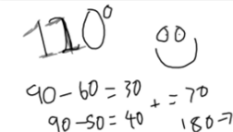
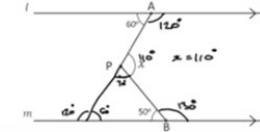
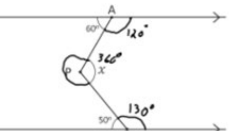
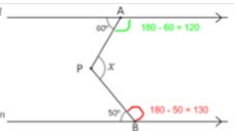
Method	Trainee responses				
A: Adding a parallel line	   				
B: Adding a perpendicular line	 Trainee 4	 Trainee 6	 Trainee 7	 Trainee 13	 Trainee 14
C: Adding a line connecting two points	 Trainee 5	 Trainee 9	 Trainee 11	 Trainee 12	 Trainee 14
D: Adding an extended line	 Trainee 3	 Trainee 8			
E: No auxiliary line	 Trainee 1	 Trainee 10	 Trainee 15		

Table 4.1 Different methods found by trainee teachers to solve the problem

Though the methods in Table 4.1 are not the only possibilities, this trial confirmed it was likely that a group of mathematics teachers would find the problem interesting and would attempt a range of alternative strategies. This trial also revealed that method C (adding a line connecting two points) may be difficult for teachers to use to gain a correct answer. Trainee 3 and trainee 8 reveal a misconception that points A and B are vertically opposite each other. This problematic approach became an important feature of the RAM session 1 design as a potential moment of variation which would challenge the thinking of teachers.

Discussion with an experienced colleague

To gain further insight into the mathematical problem, I asked a colleague to attempt two different versions of the problem and discuss their thinking with me. The two versions of the problems are shown in Figure 4.3, one with angles of 60° and 50° and the other with angles of 30° and 50° . The 30° - 50° version of the problem was chosen deliberately to see whether the location of point P, and therefore the presentation of the problem, would affect solution methods.

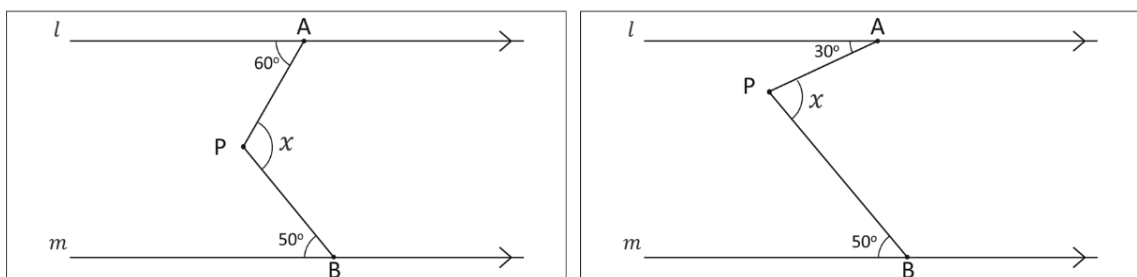


Figure 4.3 Two alternative versions of the problem

The discussion with my colleague firstly led to the confirmation that the solution methods could be grouped according to the nature of the auxiliary line added. Additionally, we also discussed the possibility of solving the problem by creating a reflection at various locations, such as illustrated in Figure 4.4. After discussion, we concluded this solution could also be classified under method B as ‘adding a perpendicular line’ though recognise it differs slightly in nature to the other strategies seen in the method B row. Again, this would be considered as a potential moment of variation for teachers in the design of the RAM resources.

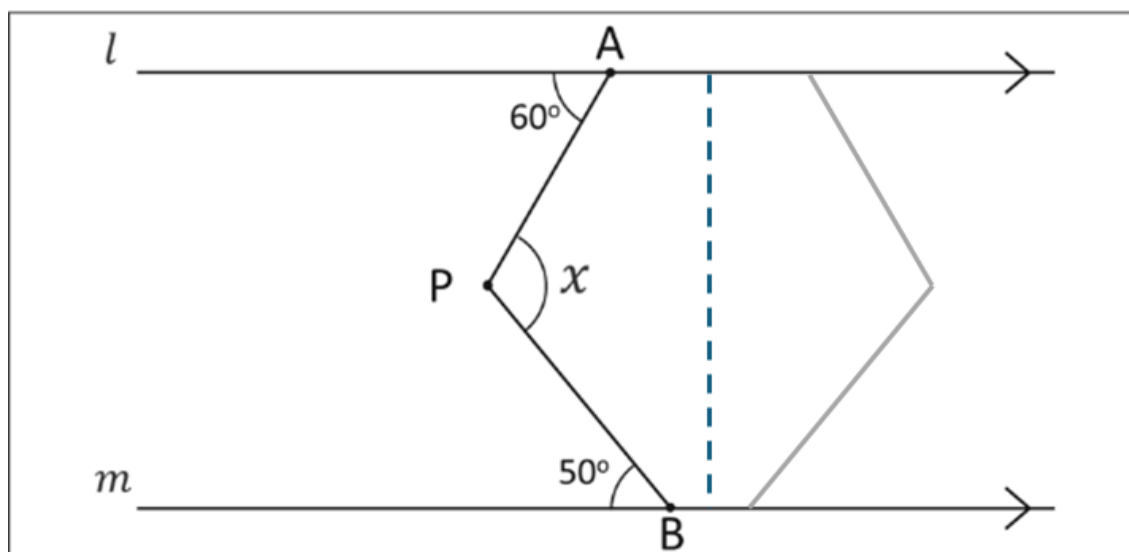


Figure 4.4 Solving by reflecting in a perpendicular line

We also agreed the 60°-50° version of the problem lent itself to finding more methods than the 30°-50° version. For example, in the 60°-50° version of the problem, we felt the line

segment AP could be extended to the parallel line m, but this is not possible without extending line m in the 30°-50° version of the problem. Therefore, after briefly considering minor adjustments to the angles, I decided to retain those provided in the original Japanese version of the problem.

4.3.4 Research lesson design

With confidence in the suitability of the problem for teachers, I created a research lesson plan according to the structure of TTP (Takahashi 2021). In creating the lesson plan, I utilised my personal notes of the lesson I witnessed in 2015, and the TIMMS 1999 resources (National Center for Education Statistics 1999). I recognise that in creating an interpretation of a Japanese lesson, some elements of the original design may be lost or modified. However, my experience of the lesson gives me confidence that I retained and highlighted some important contrasting features for teachers in England. A summary of the lesson is shown in Table 4.2 which is based on the full research lesson plan shown in Appendix 8, and the accompanying lesson slides shown in Appendix 1.

Phase	Phase title	Approximate length of phase (minutes)	Summary of phase
1	Review previous learning	5 – 10	Whole class facilitated recall of angle rules. To include recall of angles on a straight line, angle sum for polygons, angles between parallel lines.
2	Pose a new problem	5 – 10	Presentation and explanation of the mathematical problem through step-by-step construction with students.
3	Individual student problem solving	10 - 15	Students individually solve the problem, trying to find as many methods as possible with limited teacher intervention. At the same time, the teacher notes down different methods attempted by students.
4	Teacher led comparison and discussion	15 – 20	Teacher facilitates comparison of student methods, with all methods displayed on the classroom whiteboard.
5	Highlight key concepts	10 – 15	Teacher draws attention to the key points, including: • the addition of auxiliary lines, • grouping of methods according to the auxiliary line, • angle facts which are enabled through the addition of the auxiliary line. Extension problems provided if necessary.

Table 4.2 Overview of the RAM research lesson plan

The summary of phase column in Table 4.2 provides a brief description of the teacher action column shown in the research lesson plan in Appendix 8. Further discussion of the other two columns in the research lesson plan of Appendix 8 (decisions about teacher actions and decisions about student actions) takes place later in this chapter.

4.4 The design of the RAM programme

Following the creation of a research lesson, I sought to design the RAM programme based on an adjusted form of lesson study as discussed in the previous chapter. In this section I provide an overview of the programme and discuss how I designed resources to include a consideration of beliefs about mathematics education, and to include potential moments of variation.

4.4.1 An outline of the RAM programme

The RAM programme consists of an introductory session, three departmental sessions, and a research lesson to be taught by each participating teacher. A pictorial summary of this adjusted lesson cycle is shown in Figure 4.5 and is based on the Japanese influenced Collaborative Lesson Research cycle suggested by Takahashi and McDougal (2016). Takahashi and McDougal's (2016) elements of selecting a theme and topic, carrying out mathematical understanding, developing a research lesson plan, teaching a research lesson, and conducting a post lesson discussion correspond to each of the five stages of my adapted cycle.

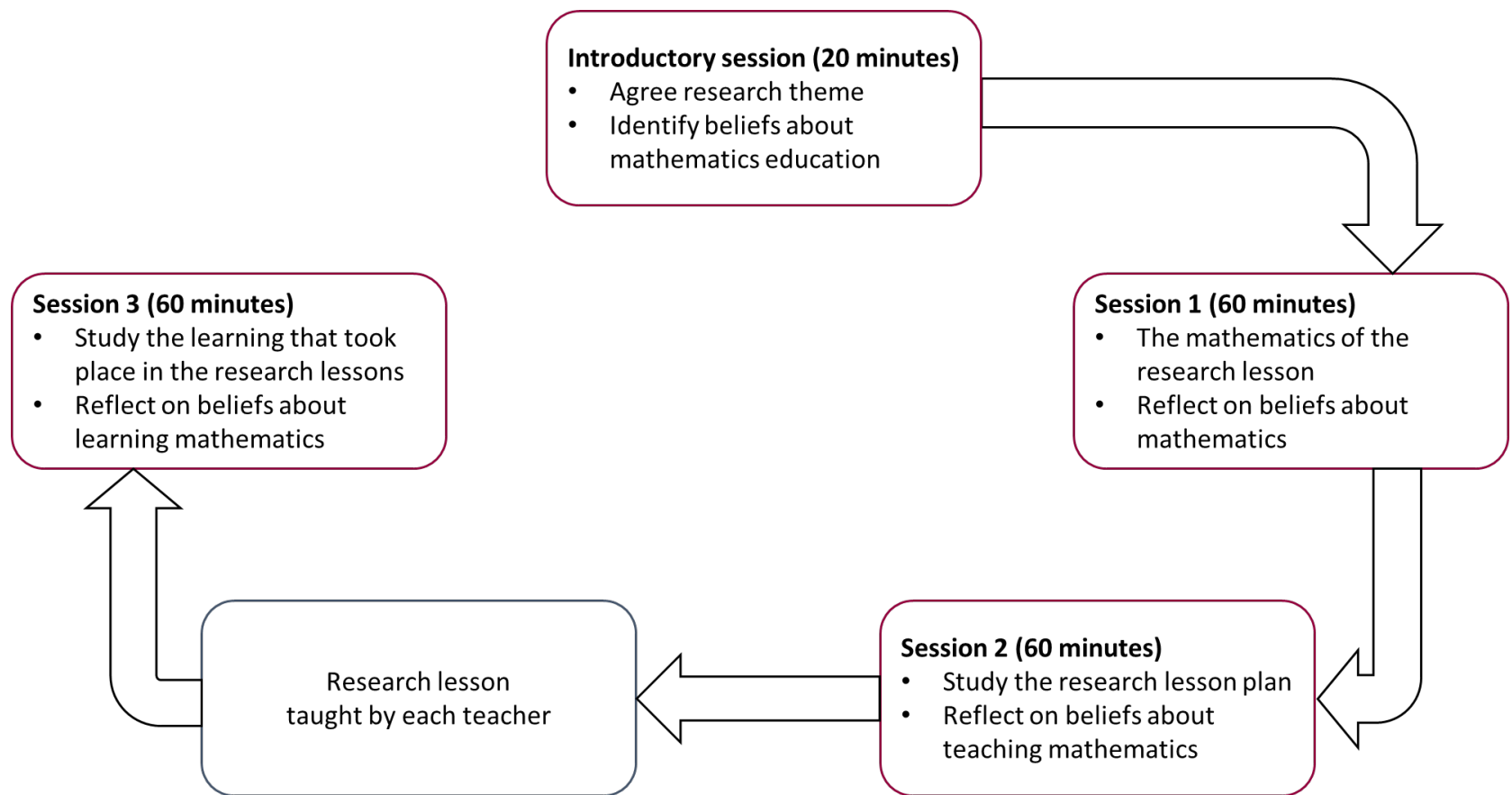


Figure 4.5 Overview of the RAM programme

To provide an understanding of each of the RAM sessions, I have chosen to summarise my design using a set of tables. This decision has been taken to balance an explanation of complex design decisions with a need for readability. Below, I provide summary tables for the introductory session (Table 4.3), and then for each of sessions 1 to 3 (Tables 4.4 to 4.6). Within each of these tables I also reference the supporting slides used in the sessions which can be found in appendices 2 to 5. A complete copy of the associated participant resources for each session can also be found in appendices 6 to 9.

Stage	Stage descriptor	Approximate duration (minutes)	Summary of stage	Presentation slides (Appendix 2)
1	Introduction	2	Introduction to the programme, explanation of the lesson study model, emphasis placed on developing beliefs about mathematics education.	1 – 4
2	Agree a research theme	10	Develop a mathematics department research theme for the lesson study process. As discussed in Woodford (2024), the research theme is agreed by linking a school aim to an aim of the mathematics national curriculum in England.	5
3	Beliefs about mathematics education	5 - 10	Participants provide their personal weightings for their beliefs about mathematics, teaching mathematics, and learning mathematics. Using the domain weightings, participants calculate, and plot, their mean average beliefs about mathematics education values.	6 - 8

Table 4.3 A summary of RAM introductory session

Stage	Stage descriptor	Approximate duration (minutes)	Summary of stage	Presentation slides (Appendix 3)
1	Introducing the problem	5	Introduction to the focus of the session as being on mathematical knowledge and beliefs about mathematics. Explanation of the angle between parallel lines problem.	1 – 2
2	Individual problem solving	8 - 10	Participants individually solve the main problem in as many ways as possible.	3 – 4
3	Small-group work	20 - 25	Participants work in small groups to discuss and classify solution methods using sorting cards.	5
4	Whole-group discussion	15	Whole group facilitated discussion of the classifications, and a consideration of difficult/incorrect/inefficient approaches. Emphasis placed on mathematics that can be used once an auxiliary line has been added.	6 – 12
5	Reflecting	5	Participants individually complete a belief questionnaire on the beliefs about mathematics they thought underpinned the design of the session.	13

Table 4.4 A summary of RAM session 1

Stage	Stage descriptor	Approximate duration (minutes)	Summary of stage	Presentation slides (Appendix 4)
1	Introducing the lesson plan	10 - 15	Introduction to the phases and aims of Japanese TTP style lessons. An overview and justification of the research lesson plan under the TTP heading phases. Consideration of an example of board development for the research lesson.	Slides 1 - 9
2	Understanding the lesson plan	35 - 45	Participants work in small groups to understand and discuss the research lesson plan. They are encouraged to commit to the suggested actions but have the option to make justified changes if required.	Slide 10
3	Reflecting	5	Participants individually complete a belief questionnaire on the beliefs about teaching mathematics they thought were shown in the design of the research lesson plan.	Slides 11 - 12

Table 4.5 A summary of RAM session 2

Stage	Stage descriptor	Approximate duration (minutes)	Summary of phase	Presentation slides (Appendix 5)
1	Introduction	5	Introduction to reflecting on the student learning that followed from teacher actions in the research lesson.	Slides 1 – 3
2	Reviewing the research lesson	45	Structured small group discussion going through each phase of the lesson. Reflections structured around the teacher action questions shown in the research lesson plan.	Slides 4 – 9
3	Reflecting	10	Participants individually complete a questionnaire on the beliefs about learning mathematics they thought were seen in the research lesson they taught.	Slides 10 – 11

Table 4.6 A summary of RAM session 3

Within each of the RAM sessions I sought to balance the introduction of knowledge with providing opportunities for reflection around beliefs about mathematics education. For instance, in session 1, I had a design intention to equip teachers with knowledge of different mathematical approaches to solve the problem. However, there was no intention to tell teachers what their beliefs about mathematics should be. Instead, teachers were asked to reflect on what beliefs about mathematics they felt were seen in the session. In this thesis I do not emphasise the mathematical knowledge I sought to develop through the design of the RAM programme. Instead, I focus on the design decisions that were made around beliefs about mathematics education. These decisions should be understood in relation to the theory of learning and the agents of change discussion for mathematics DPD as summarised in Table 3.3.

4.4.2 Theory of learning: Designing for the examination of beliefs about mathematics education

In Table 3.3, I suggested a theory of learning for mathematics DPD based on belief formation through seeing contradiction in beliefs about mathematics education in teaching approaches. This intention to focus on beliefs about mathematics education begins in the introductory session with a clear statement that the purpose of the programme is the ‘consideration of our beliefs, and therefore approach, toward mathematics teaching’ (appendix 2 slide 2). To bring further clarity a statement is included specifying the intention is ‘not to try and force you to teach in one particular style’ (appendix 2 slide 2).

Still within the introductory RAM session, participants are asked to complete an adjusted version of Swan's (2006b) belief questionnaire. A copy is shown in Figure 4.6 and weightings are requested in the domains of beliefs about mathematics, teaching mathematics and learning mathematics. In line with the discussion of Chapter 2, participants are asked to ensure the weightings sum to 100% based on how strongly they identify with each statement. In addition, the names of the belief categories (Transmission, Discovery and Connectionist) have been replaced with a letter (A, B, and C) to minimise any effects the names may have on the weightings provided.

Once the weightings within each of the domains are completed, teachers are provided with an explanation to calculate their mean average for each category across the domains. This indication of their beliefs about mathematics education are marked on a triangular plot (Graham and Midgley 2000) as shown in Activity 4-1 of Figure 4.6. Through this activity it is intended the importance of current beliefs about mathematics education are highlighted, an awareness of the differing beliefs of others created, and the existence of conflicting (or contradictory beliefs) introduced.

Activity 1-1: Beliefs about Mathematics

Weight the following belief statements according to your view of the nature of mathematics.

Belief	Mathematics is ...	Percentage
A	... a given body of knowledge and standard set of procedures. A set of universal truths and rules which need to be conveyed to students.	----- %
B	... a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	----- %
C	... an interconnected body of ideas which the teacher and the student create together through discussion	----- %
		100%

Give each belief statement in the table a weighting to reflect how strongly you agree with it by making **the three scores sum to 100%**.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

1:1:1

Activity 2-1: Beliefs about Teaching

Weight the following belief statements according to your view of the nature of teaching.

Belief	Teaching is ...	Percentage
A	... structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.	----- %
B	... assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	----- %
C	... a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	----- %
		100%

Give each orientation statement in the table a weighting to reflect how strongly you agree with it by making **the three scores sum to 100%**.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

3:1:1

Activity 3-1: Beliefs about Learning

Weight the following belief statements according to your view of the nature of learning.

Belief	Learning is ...	Percentage
A	... an individual activity based on watching, listening, and imitating until fluency is attained.	----- %
B	... an individual activity based on practical exploration and reflection.	----- %
C	... an interpersonal activity in which students are challenged and arrive at understanding through discussion.	----- %
		100%

Give each orientation statement in the table a weighting to reflect how strongly you agree with it by making the three scores sum to 100%.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

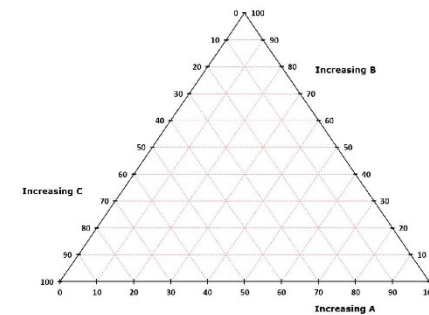
3:1:1

Activity 4-1: Beliefs about mathematics education

Calculate your mean average weightings across the three domains A, B and C.

Belief	Percentage
A	----- %
B	----- %
C	----- %
100%	

Plot the position of your beliefs about mathematics education on the axes provided below.



4:1:1

Figure 4.6 The questionnaire used to capture personal beliefs about mathematics education

The focus on beliefs about mathematics education is maintained throughout RAM sessions 1, 2, and 3. At the end of each session, participants are asked to provide domain specific beliefs weightings for an element of the RAM session. In session 1, participants are asked to provide weightings around the beliefs about mathematics they felt underpinned the whole session. In session 2, participants provide weightings for the beliefs about teaching mathematics they felt underpinned the design of the research lesson plan. Finally, in session 3, participants provide weightings for the beliefs about learning mathematics they saw in the research lesson they taught. Later in this thesis I refer to these belief documents as asking participants to reflect on RAM elements 1, 2, and 3 respectively. Copies of the three sheets used in the three sessions to capture the participants views are shown in Figure 4.7.

Activity 1-3: Session Reflection

As a final reflection, weight the beliefs that you think have underpinned the design of the session.

Belief	Mathematics is ...	Percentage
A	... a given body of knowledge and standard set of procedures. A set of universal truths and rules which need to be conveyed to students.	----- %
B	... a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	----- %
C	... an interconnected body of ideas which the teacher and the student create together through discussion	----- %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

1:3:1

Activity 2-4: Session Reflection

As a final reflection, weight the beliefs that you think have underpinned the design of the research lesson.

Belief	Teaching is ...	Percentage
A	... structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.	----- %
B	... assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	----- %
C	... a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	----- %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

2:3:1

Activity 3-4: Session Reflection

As a final reflection, weight the beliefs that you think were apparent in the learning that you saw take place in the research lesson.

Belief	Learning is ...	Percentage
A	... an individual activity based on watching, listening, and imitating until fluency is attained.	----- %
B	... an individual activity based on practical exploration and reflection.	----- %
C	... an interpersonal activity in which students are challenged and arrive at understanding through discussion.	----- %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

3:4:1

Figure 4.7 Domain specific belief questionnaires designed to capture participant views of RAM

4.4.3 Agents of change: Designing for moments of variation

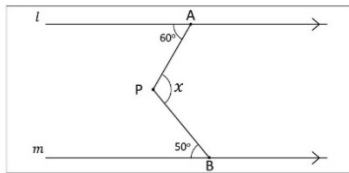
In Table 3.3, I summarised my conception of agents of change as seen in mathematics DPD. Whilst recognising teachers as agentic, I suggest McGowan's (2019) conception of the divided self means I see conscious desires and unconscious actions as revealing where a teacher is finding true satisfaction. Therefore, I seek to design potential moments of variation into the RAM programme where I can anticipate participants will have the opportunity to reveal their unconscious desires. Rather than demand conformity I seek to notice, and design for, moments where participants may show signs of being challenged (session 1), where they may make planned changes (session 2), and where they make unplanned changes (the research lesson and session 3). These design decisions are intended to provide opportunities for moments of variation which supports my interpretation of unconscious actions in my analysis of Chapter 7.

Designing moments of variation (opportunities for challenge) in session 1

In RAM session 1 the first anticipated moment of variation, in the form of opportunities for challenge, is designed to take place in when participants are asked to individually solve the mathematics problem. Participants are given a copy of the sheet shown in Figure 4.8 and given eight minutes to find as many methods as possible to solve the problem as they can. The number of diagrams provided for attempts (six), the amount of time given, the request to work individually, and the request to show working provide possible sources of discomfort seen in teacher's actions.

Activity 1-2: Missing Angle Problem

For the following problem, find the value of angle x .



Use the diagrams below to trial different approaches and write down any calculations in the first column.

1.	
2.	

1:2:1

3.	
4.	
5.	
6.	

1:2:2

Figure 4.8 The individual problem-solving sheet

A second potential moment of variation is designed when participants are asked to work in small groups to discuss the methods they found and consider additional methods together. Participants work in small groups to discuss a set of twelve sorting cards (see appendix 3 slide 9) which show the original problem with the addition of an auxiliary line hinting at a solution method. The small groups of teachers are asked to explain how each card may lead to a solution, and then to try and classify the methods into groups. There is potential for signs of being challenged as teachers work with others, share their solutions, and seek to provide mathematical reasoning behind the methods. It is anticipated participants, as per the trainee mathematics teacher trial, will have difficulty explaining the card showing points A and B joined by an auxiliary line.

A final potential moment of variation designed for RAM session 1 is when the facilitator orchestrates a whole group discussion around the methods which includes a consideration of incorrect methods (see appendix 3 slides 7 - 8 and 10 - 12). Once again, the problematic method of adding an auxiliary line connecting points A and B, is discussed to ensure an emphasis on accurate reasoning. After listening to classifications suggested by the small groups, the facilitator then explains the classification of methods based on the nature of the auxiliary line. Participants may show signs of being challenged at being told a classification that differs to theirs, and then further signs when the emphasis is placed on the angle facts that the addition of an auxiliary line enables.

Designing moments of variation (planned changes) in session 2

In RAM session 2, the main moment of variation designed for was to provide teachers with opportunities to make planned changes to the research lesson plan. However, I also note at the start of the session there is another potential moment of variation around a moment of challenge. Participants are informed of the claim from Takahashi (2021) that Japanese style TTP lessons lead to better results in international tests (see appendix 4 slide 5). Data from Mullis *et al.* (2020) is shared suggesting that despite covering a similar percentage of mathematics topics, Japanese students score higher than students in Australia, Ireland, the United States and England in international tests. This moment is used to provide the justification for cultural transposition (Mellone *et al.* 2019) of resources, and teachers are encouraged to consider the claim.

To design for moments of variation around planned changes, participants are first shown the structure of the research lesson according to the phases of TTP (appendix 4 slides 6 - 7). For ease, the table from appendix 4 slide 7 has been recreated in Table 4.7 below. As can be seen, participants are provided with a clear summary of the important actions that are intended to take place in the research lesson. In addition, a brief verbal explanation is given for each action to help participants understand the intention behind the requests. For instance, construction of the problem is encouraged to help provide student insight into the problem. In particular, this reveals that since point P is equidistant between line l and m then points A and B cannot be vertically opposite.

Phase	Approximate duration (minutes)	Key actions
Present the problem	5	Construct the problem on the board and students copy to give insight into the problem.
Individual problem solving	10	Hand out the Student Worksheet. Students work individually on their worksheet. Teacher completes the Method Recording Sheet and quietly observes students.
Whole class discussion of solution methods	15	Teacher selects students to share their methods at the board. Avoid saying whether the answers are correct. Each method and reasoning should be written on a separate diagram on the lesson PowerPoint slide.
Highlight the major points	10	Major points: • Add an auxiliary line. • Lines may be {perpendicular, parallel, extensions or connecting two points}. • Different auxiliary lines allow different angle facts to be used.
Extend	15	Ask students to modify the problem in pairs and allow them to solve.

Table 4.7 Overview of the RAM lesson given to participants

In addition to providing an overview of the lesson, participants are shown an example of the order of selecting students' methods (appendix 4 slide 8) and how they could be displayed on a board. This process may be unfamiliar to teachers and is therefore justified as allowing students to simultaneously see all methods and enable the drawing of comparisons.

Continuing with RAM session 2, participants are asked to work in small groups to review the detailed research lesson plan. Participants are encouraged to look at the teacher action column of the research lesson plan, and to either commit to the action or make a planned adjustment. These adjustments are made individually, following discussion with the small group, and in the context of the justifications shown in Table 4.7. If adjustments are made, then participants are asked to provide an explanation of their changes. In Table 4.8, I summarise the actions that teachers are asked to consider, and provide the corresponding wording taken from the teacher action column of the research lesson plan.

Phase	Summary of the teacher actions given in the research lesson plan	Question prompting decision around teacher actions given in the research lesson plan
1	Teachers asked to facilitate a whole class recall activity of angle rules.	Will you do this phase as a whole class recall activity?
2	Teachers asked to facilitate a step-by-step construction of the diagram with students.	Will you ask students to make an accurate copy of the diagram in their book?
3	Teachers asked to give students 10 minutes to work individually on the problem.	Will you ask students to work individually for 10 minutes?
	Teachers asked to stress the importance of asking students to show their reasoning.	Will you stress the importance of students writing their reasoning on the sheet?
	Teachers asked to record methods whilst students work and avoid helping students.	Will you record student methods with minimal interaction?
4	Teachers asked to base the discussion on student methods seen.	Will you only introduce methods that students have found?
	Teachers asked to anticipate what their final board might look like.	What do you anticipate your board to look like?
5	Teachers asked to ensure students make notes.	Will you ask students to write notes?

Table 4.8 Key decisions from the research lesson plan

The summary of the teacher actions column is based on my experience of seeing this, and other similar lessons, taught in Japan. They highlight some of the key teacher actions that I anticipate may provide moments of variation for teachers in England. For example, in phase 3 teachers are asked to provide students with 10 minutes of individual problem solving for one problem. In my experience, this is uncommon in England and so has the potential to cause reflection if carried out, and reflection if a planned adjustment is made. Therefore, the questions on the research lesson plan (and replicated in the third column of Table 4.6) are intended to provoke thought followed by commitment or adjustment to the action.

Designing moments of variation (unplanned changes) in session 3

In RAM session 3, teachers are asked to consider the learning that took place in their research lesson, and to reflect on beliefs about learning mathematics. To do this, participants are asked to reflect on their actions in their version of their taught research lesson and consider the impact it had on student learning. Of particular interest here is whether participants carried out the actions they committed to in their version of the research lesson plan, or whether they made unplanned changes. It is these unplanned changes which I consider as potential moments of variation. During the RAM session, participants are placed in small groups for discussion and asked to individually complete a copy of the sheet shown in Figure 4.9. This sheet is split by research lesson phase and asks teachers to discuss the learning that took place following the key decisions around teacher actions made in RAM session 2. For example, the second question asks participants to reflect on the learning that took place following the construction of the problem with the class.

Activity 3-2: Reviewing the research lesson

Phase 1: Review previous learning

How did you manage the whole class recall activity or an alternative? What learning took place?

Phase 2: Present Problem

Did you draw a careful copy of the problem and ask students to draw it or an alternative? What were the consequences of your decision? What learning took place?

Phase 3: Individual Work

Did you get students to work individually in silence for 10 minutes whilst you noted solutions or an alternative? What learning took place?

Phase 4: Discussion

Did you base only introduce methods that students found or follow an alternative? What learning took place?

Phase 5: Drawing together

Did you ask students to write notes or follow an alternative? What learning took place?

3:2:1

Figure 4.9 Sheet used to reflect on the research lesson

4.4.4. Summary of the RAM programme design

The RAM programme has been designed based on an adjusted form of lesson study on a Japanese TTP style research lesson. An introductory session explains the process to participants and provides an opportunity for an initial consideration of personal beliefs about mathematics education. Beliefs about mathematics education are further utilised in each of the RAM sessions when participants are asked to provide their views on the beliefs seen in an element of the programme. Participants are asked to give their views on the whole session, the original research lesson plan, and the taught research lesson following RAM sessions 1, 2, and 3 respectively. In addition, the design of the RAM programme includes opportunities for reaction and adjustment which I describe as potential moments of variation. These are points within the programme where I anticipate teachers' actions may reveal the unconscious undermining of their conscious desires around professional development.

4.5 Summary of Chapter 4

In this chapter I have provided an overview of the RAM programme which has been designed to exemplify the principles of mathematics DPD. I began by showing a careful consideration of the mathematics problem I elected to use in the research lesson, and its suitability for use with teachers in a professional development setting. The research lesson plan has been influenced by a Japanese lesson in the anticipation that 'cultural transposition' (Mellone *et al.* 2019, p. 199) may prompt reflection on beliefs about mathematics education. Whilst I have drawn on Japanese resources, the specific country is less important than the opportunity to prompt thought.

The RAM programme has been explained as an adjusted form of lesson study, with a focus on beliefs about mathematics education running throughout it. Design decisions within the RAM programme include the sharing of knowledge, but do not include attempts to tell teachers what to believe. This means the primary purpose of mathematics DPD can remain as the formation of beliefs about mathematics education through an understanding of contradiction. In Table 4.9, I summarise the documents referenced in this chapter that were used to generate data for this study. The wider process of data collection and my methodology are described in more detail in Chapter 5.

Document	Session the document completed in	Summary	Reference
Activities 1-1, 2-1, 3-1	Introductory	Initial beliefs about mathematics education in the form of weightings in each of the domains of mathematics, teaching mathematics, and learning mathematics.	Figure 4.5
Activity 1-2	1	Missing angle problem	Figure 4.7
Activity 1-3	1	Beliefs about mathematics that underpinned RAM session 1.	Figure 4.6
Activity 2-3	2	Annotated research lesson plan.	Appendix 8
Activity 2-4	2	Beliefs about teaching mathematics that underpinned the design of the research lesson plan.	Figure 4.6
Activity 3-2	3	A review of the research lesson.	Figure 4.8
Activity 3-4	3	Beliefs about learning mathematics that were seen in the taught research lesson.	Figure 4.6

Table 4.9 Summary of documents from which data has been generated

Furthermore, within this chapter I have explained how I used a contradiction informed theory of learning, and a contradiction informed understanding of agents of change to influence my design. Suggesting professional learning takes place through reconciliation to contradiction in beliefs about mathematics education means I included opportunities to consider beliefs in each session. Participants are asked to identify their initial beliefs about mathematics education in the introductory session, and to reflect on domain specific beliefs in each of sessions 1, 2, and 3. Finally, a contradiction informed understanding of the conscious and the unconscious meant I have sought to design for potential moments of variation. Through applying my reading of McGowan (2019), I will interpret moments of variation as occurring when participants' actions contrast with their stated desires. Broadly speaking, I suggest there are potential moments of variation which may be seen in actions following opportunities for challenge, in planned changes to the research lesson plan, and in unplanned changes during the research lesson.

Chapter 5.

Methodology and research design

5.1 Synopsis of Chapter 5

In Chapter 4, I discussed my design of the Rethinking Approaches in Mathematics (RAM) programme as an exemplification of mathematics Dialectic Professional Development (DPD). In this chapter, I provide the research design and methodology I have adopted to answer the research question introduced in Chapter 1. Hence, my explanations in this chapter are intended to provide understanding into how I have approached the generation and analysis of data. Moreover, my explanations are intended to provide clarity around the claims I subsequently make regarding a contradiction informed interpretation of the RAM programme. I explain the underpinning ethical approach and thinking that has shaped my work and has led to my adoption of a ‘revelatory’ (Yin 2018, p 50) case study approach. I identify my case as the one-year RAM programme as experienced by mathematics teachers in the context of a secondary school in the East Midlands of England. Hence, in this chapter I also introduce the school I worked with and introduce three teachers who contributed RAM documents and semi-structured interviews to my study. The generation of data from these sources is explained in detail before I introduce my approach to analysis. To this end I discuss the framework of Reflective Thematic Analysis (Braun and Clarke 2021a), and illustrate its application to my data. This leads to the identification of three themes which are discussed in each of the subsequent three chapters.

5.2 Ethics

In seeking to gain an understanding of teachers' beliefs and actions I have sought to act with an 'ethic of respect' (British Educational Research Association [BERA] 2024, p. 9). This has meant I endeavoured to make 'actively deliberative, ongoing and iterative' (BERA 2024, p. 8) decisions throughout my work. Furthermore, as noted in the BERA (2024) guidelines, I account for how ethical thinking has influenced my approach to educational research, to my decision making, to seeking formal approval, and to my research design more generally.

5.2.1 Ethical consideration of research

My ethical consideration of participants in this study should be understood in the context of my views on education research in England over recent years. Funding for educational research has been dominated by the Education Endowment Foundation (EEF) and their preference for utilising randomized control trials (RCTs) (Inglis 2018). An RCT is predicated on the view that effect can be attributed to an intervention when differences between the control and intervention groups has been minimized through randomization (Burnett and Coldwell 2021). This preference for RCTs has led to a growing culture in the United Kingdom of seeking to identify causal links into 'what works' (Gorard, See and Siddiqui 2017, p. 101) to bring about educational improvement (Burnett and Coldwell 2021).

This idea of subscribing value to educational research based on what works has led to Gorard, See and Siddiqui (2017, p. 101) claiming 'theoretical explanations appear satisfying

but are unnecessary'. There has been a marginalisation of other approaches to education research (Burnett and Coldwell 2021), but I take the perspective theoretical explanations are valuable in helping to understand why something works. As Inglis (2018, p. 319) suggests, there should be a place in education research to both catalogue what works, and to help '*predict* what will work'. I would go further and suggest there should also be space to question the status quo, space to examine the side effects of a what works approach, and space to consider why something works.

Central to my thinking in this study is the view that teachers 'act on the world, change it, and are changed by the consequences of their actions' (Brinkmann and Kvale 2015, p. 259). Furthermore, my understanding of contradiction as an ontological insight (McGowan 2019) extends to seeing this social development as taking place through contradiction. Therefore, I am drawn to investigate how 'the real contradictions of the social situation' (Brinkmann and Kvale 2015, p. 259) influence the experience of change. Consequently, I view purely empirical methods, which can seek to exclude contradiction, as limited in 'uncovering a contradictory social reality' (Brinkmann and Kvale 2015, p. 259). Thus, my work in this study is guided by an additional desire to employ non-empirical methods to support the answering of my research question.

Implications for the role of participants

My desire to generate and interpret data about the RAM programme whilst simultaneously valuing the experiences of teachers leads to a tension. I value teachers'

perspectives but also seek to provide my interpretation of what they say and do. My study is not simply an attempt to tell the stories of participants, but to share their insights alongside my interpretation. I position myself as ‘coproducing’ (Brinkmann and Kvale 2015, p. 201) data alongside participants whilst retaining the responsibility for ‘(co-)constructing meanings’ (Finlay 2021, p. 105). As Brown (2022, p. 200) suggests, a ‘fully egalitarian’ research process is not a necessary requirement of participant wellbeing. So, whilst it is the teachers’ experiences that enable the generation of data, I undertake the ‘constructivist crafting’ (Finlay 2021, p. 105) of interpretation.

Despite retaining the primary role in co-constructing meaning, I believe I also have an ethical responsibility to share my research findings with participants (Hintz and Dean 2020). This responsibility is guided by a consideration of the principles of non-maleficence, respect for persons and beneficence (Hintz and Dean 2020). I recognise the philosophical position I adopt in this study is relatively rare and may not be the interpretation the participants themselves would suggest. However, I believe I have a responsibility to suggest a ‘new road’ (Stake 2010, p. 102) for mathematics teacher professional development. As per the BERA (2024, p. 13) guidelines, on completion of my themes, I organised a group ‘debriefing’ at the mathematics departmental meeting where my case study took place to explain my ideas. As part of a departmental development meeting, I briefly explained my philosophical perspective and demonstrated the development of my first and third themes around teacher beliefs using data from every member of the department that took part in the programme. I interpret the teachers’ reactions to my approach as indicating they saw my rationale and patterns that emerged as reasonable and fair. In addition, as discussed later in this chapter, three teachers

contributed additional data to this study in the form of semi-structured interviews. I did not feel it was appropriate to discuss their data, and my analysis of actions, with the whole department. Therefore, I offered the three main teachers from the study an opportunity to meet to discuss my third theme privately. Whilst two teachers declined, one teacher took up this offer, and some of their thoughts about my interpretation are recorded in Chapter 7. For the two teachers that declined the opportunity to discuss my interpretation I have still provided a possible alternative interpretation in my discussion in Chapter 7.

5.2.2 Ethical decision making in the design of the RAM programme

As discussed in Chapter 3, I view mathematics DPD as having a different purpose to other forms of professional development. I suggest mathematics DPD, and therefore the RAM programme, should support the formation of beliefs through an understanding of contradiction. I therefore felt compelled to design a programme that would align to the purpose of mathematics DPD through a consideration for participating teachers. Thus, both my desire to create the RAM programme, and my subsequent desire to facilitate the sessions were borne of an ethical concern.

My role as designer

I acknowledge I have personal views on pedagogy, personal beliefs about mathematics education, and personal practices which I have adopted in the classroom over many years. There remains a temptation to want to tell people my ‘correct’ way to teach through the RAM programme. However, in my design I have sought to separate my current personal views

about mathematics education from my views around teacher professional development. I have not designed the lesson within the RAM programme to reflect the way I expect everyone to teach. Instead, I have sought to develop a research lesson which facilitates engagement and a professional development programme that values opportunities for teacher reflection. To support this, I have drawn on previous experience of leading professional development and my design work as part of the Maths-for Life (Education Endowment Foundation [EEF] 2019) programme. I have designed with the intention of allowing participants to make choices within the programme, and to reflect on both the experiences they have had, and the beliefs underpinning the examined approach.

My role as facilitator

Not only did I design the RAM programme, but I also took the decision to facilitate the sessions. In an earlier iteration of my RAM programme development, I created presentation slides with a recorded voice over explaining what to do. At the time, I thought this would ensure I created a resource with clinical fidelity which might allow me to identify causal links into what works. The draw of wanting to create an intervention that would be replicable on a larger scale influenced my initial thinking. As my ideas began to develop, I moved away from recordings and created a potential script for an independent facilitator to run the session. However, both the recordings and the facilitator scripts risked diminishing the participant experiences and increased the sense of being put through a programme. For me, the experience of people is an ethical matter, and it is these experiences that I want to understand. I therefore decided to facilitate all sessions and react to what was taking place. My decision

inevitably meant I shaped teachers' experiences through my actions, but it also enhanced my understanding of what was taking place. I therefore seek to ensure I provide a transparent account in this study, recognising the context of the school and the influences I may have had.

5.2.3 Ethical approval of the study

In December 2020, when gaining ethical approval from the Nottingham Trent University Schools of Business, Law and Social Sciences Research Ethics Committee, I aligned my application to the BERA (2018) guidelines. My successful application formed the foundations of my approach in this study as I endeavoured to be clear on my intentions and underpin my actions with an 'ethic of respect' (BERA 2018, p. 5). This included, though not limited to, gaining informed consent from participants and my approach to handling of data.

Informed consent

Informed consent (BERA 2018) for this study was gained from the trainee mathematics teachers and colleague mentioned in Chapter 4, along with seven mathematics teachers from the school that participated in the RAM programme. For six of these mathematics teachers, consent was gained at the introductory RAM session, and the seventh teacher provided consent prior to session 3, which was the only one they attended. I explained the programme design, provided copies of the project information sheet, and received signed consent forms from all participants. Copies of both the project information sheet and the consent form are shown in appendix 14. In addition, during the RAM introductory session, I

verbally re-iterated my request to access all materials completed during the RAM sessions, and to carry out interviews after each session.

Handling of data

Data for this study has been generated from completed RAM documents and from semi-structured interviews, both of which are discussed in detail later in this chapter. The RAM documents were completed by participants during the sessions and then filed in their own binder which I provided. These binders were collected in at the end of each session and stored in a locked room on Nottingham Trent University premises. Digitized copies of the documents were made and saved to a secure password protected server, before being returned to the participants at the next RAM session. At the end of the programme, all binders and materials were returned to the participants to keep.

Alongside written documents, I also generated data through semi-structured interviews with three participants. Brinkmann and Kvale (2015) note a tension can exist in carrying out interviews in trying to balance the gaining of knowledge with respecting the interviewee. I sought to mitigate this dilemma by utilising an interview guide alongside being responsive (Brinkmann and Kvale 2015). Once again, this process is discussed in more detail in the remainder of this chapter. In all documents which were created, the names of teachers, schools and locations have been pseudonymised (European Union 2016) to protect confidentiality.

5.3 Utilising case studies

In the context of my views on educational research, I chose to utilise a case study approach to answer my research question. As discussed in Chapter 3, I recognise professional learning is complex, and mediated by the educational environment, the school environment, and teachers' current beliefs. The complex contextual conditions of a 'real-world' (Yin 2018, p. 51) phenomenon led to my view that a case study approach is valuable (Hamilton *et al.* 2013, Yin 2018). Indeed, attending to context distinguishes case studies from many other forms of research (Yin 2018) as I seek to explain 'what others have not yet seen' (Stake 1995, p. 136). Stake (1995) additionally suggests adopting a case study approach positions me as a teacher, an interpreter, and an advocate. I aim to be a teacher in providing 'good raw material' (Stake 2010, p. 102), an interpreter in creating new meaning, and an advocate when suggesting a new road to follow in mathematics professional development.

5.3.1 Defining the case

For this study I define the bounded case (Stake 1995) as the 1-year, RAM programme, taking place in a single secondary school in the East Midlands of England. This programme comprises of the introductory session, three 1-hour professional development sessions, and the teaching of a research lesson as described in Chapter 4. Furthermore, my case study follows an embedded single-case design (Yin 2018) through my intention to gain insight into the RAM programme through the experience of three mathematics teachers at the chosen school.

Yin (2018) notes a danger exists in single-case designs when the subunits (in this instance the participating teachers) become the focus of the study, and the case become synonymous with the context. I have some reticence in describing the three teachers using the dehumanized language of ‘subunits’, and stress I am interested in their stories ‘because they are of worth’ (Seidman 2019, p. 2019). However, Yin’s (2018) warning is helpful, and I view my single-case design as a ‘revelatory case’ (Yin 2018, p 50) about a programme, informed by the valuable experiences of people. This is a distinction I have wrestled with throughout the study and therefore seek to identify commonalities across the experiences of the individual teachers and relate these back to the RAM programme. I make a slight departure from Yin’s (2018) positivist-influenced approach in my ambition to develop educational theory through an understanding of the case. My perspective aligns more closely with Stake (1995), who categorises an instrumental case study as one aiming to provide insight into an issue. I therefore summarise my methodology as an instrumental, embedded single-case design which seeks to understand beliefs about mathematics education and actions within an instance of mathematics DPD.

5.3.2 Criticism of case studies

Yin (2018) recognises a common criticism of the case study approach is that a focus on individual instances make generalisation difficult. However, Simons (1996) argues such criticisms stem from a view of research that polarizes the relationship between the particular and the universal. Instead, Simons (1996) claims the relationship should be understood as the particular providing insight into the universal. In a similar fashion, though again drawing on

positivist influenced approaches, Yin (2018) makes a distinction between analytic and statistical generalisations. Rather than a statistical generalisation from a sample to a population, Yin (2018) suggests a case study creates an analytic generalisation from a case to wider theory. Whether this is a generalisation is debatable, but I do conceive of my case study approach as seeking to apply understanding from the single case to the development of wider theory. In doing this I believe it is possible to ‘challenge certainty’ (Simons 1996, p. 238) and offer a new perspective on teacher professional development.

A second criticism of case study research is that it is shaped by personal interpretation and influenced by subjective values (Yin 2018). However, I see interpretation as inevitable in research, and transparency about values as a strength. As Stake (1995, p. 95) suggests ‘research is not helped by making it appear value free’. Instead, I seek to provide ‘a convincing account of the meanings of the dataset and explain why these meanings matter’ (Braun and Clarke 2021b, p. 175). It is my responsibility to embrace and interrogate my own position as I seek to interpret, interrupt and challenge current thinking (Denzin and Lincoln 2018).

5.3.3 The case study context

As already mentioned, the importance of context is significant in the choice of a case study methodology. I designed the RAM programme to be experienced by a group of teachers from a mathematics department in a secondary school and therefore sought a school that would host the programme through a self-selecting convenience approach (Cohen, Manion

and Morrison 2018). I noted Stake's (1995) suggestion to consider case selection based on ease of access, hospitableness, and willingness to participate and so approached the heads of mathematics at two secondary schools based in the East Midlands of England. The first school was near my place of work but with no existing relationship, and the second school further away with a head of mathematics I had previously worked with. It was the hospitableness and enthusiasm of the head of mathematics at the second school which led to the programme running in with their department. Therefore, I attended a departmental after-school session in October 2021 at Victoria School to run the RAM introductory session.

Victoria School

Victoria School is an 11 to 18 secondary school in the East Midlands of England with approximately 25% of students eligible for free school meals at the time of the study. In 2018 the school was inspected by the national education inspectorate agency in England, Ofsted, and categorised as good. The school is a single academy trust with a stable leadership team who were conscious their next Ofsted inspection was due. This concern about Ofsted had led to the school leadership team introducing a school-wide lesson planning structure, called the Task Ladder, in the academic year before my case study took place. Every teacher in the school, and therefore all teachers in the mathematics department, were expected to plan their lesson around the five steps of the Task Ladder. In addition, these five steps of recall, new information, checking of understanding, practice, and review had to be displayed in the classroom for students to see in every lesson.

I have written elsewhere about teachers' views of the Task Ladder, with one teacher explaining it had been enforced on them through repeated drop-in lesson visits by members of the senior leadership team (Woodford, Clapham and Serret 2023). The teacher commented the senior leadership team at the school had 'managed to change my practice here, but only through policy and mandating things' (Woodford, Clapham and Serret 2023, p. 1155). However, more generally, I summarise teachers' attitudes to the Task Ladder as initial scepticism leading to seeing it as a helpful, simplifying structure (Woodford, Clapham and Serret 2023).

5.3.4 The RAM sessions at Victoria School

The introductory session, the three main sessions, and the research lesson element of the RAM programme, were all completed at Victoria School between October 2021 and May 2022. Each of the sessions took place on the school site, at the end of the school day, and as part of the departmental development time. A summary of the completed sessions is shown in Table 5.1.

Session	Date	Duration	Summary
Introductory	October 2021	20 minutes	Aims and timetable of the RAM programme introduced. A department research theme agreed of: <i>how does a lesson that provides opportunities to develop reasoning through engaging with alternative explanations help to develop respect in students?</i> Initial beliefs about mathematics education documents completed by all teachers.
1	January 2022	60 minutes	Teachers individually solved the mathematics problem and placed in groups of three to classify solution methods. Whole group discussion of naming methods and examination of mathematics used. Teachers provided their views of the beliefs about mathematics seen in RAM session 1.
2	March 2022	60 minutes	Teaching Through Problem solving (Takahashi 2021) explained as a lesson strategy utilised in Japan. Overview of the research lesson provided. Teachers placed in a group of three to discuss the research lesson plan and annotate their copy. Teachers provided their views of the beliefs about teaching mathematics seen in the original research lesson plan.
Research lesson	Teacher decision		Five teachers confirmed they taught the research lesson to their classes.
3	May 2022	50 minutes	The time for this session was slightly reduced as members of the senior leadership team attended the first 10 minutes to discuss the departments' completion of assessment data. Teachers placed in a group of three to reflect on their research lesson. Teachers provided their views of the beliefs about learning mathematics seen in their taught research lessons.

Table 5.1 Summary of the RAM sessions that took place at Victoria School

As shown in the summary of the introductory sessions, a departmental research theme was developed as part of the lesson study cycle. We agreed to study the research question of: how does a lesson that provides opportunities to develop reasoning through engaging with alternative explanations help to develop respect in students? This theme was developed through discussion with the department by linking the school aims to the aims of the mathematics National Curriculum in England (Woodford 2024). I make little further reference to the departmental research question in this study, though it provides helpful contextual information for discussions that took place in the third RAM session, and potentially in the semi-structured interviews.

Six of the seven teachers that made up the mathematics department at Victoria School attended all the RAM sessions. One teacher, Yesim, was only able to attend the final session at which they were provided with their own copy of the RAM materials. During this session Yesim participated by being part of the discussion around other teacher's experiences of the taught research lessons. Contextual information about the seven teachers is shown in Table 5.2 under pseudonymised names.

Name	Role	Age range	Number of complete years teaching	Highest mathematics qualification	Dominant belief about mathematics education
Alex	Deputy head of school	51 – 55	15	O-level	Transmission
Drew	Head of department	36 – 40	3	Degree	Discovery
Chris	Assistant head of department	31 – 35	8	Degree	Connectionist
Jo	Classroom teacher	26 – 30	2	Degree	Transmission
Ronnie	Classroom teacher	46 – 50	18	A-level	Connectionist
Val	Classroom teacher	51 – 55	24	A-level	Connectionist
Yesim	Classroom teacher	36 – 40	11	GCSE	Unknown

Table 5.2 Teachers at Victoria School who participated in the RAM sessions

As can be seen in Table 5.2, there is a variety of ages, years of teaching experience, and mathematics qualifications within the department. The final column of Table 5.2 also displays the dominant belief about mathematics education for each participant. These categories were introduced in Chapter 2 and were calculated following the completion of activities during the introductory session. A precise explanation of this process is described later in this chapter.

Teacher dominant belief data is provided here since it formed the basis of purposive sampling (Denzin and Lincoln 2018, Obilor 2023) I employed to select three teachers to take part in semi-structured interviews. As discussed in Chapter 2, Swan (2006b) claimed teachers' belief trajectories in a professional development experience depend on their initial beliefs about mathematics education. Therefore, the three teachers were selected with the intention of ensuring 'different and important views about the ideas and issues at question' (Campbell *et al.* 2020, p. 654) were included in my consideration of the case. Thus, based on their beliefs about mathematics education, I invited Drew (Discovery), Jo (Transmission), and Ronnie (Connectionist) to participate in the study interviews. Yin (2018, p. 111) admits, 'there is no clear cutoff point' about how many participants should be considered for a case study. However, I perceived the three teachers, with different dominant initial beliefs about mathematics education, as providing sufficient data from differing perspectives. I anticipated that semi-structured interviews, alongside completed RAM documents, would provide an 'architecture of corroborating data' (Freeman *et al.* 2007, p. 27) that would be sufficient to provide insight into the experiences of teachers in the RAM programme.

5.3.5 Pen portraits of participating teachers

In this sub-section I provide brief pen-portraits of the three teachers (Drew, Jo and Ronnie) who were selected to discuss their experiences of the RAM programme. The comments written here reflect my subjective opinions and are drawn from my experience of the RAM sessions, comments made during interviews, and from meeting the teachers when attending the school to facilitate the RAM sessions.

Drew (dominant belief about mathematics education of Discovery)

Drew entered the teaching profession in 2018 as a career changer following successful roles as a chef and a flower shop owner. They completed a mathematics degree with the Open University and then completed a post graduate teaching qualification in mathematics teaching. Drew came across as an enthusiastic learner who enjoyed practical applications of mathematics and had an ability for lateral thinking to solve problems. Drew became the acting head of mathematics at Victoria School in their second year of teaching, and the permanent head of mathematics during their third year.

As a new head of department, Drew was determined to influence the style of teaching across the department but also recognised the need to follow requirements of the school leadership team. Indeed, this tension was clear when they spoke of not feeling personally aligned with the school Task Ladder lesson structure but wanted to ensure it was followed by members of the mathematics department. In terms of their teaching, Drew expressed a personal preference for creating opportunities for students to problem solve and helping

students to apply mathematics. These characteristics would be typical of someone who fits within the Discovery category for beliefs about mathematics education. Furthermore, Drew was an enthusiastic contributor to the RAM sessions and could always be relied on to offer supportive and thoughtful ideas.

Jo (dominant belief about mathematics education of Transmission)

Jo accepted the position of teacher of mathematics at Victoria School one year after Drew had started. This was Jo's first job, taken after completing a joint honours degree in mathematics and education at university followed by a post graduate teaching qualification. During one interview Jo acknowledged their teaching style generally involved careful demonstration followed by students repeating similar problems. Such an approach would be typical of a teacher with a dominant belief about mathematics education of Transmission. Furthermore, Jo had welcomed the school policy of the Task Ladder lesson structure as they saw it as aligning well with their beliefs about mathematics education. As a young teacher, Jo often came across as quiet and less confident than others in the department. They tended to let others speak first in discussions but would gradually get more involved as sessions progressed.

Ronnie (dominant belief about mathematics education of Connectionist)

Ronnie had taught for twenty years, including seventeen in London, prior to joining Victoria School. They claimed they had never intended to get into teaching but fell into it

through poor personal management and a lack of ambition. Ronnie was proud of the results achieved with exam classes and enjoyed helping students make sense of mathematics. This typically Connectionist belief about mathematics education reflected Ronnie's thoughtful approach to working in the classroom. During one conversation Ronnie explained how they had created a persona in the classroom but felt, over time, they were becoming the character they had created.

During one conversation, Ronnie also expressed disappointment at being overlooked for several jobs as a head of mathematics. I often got the sense Ronnie felt uncomfortable with who they were, and these insecurities could come out in their interactions with others. Ronnie adopted a sardonic demeanour during the RAM sessions but was always prepared to make contributions. After some initial reservations, followed by repeated lesson visits from senior management, Ronnie had adopted the Task Ladder lesson structure imposed by the school and now saw it as a helpful, simplifying approach. However, this experience had left a mark on Ronnie, and they displayed a level of distrust regarding the school leadership team, and around education authority figures in general.

5.4 Generation of data

Having explained my case study approach and provided details about Victoria School and the participants, I now explain the data generated for this work. As a complex case study, I felt it would be appropriate to adopt multiple, flexible, data collection methods (Stake 1995, Pearson, Albon and Hubball 2015, Yin 2018). Therefore, I have employed collection methods to gather evidence from documents completed during the RAM sessions, and semi-structured interviews completed after each session. This evidence was then used to generate data for analysis in the form of electronic documents and transcripts. Together, these multiple data files provide a way to triangulate (Denzin 2017) and provide a convergence of thought (Yin 2018) that is intended to render the ‘participant’s perspective accurately’ (Yin 2018, p. 129) within my interpretation.

A timeline of the RAM programme, the evidence, and the corresponding generated data is shown in Figure 5.1. The seven activities shown in pink in Figure 5.1 are documents completed across the four RAM sessions, and copies can be seen in appendices 6 to 9. This evidence was processed to generate a single document per teacher for the purpose of data analysis called ‘Beliefs [name]’. The three documents shown in green in Figure 5.1 were completed during the RAM sessions 1, 2 and 3 respectively. Copies can be seen in appendices 7 to 9 and digitised copies were made for analysis under the filenames Problem [name], Lesson Plan [name] and Lesson Reflection [name]. Finally, the four interview recordings shown in blue were converted to transcripts for analysis under the filename Interview (number) [name].

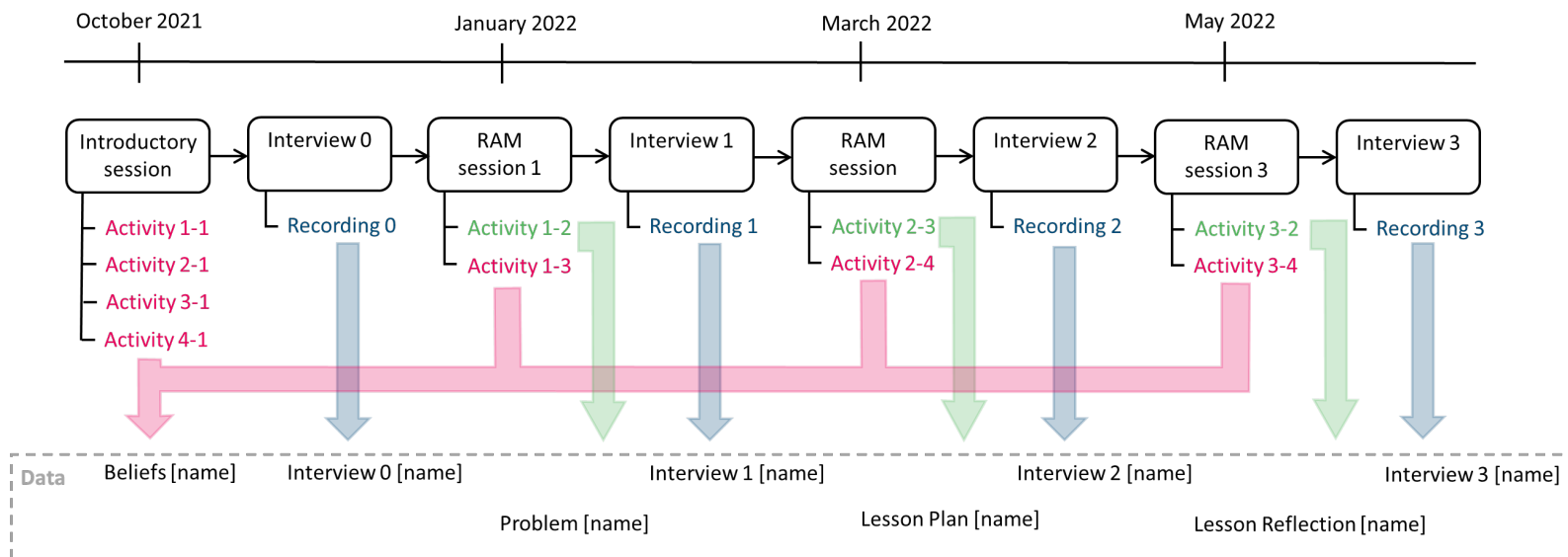


Figure 5.1 Evidence and corresponding generated data

Each of the pieces of evidence shown in Figure 5.1, and how they were used to generate data, is explained in the following three sub-sections. In summary, for each of the three teachers, eight pieces of data were generated for analysis, giving a total of $n=24$ documents.

5.4.1 RAM programme activities providing data documents directly

In each of sessions 1, 2, and 3, participants completed RAM materials which can be seen in appendices 7, 8 and 9 under the headings of Activities 1-2, 2-3 and 3-2. Once completed, and digitised, these materials (shown in green in Figure 5.2) provide ‘documentary information’ (Yin 2018, p. 113) of the teachers’ thoughts during the RAM sessions. Both Yin (2018) and Stake (1995) caution that a researcher should show an awareness of the reasons for the completion of documents to mitigate against potential misinterpretation. Therefore, I note Activity 1-2 (appendix 7) provides a record of the individual methods attempted by the teachers to solve the geometry problem. Activity 2-3 (appendix 8) provides a record of the teachers’ thoughts about the research lesson plan and a record of their intended adjustments. Activity 3-2 (appendix 9) provides a record of the teachers’ reflections about their taught research lesson. As such, these three documents provide data which are ‘records of activity the researcher could not observe directly’ (Stake 1995, p. 68) as the sessions unfolded.

All three activity sheets were completed individually in sessions 1, 2 and 3 respectively. Activity sheet 1-2 was completed individually, whilst activity sheets 2-3 and 3-2 were completed individually but as part of small-group discussions. In each of RAM sessions

1, 2, and 3, Drew, Jo and Ronnie were placed in a small group to work together to provide a relatively consistent experience. The participants were responsible for making their own decisions, making their own annotations, and making their own reflections. I recognise the written records only provide a snapshot of the full discussions, and a snapshot of each participant's thinking. Though teachers were seeking to complete the documents as part of the respective sessions, they did know the activities might be analysed after giving their informed consent in the RAM introductory session. The completed documents provide a relatively unobtrusive source of evidence (Yin 2018, Tight 2019) which were intended to be useful for the session rather than merely being carried out for the purpose of analysis.

Copies of the completed sheets for activities 1-2, 2-3 and 3-2 for one participating teacher can be seen in appendices 11 and 12 and 13 respectively. Looking at these completed documents highlights two issues – firstly, they only capture a snapshot of thinking, and secondly, they necessarily require some degree of interpretation. For example, in Appendix 12, in response to whether students would be asked to make accurate copies of the diagram, the teacher writes on three separate lines 'Draw out As explaining But give them copies'. This is a very brief conclusion that only partially captures the conversations that took place during the session. Therefore, I necessarily make some informed decisions about how to interpret what is written. In this case, drawing on my facilitation of the session and subsequent interviews, I believe the teacher was saying they intended to draw the diagram in front of the students and explain the key features as they go. Alongside this, they intended to provide the students with a printed copy of the problem to refer to. An alternative interpretation of the

annotations could be made, but I seek to provide as authentic an account as possible of this co-produced data. To support this, I seek to ensure my interpretations of documents are ‘corroborated by other evidence’ (Coe 2021, p. 47). For example, with respect to asking students to copy the diagram, the participant also referred to the event in their research lesson reflection and in their fourth interview. Therefore, as Yin (2018, p. 86) suggests, a researcher is responsible for making convergent inferences based on both documents and ‘some unspecifiable element of common sense’.

At the end of each of the RAM sessions the participants filed their documents in their personal binders. These binders were collected in, and photographic copies of the documents made before being loaded into the qualitative data analysis software NVivo 14. The files were saved using a convention of ‘Problem [name]’, ‘Lesson Plan [name]’ and ‘Lesson Reflection [name]’ for each of the three teachers. Thus, with three documents for each of the three teachers, a total of $n=9$ documents were generated directly for analysis.

5.4.2 RAM programme activities providing data documents following processing

The seven activities shown in pink in Figure 5.2 provided evidence which was processed to generate data rather than being used directly. Activities 1-1, 2-1, 3-1 and 4-1 (appendix 6) provided a record of teachers’ initial beliefs, whilst activities 1-3, 2-4 and 3-4 provided the teachers’ views of beliefs seen in a specified element of the RAM materials (as discussed in Chapter 4). These activities were completed as part of the RAM programme with the intention of allowing teachers to articulate their beliefs about mathematics education and

their views on the beliefs underpinning the RAM materials. Following the informed consent of the introductory session, the teachers were aware the activity sheets would be used in this study. The data documents used for analysis were generated from the activity sheets in three stages of tabulating, graphing and commenting. Examples of the participant completed RAM activity sheets that contributed to the documents for analysis can be seen in appendices 10 to 13. Once my comments had been added, the documents described here were imported into NVivo 14 using a filename convention of 'Beliefs [name]'. In total, one belief document was created for each teacher giving a total of $n=3$ documents for analysis.

Tabulating the weightings

For each teacher, the weightings from the six RAM activity sheets were transferred to a table in the Beliefs [name] document based on the format shown in Table 5.3. Rather than transcribe the participant calculated values from activity sheet 4-1, I chose to carry out the calculation myself as shown in cells 10, 11 and 12 in Table 5.3. The two rows shown in bold were calculated as the mean average weighting of the three cells directly above them. For instance, cell 10 provides the calculated mean average Transmission weighting across the domains of mathematics (cell 1), teaching mathematics (cell 4), and learning mathematics (cell 7). Together, cells 10, 11 and 12 provide the mean average weightings for the teacher's cluster of beliefs about mathematics education. These calculated mean average of beliefs about mathematics education figures were compared against the teacher completed Activity sheet 4-1 to check for any potential mistakes.

Activity document providing the evidence	Domain	Transmission weighting	Discovery weighting	Connectionist weighting
1-1	Mathematics	Cell 1	Cell 2	Cell 3
2-1	Teaching mathematics	Cell 4	Cell 5	Cell 6
3-1	Learning mathematics	Cell 7	Cell 8	Cell 9
	Mean average of beliefs about mathematics education	Cell 10 (Mean average of cells 1, 4 , 7)	Cell 11 (Mean average of cells 2, 5 , 8)	Cell 12 (Mean average of cells 1, 4 , 7)
1-3	Mathematics (seen in RAM element 1)	Cell 13	Cell 14	Cell 15
2-4	Teaching mathematics (seen in RAM element 2)	Cell 16	Cell 17	Cell 18
3-4	Learning mathematics (seen in RAM element 3)	Cell 19	Cell 20	Cell 21
	Mean average of RAM elements	Cell 22 (Mean average of cells 13, 16 , 19)	Cell 23 (Mean average of cells 14, 17 , 20)	Cell 24 (Mean average of cells 15, 18, 21)

Table 5.3 Tabulating teachers' beliefs

Graphing the weightings

The second stage of creating the data documents was to represent combinations of the weightings shown in Table 5.2 on four triangular plots (Graham and Midgley 2000). Swan (2006b) utilised triangular plots as a convenient way to display three pieces of belief data that sum to 100%. I illustrate this on Figure 5.3 with notional weightings of 55%, 25% and 20% for the Discovery, Transmission and Connectionist categories respectively. In addition, dotted lines have been added to provide clarity in reading the values from the axes.

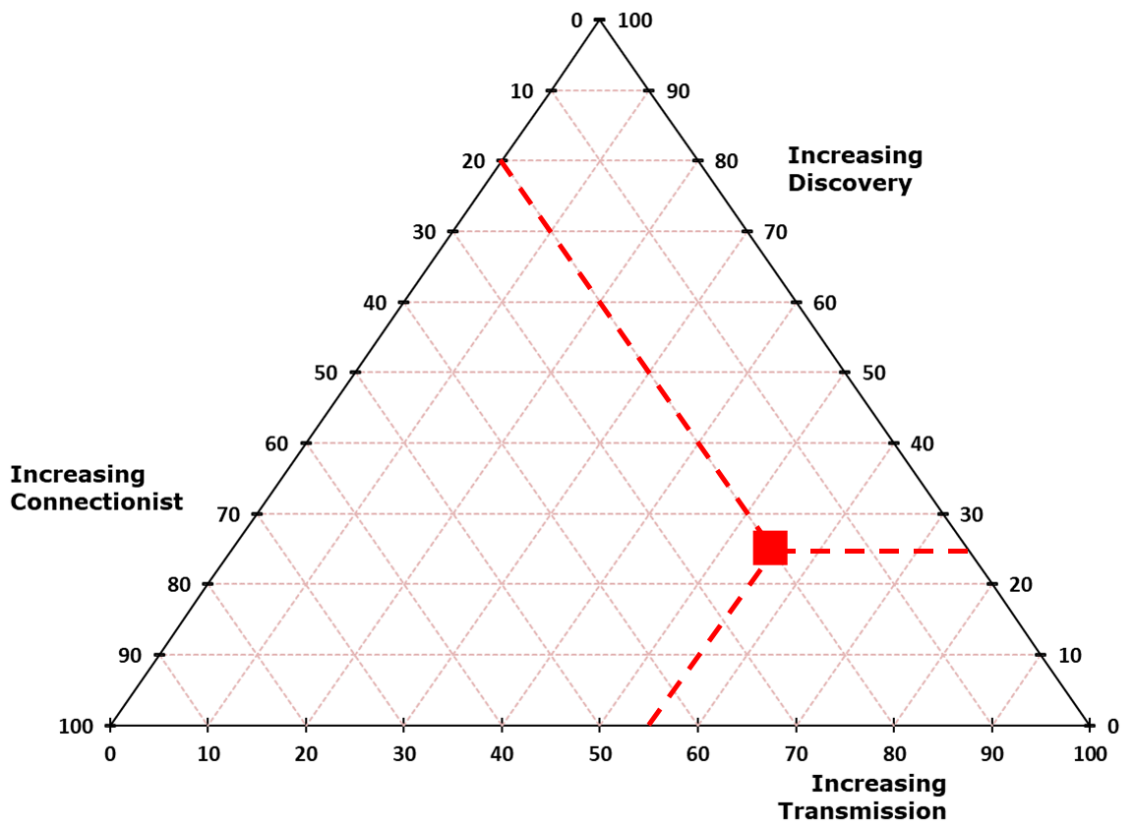


Figure 5.2 A triangular plot of belief data

I intended to use these triangular plots to facilitate an interpretation of the individual points, and to also allow relative comparisons. Therefore, for each of the three teachers, I created four triangular plots (with two points plotted on each one) as summarized in Table 5.4.

Plot number	What the plot shows	Data from Table 5.3 being plotted (Transmission, Discovery, Connectionist)
1	Personal beliefs about mathematics and beliefs about mathematics seen in RAM element 1.	(cell 1, cell 2, cell 3) and (cell 13, cell 14, cell 15)
2	Personal beliefs about teaching mathematics and beliefs about teaching mathematics seen in RAM element 2.	(cell 4, cell 5, cell 6) and (cell 16, cell 17, cell 18)
3	Personal beliefs about learning mathematics and beliefs about learning mathematics seen in RAM element 3.	(cell 7, cell 8, cell 9) and (cell 19, cell 20, cell 21)
4	Mean average of cluster of beliefs about mathematics education and mean average of RAM elements.	(cell 10, cell 11, cell 12) and (cell 22, cell 23, cell 24)

Table 5.4 Triangular plots generated for each teacher

An example of a triangular plot that might be produced from this process is shown in Figure 5.3. Here the personal beliefs weightings (shown in red) can be compared to the view of the RAM material (shown in blue). In addition, on these triangular plots I added arrows to the axes to indicate the relative difference. As can be seen, I use a black arrow to indicate a participant saw less of the belief in the RAM material compared to their personal beliefs. Similarly, I use a green arrow to indicate a participant saw more of the belief in the RAM material compared to their personal belief.

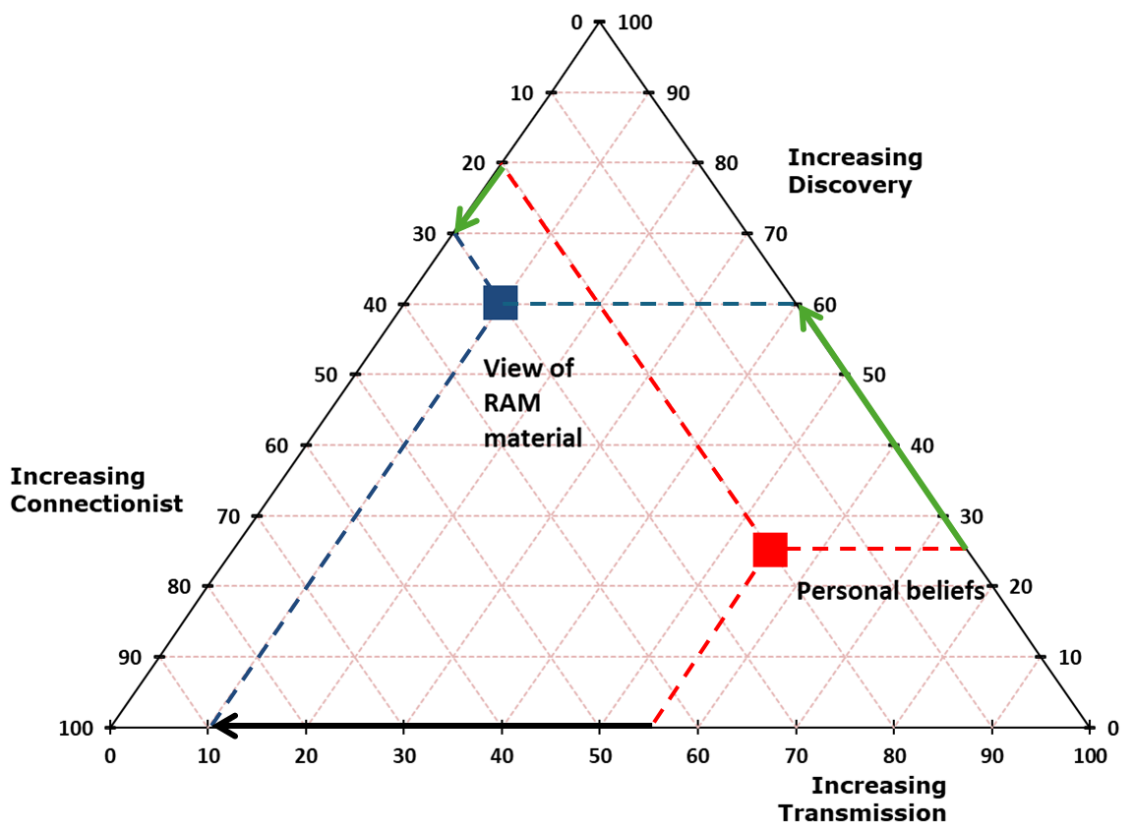


Figure 5.3 A triangular plot showing relative belief weightings

Commenting on the weightings

As a final step in generating data, I annotated the triangular plots and wrote summary comments of what I noticed for each teacher. These annotations and descriptions were completed with the intention of stating observations and facts, rather than straying into interpretation and analysis. However, I recognise there is some overlap and that my observations may have been influenced by my contradiction informed perspective and my embryonic ideas around interpretation.

My approach to the triangular plots differs to that adopted by Swan (2006b). Firstly, Swan (2006b) took the approach that a teacher can only be classified in one of the belief categories at a time. For instance, weightings of 55%, 25% and 20% for the Transmission, Discovery and Connectionist respectively would mean the teacher is classified as a teacher with Transmission beliefs about mathematics education. This approach is emphasised by how Swan (2006b) utilised the triangular plots. As illustrated in Figure 5.4 Swan split triangular plots into three regions of Transmission (bottom right), Discovery (top), and Connectionist (bottom left) to help identify the teacher's dominant belief.

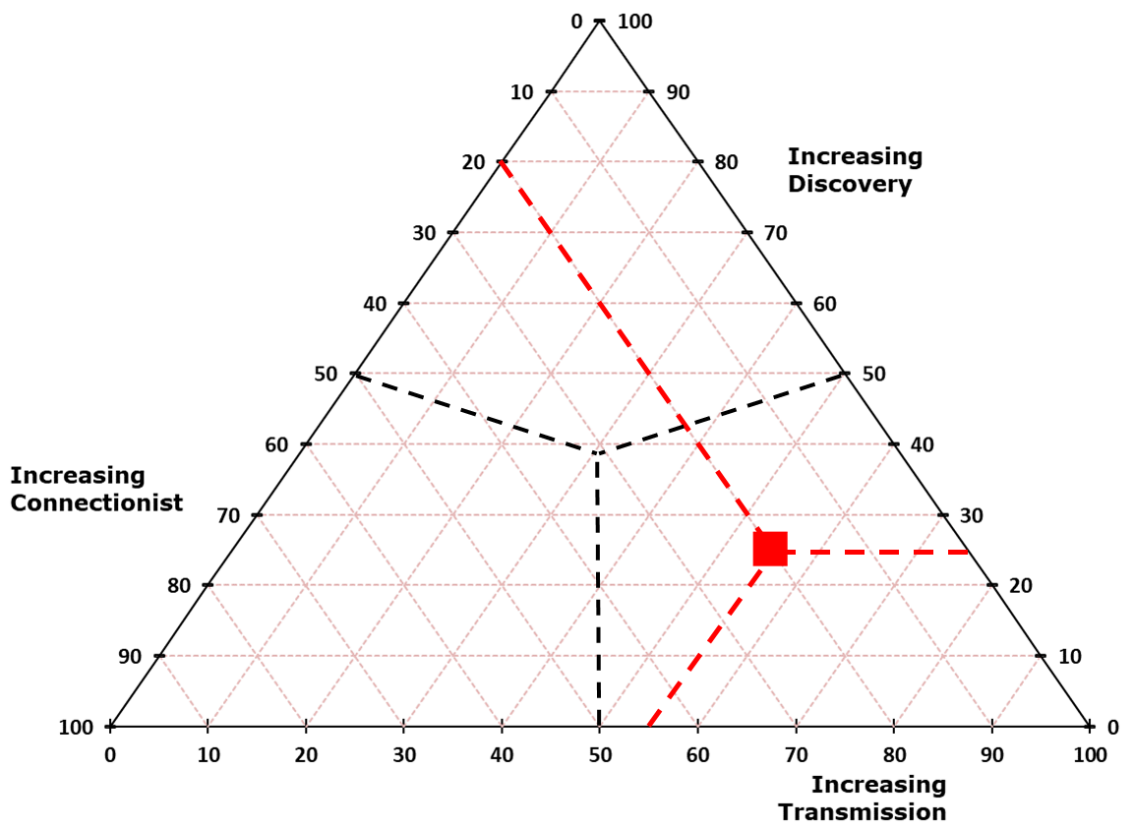


Figure 5.4 A triangular plot of belief data with regions

Using Figure 5.4, the teacher's dominant belief about mathematics education would be identified as Transmission. In contrast, since I anticipate teachers' beliefs about mathematics education as being contradictory, I do not replicate the idea of regions on the Beliefs [name] documents. In addition, I also note the black dotted lines which create the three regions indicate where equal belief weightings, in two or more of the belief categories, would be plotted. I address my understanding of these diagrams, and the boundary lines, in my interpretation of the data regarding teachers' beliefs about mathematics education in Chapter 6.

The second difference in my approach compared to Swan (2006b) stems from the nature of the data I collected following the professional development sessions. Swan (2006b) sought to collect data on participants' post professional development programme beliefs to measure change. In contrast, I seek to understand participants views on the beliefs about mathematics education they see as underpinning the RAM materials. My approach facilitates the contrasting of beliefs about mathematics education (by domain) with participants' views of the beliefs underpinning each RAM element. I am less interested in measuring short term claims of change and more interested in understanding teachers' views of the beliefs about mathematics education they see in the RAM programme.

5.4.3 Semi-structured interviews providing transcribed data documents

Recordings 0, 1, 2, and 3 (shown in blue on Figure 5.1) were taken from semi-structured interviews (Brinkmann and Kvale 2015) that were completed with each of the three teachers within one week of the corresponding RAM session. All participants were offered the choice of an online interview via Microsoft Teams or a face-to-face meeting. All three teachers elected to carry out the interviews virtually due to the ease and convenience of an online meeting. In Table 5.5, I provide a record of the length of the recordings for the twelve online interviews that took place.

Interviewee	Length of interview recording (minutes:seconds)			
	Interview 0	Interview 1	Interview 2	Interview 3
Drew	31:20	27:33	22:55	21:14
Jo	18:30	15:08	27:18	23:25
Ronnie	27:09	30:52	17:57	24:08

Table 5.5 Summary of interview lengths

All interviews took place with cameras enabled so both participants were able to read verbal and non-verbal cues from one another (Mears 2021). Deakin and Wakefield (2014) note there are some potential drawbacks with on-line interviews such as higher absenteeism and difficulties with building working relationships. However, my personal experience of the virtual interviews accords with Archibald *et al.* (2019) who suggest interviewees prefer an on-line experience as it is convenient, interactive and enables personal connections. The participants took part in the interviews from desk top computers situated in their own classrooms either during a free period, or after school.

The semi-structured interview provides a way to understand the experiences of others (Mears 2021), provides deeper insight into participants' perspectives (Stake 1995) and provides insight into critical events (Yin 2018). I align with Parker (2005) who suggests most interviews could be considered semi-structured since the interviewer and the interviewee are likely to have different aims. Therefore, to help ensure my aims were clear to me, I prepared

interview guides (Brinkmann and Kvale 2015) which were intended to balance focus with responsiveness to the interviewee.

My use of interview guides

Copies of the interview guides designed for interviews 0, 1, 2 and 3 are provided in Appendix 16. Brinkmann and Kvale (2015) identify semi-structured interviews to be most purposeful when evidence is sought around chosen themes and informed by a consideration of worthwhile knowledge. As discussed in the design of the RAM resources in Chapter 4, there were several features I thought would be helpful to gain participants' insights on. Firstly, I included questions around beliefs about mathematics education to provide insight into the theory of learning I have adopted for mathematics DPD. Secondly, I included questions around the potential moments of variation I designed to provide insight into the agents of change approach I have adopted.

Regarding beliefs about mathematics education, I planned to ask participants to comment on their personal belief weightings by domain, and their views of the RAM elements. For instance, in interview 1 I planned questions on participants' personal weightings provided for the domain of beliefs about mathematics in the introductory session, and their views of the beliefs about mathematics seen in RAM element 1. Similarly, for interviews 2 and 3, I planned questions around personal beliefs and views of beliefs underpinning RAM elements 2 and 3 in the domains of teaching mathematics and learning mathematics respectively. Regarding my understanding of agents of change, I included questions in the

interview guides around the events I anticipated might lead to moments variation. For instance, as discussed in Chapter 4, I anticipated potential moments of variation may occur in RAM session 1 when participants solve the problem individually, when they work in small groups, and when a potential classification is shared. In the same way, in interviews 2 and 3 I prepared questions to support my interpretation of moments of variation regarding planned changes to the lesson plan, and unplanned changes in the live research lesson.

My approach during semi-structured interviews

In planning my approach to the semi-structured interviews, I was clear my aim was to develop knowledge rather than bring change to the interviewee. Brinkmann and Kvale (2015, p. 48) recognise qualitative interviews have been influenced by psychoanalytic techniques but are clear any changes to the interviewee should be considered a ‘side effect’ rather than the aim. Some of the interviewees were very open about their work lives and their professional frustrations. For example, one interviewee spoke of their recent disappointment at being turned down for a job at another school. In such situations my role was not to seek to resolve this issue, but to listen to the concern, before trying to return to my focus. Without adopting a psychoanalytic interview approach, I do think there is significance in being aware of what an interviewee chooses to talk about (Mayo 2000). I therefore aimed to balance following a line of enquiry with allowing a participant to express their thoughts (Parker 2005). Thus, whilst steered by my interview guides, I sought to listen to what was said, listen to what was not said, and listen for what cannot be said without assistance (Mayo 2000).

In practice, I was conscious my use of an interview guide could create the perception of a power imbalance (Brinkmann and Kvale 2015) for interviewees. Mayo (2000, p. 64) offers some guiding principles which he developed to move away from a ‘question-and-answer type of interview’. I see this as a necessary part of a semi-structured interview to mitigate against ‘an instrumentalization of the process’ (Brinkmann and Kvale 2015, p. 37). Therefore, during interviews I planned to give my attention to the interviewee, listen rather than talk, avoid arguing, and summarise what was said (Mayo 2000). I endeavoured to create an environment whereby meaningful data would be generated through the interviewee feeling valued, rather than create an atmosphere that might suggest I was only interested in scripted questions. Whilst Mears (2021) suggests this is a contributing factor in developing rapport, I am a little more cautious in making this claim. Claims of creating rapport are ‘necessarily one sided’ (Parker 2005, p. 66) and neglect to consider the perspective of the interviewee. For me, the conversations that took place were not about rapport, but about an attempt to generate knowledge about ‘a particular account on a particular occasion’ (Parker 2005, p. 67).

My transcription of data

When transcribing recordings Hammersley (2020, p. 378) suggests there are ‘no simple rules to be followed’. Decisions must be taken around whether to transcribe all the recordings; whether to adjust transcribed words to reflect the spoken sound; whether to include non-words such as ‘urr’; whether to include silences and pauses; and whether to label the speakers (Hammersley 2020). I began with an automated transcription of each interview generated from Microsoft Teams. This made possible a ‘direct coding on the sound’

(Brinkmann and Kvale 2015, p. 206) with a reasonable degree of accuracy. As McMullin (2021) notes, automated transcription provides an acceptable first draft, but the software may be unable to accurately determine punctuation, hesitation words, and nonverbal cues. Therefore, for each of the transcriptions, I sought to ensure confidence in what had been generated through repeated listening to the recordings and adjusting punctuation or word errors according to what I heard. In doing this I did not seek to reflect pronunciation or dialect but to create a transcript on the naturalized-denaturalized spectrum, between ‘intelligent verbatim’ (McMullin 2021, p. 141) and ‘full verbatim’ (McMullin 2021, p. 141).

In addition to adjusting some of the automatically transcribed words to what I heard, I also removed ‘back-channel noises’ (Hammersley 2020, p. 375) such as laughter and ‘urr’. Similarly, no record of non-verbal communication such as ‘posture and gesture’ (Brinkmann and Kvale 2015, p. 204) were included. Whilst I considered adding this information, I felt it would require additional subjective decisions to be made (Brinkmann and Kvale 2015) and so recognise a potential source of data may be ‘lost in transcription’ (Bourdieu and Ferguson 1999, p. 622). However, in the approach I planned to adopt for analysis, I consider the completed documents and written language as sufficient. I believe the ‘language we use is woven into reality’ (Parker 2005, p. 17) and therefore elected to focus only on transcribed words.

Some re-formatting of the Microsoft transcriptions also took place. As can be seen in the sample excerpt of Appendix 17, I used two different fonts to show when I spoke and when

the interviewee spoke. Parker (2005, p. 54) suggests the interviewer's words should be 'treated with as much care' as those of the interviewee, highlighting the interviews took place in the context of a conversation. An excerpt from an interview 1 transcript is shown in Appendix 17 and illustrates how I attempted to balance using my interview guide with in-the-moment responses. For instance, according to my guide for interview 1, I had planned to begin by asking for comments on personal weightings regarding beliefs about mathematics and then ask for comments about the session. However, in practice I found it easier to talk about the shared experience of the first RAM session, before going back to ask about weightings.

Once satisfied with the accuracy of the transcriptions, as recommended by Mero-Jaffe (2011), I sent a copy to each of the interviewees. It is suggested this empowers interviewees, removes any potential embarrassment about seeing their words written down (Mero-Jaffe 2011) and gives the opportunity for corrections to be made (Brinkmann and Kvale 2015). Whilst receiving acknowledgement of receipt, none of the interviewees suggested any changes to the transcripts. The transcribed files were then loaded into NVivo 14 and saved using the naming convention of 'Interview (number) [name]'. With four interviews for each of the three teachers, this process generated n=12 documents for analysis.

5.5 Analytic framework

With the generation of data described, I now turn to explain the analytic framework I utilise in this study. To answer my research question, I needed a framework that would

support the finding of commonalities in participants' experiences of the RAM programme. Therefore, I felt the general approach of Thematic Analysis (TA), which seeks to 'identify patterns and meanings within data' (Finlay 2021, p. 103), would be appropriate. Within the broad spectrum of TA techniques, Finlay (2021) suggests there exists two overlapping categories of scientifically descriptive and artfully interpretive. Furthermore, Finlay (2021) proffers a description of a researcher who resides in the overlap as one who embraces creativity, sees meanings as fluid, and is rigorous in generating and evidencing themes. These resonating principles led me to Reflexive Thematic Analysis (RTA) (Braun and Clarke 2021b), which prioritises the interrogation and transparency of 'the researcher's role' (Finlay 2021, p. 107) in the production of knowledge. Indeed, three features of RTA make it particularly relevant to my study in terms of creative reflexivity, compatibility with deductive analytic orientations, and a rigorous framework (Braun and Clarke 2021b).

5.5.1 Utilisation of creative reflexivity

Braun and Clarke (2021b) suggest the term reflexivity captures a recognition of personal subjectivity and a willingness to interrogate it. It is this reflexivity which becomes a creative '*resource* for doing analysis' (Braun and Clarke 2021b, p. 8) and 'the fuel that drives the engine' (Braun and Clarke 2021b, p. 12) of analysis. I recognise, in my role as the researcher, I am central to the production of knowledge (Terry and Hayfield 2020) and therefore am responsible for showing 'critical self-awareness' (Finlay 2021, p. 107). Braun and Clarke (2021b) identify this critical self-awareness as including reflecting on disciplinary assumptions, theoretical assumptions, personal assumptions, and design choices. I have sought

to adopt this position of reflexivity throughout my work and include a final reflection in Chapter 9.

I suggest my utilisation of reflexivity is complex and likely to only be ‘partial, tentative, [and] provisional’ (Finlay 2002, p. 542). The requirement for reflexivity rests upon being ‘critically self-conscious’ (Finlay 2002, p. 542) and considerate of subjective feelings. Furthermore, Parker (2005, p. 28) cautions against reducing reflexivity to a self-indulgent ‘feeble confession’ in an attempt justify the knowledge prioritised. Instead, reflexivity should include a consideration of ‘the social relations that have enabled someone to experience themselves as an individual in relation to others’ (Parker 2005, p. 29). A similar sense of wider interaction is captured by Berger (2015), who claims a researcher must take responsibility for the effect they may have on people, the interviews, the data in general and its interpretation.

Personal reflexivity

In Chapter 1, I outlined both my personal experiences and philosophical underpinnings that have led to this work. I see these two factors as contributing to the knowledge I generate in this study but recognise these ideas have developed in relation to others. Indeed, the education experiences I shared in Chapter 1 provide some understanding of ‘how it might have come to be that I felt this’ (Parker 2005, p. 29). My experiences have led to a desire to deepen my understanding of teacher professional development, and to value the experiences of people. I believe contradiction potentially provides a basis for gaining this deeper insight,

and a ‘basis for freedom, equality and solidarity’ (McGowan 2019, p. 10). My ontological perspective has been particularly shaped through books by McGowan (2016, 2019) and Žižek (2012, 2023). Their Hegelian frameworks, and employment of psychoanalytic thinking, provides a perspective which I believe is under-utilised in education circles.

This study is also shaped by my ‘social relations’ (Parker 2005, p. 29) with the teachers who participated in the study. For instance, as mentioned earlier, prior to this study Drew had been part of a mathematics teacher training programme I led. Though this may imply some power imbalance I suggest it is mitigated by Drew being a confident, mature trainee close to my age. The first time I met the rest of the department at Victoria School was at the RAM introductory session. I am aware there is a potential power imbalance with Jo who was in an early stage of their teaching career and lacked mathematical confidence. Additionally, during my first meeting with Ronnie I felt they revealed a cautious and slightly suspicious approach to me as a university researcher. Each of these over-simplified perspectives may have had some effect on the completion of documents within sessions and may have affected the direction of the semi-structured interviews.

5.5.2 Compatibility with deductive analytic orientations

A second feature of RTA is its compatibility with a deductive orientation to analysis (Braun and Clarke 2021b). Within this orientation there is acceptance that analysis can be ‘shaped by existing theoretical constructs’ (Braun and Clarke 2021b, p. 10). Whilst recognising the potential for a ‘a monopoly of interpretation’ (Brinkmann and Kvale 2015, p.

38), I seek to present a transparent account of my decisions. This work is not a study of teachers, but nor is it an attempt to merely share their stories. Instead, I work alongside the teachers to gain their insights whilst retaining my responsibility for analysing and interpreting the data. In my analyst-driven deductive approach (Byrne 2022) I utilise two theoretical constructs to inform the development of codes and themes. Firstly, in respect to beliefs I draw on Swan's (2006b) framework of beliefs about mathematics education as discussed in Chapter 2. Secondly, I seek to draw on McGowan's (2019) contradiction informed understanding of human nature and take note of moments of variation as discussed in Chapter 3. These influences are explored, and exemplified, in greater detail in the next section. Whilst placing myself on the deductive side of the analyst spectrum, I also briefly discuss an alternative thematic analysis approach influenced by dialectics.

Thematic analysis influenced by dialectics

MacCarthy (2021) commends Dialectic Thematic Analysis (DTA) as an approach to analysis which facilitates the concurrent use of both analytic and deductive approaches. MacCarthy (2021) claims DTA follows a process whereby data is simultaneously analysed inductively and deductively, before being brought together to produce new knowledge. For me, this epistemological process echoes the approach of determinate negation discussed in Chapter 1. The inductive analysis acts as the thesis, the deductive analysis acts as the antithesis, and a synthesis between the two is sought. Again, I believe dialectics has been reduced to a misunderstood epistemology, and consequently the potential ontological insight it provides is lost. DTA sets the analytic and deductive approaches against each other in seeking

to reveal the contradiction rather than see the contradiction that exists within. Instead, I believe:

at every turn, contradiction manifests itself, even when one attempts to articulate the simplest proposition. If contradiction isn't just an error of thought but a prerequisite of being, then it becomes impossible to avoid (McGowan 2019, p. 63).

This impossibility of avoiding contradiction has led to it informing the generation of my themes and my interpretation of data, as shown in the next section. It is the ontological insight of dialectics, and not simply an epistemological process, that shapes the 'thoroughly partial 'reading'' (Braun and Clarke 2021b, p. 174) I seek to tell.

5.5.3 A rigorous framework

The third feature of RTA I find helpful is the clear guidance provided around 'developing, analysing and interpreting patterns' (Braun and Clarke 2021b, p. 4). This guidance takes the form of 'robust, process guidelines, not rigid rules' (Braun and Clarke 2021b, p. 10). Or, as Terry and Hayfield (2020, p. 434) suggest, the framework phases of RTA are intended to provide 'ever-increasing engagement' rather than simple steps of a recipe. Braun and Clarke (2021b) identify six phases of RTA – familiarisation, coding, generating themes, developing themes, naming defining themes, and writing up. I utilise this framework in my discussion of the next section and exemplify its use in my study.

5.6 My application of Reflexive Thematic Analysis

In this section I explain the journey I experienced in utilising RTA for the data generated from my study according to the phases of RTA suggested by Braun and Clarke (2021b). However, I will describe the phases of generating themes and naming themes together as they did not fit into two clearly distinguishable experiences for me. In addition, the write up phase will take the form of the subsequent chapters where I share my themes and seek to create a persuasive story that draws on ‘vivid data’ (Finlay 2021, p. 107).

5.6.1 Familiarisation with the data

Braun and Clarke (2021b) recommend an initial immersion in the data which involves re-listening to recordings, reading transcripts and studying other data. Indeed, my familiarisation process began in this way to ensure I was satisfied with the data before loading it into NVivo 14. Once the data had been loaded into NVivo 14 (see Appendix 18), I sought to purposefully review all documents again and create summary notes by documenting initial insights (Terry and Hayfield 2020, Braun and Clarke 2021b). The resulting document can be seen in Appendix 19 and provided a helpful way for me to gain an overview of a large amount of data. To create the summary document, I worked through the data documents for Drew, Jo and Ronnie in turn. As with all the phases of RTA in my study, my analysis included consideration of both qualitative data, and more quantitative data in the form of tables of beliefs weightings and triangular plots. Following the creation of these summary notes, I embarked on a phase of initial ‘critical engagement’ (Braun and Clarke 2021b, p. 44) by making initial observations of commonality and possible reasons. My summary document

reveals initial considerations around Task Ladder lesson structure, thoughts around effective professional development, beliefs about mathematics education, and views of the RAM programme.

5.6.2 Coding the data

Following familiarisation with the data, I generated ‘pithy, analytically-meaningful’ (Braun and Clarke 2021b, p. 35) codes in a ‘systematic way’ (Finlay 2021, p. 107). The codes I created were informed by the familiarisation phase, but also by the theory that has been informing this work. Hence, I took notice of data that could be seen as indicating Transmission, Discovery, and Connectionist beliefs according to the work of Chapter 2. Similarly, my interest in Hegelian contradiction encouraged me to pay attention to moments of variation in terms of discomfort and adjustments. In Chapter 3, I defined moments of variation as when a teacher’s conscious desires contrasted with their unconscious actions. Therefore, in coding the data I sought to take notice of teacher’s claims and actions in terms of beliefs about mathematics education.

Utilising the functionality of NVivo 14, I initially coded the data for each teacher beginning with Drew, then Jo, then Ronnie. My intention here was to create a set of codes from the first teacher, which could then be added to with each subsequent teacher. To then disrupt this pattern, and to help develop a more evenly coded dataset (Braun and Clarke 2021b), I repeated the coding process by looking at the equivalent documents for each of the three teachers. The initial set of codes I generated can be seen in Appendix 20 and I

experienced some doubt around whether I was duplicating codes or creating needless additional codes. For example, after coding some extracts as 'Connectionist', I then coded other extracts as 'Connectionist view of mathematics', 'Drew Connectionist view of mathematics', and 'Connectionist linked to other beliefs'. However, Braun and Clarke (2021b, p. 64) advise to 'keep all potentially relevant ideas 'in play'' with a view toward gradual refinement.

To help structure the refinement of codes, I firstly re-coded data that was relevant to teachers' beliefs about mathematics education. Secondly, I re-coded data relevant to teacher's views of the beliefs about mathematics education underpinning the RAM programme. Thirdly, I re-coded data that was relevant to what teachers claimed about their development and their actions within the RAM programme. This re-coding included promoting codes (Terry and Hayfield 2020), removing codes, re-coding the data within (Finlay 2021), and clustering codes (Braun and Clarke 2021b). The final set of codes for my dataset can be seen in Appendix 20.

Following the creation of my codes I spent time reflecting on the process I had adopted, and the fact my largely deductive approach meant some data had been ignored. I considered whether I was selecting data to support my thinking or even misrepresenting the dataset. For example, two teachers spoke to me about their experiences in applying for jobs, but I elected not to make use of these sections since they did not relate to beliefs about mathematics education or moments of variation. Braun and Clarke (2021b, p.101) acknowledge this concern and recognise analysis may only be based on 'part of the dataset'.

With the large amount of data I had, I felt it necessary to make judgement calls on whether the data simply reflected the different aims of the interviewer and the interviewee (Parker 2005). Eventually, I felt I had a set of codes that satisfied my desire for a diversity of meaning, and an indication of my ‘analytic take on things’(Braun and Clarke 2021b, p. 71).

5.6.3 Generating and naming themes

Following the creation of a set of codes, I sought to generate meaningful themes that would ‘describe patterns in the data’ (Finlay 2021, p. 107). These themes were influenced by a desire to answer my research question, alongside the ideas of contradiction introduced in Chapter 1. Therefore, I sought to create names that would signal both ‘meaning and analytic direction’(Braun and Clarke 2021b, p. 112) whilst articulating ‘the central organising concept’ (Terry and Hayfield 2020, p. 439). In response to seeking to understand how mathematics teachers’ experiences of a mathematics DPD programme can be interpreted through the lens of contradiction I have suggested three themes. I claim beliefs are interdependent, moments of variation are acts of self-sabotage, and belief trajectories show journeys not destinations. The simple act of listing my theme titles belies the time invested in this stage, and the re-writes that took place. In addition, I am conscious the inclusion of my analytic direction in the theme titles means there is still work to be done in conveying the ‘scope, boundaries and core concept’ (Braun and Clarke 2021b, p. 108). Indeed, I seek to convey this meaning through summaries, extended data extracts, and my analytic interpretations in the next three chapters. By addressing each theme in a separate chapter, I combine ‘the ‘results’ and ‘discussion’

sections' (Braun and Clarke 2021b, p. 131) with the intention of avoiding repetition and providing clarity.

5.7 Summary of Chapter 5

In this chapter I have summarised the methodological approach and research design I have adopted in this study. I have sought to act ethically throughout, and my approach has been positioned within an epistemological and ontological understanding of contradiction. I have adopted a case study approach with the intention of answering my research question around how mathematics teachers' experiences of a mathematics DPD programme can be interpreted through the lens of contradiction. The case has been clearly defined as the one-year RAM programme, described in Chapter 4, taking place at Victoria School. Insight into this case is gained through the experiences of three teachers who provide completed RAM documents and semi-structured interviews. Data has then been generated from these sources and examined through Reflexive Thematic Analysis (Braun and Clarke 2021b). Finally, in this chapter I have detailed my application of RTA to this study and my utilisation of NVivo 14 to support this. I have provided details on my deductive approach, informed by contradiction, a framework of beliefs about mathematics education, and my understanding of moments of variation. With my approach established, in the next three chapters I provide the results and analysis to support each of my three themes.

Chapter 6.

Analysis of theme 1: beliefs are interdependent

6.1 Synopsis of Chapter 6

In this chapter I present the first of three themes which I developed from my data through Reflexive Thematic Analysis. The theme name of *beliefs are interdependent* is simple, and only hints at the claims I make in this chapter. To present these claims I share the three participating teachers' personal belief weightings in the domains of mathematics, teaching mathematics, and learning mathematics. These weightings were completed in the Rethinking Approaches in Mathematics (RAM) introductory session and provide a measure for each teachers' cluster of beliefs about mathematics education. In addition to considering the weightings, I interpret supporting data taken from interview transcripts following semi-structured interviews. Once the results have been presented, I discuss my interpretation of this data through a form of dialectic thinking. I highlight the contradiction I see as inherent in Swan's (2006a, 2006b) original interpretation of the data, before suggesting a contradiction informed alternative. Finally, I translate the three examples of dominant beliefs in this study to suggest the universal categories of beliefs about mathematics education are contradictory.

6.2 Introduction

Within this chapter, and the next two, I have decided to present my analysis by examining the results first followed by my interpretation. In doing this, I acknowledge the results are not a neutral presentation of data since I have made decisions on what to include,

what not to include, and how to organise the sections. Therefore, even in presenting my results I am providing an initial ‘interpretive account’ (Braun and Clarke 2021b, p. 132) of the data. I also note that in providing these interpretive accounts it may appear at times that I focus on teachers. However, I view the teachers as providing data from three contrasting perspectives regarding the case of the RAM programme. Once the data has been shared, I then seek to make ‘theoretical, scholarly and wider contextual interconnections’ (Braun and Clarke 2021b, p. 132) in my discussion.

My results are organised under headings that indicate the analytic interpretation I will make in the second half of this chapter. I share data to suggest the teachers have dominant beliefs about mathematics education, but that they can also be interpreted as being interdependent with non-dominant beliefs. I use the word interdependent to capture the sense that each belief category is dependent on the other belief categories, or as Engley (2023, p. 747) states ‘on their own internal opposition’. This theme was developed from the personal belief weightings provided by the participating teachers in the RAM introductory session, and from interview comments. As described in Chapter 5, the personal belief weightings were recorded in the documents titled Beliefs [name] along with the corresponding triangular plots. Interview comments were recorded in the documents Interview (number) [name], and provide the teacher’s own descriptions of their teaching, and their discussions of the weightings they provided. In the development of this theme, I included the codes shown in Table 6.1.

Codes
My main belief about mathematics
My main beliefs about teaching mathematics
My main belief about learning mathematics
My main belief about mathematics education
I also believe this
Simultaneous beliefs
External pressure on beliefs

Table 6.1 The codes that contributed to the first theme

6.3 Results supporting the theme of beliefs are interdependent

To provide an overall indication of the teachers' beliefs, I begin by sharing data on the calculated cluster of beliefs about mathematics education. These weightings are supported by teacher comments taken from Interview 0 [name] made following the RAM introductory session. I then take a detailed look at the domain level beliefs which make up the beliefs about mathematics education for each teacher. Hence, I share results at the domain levels of beliefs about mathematics, teaching mathematics, and learning mathematics separately and support each of these with results taken from the corresponding interview transcripts.

6.3.1 The cluster of beliefs about mathematics education

The weightings for the cluster of beliefs about mathematics education, which were calculated following the introductory session, are shown in Table 6.2. A bold typeface has been used to draw attention to the dominant belief category for each teacher.

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	12	65	23
Jo	50	28	22
Ronnie	11	12	77

Table 6.2 The cluster of beliefs about mathematics education for Drew, Jo and Ronnie

Additionally, the data from Table 6.2 is displayed in the form of a triangular plot in Figure 6.1, with dotted lines used to highlight each teachers' dominant belief category.

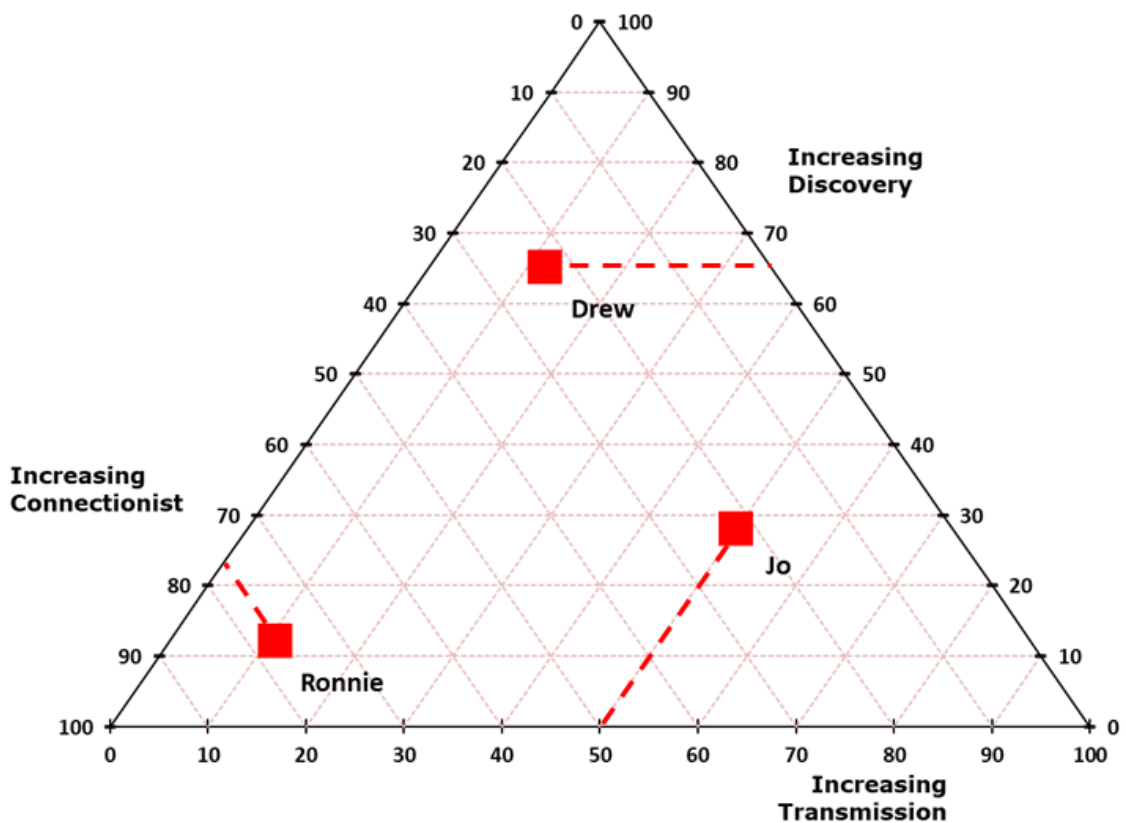


Figure 6.1 A triangular plot of the cluster of beliefs about mathematics education for Drew, Jo and Ronnie

As shown in Table 6.2 and Figure 6.1, Drew had a dominant belief of Discovery, Jo had a dominant belief of Transmission and Ronnie had a dominant belief of Connectionist for the cluster of beliefs about mathematics education. My interpretation of the reasonableness of this data is supported by teacher comments recorded in Interview 0 [name]. These comments were made prior to the start of the RAM programme and were given in response to the

planned interview guide request to describe their teaching, alongside subsequent follow up questions.

Drew's dominant Discovery belief interdependent with the other categories

As seen in Table 6.2, Drew had weightings of 12%-65%-23% for Transmission-Discovery-Connectionist respectively for beliefs about mathematics education. Drew's dominant Discovery weighting was reflected in the importance they placed on problem solving and seeing mathematics as a tool. Drew stated (Interview 0 [Drew], 00:10:59) "what's the point in having a tool if you don't know how to use it?" and "what they [the students] really need to do is problem solve". Drew additionally spoke of a desire to create stimulating problems for students to solve in lessons. Drew (Interview 0 [Drew], 00:12:38) illustrated this with two recent lesson foci where students were asked to independently discover "how long to drive to the moon" and "how many cans of coke would we need to empty into this room to fill it".

Though indicators of the dominant belief of Discovery were visible in Drew's interview, so too were comments reflecting the importance of the Connectionist weighting of 23%. For example, Drew (Interview 0 [Drew], 00:11:59) revealed how they saw problem solving (reflecting a Discovery belief) as ultimately helping to reveal mathematics is an interconnected body of ideas since "multiplication - it's a ratio problem really". Similarly, Drew (Interview 0 [Drew], 00:11:59). argued problem solving required making connections since it involved "classifying and linking". Interdependence with Transmission beliefs (given

a weighting of 12%) could also be seen when Drew discussed their use of the school wide Task Ladder lesson structure. Drew (Interview 0 [Drew], 00:12:38) confessed they used recall starters before engaging with problem solving to encourage practice and fluency despite it making them “feel a little bit queasy”. Drew (Interview 0 [Drew], 00:12:38) explained this discomfort existed because recall starters are “quite effective”, “boring” and “good at the same time”.

Jo’s dominant Transmission belief interdependent with the other belief categories

As seen in Table 6.2, Jo had weightings of 50%-28%-22% for Transmission-Discovery-Connectionist respectively for beliefs about mathematics education. Support for a dominant Transmission perspective could be seen when Jo spoke of their teaching as being “very structured” (Interview 0 [Jo], 00:01:15). Furthermore, Jo explained how the school’s Task Ladder lesson structure matched closely to their beliefs. Using the words of the Task Ladder structure, Jo described their teaching as:

check their understanding, can they remember what we did before? Introduce something new, do you get it? Let's have a practice. Let's just do an assessment. Bang, you're done (Interview 0 [Jo], 00:08:15).

However, Jo also provided interview comments that revealed interdependence on the Connectionist category (given a weighting of 22%). Jo (Interview 0 [Jo], 00:01:15) revealed, “I like students to come up and have a go on the board and I like students to explain”. Jo adopted an approach where teacher transmission of information and class involvement were interdependent with each other. For example, Jo (Interview 0 [Jo], 00:01:15) explained “I like

everybody to be involved”, “we bat it around”, and “it's very much about making it they're part of the classroom”. Alongside their Transmission and Connectionist beliefs, Jo also provided a significant Discovery weighting of 28%. However, there was no clear comments made in this interview that I interpret as reflecting Discovery beliefs.

Ronnie's dominant Connectionist belief interdependent with the other belief categories

Table 6.2 shows Ronnie had weightings of 11%-12%-77% for Transmission-Discovery-Connectionist respectively for beliefs about mathematics education. These numbers align well with comments made by Ronnie in their first interview. For example, when asked to describe their approach to teaching, Ronnie (Interview 0 [Ronnie], 00:03:31) stated, “I'm a big one for thinking about connections within maths” and concluded that, “rather than being compartmentalised, it's important to look at the going across”. Ronnie further confirmed the reasonableness of a dominant Connectionist weighting by speaking of a poster they once saw showing links across school mathematics topics.

Algebra was all blue, and like statistics and probability was all red, and it connected all of the things, and then the ones that were like kind of both, so involved both algebra and number whatever, were like the mixture of the two colours. And it was not some, just like five different colours of bars, it was like literally a complete rainbow (Interview 0 [Ronnie], 00:03:31).

Ronnie provided very low weightings for the Discovery and Transmission categories of 11% and 12% respectively. Accordingly, very few comments made in the initial interview could be used to infer these beliefs. However, Ronnie still talked about the value they placed on the school Task Ladder structure, which I interpret as reflecting Transmission beliefs

toward mathematics education. Ronnie began by describing the Task Ladder as “a way to try and make sure that teachers do what they’re supposed to do in a classroom” (Interview 0 [Ronnie], 00:15:48), a “non-negotiable” (Interview 0 [Ronnie], 00:17:57), and that “you can’t argue with it, just do it” (Interview 0 [Ronnie], 00:17:57). Following this, Ronnie (Interview 0 [Ronnie], 00:23:12) went on to admit “but, you know, I quite like the Task Ladder now, yeah, I’m, I’m not, I’m not opposed to it”. When asked to expand on this, Ronnie (Interview 0 [Ronnie], 00:23:12) confirmed ‘I am the expert in the room so sometimes I need to show students what to do’. Despite their initial frustration, Ronnie had accepted that helping students make connections in mathematics also sometimes required the teacher to transmit mathematics to students.

6.3.2 Beliefs about mathematics

The first domain which contributes to the cluster of beliefs about mathematics education is that of beliefs about mathematics. Here I examine the weightings for each of the three teachers in the domain of beliefs about mathematics taken from the documents Beliefs [name]. In addition, I support these weightings with comments recorded in the documents Interview 1 [name]. These comments were made following the first RAM session, but in reference to the weightings the participants provided during the RAM introductory session.

The teacher’s weightings for the domain of beliefs about mathematics are shown in Table 6.3, with a bold typeface used to highlight the dominant category

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	10	70	20
Jo	60	30	10
Ronnie	5	5	90

Table 6.3 Beliefs about mathematics for Drew, Jo and Ronnie

The data from Table 6.3 is displayed in the form of a triangular plot in Figure 6.2, with dotted lines used to highlight the dominant belief category for each teacher.

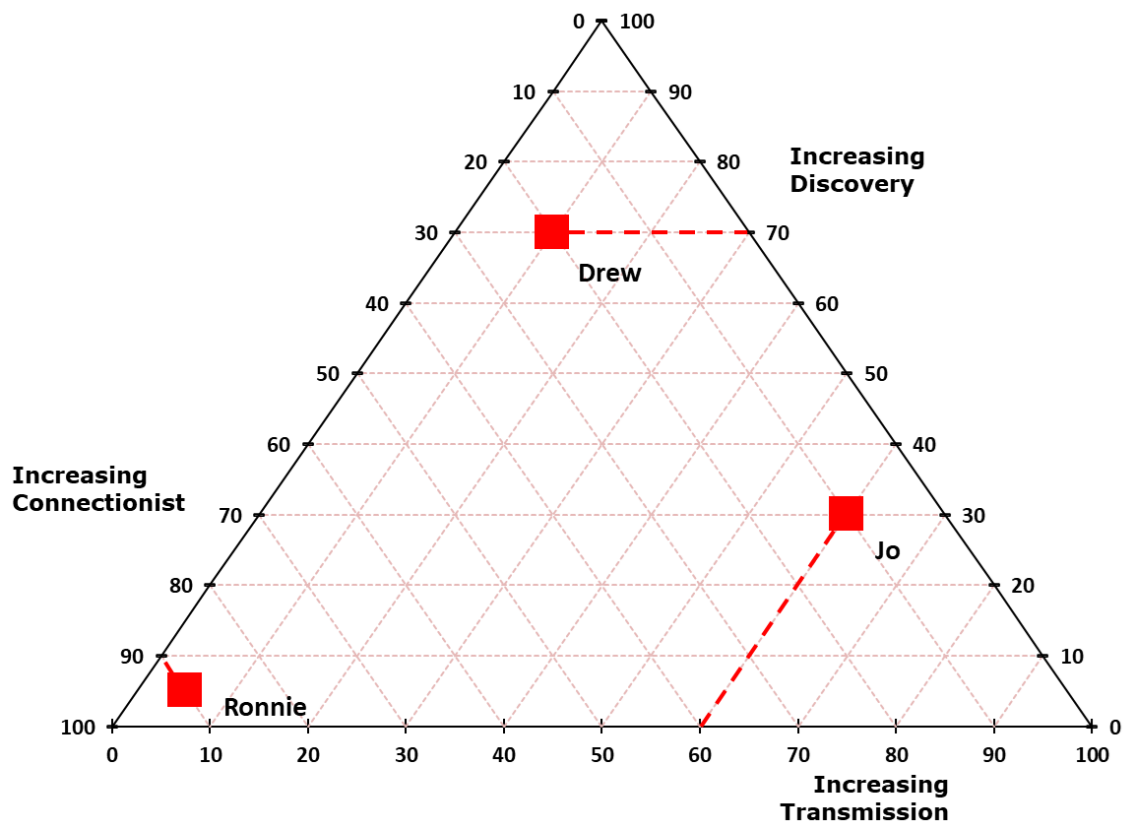


Figure 6.2 A triangular plot of beliefs about mathematics for Drew, Jo and Ronnie

During interview, the teachers confirmed, and justified, the weightings they had provided regarding beliefs about mathematics. In general, the teachers tended to begin by affirming their dominant weighting, before explaining their non-dominant weightings as being influenced by their view of national mathematics examinations.

Initial affirmation of dominant beliefs

As shown in Table 6.3, Drew, Jo and Ronnie provided dominant weightings of Discovery (70%), Transmission (60%), and Connectionist (90%) respectively. Drew (Interview 1 [Drew], 00:04:34) supported their Discovery belief by stating “kids enjoy maths when they see it answers questions”. Drew even explicitly stated their perspective that Discovery is better than the other two belief categories since “to be honest I think B [Discovery] is the zenith. If you can do B [Discovery] then you’re a good teacher” (Interview 1 [Drew], 00:09:58).

Jo (Interview 1 [Jo], 00:03:59) similarly affirmed their dominant perspective of Transmission by stating, “I am largely believing that, at the core, the core of maths, it is algorithms and procedures and rules”. Additionally, Jo explained:

there are loads of different concepts that involve, that have like so many different methods that you could solve something with, so we could teach them [students] one method, that general method that's usually used, or the one we use, or a set method must be known (Interview 1 [Jo], 00:06:37).

Finally, Ronnie (Interview 1 [Ronnie], 00:10:13) affirmed their dominant Connectionist belief weighting by saying, “I think all we do is really give them [students] the

language to connect up the maths”. Ronnie (Interview 1 [Ronnie], 00:10:13) continued by explaining how language enables creation, since “maths doesn't really get discovered. What happens is people develop the language to explain things”.

Interdependence of dominant and non-dominant beliefs

Each teacher also revealed the interdependence of their dominant beliefs with their non-dominant beliefs by making a link to national mathematics examinations. Drew explained a Connectionist weighting was necessary because “ultimately the qualification is not in discovering mathematics, the qualification is in applying mathematics” (Interview 1 [Drew], 00:05:20). Notably, Drew (Interview 1 [Drew], 00:05:20) explicitly stated, “I don't think A [Transmission] and C [Connectionist] are contradictory to B [Discovery] at all”. Instead, Drew saw all three belief categories as necessary and complementary to one other.

When Jo was asked to explain their different weightings for beliefs about mathematics, they explained how a Transmission perspective alone is insufficient for exams. Jo (Interview 1 [Jo], 00:06:22) stated rules are “not enough to do well in exams”, and so the teacher has responsibility to show how to “apply the rules in unfamiliar contexts as well”. Jo (Interview 1 [Jo], 00:06:22) re-iterated this view by explaining how exam requirements meant “you have to help students to want to tackle different problems”.

Ronnie also revealed seeing the interdependence of belief categories by explaining the influence of examinations on their approach. Ronnie suggested, “our responsibility is to get

them [students] through exams” (Interview 1 [Ronnie], 00:11:58). This led Ronnie (Interview 1 [Ronnie], 00:11:58) to the reflection that school can be seen as an exam production line since “we’re a factory. Yeah, we are. We are a factory”. Hence, Ronnie (Interview 1 [Ronnie], 00:11:58) claimed “my role is not just making connections, but teaching the kids how to communicate with the examiner”. Ronnie inferred national examinations show the need for more than only Connectionist beliefs about mathematics:

because that's what really matters. I mean, ultimately at the end of the day. You know, what are you getting tested on? You're getting tested on your ability to sit at a single desk and answer maths questions by yourself (Interview 1 [Ronnie], 00:11:58).

6.3.3 Beliefs about teaching mathematics

The second domain I take a detailed look at is that of beliefs about teaching mathematics. Participants completed their weightings in the introductory session but were asked to provide comments about these weightings in interviews following the second RAM session. Their responses, alongside responses to follow-up questions, are recorded in the documents Interview 2 [name]. The belief weightings taken from the documents Beliefs [name] are shown in Table 6.4, with a bold typeface used to highlight the dominant category.

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	15	45	40
Jo	50	25	25
Ronnie	19	1	80

Table 6.4 Beliefs about teaching mathematics for Drew, Jo and Ronnie

The data from Table 6.4 is displayed as a triangular plot in Figure 6.3, with dotted lines used to highlight the dominant belief category for each teacher.

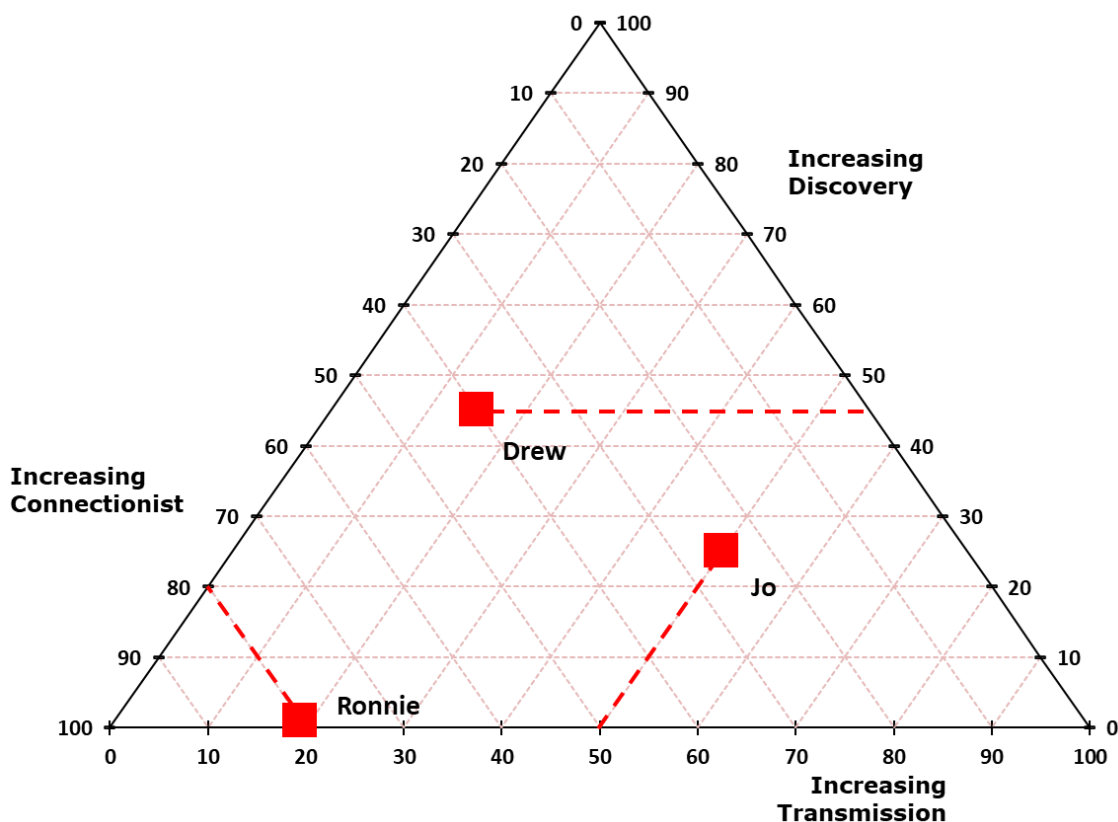


Figure 6.3 A triangular plot of beliefs about teaching mathematics for Drew, Jo and Ronnie

During interview, the three teachers confirmed, and justified, the weightings they had provided regarding beliefs about teaching mathematics. In general, the teachers tended to begin by affirming their dominant belief weighting, before explaining their non-dominant belief weightings had been influenced by Victoria School's Task Ladder lesson structure.

Initial affirmation of dominant belief

As shown in Table 6.4, Drew, Jo and Ronnie provided dominant weightings of Discovery (45%), Transmission (50%) and Connectionist (80%) respectively in the domain of teaching mathematics. Drew confirmed their dominant Discovery weighting by explaining their preference for teaching using interesting problems, and that their ideal would be:

a two hour lesson a week where we just go pick a problem. I don't know, pick some maths, there's a big thing of maths in the corner, interesting problems. Go pick one, try and solve it (Interview 2 [Drew], 00:03:20).

Drew (Interview 3 [Drew], 00:02:20) explained how they recently adjusted "quite a didactic lesson" by asking students "to investigate the statement, there's no such thing as division it's just multiplication by the reciprocal". Drew then added another example where, for a lesson on 3-dimensional shapes, they asked students to consider the statement "the planet has two poles, so it must have two vertices" (Interview 2 [Drew], 00:02:20).

Jo provided a justification of their dominant weighting of Transmission by describing how teaching required students to be shown a method followed by a period of practice:

well, I feel like the teacher conveys the basics, the foundation, and then the students then need to use that to, I don't know. I'm trying to word it, but, yeah, we convey the foundations and then they then use it to solve problems, almost like" (Interview 2 [Jo], 00:00:18).

Finally, Ronnie explained their dominant weighting of Connectionist by focusing on the value they place on discussion. Ronnie explained:

I never really leave the kids to it, I'm always talking to them. That's the way I teach. I do a lot of talk, and probably do far more talk than other teachers. I think you working with kids makes it more interesting, makes it more interesting, and it, it just engages more (Interview 2 [Ronnie], 00:00:43).

Ronnie (Interview 2 [Ronnie], 00:07:51) continued by explaining, "am I actively teaching them how to talk to each other about their maths? Yes, I'm modelling it I would have said". For Ronnie (Interview 2 [Ronnie], 00:07:51) this meant they "play basketball" with student questions by asking for ideas from other students. When a student asked a question, Ronnie explained how they moved the discussion between students rather than directly answer it themselves:

what I try and do is try and go right, that goes to that student, then I'm trying to manage that, manage the reply from that student, and send it to that other student. Rather than it coming back to me and sending it back to that person. I want to send it to that person to send it to that person, to send it to that person (Interview 2 [Ronnie], 00:07:51).

Ronnie (Interview 2 [Ronnie], 00:07:51) concluded this explanation with the statement "now I believe that's C [Connectionist]".

Interdependence of dominant and non-dominant beliefs

The three teachers revealed how they saw their dominant belief as interdependent with their non-dominant beliefs through reference to the school Task Ladder. Regarding the non-dominant weighting of 15% for the Transmission perspective, Drew acknowledged:

I have a job to do and I am paid by somebody else who tells me, not how to teach, but that there are minimum standards and things that I have to do and they fall more into A [Transmission] (Interview 2 [Drew], 00:07:32).

Drew (Interview 2 [Drew], 00:07:32) further described the structure of the Task Ladder as “incongruous” to their dominant belief of Discovery, but that at the same time it was helpful since “it can be an effective tool”. This awareness of interdependence of beliefs is further heightened when Drew (Interview 2 [Drew], 00:07:32) admitted, “I hate myself for when it [a chalk and talk style lesson] goes well”. Despite worrying this style of lesson would be “letting them [the students] down” (Interview 2 [Drew], 00:08:59), Drew recognised the transmission of information is sometimes necessary to allow students to get to the problem-solving aspect, which is what “they really need to do” (Interview 2 [Drew], 00:08:59).

Similarly, Jo (Interview 2 [Jo], 00:00:55) affirmed the interdependence of their Transmission beliefs on their non-dominant beliefs by linking to the school Task Ladder structure. Jo (Interview 2 [Jo], 00:00:55) began by stating, “ultimately I use the Task Ladder structure to show students the new learning, and then they need to practice so I can check understanding”. However, Jo (Interview 2 [Jo], 00:00:55) then acknowledged interdependence since, “I’m also believing that students need to learn how to problem solve and they need me

to help that happen”. Notably, Jo phrased this view of problem solving as being dependent on the teachers’ actions.

Ronnie also revealed interdependence between beliefs by referring to the Task Ladder. Despite strong Connectionist beliefs, Ronnie (Interview 2 [Ronnie], 00:09:35) accepted the Task Ladder as being “based on some Rosenshine principle of something or other, I sort of understand why we do it”. Ronnie (Interview 2 [Ronnie], 00:09:35 confirmed how they saw “breaking explanations into small steps” as providing a basis for students to discuss mathematics. Ronnie (Interview 2 [Ronnie], 00:10:58) went on to explain how “you have to incorporate it [the Task Ladder] to make it work for you”.

6.3.4 Beliefs about learning mathematics

The final set of data that contributed to the cluster of beliefs about mathematics education is in the domain of beliefs about learning mathematics. In interviews following the third RAM session, participating teachers were asked to comment on the weightings they provided at the RAM introductory session. Their responses, alongside responses to follow-up questions, are recorded in the documents Interview 3 [name]. The belief weightings taken from the documents Beliefs [name] are shown in Table 6.5, with a bold typeface used to highlight the dominant category.

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	10	80	10
Jo	40	30	30
Ronnie	10	30	60

Table 6.5 Beliefs about learning mathematics for Drew, Jo and Ronnie

The data from Table 6.5 is displayed as a triangular plot in Figure 6.4, with dotted lines used to highlight the dominant belief category for each teacher.

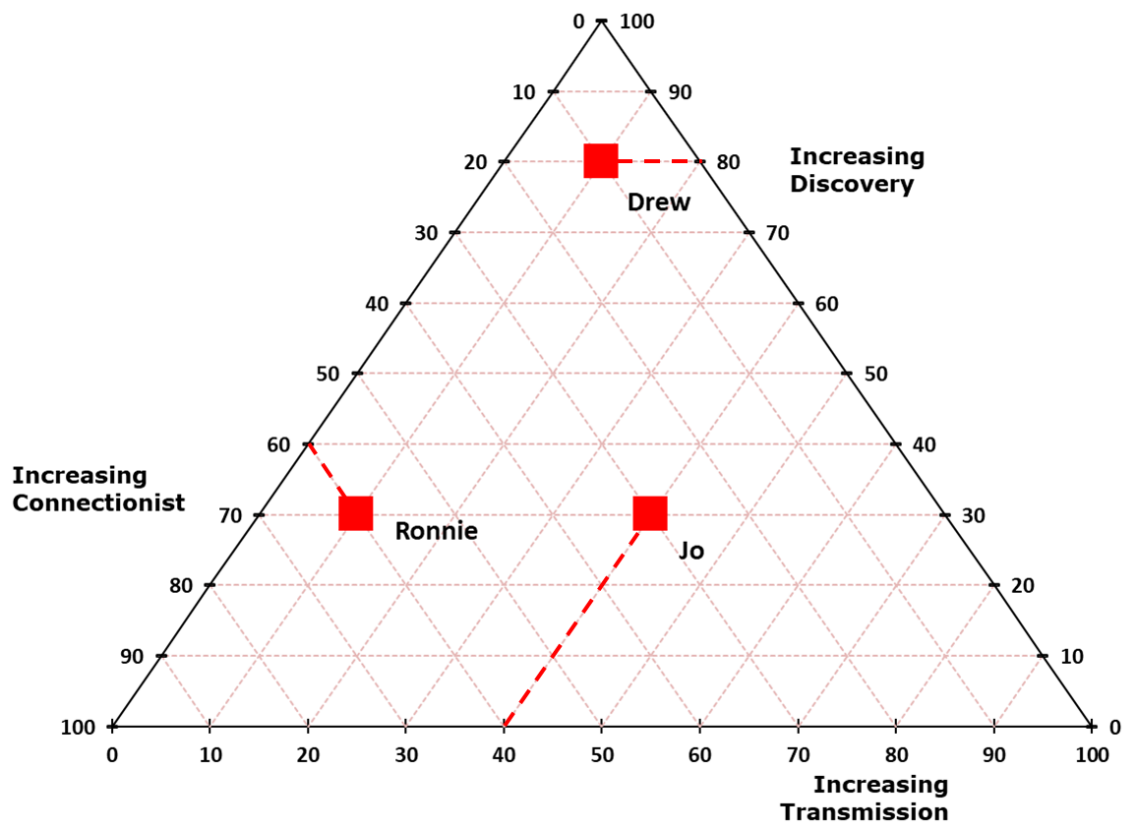


Figure 6.4 A triangular plot of beliefs about learning mathematics for Drew, Jo and Ronnie

During interview, the three teachers confirmed and justified the weightings they had provided regarding beliefs about learning mathematics. In general, the teachers tended to begin by affirming their dominant belief weighting, before explaining their non-dominant belief weightings as being influenced by their classroom experiences.

Initial affirmation of dominant belief

As shown in Table 6.5, Drew, Jo and Ronnie provided dominant weightings of Discovery (80%), Transmission (40%) and Connectionist (60%) respectively in the domain of beliefs about learning mathematics. Drew (Interview 3 [Drew], 00:04:50) justified their dominant belief of Discovery by stating “students need to be interested in the maths to learn it”. Similarly, Jo (Interview 3 [Jo], 00:00:30) justified their dominant belief of Transmission by referring to how “the Task Ladder structure in my lessons reflects how I think pupils learn”. Jo (Interview 3 [Jo], 00:00:30) expanded on this by explaining “they [students] need to be shown what to do, I have to model it clearly, and then they, they need to practice so I can check they get it”. Finally, Ronnie (Interview 3 [Ronnie], 00:00:18) confirmed their dominant belief of Connectionist by stating “do I believe learning takes place through discussion? Yes”. Ronnie (Interview 3 [Ronnie], 00:00:18) added that they hold this view “because that’s how I learnt at school”.

Interdependence of dominant and non-dominant beliefs

The three teachers revealed interdependence between their dominant and non-dominant beliefs by referring to other experiences of student learning in the classroom. In all three teachers' answers, there was a sense of them accepting their dominant beliefs were not always sufficient in helping students learn. When asked about their Transmission weighting of 10% Drew (Interview 3 [Drew], 00:06:20), referring to students, suggested "they've got to, they've got to be given, they've got to be pointed in the right direction" and "you have to know the rules of the game you're playing". Drew expanded on this link between Discovery and Transmission by stating:

I think, I think there is room sometimes, because you can't just go, if it was all B [Discovery], you could just go into a room of 5-year-olds and say, find the volume of this shape. You need at some point to say, right, well this is the general idea, you know, you need a little bit of that to be honest with you (Interview 3 [Drew], 00:09:02).

Jo accounted for their non-dominant Discovery belief weighting by linking to a Discovery belief of problem solving:

I feel like, you [learners] need to be creative in your problem solving because like you, you can be faced with a problem in maths and you haven't been taught how to how to solve that particular problem (Interview 3 [Jo], 00:02:14).

In a similar fashion, Ronnie linked their non-dominant weightings to their dominant Connectionist weighting by suggesting students found learning through discussion difficult. Ronnie (Interview 3 [Ronnie], 00:00:18) began by stating, "they've got to want to learn, and no matter how hard I try to make them discuss, at the end of the day they've got to take

responsibility”. However, Ronnie (Interview 3 [Ronnie], 00:00:18) added, “I think nowadays the skills of discussion are sort of disappearing. The children generally are losing those skills, the ability to sort of talk to each other properly”. This led Ronnie to reveal how their personal classroom experiences meant they saw a requirement for Transmission beliefs about learning mathematics:

we don't have this sort of nice free three-hour lesson, where kids can come in and just relax and enjoy some maths. It's part of the problem with the education system as it is, so that's why it's best just show them [the students] the routine” (Interview 3 [Ronnie], 00:08:22)

Ronnie further explained how they saw practice as an important part of learning in the classroom. Ronnie (Interview 3 [Ronnie], 00:10:30) admitted, “kids need to practise maths. You get better at the things you pay attention to. And David Beckham didn't get good at kicking the football by not kicking a football”.

6.4 Discussion around the theme of beliefs are interdependent

The results provided in the previous section contribute to my first theme entitled beliefs are interdependent. As Braun and Clarke (2021b) suggest, a title alone is insufficient and an accompanying discussion of the ‘scope, boundaries and core concept’ (Braun and Clarke 2021b, p. 108) is required. Hence, my claim is that a contradiction informed interpretation of the RAM programme reveals teachers’ beliefs about mathematics education are interdependent. The word interdependent is used to convey my interpretation that the participants see the categories of beliefs about mathematics education as being mutually dependent. To explain my argument, I begin by aligning to the original interpretation of

dominant beliefs about mathematics education used by Swan (2006a, 2006b). From this initial position I employ a form of dialectic thinking to reveal successive contradictions and move to a new contradiction informed interpretation of beliefs about mathematics education.

6.4.1 Understanding beliefs based on dominant weightings

Swan (2006a, 2006b) sought to create a model that would quantify teachers' beliefs about mathematics education and allow changes to be measured. By calculating the cluster of beliefs about mathematics education, Swan (2006b) categorised teachers according to their dominant belief weighting only. Therefore, referring to Table 6.2, Drew would be classified as a teacher with Discovery beliefs (65%), Jo as Transmission beliefs (50%), and Ronnie as Connectionist beliefs (77%). Therefore, once a teacher has been assigned to a category, their views can be summarised using the associated descriptor. So, in Table 6.6, I summarise the three teachers according to the descriptors of beliefs about mathematics education which were stated in Chapter 2.

Name	Dominant belief category	Dominant belief description
Drew	Discovery	Views mathematics as a human creation and encourages students to learn through individual exploration and reflection, while the teacher adopts a reactive, facilitating role.
Jo	Transmission	Views mathematics as a series of ‘rules and truths’ that must be conveyed to students and teaching as ‘chalk and talk’ followed by individual practice until fluency is attained.
Ronnie	Connectionist	Views mathematics as a network of ideas that the teacher and student must construct together through collaborative discussion. Here the teacher has a proactive role in challenging students

Table 6.6 The dominant belief descriptors of Drew, Jo and Ronnie

These simple descriptions of teachers would apply to any teacher whose dominant belief weightings fall within a range. Conceivably, a teacher could be allocated to one of these categories if they had a dominant belief weighting between 34% and 100%. This same approach of classification according to a dominant belief can even be seen in Swan’s (2006b) use of the triangular plot where he created three regions of Transmission, Discovery, and Connectionist. In Figure 6.5, I have recreated the triangular plot from Figure 6.1 for the three teachers in my study, with the addition of Swan’s (2006b) regions.

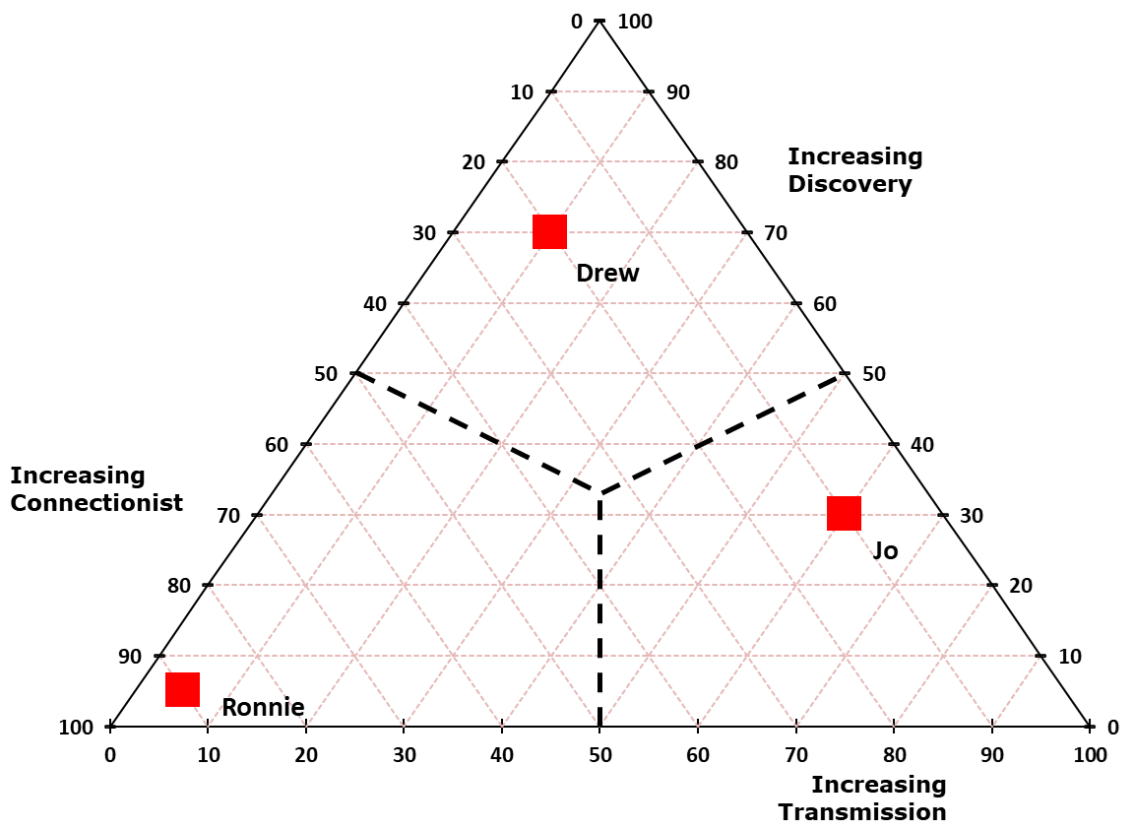


Figure 6.5 Belief about mathematics education with the addition of domain regions

For Swan (2006b), the region the teacher's plotted point falls into reveals the dominant belief category of the teacher. Drew falls in the Discovery region, Jo in the Transmission region, and Ronnie in the Connectionist region.

Swan (2006b, p. 67) validated his belief model against interview comments made by teachers and found 'consistencies' between the dominant belief weighting given by teachers and their explanations for their different practices. In a similar fashion my data, shared in this chapter, can be used to affirm the dominant belief categories according to the teachers'

descriptions. Drew did indeed make comments, prior to the RAM programme starting, that could be used to infer they are accurately described by the Discovery descriptor. Similarly, I have provided data that could be used to support the view Jo is accurately described using the Transmission descriptor, and Ronnie by the Connectionist descriptor. Not only do teachers' comments before the RAM programme align with the dominant belief category, so too do comments made during the RAM programme. In paragraphs in this chapter under the headings of *initial affirmation of dominant beliefs*, participating teachers confirmed and justified the dominant weightings they had provided across all domains. Sub-sections 6.3.2, 6.3.3, and 6.3.4, provide a record of how participants felt their dominant weightings in each of the domains making up the cluster of beliefs about mathematics education were accurate. It could be inferred that both within domains, and at the cluster level, the teachers saw the dominant belief categorisation as an accurate reflection of their views.

However, categorising teachers according to their dominant belief category creates a potential issue when there is more than one dominant belief weighting. This would manifest as two or three domains with the same highest value, or where a teacher's plotted point lay on the region boundaries of Figure 6.5. Although this was not an issue at the level of beliefs about mathematics education for the three teachers in my study, the situation did occur in Swan's (2006a, 2006b) studies. In this approach to classification, the beliefs about mathematics education are understood as mutually exclusive. It makes little sense to suggest, for example, that a teacher believes mathematics is simultaneously discovered (Discovery) and created (Connectionist). Therefore, to resolve this issue, Swan (2006b, p. 64) 'excluded

from this analysis' any teacher that showed no overall belief preference. For example, Swan (2006a) excluded a teacher with weightings of 40%-40%-20% for Transmission-Discovery-Connectionist respectively from his analysis since they could not be classified into a single belief category. The contradiction is therefore revealed, in that seeking to classify teachers according to their beliefs leads to some teachers being unclassified and excluded from the dataset.

6.4.2 Understanding beliefs as a set of alternative weightings

A potential solution to the problem of excluding teachers with no overall preference is to describe beliefs toward mathematics education using an ordered combination of weightings. So, for example, using the data from Table 6.2, Drew would be described as a teacher with a combination of Discovery (65%) Connectionist (23%) Transmission (12%) beliefs. On a triangular plot, acknowledging all three weightings promotes an understanding of a point as a position in three dimensions rather than focusing on the region it lays in. Drew's dominant belief of Discovery is clear on Figure 6.5, and so too their proximity to the Connectionist region indicating a stronger weighting than the Transmission weighting. Similar interpretations can be made for both Jo and Ronnie and would mean no teacher is excluded from consideration. Furthermore, the data I provided in section 6.3, could be interpreted as indicating each teacher confirmed their beliefs to be more complex, and more nuanced than the simple descriptors of Table 7.1.

Alternative weightings as indicating the switching of beliefs

Interpreting the meaning of an ordered set of beliefs remains problematic. Once again, if the three belief categories are considered mutually exclusive then it is difficult to conceive of how two contradictory beliefs can be held at the same time. Swan (2011, p. 60) offers a consideration of this when suggesting a teacher switches their beliefs ‘according to the values and purposes’ they apply to that lesson. In effect, it is suggested beliefs about mathematics education are turned on and off based on teacher circumstances. For instance, a teacher might hold Connectionist beliefs when working with a top set but switch to their Transmission beliefs when working with a bottom set. However, in my study the teachers made no indication they switched beliefs according to location or time. The mathematical ability of a class was never provided as a reason for teachers to switch beliefs, nor the age of the students, nor the location of the school. Similarly, within each domain the teachers recognised the interdependence of their dominant and non-dominant beliefs. In line with McGowan (2019), I suggest the idea of switching beliefs depending on circumstances allows contradiction to be disguised as difference. Instead, I interpret the three teachers as simultaneously holding interdependent beliefs about mathematics education.

6.4.3 Understanding beliefs as interdependent

I have argued that classifying teachers by their dominant belief is insufficient, and that an ordered set of alternative beliefs neither aligns with my data nor with my philosophical perspective. Therefore, I suggest contradiction can support an alternative interpretation that teachers’ beliefs about mathematics education are interdependent. Such an argument is

coherent with McGowan's (2019) contradiction informed explanation of an identity statement. For McGowan (2019, p. 27), any attempt to describe an identity based on what they are 'always leads down the path of misidentification'. Instead, if contradiction is an ontological feature, then any statement must include both identity and non-identity since it is 'dependent on what it negates' (McGowan 2019, p. 27). Therefore, I suggest the teachers do not see themselves as holding one belief, or as switching beliefs, but as simultaneously holding beliefs that require each other.

Interdependence within beliefs about mathematics

In the domain of beliefs about mathematics each teacher recognised the interdependence of their dominant and non-dominant beliefs based on their views of national mathematics examinations. As shown in sub-section 6.3.2, Drew had a dominant belief of Discovery but used examinations to explain that school mathematics also required the application of rules. Jo, with a dominant belief of Transmission, used examinations to justify that school mathematics also required creation through problem solving. Finally, Ronnie, with a dominant belief of Connectionist, used examinations to justify that school mathematics also required procedures and routines. Each teachers' words provided a sense of them recognising their dominant belief required their non-dominant beliefs for students to be successful in national exams.

Contradictions within beliefs about teaching mathematics

Within the domain of teaching mathematics, irrespective of their dominant belief, each teacher confirmed interdependence with their non-dominant beliefs. There appeared to be some commonality in teachers' reasons for this interdependence, with each teacher referencing the influence of the school enforced Task Ladder lesson structure. Indeed, each teacher gave their highest Transmission weighting in this domain compared to their other two domains. I interpret this as providing an indication that the school's Task Ladder lesson structure had a significant effect in revealing the interdependence of beliefs about teaching mathematics. Both Drew and Ronnie spoke of how they had accepted teaching mathematics also required some degree of telling procedures to students by drawing on their teacher expertise. Even Jo, whose dominant Transmission belief aligned well with the school structure, used the Task Ladder to illustrate how they saw it as an insufficient model for teaching mathematics. Therefore, Jo argued whilst teaching mathematics does include demonstrating procedures, it should also lead to problem solving. Collectively the Task Ladder structure revealed the interdependence of each teacher's dominant belief about teaching mathematics with their non-dominant beliefs.

Contradictions within beliefs about learning mathematics

In the domain of learning mathematics, the three teachers confirmed the interdependence of their dominant and non-dominant beliefs by referring to previous experiences in the classroom. Drew argued their personal classroom experiences led them to

see Discovery beliefs as stemming from Transmission beliefs around students listening to an explanation of procedures. Jo, with a dominant Transmission weighting, believed students learnt through rehearsing given methods, which would lead to students developing their own methods. Similarly, Ronnie revealed interdependence when suggesting learning took place through discussion which has been founded on students rehearsing given methods. I interpret all three teachers as revealing their classroom experiences had revealed the interdependence of their dominant and non-dominant beliefs.

6.5 Implications of this theme

I have argued teachers' beliefs about mathematics education, as revealed in the RAM programme, can be interpreted as being interdependent. Hence, the three weightings for Transmission, Discovery and Connectionist should be read as an acknowledgement of simultaneous interdependence of beliefs. In addition, I suggest the particular examples of this study could provide insight into the universal categories of beliefs about mathematics education. Drew, Jo and Ronnie, as particular examples of different dominant beliefs, indicate something about the universal belief categories of Discovery, Transmission and Connectionist respectively. I suggest my results provide an indication that a belief category is inherently contradictory since it 'is also what it is not and has its identity in what negates it' (McGowan 2019, p. 85). My theoretical perspective suggests this logic is appropriate, and I interpret the data as confirming the potential for this interpretation. External influences (which in this study were national examinations, lesson structures, and personal experiences) reveal the existence of inherent contradiction in the belief categories. The belief categories of Transmission,

Discovery and Connectionist can only be understood in relation to their interdependence on one another.

In Chapter 3 section 3.3, I suggested professional development founded in a dualist approach is characterised by conformity (Strom and Viesca 2021). I also inferred Swan's approach to professional development as aligning more closely with an underpinning perspective of multiplicity, characterised by difference. Therefore, I understand Swan as creating a model of beliefs about mathematics education that aligned with his approach to professional development of seeking to support teachers develop other teaching approaches. However, for me both dualism and multiplicity as underpinning philosophies risk leading to teachers feeling pedagogically confused, disappointed in the compromises they make to their beliefs, and angry at the practices they have been encouraged to adopt (Woodford, Clapham and Serret 2023). Therefore, I suggested mathematics Dialectic Professional Development (DPD) should be characterised by the formation of beliefs through an understanding of contradiction. Hence, the model of beliefs about mathematics education I have suggested is not intended to facilitate the measurement of belief change. Instead, my model of beliefs can be used within a professional development programme to contribute to the formation of beliefs about mathematics education. I envision the model as being used to help reveal the interdependence of the different beliefs categories, rather than portray them as in conflict with one another. By recognising contradiction in the form of interdependence I argue the risks of confusion, disappointment and anger for teachers is mitigated. Hence, the reflection element

of mathematics DPD, which contributes to the formation of beliefs, should include opportunities for participants to reflect on the revelation of belief interdependence.

6.6 Summary of Chapter 6

In this chapter I provided results and discussion to support my first theme of beliefs are interdependent, developed in response to my research question. Beginning with Swan's (2006a, 2006b) original model of beliefs about mathematics education I have confirmed how the data in this study supports his work. Teachers participating in the RAM programme confirmed in interview the reasonableness of the dominant belief categories assigned to them. However, through a form of dialectic reasoning I have argued Swan's system potentially leads to the exclusion of teachers with multiple dominant beliefs. One solution to this contradiction is to suggest teachers switch their dominant belief depending on their circumstances. However, I do not interpret the data in this study as supporting this approach. Instead, through a lens of contradiction, participating teachers revealed they hold interdependent beliefs. Therefore, I suggest Swan's model can be understood as providing an indication of a teacher's view of belief interdependence. Moreover, this description of interdependent beliefs is understood as being simultaneous, rather than as indicating the existence of belief switching.

I concluded the chapter by suggesting the particular instances of the three participating teachers may suggest something about the universal categories of Transmission, Discovery and Connectionist. The participating teachers explained how the national exam system, the school enforced lesson structure, and their personal classroom experiences revealed the

interdependence of beliefs in my study of the RAM programme. I interpret this as a potential indicator of the validity of the philosophical perspective of contradiction I have adopted. My theory suggests beliefs about mathematics education could be interpreted as also including that which negates them. Therefore, I suggest a contradiction informed understanding of beliefs is compatible with the purpose of mathematics DPD. Furthermore, it suggests an important component of mathematics DPD should be the examination of interdependence in personal beliefs about mathematics education.

Chapter 7.

Analysis of theme 2: moments of variation are acts of self-sabotage

7.1 Synopsis of Chapter 7

In Chapter 6, I suggested data within my study could be used to support the interpretation that beliefs about mathematics education are interdependent. In this chapter, I suggest a second way in which contradiction can support an interpretation of events in the Rethinking Approaches in Mathematics (RAM) programme. Under the theme title of *moments of variation are acts of self-sabotage*, I provide results around teachers' actions in the potential moments of variation described in Chapter 4. The teachers' actions were anticipated as occurring in designed opportunities for challenge, in opportunities to make planned changes to the research lesson plan, and during unplanned changes made to the live research lesson. I interpret the teachers' actions in line with McGowan's (2019) explanation of the divided self as discussed in Chapter 3. Through my interpretation, I suggest some teacher actions can be understood as moments of self-sabotage, suggesting their satisfaction is not found in eliminating contradiction. This ultimately leads to my claim that teachers find satisfaction in the experience of a professional development programme, rather than in achieving a particular style of teaching.

7.2 Introduction

In Chapter 4, I detailed the design of my RAM programme with the intention of exemplifying mathematics Dialectic Professional Development (DPD). I identified where I

designed for potential moments of variation within the programme in the form of opportunities for challenge, opportunities for planned changes, and opportunities for unplanned changes. A summary of the activities, taken from Chapter 4, which were designed as potential moments of variation is shown in Table 7.1.

RAM session	Teacher actions as potential moment of variation
1	Reactions to individual solving of the mathematical problem
1	Reactions to small group explanations of methods
1	Reactions to whole group discussion of strategies
2	Reactions to Teaching Through Problem-solving style lessons
2	Planned changes made to the research lesson plan
3	Unplanned changes made to the live research lesson

Table 7.1 Potential moments of variation in the RAM sessions

The teacher actions shown in Table 7.1 offer potential moments of variation which I sought to take notice of when analysing the teachers' comments and completed documents. The codes which subsequently contributed to the theme title of moments of variation are acts of self-sabotage are shown in Table 7.2.

Codes
My development as a teacher
Positive features of professional development
Negative features of professional development
Negative reaction to an element of the PD
Planned changes to the research lesson plan
In line with original research lesson plan
Unplanned changes to the live research lesson
In line with own research lesson plan

Table 7.2 The codes that contributed to the second theme

To structure this chapter, I first share results in terms of teachers' conscious desires around professional learning, before discussing my interpretation of unconscious actions, taking place in moments of variation, are acts of self-sabotage.

7.3 Results supporting the theme of moments of variation are acts of self-sabotage

Throughout my sharing of results in this section I provide headings which refer to teachers' conscious desires and unconscious actions. In doing this I am indicating the analytic interpretation I take in my discussion and providing an initial 'interpretive account' (Braun and Clarke 2021b, p. 132) of the data. Hence, my use of the language of conscious and unconscious signifies my intent to draw on McGowan's (2019) thinking around the divided self, introduced in chapter 3. In summary, in my discussion I use the data to suggest teachers unconsciously seek to 'avoid eliminating contradictions in order to continue to draw satisfaction from them' (McGowan 2019, p. 53). Therefore, potential moments of variation

will be interpreted as actual when I see a teacher's unconscious actions conflicting with their conscious desires around professional learning.

7.3.1 Teachers' professional learning desires

Following the RAM introductory session, I asked Drew, Jo and Ronnie to describe their hopes for the RAM programme, their previous experiences of professional development programmes, and how they wanted to develop as mathematics teachers. Their answers during this discussion are recorded in the documents Interview 0 [name] and are broadly structured here in relation to their views around mathematics, around teaching mathematics, and around learning mathematics.

Drew's conscious desires around professional learning

Drew, as the head of department, revealed some of their conscious desires through how they wanted other teachers to develop. For instance, Drew (Interview 0 [Drew], 00:00:46) identified a desire to develop colleague's abilities to assess student progress in lessons and move away from "cursory AfL [Assessment for Learning]". Drew believed this would be possible if teachers saw the importance of a thorough understanding of the mathematics involved. However, Drew worried many of the members of the department lacked mathematical confidence and it would be hard to "change the dynamic" (Interview 0 [Drew], 00:05:36) when seeking to help them "find their voice" (Interview 0 [Drew], 00:05:36). Regarding their own teaching, Drew felt they were "in a rut" (Interview 0 [Drew], 00:05:32) and had become reliant on the Task Ladder lesson structure. Drew expressed a

desire to develop problem-solving style lessons that would “make a difference in the classroom” (Interview 0 [Drew], 00:05:32). Furthermore, by adopting a different lesson approach, Drew (Interview 0 [Drew], 00:05:32) hoped students would begin to take responsibility to “understand the maths” and not just “go through the motions of answering questions”.

Jo’s conscious desires around professional learning

When asked to describe how they wished to develop as a teacher, Jo expressed a desire to trial problem solving strategies in lessons. Jo (Interview 0 [Jo], 00:11:24) recognised they sometimes had low “confidence in maths” and hoped the RAM programme would help improve “what I do in the classroom”. Developing thoughts around teaching approaches, Jo (Interview 0 [Jo], 00:10:38) mentioned feedback from a recent lesson observation had left them “frustrated” and “angry”. The feedback, in which Jo had attempted to follow the Task Ladder lesson structure, critiqued the lesson as being “all driven by me” (Interview 0 [Jo], 00:12:15). Following the feedback Jo had arranged to watch an art lesson and spoke of this as a positive experience. In contrast to teacher-led learning, Jo (Interview 0 [Jo], 00:13:01) spoke admiringly of how the art teacher was able to calmly listen to student ideas and “bring everything together”. Finally, in relation to student learning Jo (Interview 0 [Jo], 00:10:38) commented how the lesson feedback they had received noted “students were busy” working, but also suggested students did not really understand the mathematics. Whilst Jo (Interview 0 [Jo], 00:10:38) felt “talked down to” during the feedback, they acknowledged their hope that

they could explore helping students take responsibility for their thinking during the RAM programme.

Ronnie's conscious desires around professional learning

In interview, Ronnie (Interview 0 [Ronnie], 00:11:29) expressed very general professional learning desire of having “someone to tell me how to improve”. Ronnie (Interview 0 [Ronnie], 00:11:29) recognised this request may sound “arrogant”, but claimed to be “looking for one thing that will change what I do”. According to Ronnie, (Interview 0 [Ronnie], 00:11:29), “no one has ever shown me anything better” and “what I do is better”. Ronnie recounted the only time they had changed their teaching approach was when the school leadership team enforced the use of the Teaching Ladder lesson structure. Ronnie (Interview 0 [Ronnie], 00:21:17) acknowledged the Task Ladder lesson structure meant lessons had developed to have greater emphasis on students “completing work” rather than understanding the mathematics. Ronnie (Interview 0 [Ronnie], 00:21:17) reflected, “they’ve managed to change my practice here, but only through policy and mandating things”. In this interview, Ronnie made little comment about how they hoped the RAM programme would develop personal views around student learning. However, Ronnie did recognise being the dominant character in the classroom who did most of the talking. Drawing on a later interview, Ronnie (Interview 3 [Ronnie], 00:00:18) explained that student learning through discussion was important “because that’s how I learnt at school”. Similarly, Ronnie (Interview 2 [Ronnie], 00:07:51) revealed a desire to model mathematical discussion to students by

explaining, “am I actively teaching them [students] how to talk to each other about their maths? Yes, I’m modelling it I would have said”.

7.3.2 Potential moments of variation from RAM session 1

I now move on in this sub-section to describe some of the actions which took place in the potential moments of variation of RAM session 1. RAM session 1 involved participants working individually, in small groups, and as a whole group with the intention of gaining a deep understanding of the mathematical problem from the research lesson and reflecting on beliefs about mathematics. Data is taken from the documents Problem [name] and from comments recorded in Interview 1 [name].

Reactions to individual solving of the mathematical problem

During RAM session 1, each of the participating teachers attempted to solve the main mathematical problem of the research lesson shown in Figure 4.2. I asked participants to work individually and to record as many different methods as they could in eight minutes. Their methods provide a foundation for understanding the rest of the session and contribute to understanding my interpretation of moments of variation. Drew found six correct methods which are shown in Table 7.3, Jo attempted two methods which are shown in Table 7.4, and Ronnie attempted five methods which are shown in Table 7.5. The images are taken from the documents Problem [name] and are grouped into tables according to the four anticipated strategies discussed in Chapter 4. Therefore, the order they appear in the tables does not necessarily correspond with the order in which they occurred to Drew, Jo, and Ronnie.

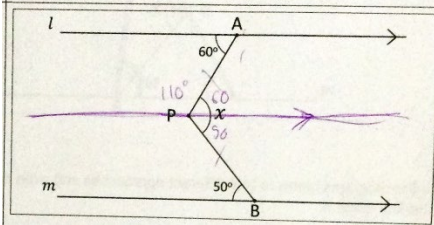
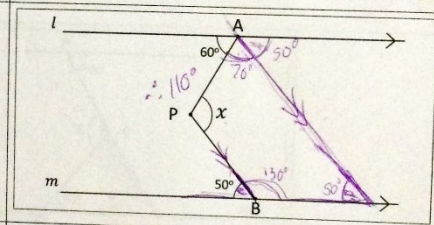
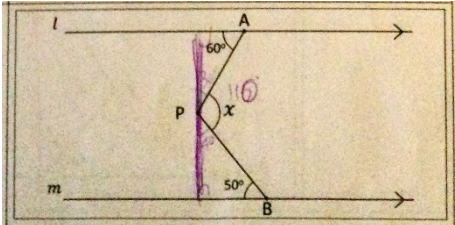
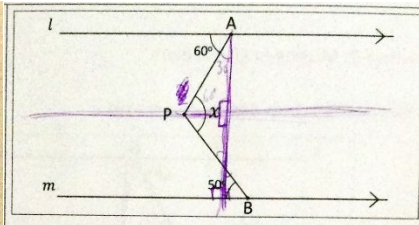
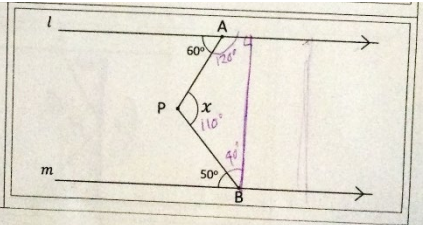
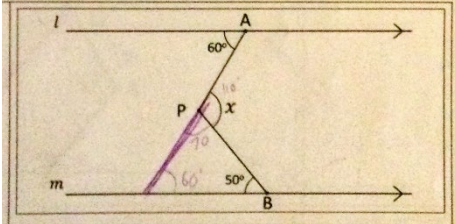
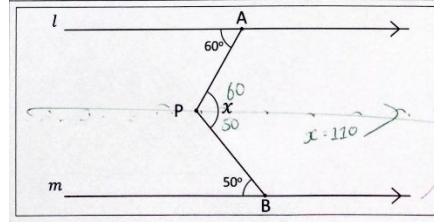
Strategy	Drew's methods
A: Adding a parallel line	 
B: Adding a perpendicular line	  
C: Adding a line connecting two points	
D: Adding an extended line	

Table 7.3 Drew's six methods for solving the geometry problem (Problem [Drew])

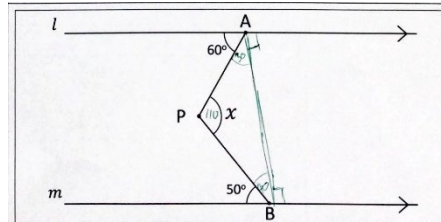
Strategy	Jo's methods
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A: Adding a parallel line



B: Adding a perpendicular line

C: Adding a line connecting two points



D: Adding an extended line

Table 7.4 Jo's two methods for solving the geometry problem (Problem [Jo])

Strategy	Ronnie's methods
A: Adding a parallel line	
B: Adding a perpendicular line	
C: Adding a line connecting two points	
D: Adding an extended line	

Table 7.5 Ronnie's five methods for solving the geometry problem (Problem [Ronnie])

Looking at Tables 7.3 and 7.5, I suggest Drew and Ronnie found the task of finding multiple methods relatively straightforward with six methods and five methods respectively. To support this perspective, both teachers confirmed they found it an enjoyable activity stating:

You just gave me a load of sheets of paper with a problem on it, and go mental, you know, and I like that. (Interview 1 [Drew], 00:21:17).

I just, I really liked it. It was the kind of thing where I could just kind of have a go at it and just get into it (Interview 1 [Ronnie], 00:19:32).

However, Jo found the task quite uncomfortable and only managed one correct method in the time limit. During interview Jo volunteered an explanation for why, two minutes into the individual problem-solving activity, they walked over to Ronnie to discuss the problem. Jo explained feeling “a bit panicked, you know, because everyone else seemed to know what to do” (Interview 1 [Jo], 00:01:31). Reflecting further on the incident, Jo (Interview 1 [Jo], 00:00:55) stated “I’m a bit of a hypocrite” for not writing down my ideas since this is what is expected of students in a classroom. Jo summed up the difficulties they had in finding different methods by explaining, “I’ve never been shown how to do a problem like that before” (Interview 1 [Jo], 00:01:31). After returning to their seat, Jo wrote down a correct method, shown in Figure 7.4 strategy A, though it is unclear whether Jo thought of this method or copied it from Ronnie (compare with Figure 7.5 strategy A).

Reactions to small group explanations of methods

During the small group phase of RAM session 1, the three teachers Drew, Jo and Ronnie, worked together to discuss different methods. In interview all three teachers referred to an extended discussion they had around method C (adding a line connecting two points). Both Jo (Table 7.4) and Ronnie (Table 7.5) appeared to have held the mistaken assumption that a 90° angle had been created between the auxiliary line and the parallel lines. Indeed, in interview, Jo (Interview 1 [Jo], 00:02:48) explained “it [the angle] looked like 90° ” and “you get the right answer”. Through interview comments it was clear Drew assumed responsibility for explaining the misconception to the other two teachers, and for providing a correct explanation. Drew shared their enjoyment of working through the method with the other teachers by saying “if I know that it is possible to get somewhere, I know that using algebra will probably get me somewhere eventually” (Interview 1 [Drew], 00:02:27).

In contrast, Ronnie recounted “Drew took over a bit” (Interview 1 [Ronnie], 00:01:11) in explaining how an algebraic approach was required to achieve the correct answer. Ronnie (Interview 1 [Ronnie], 00:01:11) went on to describe Drew’s method as a “stupidly long solution method that doesn't need to be done”. Rather than see the links between areas of mathematics, Ronnie placed an emphasis on functionality and efficiency and concluded “solve it simultaneously, why?” (Interview 1 [Ronnie], 00:01:11). Jo also referred to the small group discussion around method C and confessed “I still don’t really understand why we can’t assume that they're equal angles because you get the right answer” (Interview 1 [Jo], 00:07:32). The incident left Jo feeling inferior to the other two teachers stating:

being the, maybe, the junior member [...] the older person [Ronnie] has obviously been teaching for a lot longer, and Drew is a much more capable mathematician than myself, and I'm okay with that. As I think maybe sometimes my opinions are somewhat less valid in their eyes (Interview 1 [Jo], 00:07:32).

For Jo, despite feeling intimidated during this small group discussion, they still referred to the stage as being instrumental in making them think. Jo (Interview 1 [Jo], 00:11:28) explained “working with Ronnie and Drew really showed me that you need confidence to find different ways”. Jo (Interview 1 [Jo], 00:11:28) went on to state “I think the session showed the core is the rules, and but yeah, like I think maybe I opened my eyes up to it being more about how you apply those rules as well”.

Reactions to whole group discussion of strategies

In interview only Drew and Ronnie referred to the whole group discussion of strategies in the RAM session. During this stage I facilitated a discussion which led to the classification of methods based on the nature of the auxiliary line. Ronnie did not like the suggested classification during the RAM session and later stated in interview “I still don't agree with the classification you came up with” (Interview 1 [Ronnie], 00:15:23). In the session, Ronnie had argued for a different classification based on the shapes that were created when an auxiliary line was added. So, according to Ronnie (Interview 1 [Ronnie], 00:15:23), the categories would include “create a pentagon or a quadrilateral” since “I think that's what the kids would spot, rather than what line has been added” (Interview 1 [Ronnie], 00:15:23). Ronnie explained this approach would place an emphasis on “areas not lengths” (Interview 1 [Ronnie], 00:15:43). Reflecting on the whole session, Ronnie (Interview 1 [Ronnie], 00:28:41)

admitted it had been “very interesting for me”, but was “absolutely useless for the classroom”. When asked to expand on this, Ronnie suggested it would not be a practical classroom activity due to the “time constraints” of looking at so many different methods. Ronnie (Interview 1 [Ronnie], 00:28:50) further supported this by suggesting students “don't have my fluency” and “are not capable of finding all the solutions”. When asked if finding all the solutions was important and whether that was what the students needed, Ronnie [Interview 1 [Ronnie], 00:30:02] stated, “yes, ultimately”.

Drew also mentioned the whole group discussion in interview as being where they developed a deeper understanding of the problem. As Drew (Interview 1 [Drew], 00:11:27) explained, “it was nice to argue about the classifications. That was the first part of the session where I saw something I didn't already know”. Drew (Interview 1 [Drew], 00:11:51) concluded this is “what I really liked”, since “it was about the family of, of putting things into groups and sets. That's what is really important, for me”. Drew also revealed a Connectionist influenced belief in noticing my facilitation of the stage brought out connections:

the important bit was your facilitation. It was only when we classified the methods using your classification that I really felt I had a proper understanding of the problem [...] ultimately, ultimately you knew where, what you wanted us to get from it” (Interview 1 [Drew], 00:22:43).

7.3.3 Potential moments of variation from RAM session 2

In RAM session 2, teachers were introduced to the Teaching Through Problem-solving (TTP) style lesson, shown the research lesson plan, and considered beliefs about teaching

mathematics. Here I share some details around the potential moments of variation which took place in reaction to the TTP introduction, and as planned changes to the research lesson plan. Data for this section is taken from the documents Lesson Plan [name] and Interview 2 [name].

Reaction to Teaching Through Problem-solving style lessons

At the start of RAM session 2, I provided a brief overview of some of the features of TTP style lessons including an explanation of the claimed benefits, and an overview of the five phases. I shared Takahashi's (2021) claim that TTP style lessons contributed to Japanese children having the confidence to tackle unfamiliar problems even though they may not have covered the required mathematical content. Drew appeared convinced, stating "that approach to teaching works" (Interview 2 [Drew], 00:14:27). Similarly, Jo (Interview 2 [Jo], 00:02:21) could see the style of lesson "where we do more problem solving might be beneficial" for students. In contrast, Ronnie came across much more cautious about the claims, since "it measures one thing in a sea of variables" (Interview 2 [Ronnie], 00:14:02). Echoing a comment made in the session, Ronnie (Interview 2 [Ronnie], 00:14:02) suggested other statistics should also be considered such as "suicide rates in young people" before any conclusion could be made about benefits.

Planned changes to the research lesson plan

Following an overview of the lesson, the three teachers worked together in a small group to discuss the research lesson plan. Drew, Jo and Ronnie were asked to discuss the rationale behind the actions and individually affirm whether they would adopt or adapt the suggestions made in the research lesson plan. Participants responded to the questions on the research lesson plan which were summarised in Table 4.8. As part of the introduction to the session, participants were encouraged to adopt actions where possible and asked to write any intended adaptations onto the research lesson plan if necessary. I summarise Drew, Jo, and Ronnie's thinking around the research lesson plan in Tables 7.6, 7.7 and 7.8 respectively. These tables are structured by lesson phase and indicate where teachers either committed to adopt the actions or suggested a planned adjustment. In addition, in Figures 7.1 and 7.2, I show the starter questions which Drew and Ronnie designed for their research lesson (taken from Interview 3 [name]). Drew planned to teach the research lesson to a high attaining Year 9 class, Jo to a low attaining year 9 class, and Drew to a high attaining Year 10 class. In short, I suggest Drew committed to remain closest to the original research lesson plan, followed by Ronnie, and then Jo.

Phase	Phase title	Drew's intentions
1	Review previous learning	Planned adjustment to give students review questions in order to align with “the Task Ladder structure” (Interview 2 [Drew], 00:16:15). A copy of the two questions Drew created, and used, are shown in Figure 7.1 (Interview 3 [Drew]), and require students to create chains of reasoning to solve the questions.
2	Pose a new problem	Committed to construct the problem ‘quickly and accurately’ (Lesson Plan [Drew]) with the students despite ‘potential question over time’. This decision was made since, “that’s what really gives insight into the problem” (Interview 2 [Drew], 00:16:55) and that it would help students understand that the two points “A and B are not opposite” (Interview 2 [Drew], 00:16:55).
3	Individual student problem solving	Committed to 10 minutes of individual student problem solving whilst recording students’ methods to help plan the next phase.
4	Teacher led comparison and discussion	Committed to this phase, with the intention of putting the problem “up onto big pieces of paper” (Interview 2 [Drew], 00:18:20) for students to write on. Planned adjustment in not considering an ideal order of solutions in advance since, “I’m not really sure what methods the students will use” (Interview 2 [Drew], 00:15:24). Instead, Drew decided knowing potential methods in advance would mean “I can just use them if they come up” (Interview 2 [Drew], 00:15:24).
5	Highlight key concepts	Committed to placing a focus on classifying methods so students could see there are “four methods, and within those methods there are lots of, like, variations” (Interview 2 [Drew], 00:19:48).

Table 7.6 Summary of Drew’s response to the research lesson plan

Phase	Phase title	Jo's intentions
1	Review previous learning	Planned adjustment to ask students to complete review questions “so then you know they can do the next part of the lesson” (Interview 2 [Jo], 00:05:41). There is no copy of the questions, but Jo (Interview 3 [Jo], 00:19:30) later described them as “a recall starter” including “a pentagon, an octagon, a straight line, and three sets of parallel lines, you know for interior, corresponding and alternate”.
2	Pose a new problem	Committed to drawing an accurate diagram in front of the students but saw this as a behaviour management technique to ensure students are “paying attention to what's going on and listening to what you're saying” (Interview 2 [Jo], 00:18:52).
3	Individual student problem solving	Planned adjustment to ‘give out prompt cards’ (Lesson Plan [Jo]) since students “might just want to sit there” (Interview 2 [Jo], 00:06:28). Jo felt prompt cards, such as the ones used in RAM session 1, would help since “it doesn't tell them the answer. It doesn't even tell them necessarily how to find the answer. It just shows them the start of the method” (Interview 2 [Jo], 00:07:02).
4	Teacher led comparison and discussion	Planned adjustment of not bringing the class together to discuss the different methods since there would be more value if the students “work in their small groups” (Interview 2 [Jo], 00:23:37) to discuss prompt cards with additional group support from the teacher.
5	Highlight key concepts	Planned adjustment of not to grouping strategies since students will “just start messing around” (Interview 2 [Jo], 00:24:39). Instead, Jo planned to ask students to think “about how you start these sorts of problems” (Interview 2 [Jo], 00:24:39). During this phase students would be expected to do ‘pens down listening’ (Lesson Plan [Jo]), where there would be ‘no key steps or rules to copy, just ideas’ (Lesson Plan [Jo]).

Table 7.7 Summary of Jo's response to the research lesson plan

Phase	Phase title	Ronnie's intentions
1	Review previous learning	Planned adjustment to provide simple review questions. A copy of the nine questions Ronnie designed, and used, are shown in Figure 7.2 (Interview 3 [Ronnie]) and illustrate a comprehensive coverage of previous learning.
2	Pose a new problem	Planned adjustment of asking students to make a 'sketch – not accurate' (Lesson Plan [Ronnie]) of the problem since "what's the point?" (Interview 2 [Ronnie], 00:16:00). Ronnie further suggested a planned adjustment of handing out copies of the problem so that students could get "onto the problem solving as quickly as you can" (Interview 2 [Ronnie], 00:16:00).
3	Individual student problem solving	Committed to 10 minutes of individual student problem solving whilst recording students' methods to help plan the next phase.
4	Teacher led comparison and discussion	Planned adjustment of not considering the ideal order of methods to be looked at, preferring instead to react to student ideas, since "I know the four different methods the students use, so I can just riff off that" (Interview 2 [Ronnie], 00:16:40).
5	Highlight key concepts	No sign of adjustment or commitment, but Ronnie doubtful of the value of this phase. Instead, Ronnie (Interview 2 [Ronnie], 00:17:22) suggested it is more important students "take some level of responsibility in learning".

Table 7.8 Summary of Ronnie's response to the research lesson plan

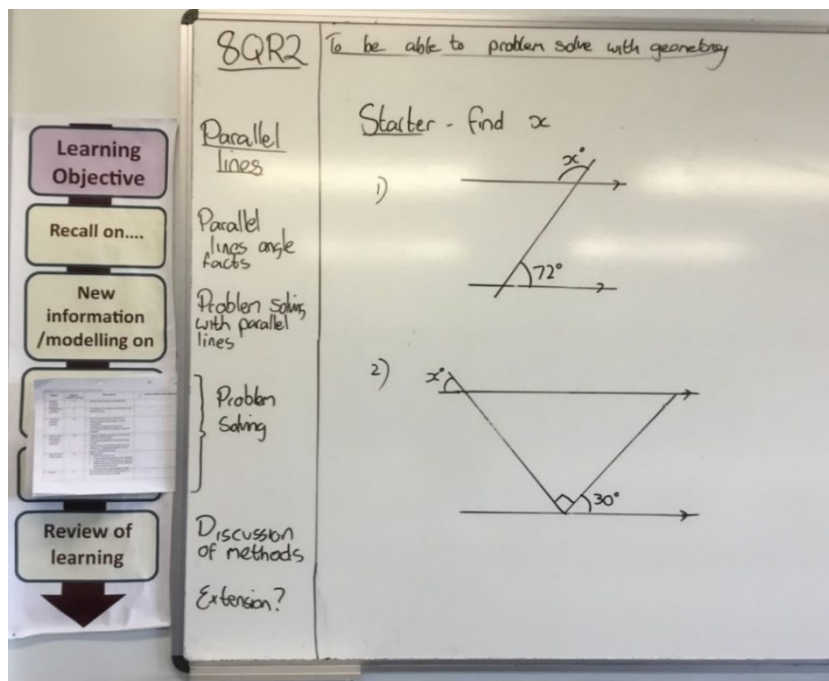


Figure 7.1 Research lesson starter questions created by Drew (Interview 3 [Drew])

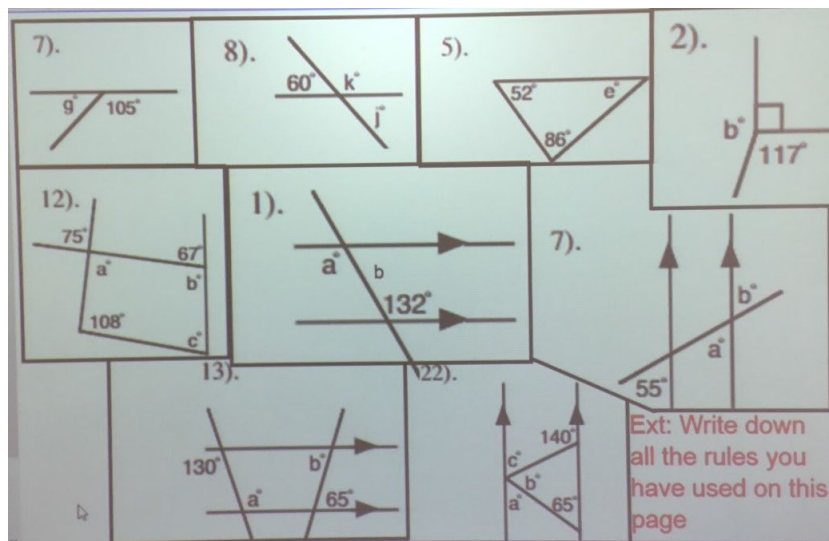


Figure 7.2 Research lesson starter questions created by Ronnie (Interview 3 [Ronnie])

7.3.4 Potential moments of variation from RAM session 3

In the third RAM session, participants were asked to discuss their taught research lessons and to reflect on beliefs about learning mathematics. In this sub-section, I share Drew, Jo and Ronnie's reflections with a focus on the unplanned changes to their taught research lessons which they revealed. Data used in this section is taken from the documents Lesson Reflection [name] and Interview 3 [name].

Drew's unplanned changes to the live research lesson

According to Drew the adjusted lesson plan they created, and summarised in Table 7.6, was followed quite closely in the live research lesson. This included the original design intention of giving students ten minutes of individual problem solving, and Drew's planned adjustment of providing starter questions. Drew explained the starter questions shown in Figure 7.1 were created as the "the first step of recall information" (Interview 3 [Drew], 00:09:46) from the Task Ladder lesson structure. Drew (Interview 3 [Drew], 00:17:32) explained, "I think I've switched so much into settler tasks as part of the recall starter now because it's part of the school structure". In addition to creating a calm classroom environment, Drew felt the two questions had "all the key, kind of, important maths facts are all in there" (Interview 3 [Drew], 00:11:01).

Drew spoke of an unplanned change to the teaching of the research lesson that took place during the phase of pose a new problem. As seen in Table 7.6, Drew had committed to constructing accurate copies of the geometry problem alongside the students. Instead, Drew

explained how during the lesson they made an unplanned change of simply asking students to sketch a copy of the projected diagram (Lesson Reflection [Drew]) on their own. Drew (Interview 3 [Drew], 00:14:49) reflected on this decision by acknowledging “I should have constructed the problem carefully” with the students since that was “the most important bit”. From both RAM session 1, and the commitment shown in Table 7.5, I infer Drew had a good understanding that constructing the problem would allow students to understand points A and B (see Figure 4.2) are not vertically opposite. However, Drew identified a concern over lesson timing meant an adjustment was made which resulted in students creating “some really confusing diagrams” (Interview 3 [Drew], 00:14:49).

A second unplanned, or perhaps unanticipated, change can be inferred when Drew spoke of how they had to “shut up, and not lead people” (Interview 3 [Drew], 00:15:42) during the lesson. This led Drew (Interview 3 [Drew], 00:16:11) to further reflect “it’s going to sound arrogant, but when I talk people listen. And you can abuse that”. Drew suggested this attitude had developed in them through messages from the school leadership team around use of the Task Ladder to control behaviour:

You’re put in the position and told this is your room, this is yours, everything you’re told is this is your classroom act like it’s yours and own it. It’s very self-centred (Interview 3 [Drew], 00:16:11).

Drew concluded the lesson had “made me question so much of what I do and know as a teacher” (Interview 3 [Drew], 00:15:45). Expanding on this Drew (Interview 3 [Drew], 00:17:32) explained “we get into habits, I guess, or routines” and the research lesson

highlighted the value of “doing things a bit different”. Finally, reflecting on students’ difficulties during the individual problem-solving phase, Drew reflected “I’ve really wobbled. I’m like, have I just been leading them the whole way, have I just been leading and they [the students] follow the whole time?” (Interview 3 [Drew], 00:16:11).

Jo’s unplanned changes to the live research lesson

As per their intended adjustment, Jo used a set of recall starter questions in their research lesson, and in interview revealed how this led to an unplanned change. Jo explained that another question was added after seeing students have difficulties solving the original set of starter questions. This question was taken from “the extension ones you had in the lesson plan” (Interview 3 [Jo], 00:05:54). A copy of the question is shown in Figure 7.3, and other than which angle is given at point A, is virtually identical to the main problem being used in the research lesson. Jo (Interview 3 [Jo], 00:05:54) explained how the problem was adjusted to make it “goal free”, by asking students “to find any angle rather than the specific angle x ”. Jo continued by explaining how they orchestrated a whole class discussion to solve this problem, in order to illustrate to students it is possible to find many angles, which in turn help you to work out more angles.

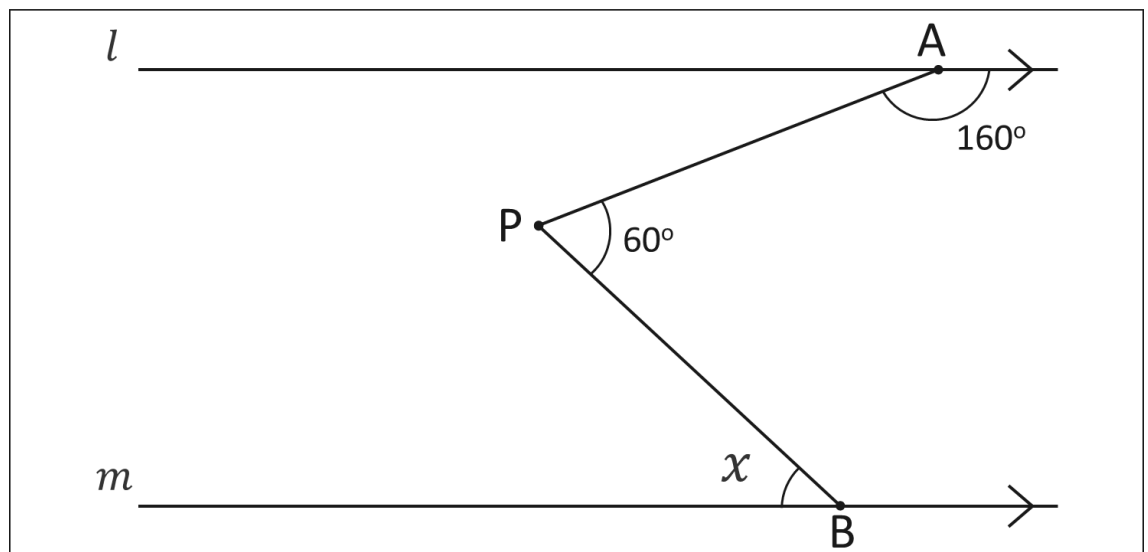


Figure 7.3 Jo’s additional starter question taken from the original RAM research lesson plan

A second unplanned adjustment occurred as Jo (Interview 3 [Jo], 00:11:21) reacted to students being very vocal about how “they didn’t know what they’re going to do” when the problem was introduced. Although this had been anticipated (see Table 7.7), Jo ended up shortening the individual problem-solving phase to “seven and a half minutes of sort of silence” (Interview 3 [Jo], 00:11:21). Commenting on this experience, Jo reflected, “my main observation, main observation, is they didn’t quite have the confidence or the resilience to take that first step” (Interview 3 [Jo], 00:04:57) and “they just didn’t have that sort of confidence to make a mistake” (Interview 3 [Jo], 00:07:09). However, I did not see Jo as trying to blame the students, but admitting some level of responsibility since, “I’m a bit, I’m a bit upset for them that they weren’t confident enough, but I understand why, because it’s first time doing something like that” (Interview 3 [Jo], 00:08:59). Jo felt their planned adjustment, shown in

Table 7.7, of handing out prompt cards at this point provided students with something else to discuss rather than focusing on being unable to access the problem (Lesson Reflection [Jo]).

Finally, Jo revealed another unplanned adjustment had been made when they taught some of the important concepts in the lesson prior to the research lesson taking place. Jo explained this had been done to try and ensure the students would be successful in the research lesson and felt that the students had “smashed it” (Interview 3 [Jo], 00:20:58). This appeared to heighten the sense of disappointment Jo felt with how difficult the students found the research lesson. Jo reflected, “part of it might be forgetting it, and then part of it might be, not, be that they had never really understood it in the first place” (Interview 3 [Jo], 00:21:19). However, the experience had made Jo think about learning mathematics suggesting, “I like definitely the element of having the just work on one big problem. I like that a lot” (Interview 3 [Jo], 00:18:00). When asked to explain why, Jo expanded that a single problem “meant the students kept building their understanding without having to worry about the right answer. That really made a difference I think” (Interview 3 [Jo], 00:18:10).

Ronnie’s unplanned changes to the live research lesson

Following the planned change of the starter questions shown in Figure 7.2, Ronnie discussed an unplanned adjustment was made to the individual problem-solving phase. Whilst students engaged in individual problem solving, Ronnie had committed to noting down different solution methods attempted by the class (see Table 7.8). However, Ronnie admitted ‘I didn’t make notes’ (Lesson Reflection [Ronnie]) because “I think it looks weird and

officious. It didn't really matter to me who was doing what. They were all doing something and that's what mattered" (Interview 3 [Ronnie], 00:14:39).

Ronnie's unplanned change led to further unplanned changes as Ronnie sought to direct the teacher led comparison and discussion phase. With no record of student methods, Ronnie decided to randomly ask students for their methods and unfortunately picked a student with a difficult to comprehend method. After initially floundering, Ronnie received help from another student in the class to explain the thinking and confessed "it was a fluke that the student put her hand up. And I think if, if she hadn't put her hand up, I'd have been a bit stuffed" (Interview 3 [Ronnie], 00:16:00). Ronnie admitted at this point in the research lesson "I was panicking on my feet" (Interview 3 [Ronnie], 00:16:00) and that "maybe [I] should have got more out of it [the teacher led comparison and discussion phase]" (Interview 3 [Ronnie], 00:20:12). This uncomfortable experience led Ronnie to make a connection to looking at the students' work during the individual problem-solving phase, and reflected:

so maybe I can see why, this is why the Japanese say to walk around clocking the methods. So that you actually orchestrate only the ones you want. Okay, I can see it" (Interview 3 [Ronnie], 00:16:00).

For the phase of highlighting the key concepts, Ronnie had expressed doubt around the value to students (see Table 7.8). When asked about this phase, Ronnie admitted to a final unplanned change during the lesson as they thought "is it worth me getting, starting this, going down that road, or is it worth me just going, stuff it we're done?" (Interview 3 [Ronnie], 00:21:05). However, in interview Ronnie acknowledged their actions as the teacher had an

impact on the learning of the students and suggested “we could have got to the grouping of methods if I’d planned that a bit better” (Interview 3 [Ronnie], 00:21:05). Ronnie linked all this back to the individual problem-solving phase and concluded, “I can see that knowing what the kids have done better would mean I can bring out the key points better” (Interview 3 [Ronnie], 00:21:05).

7.4 Discussion around the theme of moments of variation are acts of self-sabotage

In my discussion of this theme, I utilise McGowan’s (2019) explanation of the divided self, where an individual is considered as having an unconscious acting against their conscious. Therefore, I interpret unconscious actions as acts of self-sabotage which undermine the individuals’ conscious desires (McGowan 2019). Throughout this discussion I utilise the generalised structure of McGowan’s (2019) explanation of the divided self which I introduced in sub-section 3.8.2. Hence, I identify the contradiction between A and B which an individual seeks to resolve through participation in C. I then interpret an individuals’ conflicting actions as suggesting a desire to maintain the contradiction between A and B, and consequently offer the suggestion that satisfaction is found in C. Throughout this discussion, I consistently suggest C is understood as participation in the RAM programme and return to discuss this in section 7.5. Therefore, rather than focus on moments of fidelity and compliance I have provided data in this chapter on potential moments of variation for the teachers. Whilst these potential moments of variation were identified in the design of Chapter 4, I interpret them as actual moments of variation when the teachers’ actions contrast with their stated desires around professional learning. To structure my discussion, I link the three participating

teachers' conscious desires from sub-section 7.3.1 to their actions seen in RAM sessions 1, 2 and 3.

7.4.1 Inferring Drew's self-sabotage in actual moments of variation

In this sub-section I separate Drew's conscious desires around professional learning into desires around mathematics, teaching mathematics, and learning mathematics. This then allows me to link those desires to Drew's actions seen in RAM sessions 1, 2 and 3 respectively.

Drew's self-sabotage in RAM session 1 around beliefs about mathematics

From sub-section 7.2,.1, I infer Drew was consciously seeking to overcome a contradiction between developing teachers' mathematical knowledge and teachers' lack of mathematical confidence through participation in the RAM programme. However, I interpret Drew's actions during the small group discussion as an actual moment of variation since they show a conflict between Drew's unconscious actions and conscious desires. Drew dominated the small group discussion and left Ronnie feeling unimpressed and Jo feeling inadequate. Therefore, according to McGowan's (2019) explanation of the divided self, I consider Drew's actions as the unconscious self-sabotage of conscious desires. Furthermore, I suggest this interpretation indicates Drew unconsciously sustained the conditions for the contradiction to be maintained.

Drew's self-sabotage in RAM session 2 around beliefs about teaching mathematics

Regarding their own teaching, Drew identified a desire to overcome the contradiction between teaching problem-solving style lessons and the use of the Task Ladder lesson structure through involvement in the RAM programme. Therefore, I infer actual moments of variation as taking place when Drew's unconscious actions contrast with Drew's conscious desires. Out of the three teachers, Drew had intended to adhere most closely to the original research lesson plan and adopted many of the phases suggested. However, I see some of the ways in which Drew adjusted the research lesson plan as being the unconscious undermining of conscious desires. As shown in Figure 7.1, Drew created angle questions for students to complete at the start of the lesson. The inclusion of these starter questions was justified based on needing to align with the Task Ladder lesson structure, despite the research lesson plan supporting a simple verbal recap. Moreover, Drew created two questions which would allow students to see the need for multiple steps of reasoning when solving angle problems. I see Drew's actions as an attempt to pre-empt to students the approach that would be needed during the individual problem-solving phase. Furthermore, as can be seen in Figure 7.1, not only did Drew create starter question for the recall phase but Drew also summarised the whole research lesson plan into the format of a Task Ladder lesson. Therefore, I interpret Drew's actions as acts of self-sabotage which unconsciously sustain the contradiction between developing problem-solving style lessons and the Task Ladder lesson structure.

Drew's self-sabotage in RAM session 3 around beliefs about learning mathematics

Regarding learning mathematics, I suggest Drew expressed a conscious desire of overcoming the contradiction between developing mathematical understanding, and students “going through the motions” (Interview 0 [Drew], 00:05:32), through participation in the RAM programme. Drew had explained how they saw learning mathematics as not being about reproducing processes, but about learning through making sense of the mathematics. Therefore, I infer Drew's action in the form of an unplanned change to the diagram construction in the research lesson as a moment of variation. Drew elected not to construct the diagram with the students and missed the opportunity for mathematical insight. Indeed, in interview Drew recognised the decision to ask students to only sketch the diagram was a missed opportunity for students to gain mathematical understanding. In a key moment of the lesson, with an opportunity to facilitate reasoning and sense making, Drew made an unplanned change. I interpret this action as an act of self-sabotage which undermined Drew's conscious desire to develop mathematical understanding.

An alternative explanation of Drew's actions

I have suggested actual moments of variation can be inferred when Drew dominated the small group problem solving, adjusted the research lesson, and made unplanned changes to the research lesson. My claim is that Drew's actions can be explained as the unconscious undermining of conscious desires through a contradiction informed understanding of the divided self. However, I acknowledge other interpretations of these actions are possible and so

seek to ensure Drew's 'views are presented' (BERA 2024, p. 13). Following the development of this theme I met with Drew via Microsoft Teams to ask Drew to comment on the three moments of variation I identified, to explain my philosophical perspective, and to explain my interpretation. I asked Drew how they would explain the actions I have here called actual moments of variation.

After consideration of the small group discussion, Drew suggested their leading role resulted from their excitement in using a unique approach, and through wanting to help Jo see Ronnie should not always be considered "the mathematics teaching expert" (Interview 4 [Drew], 00:04:05). Drew also explained how they felt compelled to model and display the Task Ladder lesson structure for their research lesson since this was the school policy they had been passing on from the school leadership team (Interview 4 [Drew]). Finally, Drew felt that forgetting to construct the problem was simply a mistake they would not make if they tried the lesson again (Interview 4 [Drew]). After explaining the idea of the divided self, Drew (Interview 4 [Drew], 00:06:21) felt my interpretation was "not unreasonable", though "you'd need to be a psychologist" to establish whether it was correct. I acknowledge Drew's alternative explanations are plausible, and I return to consider the tension between different interpretations in section 7.5.

7.4.2 Inferring Jo's self-sabotage in actual moments of variation

In this sub-section I separate Jo's conscious desires around professional learning into desires around mathematics, teaching mathematics, and learning mathematics. This then allows me to link those desires to Jo's actions seen in RAM sessions 1, 2 and 3 respectively.

Jo's self-sabotage in RAM session 1 around beliefs about mathematics

Drawing on sub-section 7.3.1, I interpret Jo as having a desire to overcome a contradiction between developing mathematical problem-solving skills and their own lack of mathematical confidence, through participation in the RAM programme. Hence, when actions conflict with desires, I interpret some of Jo's actions during the individual problem-solving phase as actual moments of variation. During this stage Jo had an opportunity to experience mathematical problem-solving, and the time to plan out how to approach the problem. However, despite being asked to work individually and given time to find as many methods as possible, Jo elected to walk over to Ronnie to ask for help within the first few minutes of the activity. I interpret this action as Jo unconsciously undermining their stated desire and instead unconsciously maintaining the contradiction between problem-solving and personal confidence.

Jo's self-sabotage in RAM session 2 around beliefs about teaching mathematics

Drawing on sub-section 7.3.1, Jo expressed a desire to overcome a contradiction between facilitating student thinking and telling students what to do through participation in

the RAM programme. Jo's conscious desire stemmed from their observed lesson feedback, and from seeing the ways in which an art teacher facilitated student conversation. Therefore, when actions conflict with desires, I interpret some of Jo's actions as occurring in moments of variation. For instance, I interpret Jo teaching a lesson prior to the research lesson, to re-cover the topic of finding angles on parallel lines, as an act of self-sabotage. Jo explained covering this content was necessary to remind students of the basic rules but stands in contrast to the contradiction Jo hoped to overcome. Furthermore, Jo created starter questions for the research lesson to remind students of the rules that had been considered in the previous lesson. In a similar fashion, I interpret Jo planning to give out prompt cards during the individual problem-solving phase as an unconscious undermining of desires. Whilst Jo could have planned to allow students time to work on the problem a planned adjustment was made with the intention of simplifying, and shortening, the student experience. For me, Jo's actions could be interpreted as unconscious acts of self-sabotage which sustain the contradiction between facilitation and instruction.

Jo's self-sabotage in RAM session 3 around beliefs about learning mathematics

I interpret data from section 7.3.1 as suggesting Jo wished to resolve a contradiction between students taking responsibility for learning and students completing questions, through participation in the RAM programme. Consequently, I interpret some of Jo's unconscious actions within the taught research lesson as actual moments of variation. For instance, I interpret the unplanned action of using an additional starter question to smooth the student experience as an unconscious act of self-sabotage. Jo sketched the question shown in Figure

7.3, on the board, modelled the required thinking, and effectively proceduralised an approach for the individual problem-solving phase. Despite a conscious desire for students to take responsibility, Jo's actions unconsciously undermined this desire and suggest Jo wished to maintain the contradiction. The unplanned change took away almost any opportunity for students to take responsibility for their own learning.

An alternative explanation of Jo's actions

I have suggested actual moments of variation can be inferred from Jo's actions during the individual problem solving, in making planned adjustments to the research lesson plan, and in the unplanned introduction of further support during the live research lesson. In these moments of variation, I interpret Jo's actions as the unconscious undermining of their conscious desires. Jo declined the opportunity to discuss this second theme, however I feel it is appropriate to provide alternative interpretations of actions based on my other interviews and experience of Jo.

In Chapter 4, I noted Jo was a new teacher who lacked mathematical confidence and lacked teaching confidence. Indeed, in interview Jo attributed the action of speaking to Ronnie during the individual problem-solving as stemming from feeling self-consciousness around not knowing how to solve the problem. Jo explained that no one had ever shown them how to do the type of problem we were looking at before. Furthermore, Jo's lack of classroom confidence around student behaviour may also provide an explanation for the planned and unplanned research lesson adjustments. It could be argued Jo made adjustments that would

reduce the likelihood of students misbehaving by ensuring students knew how to approach the angle between parallel lines question. Finally, I think there is a plausible interpretation of Jo's unplanned research lesson adjustments as simply being a response to student needs. Jo's lesson took place with a low attaining year 9 class and so the shortening of the individual problem-solving phase, and provision of prompt cards could be seen as responsive in-lesson adjustments. I acknowledge the plausibility of these explanations and return to consider the tension between different interpretations in section 7.5.

7.4.3 Inferring Ronnie's self-sabotage in actual moments of variation

In this sub-section I separate Ronnie's conscious desires around professional learning into desires around mathematics, teaching mathematics, and learning mathematics. This then allows me to link those desires to Ronnie's actions seen in RAM sessions 1, 2 and 3 respectively.

Ronnie's self-sabotage in RAM session 1 around beliefs about mathematics

Based on section 7.3.1, I interpret Ronnie as expressing a conscious desire to overcome the contradiction between learning from others and their confidence in their current approach through participation in the RAM programme. Thus, when actions conflict with desires, I interpret some of Ronnie's actions in RAM session 1 as actual moments of variation. Firstly, Ronnie expressed discomfort in how Drew had taken over in the small group discussion when explaining the method involving connecting two points. Rather than appreciate the mathematical connections, Ronnie felt marginalised and dismissed the method

as being inefficient. Secondly, during the whole-group discussion around grouping methods, Ronnie was reluctant to accept the classification based on the nature of the auxiliary line. Ronnie argued in favour of an alternative classification which would place the focus on area by looking at what shapes were created to solve the problem. Thirdly, although it took place in RAM session 2, Ronnie was cynical of the claimed benefits of the TTP style of lesson approach. Ronnie dismissed the claims by suggesting other factors, such as high levels of youth suicide in Japan, should be considered. Despite Ronnie claiming they wanted to learn from others, I interpret Ronnie's unconscious actions as acts of self-sabotage which maintained the contradiction between learning from others and current practices.

Ronnie's self-sabotage in RAM session 2 around beliefs about teaching mathematics

Drawing on sub-section 7.3.1, I suggest Ronnie hoped to overcome the contradiction between a teacher explaining mathematics and a teacher ensuring students completed work through participation in the RAM programme. I infer this from Ronnie's recognition that the Teaching Ladder lesson structure had shifted their approach from helping students make sense of mathematics and toward ensuring there were visible signs of students completing work. Therefore, when actions conflict with desires, I interpret some of Ronnie's actions as occurring in actual moments of variation. Firstly, Ronnie adjusted the research lesson plan to include a recall starter consisting of ten questions to prepare students for the learning. As seen in in Figure 7.2, this adjustment included both single-step and multi-step questions that would provide students with a reminder of the facts they would need for the lesson. Secondly, Ronnie made a planned adjustment of not constructing the diagram in the posing a new

problem phase of the research lesson plan. Instead, Ronnie adapted the research lesson plan to include handing out copies of the diagram to get “onto the problem solving as quickly as you can” (Interview 2 [Ronnie], 00:16:00). Ronnie elected not to utilise the opportunity for student insight that careful construction of the diagram might provide. I interpret Ronnie’s unconscious actions as acts of self-sabotage around Ronnie’s conscious desire to overcome the contradiction between explaining mathematical insight and directing students to complete questions.

Ronnie’s self-sabotage in RAM session 3 around beliefs about learning mathematics

I infer Ronnie as wishing to overcome the contradiction between modelling mathematical discussion and students’ lack of ability to discuss mathematics through participation in the RAM programme. Therefore, as described in sub-section 7.3.4, I see Ronnie’s unconscious action of not noting down student methods during the individual problem solving as an unconscious act of self-sabotage. Whilst Ronnie claimed they did not look at students’ ideas to avoid looking officious, they made an unplanned decision which undermined their stated desire. Furthermore, in the lesson phase requiring the orchestration of student discussion, Ronnie randomly selected a student but was unable to make sense of the method. Ronnie’s action had further consequences later in the lesson when Ronnie decided not to take the opportunity to highlight the links between different methods. In a lesson phase ideal for modelling mathematical discussion Ronnie instead decided “stuff it we’re done” (Interview 3 [Ronnie], 00:21:05). Ronnie had previously shown their understanding of the importance of this part of the lesson and had implied their desire to show students a

classification based on area. Therefore, I suggest these actions can be understood as moments of self-sabotage.

An alternative explanation of Ronnie's actions

I have interpreted Ronnie's actions of not listening to others, the planned research lesson plan adjustments, and the unplanned research lesson adjustments as actual moments of variation. I have suggested Ronnie's actions can be interpreted as an unconscious undermining of conscious desires. Ronnie declined the opportunity to discuss the development of this theme, though I still believe it is appropriate to offer alternative interpretations in line with my knowledge of Ronnie. As explained in Chapter 5, Ronnie was an experienced teacher who was successful with the approach they adopted in the classroom and thought deeply about student learning. In addition, Ronnie had been hurt by the enforced Task Ladder lesson structure and expressed frustration with Drew's role in enforcing its use. Hence, Ronnie may have been reluctant to learn from Drew in the small group phase of RAM session 1. Similarly, Ronnie's more general distrust of authority may account for a reluctance to learn from others throughout the RAM programme. In addition, Ronnie's planned adjustments of creating starter questions and handing out a copy of the problem on a sheet may have been borne of a desire to save time. Also, Ronnie explained their decision not to look at students' methods during the individual problem-solving phase came from not wanting to appear officious and not wanting students to feel watched. Finally, the decision not to classify the different methods at the end of the lesson could have resulted from simply running out of time, or from feeling the lesson up to that point had not quite progressed as planned. This sense of worrying

about their teaching approach and a concern for students' wellbeing provides a possible alternative interpretation of Ronnie's actions.

7.5 Implications of this theme

I have explained how I have interpreted some teacher actions, seen in the RAM programme, as unconscious acts of self-sabotage. Before drawing together my thinking around moments of variation I return to make some general observations about alternative interpretations of the data I have presented.

7.5.1 Alternative interpretations of the data

In Chapter 5 I identified how I do not see this study as simply telling the story of the participants. Instead, it has been my intention to use participants' experiences to provide an interpretation of the RAM programme according to my contradiction informed perspective. I have endeavoured to ensure I have not misrepresented the participants' desires or experiences in what I have shared. Furthermore, I have seen it as my ethical responsibility to provide Drew's views of my thinking, and plausible interpretations on behalf of Jo and Ronnie. I recognise these alternative explanations are feasible, but suggest they require an alternative philosophical perspective that does not interpret significance in actions. As per McGowan (McGowan 2019, p. 43), I do see significance in actions since I consider 'what we do manifests our desire in a way that our thinking cannot'. Therefore, I have taken care to record teacher's conscious desires around professional learning and used this understanding to claim significance in actions.

Despite not sharing my philosophical views, Drew graciously accepted the possibility of the interpretation I offered. However, seeking to discuss whether an individual is unconsciously undermining their conscious desires is unlikely to be straightforward. I recognise the difficulty of my approach, but suggest it is required since:

the subject cannot know what it's doing because it is constantly thwarting its own good and finding satisfaction in this self-derailing. If one consciously thwarts oneself, one is no longer actually thwarting oneself (McGowan 2019, p. 51).

Therefore, I feel it has been incumbent on me to infer the moments of unconscious self-sabotage through the careful approach I adopted. I sought to base my interpretations on a clear definition of moments of variation, by designing potential moments of variation into the RAM resources, by examining what participants wrote and said about the potential moments of variation, and by taking note of the participants' conscious desires. Hence, I claim a contradiction informed interpretation of the divided self provides a plausible explanation for participants' actions in moments of variation in the RAM programme.

7.5.2 Commonalities in the moments of variation of the three teachers

Within my interpretation of teacher actions, I have used the structure of overcoming contradiction between A and B through C, where C has been identified as participation in the RAM programme. However, I have not yet discussed the second part of McGowan's (2019) explanation of the divided self which suggests conflicting actions imply satisfaction is found in C, rather than achieving A.

Implications of acts of self-sabotage

I argue my interpretation of actions in the RAM programme has a wider implication for the understanding of mathematics DPD. As per McGowan (2019, p. 51), I suggest acts of self-sabotage reveal an individual finds satisfaction in the ‘repetition of failure’. Hence, I interpret the existence of self-sabotage as collectively indicating participants do not find satisfaction in attaining the object of a professional development programme, but through their failure to attain it. In this study, I argue teacher satisfaction would not be found in attaining a TTP-style of teaching, but in the process of experimenting and reflecting on the approach.

McGowan (2019) illustrates the idea of the repetition of failure with the example of an individual repeatedly buying the latest version of a new car. He argues the individual’s satisfaction is unconsciously found in the repeated process of buying, rather than in attaining, the car. I suggest a similar structure is revealed in this study, where the acts of self-sabotage illustrate participants do not find satisfaction in attaining a style of teaching, but in its exploration. I interpret the participants’ unconscious actions as revealing a desire to sustain, rather than resolve, contradiction. This interpretation can be further supported by considering what would happen if any of the participating teachers had executed the perfect RAM research lesson. I suggest none of the teachers would have been satisfied, and instead, Drew, Jo and Ronnie would move on to consider other alternative teaching approaches. Applying my reading of McGowan, I suggest pedagogic satisfaction is not found ‘in the conclusion of the game but in the confrontation that it makes possible’ (McGowan 2019, p. 63). Therefore, I claim in mathematics DPD success should not be understood as whether a teacher adopts a

new belief, or whether a lesson is taught with fidelity. Instead, success should be understood as whether the approach being studied helps teachers to engage with contradiction through consideration of the beliefs about mathematics education and the moments of variation.

Whilst I think there are opportunities for consideration of actions in all moments of variation, participants' reflections following the unplanned adjustments to the research lesson were particularly important. As shared in section 7.4, it was at this stage that the three teachers made perceptive reflections on their experiences. Drew claimed to have wobbled and to have re-thought the amount they directed their class. Jo claimed to have understood the benefit of working on one problem in the lesson to deepen student understanding. Even Ronnie, who did not feel they had found something to change their approach, made a connection to realising they could have helped students discuss more in the lesson. The teachers understood the success of the RAM programme in terms of their personal progression, not in terms of reaching the same end outcome. I am conscious these insights were revealed through interview and suggest this may indicate the need to further develop how to design for reflection around moments of variation in mathematics DPD.

7.6 Summary of Chapter 7

In this chapter I provided results and discussion to support my second theme of moments of variation are acts of self-sabotage. I presented results around the actions of Drew, Jo, and Ronnie, as seen in the potential moments of variation of the RAM programme and explained in interviews. I interpreted these actions as actual moments of variation when the

actions contrasted to the teacher's conscious desires about professional learning. Therefore, I argue contradiction, in terms of the divided self, provides a reasonable interpretation of actions seen in moments of variation of the RAM programme. Furthermore, I also suggested these moments of variation may be used to collectively infer satisfaction may be found in the repetition of failure. Hence, I argue teachers gain satisfaction from the experience of professional development programmes, rather than in attaining the practices or beliefs they seek to examine.

Chapter 8.

Analysis of theme 3: belief trajectories show journeys not destinations

8.1 Synopsis of Chapter 8

In the previous two chapters I shared the themes of beliefs are interdependent, and moments of variation are acts of self-sabotage. In this chapter, I extend my interpretations to a third theme of *belief trajectories show journeys not destinations*. For this analysis, I provide results indicating the teachers' views on beliefs seen in elements of the Rethinking Approaches in Mathematics (RAM) programme. Results are provided for the three participating teachers' views on the first session (RAM element 1), the original research lesson plan (RAM element 2), and their taught research lesson (RAM element 3). In my discussion I firstly suggest the RAM programme is seen as reflecting simultaneously interdependent beliefs about mathematics education. From this, I secondly suggest there is a link between teachers' views of the RAM programme and their personal beliefs about mathematics education. I claim the participants view the RAM programme as revealing less of their dominant belief, and more of a non-dominant belief, relative to their personal beliefs about mathematics education. Thus, I argue belief trajectories reveal the journey of reconciliation to contradiction, rather than the destination of a particular belief.

8.2 Introduction

As discussed in Chapter 4, at the end of RAM sessions 1, 2, and 3, I asked teachers to provide their views on an element of the RAM programme according to a specific domain of beliefs about mathematics education. RAM session 1 focused on the mathematics needed for the research lesson, and so participants were asked to provide weightings on the beliefs about mathematics they felt were seen in the session (RAM element 1). RAM session 2 focused on teaching mathematics, and so participants were asked to provide weightings on the beliefs about teaching mathematics they felt were seen in the original research lesson plan (RAM element 2). RAM session 3 focused on learning mathematics, and so participants were asked to provide weightings on the beliefs about learning mathematics they felt were seen in their taught research lesson (RAM element 3). Within this theme I firstly identify how beliefs about mathematics education seen in the RAM programme were understood as interdependent. From this, I argue the interdependence of the beliefs seen in the RAM programme is related to the teacher's personal beliefs about mathematics education. This leads to my claim that belief trajectories reveal the journey of reconciliation to contradiction, rather than the destination of a particular belief. To this end, I share the teachers' weightings, and their comments on those weightings, made in interview following each session. The codes that contributed to the development of this theme are shown in Table 8.1.

Codes
Beliefs about mathematics seen in RAM element 1
Beliefs about teaching mathematics seen in RAM element 2
Beliefs about learning mathematics seen in RAM element 3
Beliefs about mathematics education seen in RAM elements
Less of my dominant belief
More of my non-dominant beliefs

Table 8.1 Codes contributing to the theme of the existence of belief trajectories

8.3 Results supporting the theme of belief trajectories show journeys not destinations

In this section I share data around Drew, Jo, and Ronnie's views of the beliefs about mathematics education they felt were seen in each of the three specified RAM elements. The belief weightings the participants provided were recorded in the documents Beliefs [name], and these results are triangulated with interview comments taken from Interview 1 [name], Interview 2 [name] and Interview 3 [name]. I structure my results by looking at the individual RAM elements 1, 2 and 3 before considering mean average views of the RAM programme elements.

8.3.1 Views of beliefs about mathematics seen in RAM element 1

At the end of RAM session 1, participating teachers were asked to provide weightings for the beliefs about mathematics they felt were seen in RAM element 1 (the first session). The weightings for Drew, Jo and Ronnie are shown in Table 8.2, with a bold font used to highlight the highest weighting for each teacher.

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	10	30	60
Jo	45	35	20
Ronnie	5	35	60

Table 8.2 Teachers' views of the beliefs about mathematics seen in RAM element 1

Additionally, the data from Table 8.2 is displayed in the form of a triangular plot in

Figure 8.1 with a dotted line indicating the participants' dominant belief category.

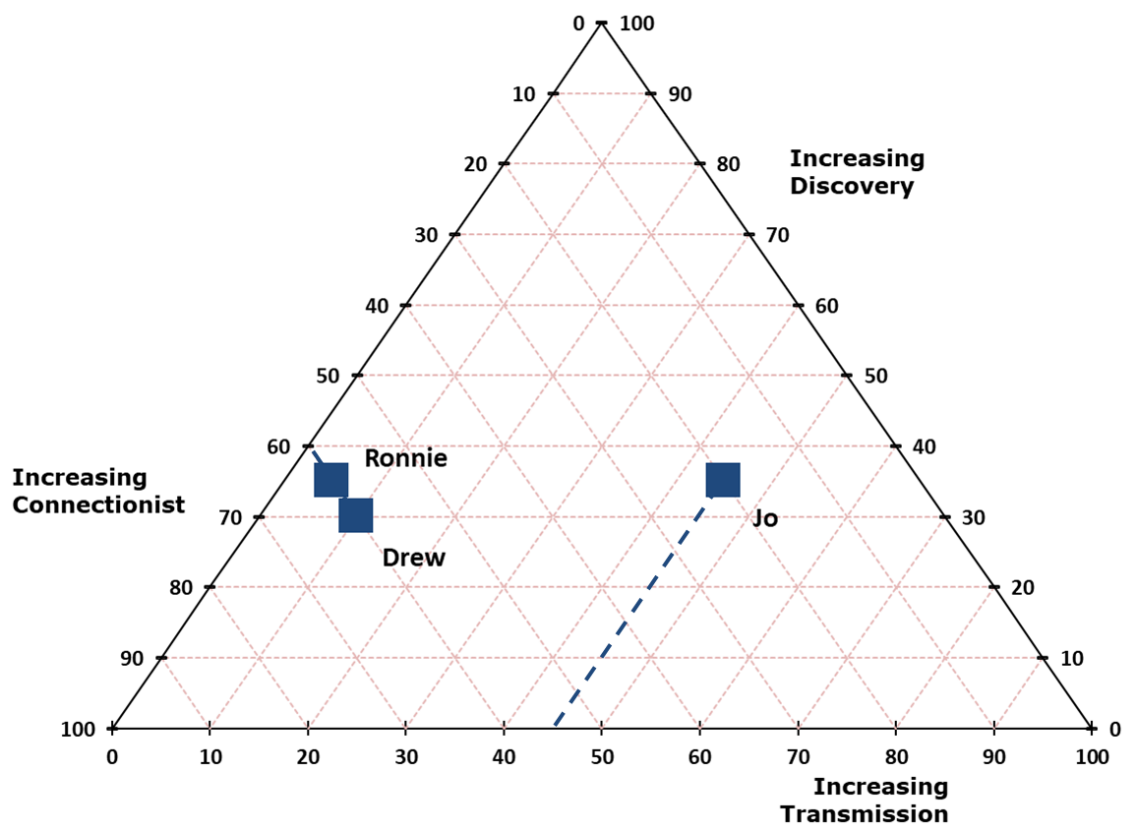


Figure 8.1 A triangular plot of views of beliefs about mathematics seen in RAM element 1

As can be seen in Figure 8.1, Drew and Ronnie had very similar views of the beliefs about mathematics seen in RAM element 1, whilst Jo's were quite different. In interview, all three teachers indicated the weightings were a fair reflection of their views and revealed how they considered the beliefs as interdependent.

Drew's views of interdependent beliefs about mathematics seen in RAM element 1

From Table 8.2, Drew gave an ordered weighting of Connectionist (60%) Discovery (30%) Transmission (10%) for RAM element 1. When asked to explain these weightings, Drew (Interview 1 [Drew], 00:19:30) began by justifying the high Discovery weighting as being a result of "us working independently, trying to create as many solutions as we could", and concluded "so, you know, that fits pretty squarely in B [Discovery]". Drew (Interview 1 [Drew], 00:19:40) then revealed how they saw beliefs as interdependent by explaining the Connectionist weighting resulted from the "doing it together" whole group discussion which "built on the independent work". Drew (Interview 1 [Drew], 00:19:40) expanded on this by reading the Connectionist descriptor from the RAM materials and exclaiming, "that's exactly what we did – the teacher and learner create together through discussion".

Jo's views of interdependent beliefs about mathematics seen in RAM element 1

Jo also provided a justification of their ordered weightings of Transmission (45%) Discovery (35%) Connectionist (20%) for RAM element 1. Jo (Interview 1 [Jo], 00:14:08) began by focussing on the Transmission weighting and stated the RAM session showed "the

core is the rules”. In addition, Jo (Interview 1 [Jo], 00:14:08) then explained how their difficulties with the individual problem solving illustrated the importance of knowing the rules since, “I didn’t know the rules so I struggled with the individual bit”. Jo then indicated an understanding of interdependence between the Transmission and Discovery categories by stating:

these are all just rules, but it's, it's amazing how many different rules you could apply to the problem and like how, how you take them, and the, like, the paths you'd go down just by applying a different rule in each situation (Interview 1 [Jo], 00:14:08).

Still referring to rules, Jo (Interview 1 [Jo], 00:14:08) then explained the discussions “opened my eyes up to it being more about how you apply those rules”. Hence, Jo (Interview 1 [Jo], 00:14:08) suggested the Discovery and Connectionist weightings reflected the value of “a flow of ideas, just from discussing our methods”.

Ronnie’s views of interdependent beliefs about mathematics seen in RAM element 1

As shown in Table 8.2, Ronnie provided ordered weightings of Connectionist (60%) Discovery (35%) Transmission (5%) for RAM element 1. Ronnie (Interview 1 [Ronnie], 00:19:44) justified their dominant Connectionist weighting by suggesting my role as the facilitator “makes it more interesting, and it, it just engages more”. Ronnie (Interview 1 [Ronnie], 00:19:34) built on this by explaining how I demonstrated a belief about mathematics that required drawing “attention to the important learning” and hence showed the connections within mathematics. Ronnie then explained how they saw my facilitation as only possible

following “us discovering for ourselves” (Interview 1 [Ronnie], 00:19:32) and hence the Discovery weighting of 35%. Finally, Ronnie justified the Transmission weighting as being the result of when I took control in my role as facilitator of the session. Once again, Ronnie (Interview 1 [Ronnie], 00:19:34) recognised the interdependence of the belief categories by saying “you told us when to move on, and that’s why I put a little bit of A [Transmission] in. Because I think you knew what you wanted us to get out of it”.

8.3.2 Views of beliefs about teaching mathematics seen in RAM element 2

Following the second RAM session, participants were asked to provide weightings for the beliefs about teaching mathematics they felt were shown in RAM element 2 (the original version of the RAM research lesson plan). The weightings for the three teachers are shown in Table 8.3, with a bold font used to highlight the highest weighting provided by each teacher.

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	10	30	60
Jo	20	20	60
Ronnie	0	50	50

Table 8.3 Teachers’ views of the beliefs about teaching mathematics seen in RAM element 2

Additionally, the data from Table 8.3 is displayed in the form of a triangular plot in Figure 8.2 with a dotted line used to highlight the dominant belief category provided by each teacher.

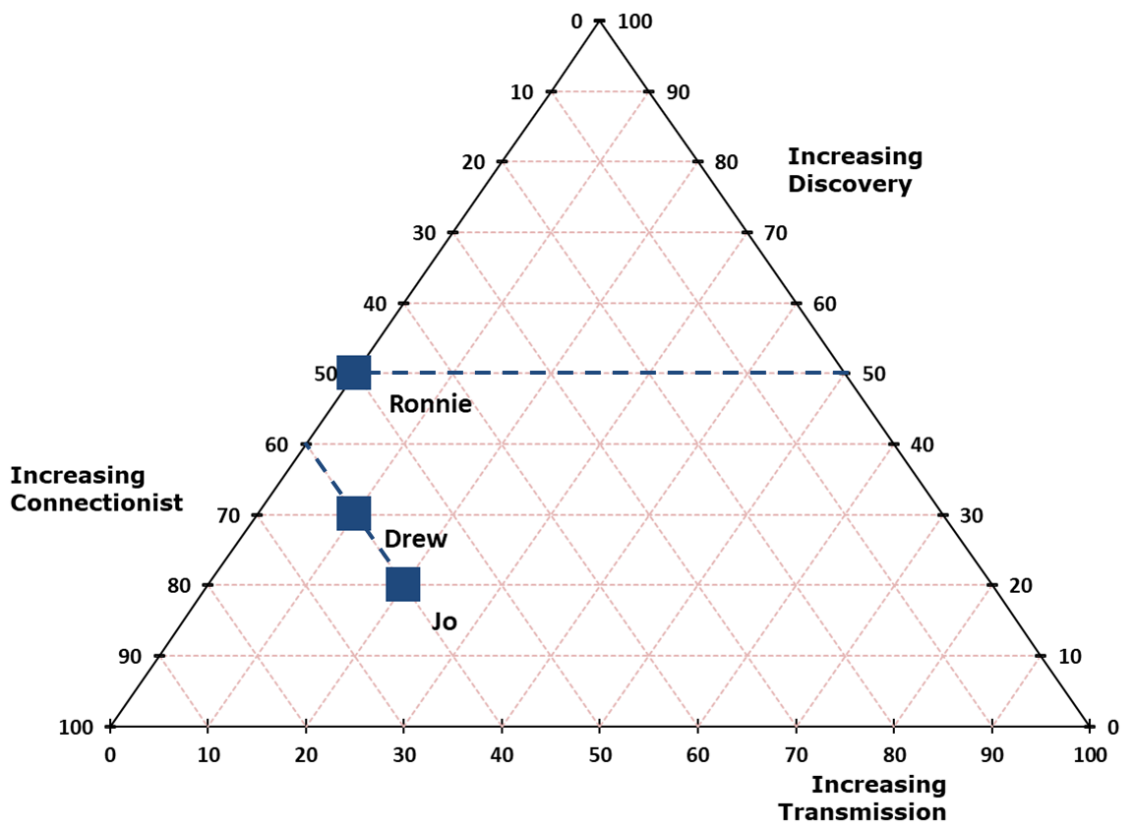


Figure 8.2 A triangular plot of views of beliefs about teaching mathematics seen in RAM element 2

As can be seen in Figure 8.2, all three teachers provided a dominant belief weighting category of Connectionist for RAM element 2. (Though Ronnie provided a joint highest weighting in both the Connectionist and Discovery categories). Once again, in interview, all three teachers accepted the weightings were a fair reflection of their views and revealed how they considered the beliefs as interdependent.

Drew's views of interdependent beliefs about teaching mathematics seen in RAM element 2

From Table 8.3, Drew gave ordered weightings of Connectionist (60%) Discovery (30%) Transmission (10%) for RAM element 2. In interview, Drew explained their weightings by noting the connections between the different roles expected of the teacher in the RAM research lesson plan. Working through the phases of research lesson plan, Drew (Interview 2 [Drew], 00:21:02) firstly identified the phase of “when you remind them of the rules, and draw the diagram” as influencing the Transmission weighting. From explaining the rules, Drew identified the next phase of “individual problem solving” (Interview 2 [Drew], 00:21:02) led to the Discovery weighting. Similarly, Drew explained the progression to Connectionist could be seen in the teacher facilitated discussion “when you bring it all together” (Interview 2 [Drew], 00:21:02). Drew (Interview 2 [Drew], 00:21:02) then explained how the Discovery weighting felt most important since it allowed the teacher to choose how to act, “for me, the absolute key to this lesson is the facilitation” and “the way the teacher draws the learning together”. For Drew (Interview 2 [Drew], 00:21:02) it is the teacher who must “make sure that connections are made, drawing out mistakes and misconceptions”.

Jo's views of interdependent beliefs about teaching mathematics seen in RAM element 2

As show in Table 8.3, Jo gave an ordered weighting of Connectionist (60%) Discovery (20%) Transmission (20%) for RAM element 2. Jo (Interview 2 [Jo], 00:25:50) emphasised the interdependence they saw on the Transmission category by explaining “this lesson is built off the recall task at the start”, and continued, “if you don’t get that right then the students can’t

do much”. Jo then explained how the Discovery weighting reflected the facilitation of individual problem solving in the research lesson plan. However, Jo (Interview 2 [Jo], 00:26:19). still related the problem solving back to the importance of the teacher creating a recall starter so students could “try and do the creative thing based on, well, what I’ve just told them I guess” Finally, Jo (Interview 2 [Jo], 00:25:50) explained the highest weighting of Connectionist was given because the research lesson plan needed the teacher “bringing it all together”. However, once again, Jo (Interview 2 [Jo], 00:25:50) noted interdependence with the Transmission category underpinned even this phase of the research lesson plan, since the teacher would “need to know the new knowledge they [the students] need”.

Ronnie’s views of interdependent beliefs about teaching mathematics seen in RAM element 2

As shown in Table 8.3, Ronnie provided an ordered weighting of Connectionist (50%) Discovery (50%) Transmission (0%) for RAM element 2. When asked about these weightings, Ronnie (Interview 2 [Ronnie], 00:11:31) immediately dismissed the Transmission category since there is “no telling in this lesson”. However, Ronnie (Interview 2 [Ronnie], 00:11:31) did recognise the interdependence of the Connectionist and Discovery categories since “one doesn’t happen without the other”. Ronnie (Interview 2 [Ronnie], 00:11:31) also indicated they did not give a higher weighting to Connectionist since they felt the research lesson plan was “a little prescriptive”. When asked to expand, Ronnie (Interview 2 [Ronnie], 00:12:00) may have indicated a Transmission belief in stating “my understanding of the C [Connectionist] descriptor is that it’s about non-linear dialogue. Well this isn’t, is it? It’s dialogue, but it’s not non-linear if I know where it’s going”.

8.3.3 Views of beliefs about learning mathematics seen in RAM element 3

Following the third RAM session, participants were asked to provide weightings for the beliefs about learning mathematics they felt were seen in RAM element 3 (their taught version of the RAM research lesson). The weightings for the three teachers are shown in Table 8.4, with a bold font used to highlight the highest weighting for each teacher.

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	15	60	25
Jo	10	45	45
Ronnie	0	70	30

Table 8.4 Teachers' views of the beliefs about learning mathematics seen in RAM element 3

Additionally, the data from Table 8.4 is displayed in the form of a triangular plot in Figure 8.3 with a dotted line used to highlight the dominant belief category of each teacher.

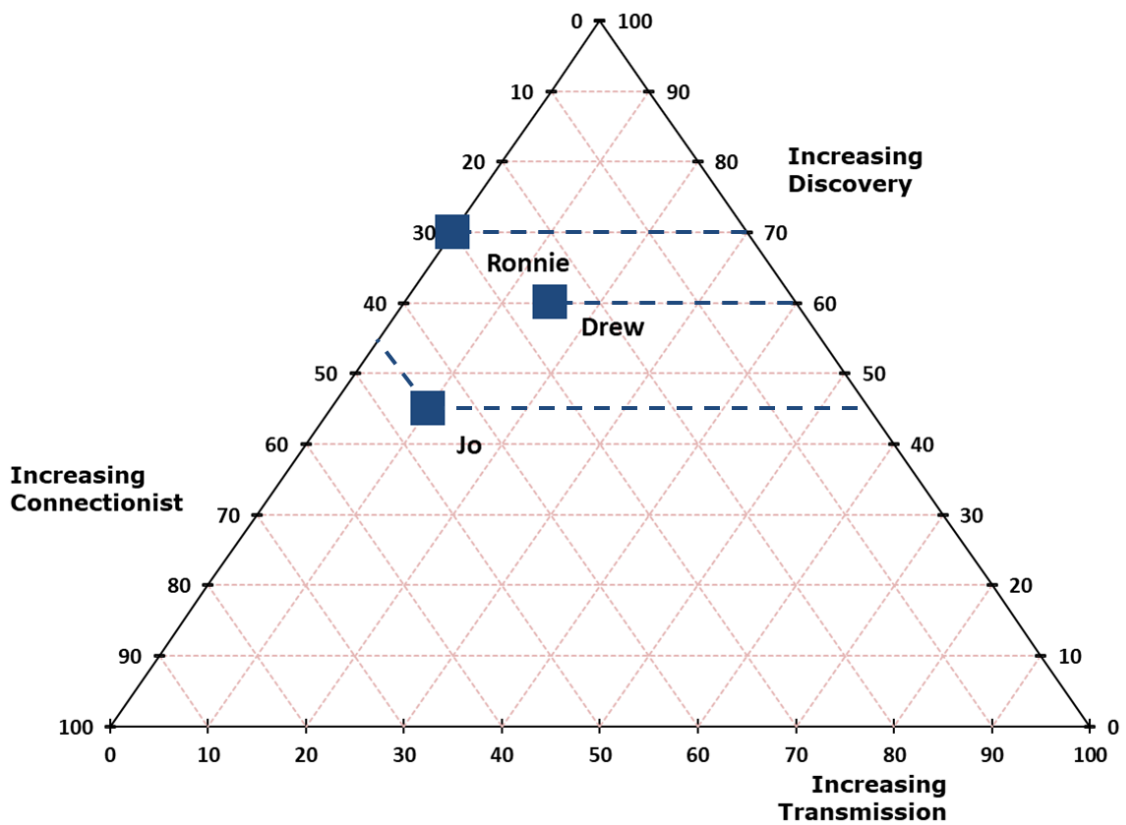


Figure 8.3 A triangular plot of views of beliefs about learning mathematics seen in RAM element 3

As can be seen in Figure 8.3, the three teachers viewed RAM element 3 as mostly displaying a Discovery belief about learning mathematics. (Though Jo provided a joint highest weighting to both the Discovery and Connectionist categories). In interview, all three teachers indicated the weightings were a fair reflection of their views and revealed how they saw the beliefs as interdependent.

Drew's views of interdependent beliefs about learning mathematics seen in RAM element 3

From Table 8.4, Drew gave ordered weightings of Discovery (60%) Connectionist (25%) Transmission (15%) for the third RAM element of their taught research lesson. When asked about these weightings, the teachers confirmed how they saw learning taking place across many parts of the lesson. For example, Drew (Interview 3 [Drew], 00:00:19:58) explained the highest weighting was given to Discovery since most student learning came from “when they were doing the problem on their own”. Similarly, Drew (Interview 3 [Drew], 00:00:19:58) explained there “should probably have been more Connectionist”, but running out of time meant there was less learning in the drawing together stage which did not happen “as well as I would’ve liked”. Drew (Interview 3 [Drew], 00:00:20:29) also added the Transmission weighting resulted from seeing student learning taking place in the recall starter where students were reminded of rules such as the meaning of “the words alternate and corresponding”.

Jo's views of interdependent beliefs about learning mathematics seen in RAM element 3

As shown in Table 8.4, Jo gave ordered weightings of Connectionist (45%) Discovery (45%) Transmission (10%) for the third RAM element of their taught research lesson. Jo explained how they saw both the individual problem-solving phase, and their adjusted small group work phase, as the main contributors to student learning. Jo (Interview 3 [Jo], 00:22:20) provided a high Discovery weighting since, “for some of them [the students] they got a lot out of trying the problem”. However, Jo (Interview 3 [Jo], 00:22:20) also recognised there was “a

significant minority that learnt very little” during this phase. Therefore, Jo (Interview 3 [Jo], 00:22:20) explained the Connectionist weighting was given since these students “got loads out of the prompt cards”. For Jo (Interview 3 [Jo], 00:22:20), the use of prompt cards reflected a Connectionist belief since it gave Jo the opportunity to “walk round and listen in and try to prompt them”.

Ronnie’s views of interdependent beliefs about learning mathematics seen in RAM element 3

From Table 8.4, Ronnie gave ordered weightings of Discovery (70%), Connectionist (30%), Transmission (10%) for the third RAM element of their taught research lesson. In interview, Ronnie (Interview 3 [Ronnie], 00:23:04) again dismissed the Transmission perspective since “everything was done through their ideas followed by discussion”. Ronnie (Interview 3 [Ronnie], 00:23:04) continued by explaining the other two weightings were based on which part of the lesson led to the most learning for students. Ronnie suggested (Interview 3 [Ronnie], 00:23:04) “I think, at the end of the day, I think they got most out of the individual problem solving”. However, Ronnie (Interview 3 [Ronnie], 00:23:04) admitted that although the teacher led discussion phase did lead to learning, “I was thinking on my feet so there were bits that weren’t as good”. Ronnie (Interview 3 [Ronnie], 00:23:51) further explained how students became confused by a student shared method, and “I couldn’t work out if it worked” so “maybe I ended up muddling things for some of them”.

8.3.4 Overall views of beliefs about mathematics education seen in the RAM elements

An indication of the overall views of beliefs about mathematics education seen in the RAM elements can be calculated. In Table 8.5, I provide the calculated mean average weightings for each belief category across the three domains taken from the previous subsections. Some caution needs to be exercised in interpreting these numbers, since the three RAM elements were very different in nature. However, I suggest it provides a helpful proxy for each teacher's overall view of the RAM programme.

Name	Transmission (%)	Discovery (%)	Connectionist (%)
Drew	12	40	48
Jo	25	33	42
Ronnie	2	51	47

Table 8.5 Mean average view of beliefs about mathematics education seen in the RAM elements

Additionally, the data from Table 8.5 is displayed in the form of a triangular plot in Figure 8.4, with a dotted line used to highlight the dominant belief category for each teacher.

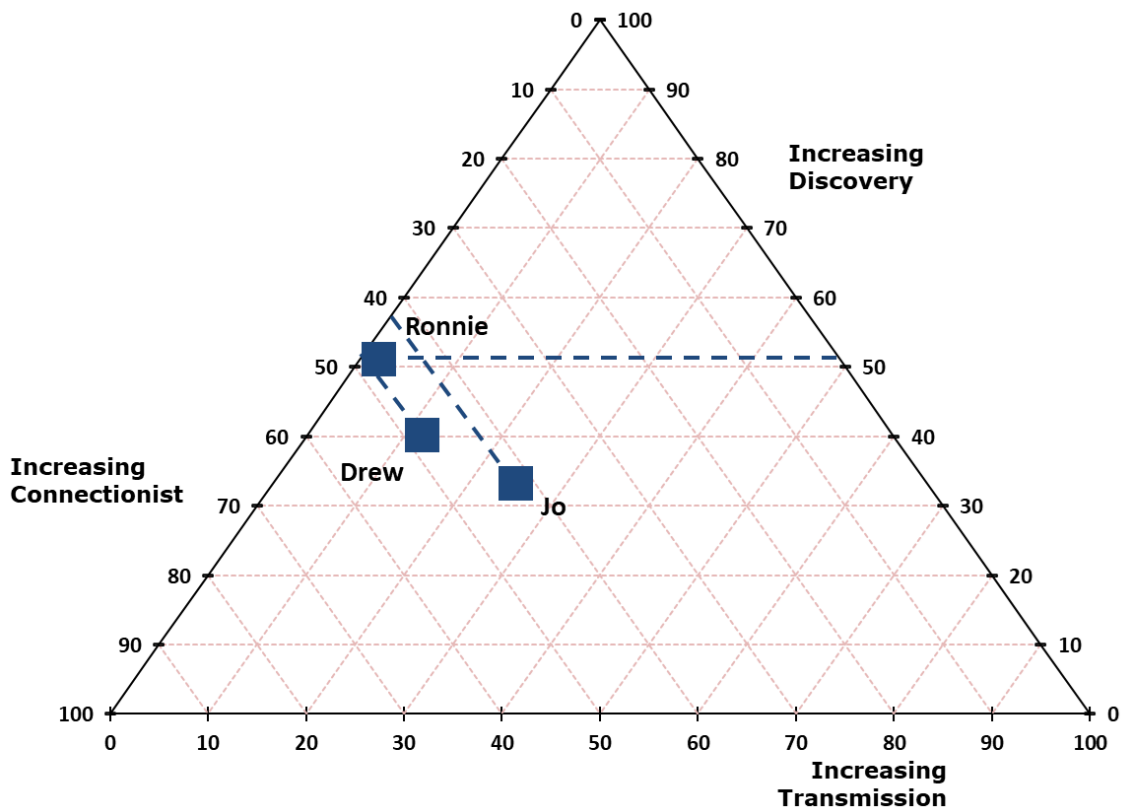


Figure 8.4 A triangular plot of the teachers' views of beliefs about mathematics education seen in the RAM programme

As can be seen in Table 8.5 and Figure 8.4, I interpret the teachers as seeing the RAM programme as simultaneously reflecting interdependent Transmission, Discovery and Connectionist beliefs about mathematics education. Transmission beliefs were viewed as lowest across the three RAM elements by all three teachers with high weightings provided for both the Discovery and Connectionist categories. Drew and Jo overall saw the RAM elements as being dominated by Connectionist beliefs, and Ronnie saw very similar levels of Discovery

and Connectionist. However, I suggest Ronnie's high Discovery weighting from RAM element 3 had a disproportionate effect on their overall dominant Discovery weighting.

8.4 Discussion around the theme of belief trajectories show journeys not destinations

Having shared data about Drew, Jo and Ronnie's views on the beliefs about mathematics education underpinning the RAM elements, I now discuss these results in two stages. Firstly, I draw attention to how the teachers viewed the RAM elements as reflecting interdependent belief categories. However, I also draw attention to the differences in their interdependent belief categories. Secondly, I seek to analyse how the differences in belief category weightings are linked to the teachers' personal (interdependent) beliefs about mathematics education.

8.4.1 Teachers' views of the RAM programme

In this first sub-section, I discuss the three teachers' views of the RAM elements according to their belief weightings and interview comments. I do this with the intention of drawing attention to the interdependence of beliefs, and to the differences in these belief weightings. In analysing this theme, I had anticipated there would be differences between the teachers' views of the beliefs seen in each of the RAM elements. I anticipated the most, second most, and third most variation would be seen in RAM elements 3, 1, and 2 respectively. This anticipated variation between a live research lesson, an RAM session, and a research lesson plan will be commented on in each of the following sub-section. This paves

the way for a discussion in the next sub-section of how I view the RAM programme beliefs as being linked to personal (interdependent) beliefs about mathematics education.

Interdependent beliefs about mathematics seen in RAM element 1

I interpret all three teachers as viewing the three categories of beliefs about mathematics as being interdependent in the whole session of RAM element 1. Drew and Ronnie gave dominant Discovery and Connectionist weightings but recognised how these two categories were dependent on each other. In addition, Ronnie recognised RAM element 1 revealed a Transmission view of mathematics as a set of shared facts to help draw the session together. Jo displayed the opposite view to Drew and Ronnie in claiming a Transmission view of beliefs about mathematics education dominated over the other two categories. However, all three teachers recognised the interdependence of beliefs were seen in the individual problem-solving stage and the small group discussion stage. Drew and Ronnie suggested these two phases revealed opportunities for discussion and connecting mathematics, whilst Jo saw them as a supporting the application of rules.

The differences in the weightings given by the three teachers are also of interest. Prior to the development of this theme, I had anticipated there would be some variation between the teachers in their views around beliefs about mathematics seen in RAM element 1. As shown in Table 8.2 and Figure 8.1, Drew and Ronnie provided very similar ordered weightings of Connectionist-Discovery-Transmission, whilst Jo had an ordered weighting of Transmission-Discovery-Connectionist. Despite having very similar experiences of the session, and even working together in the small group discussion, Jo saw the beliefs about mathematics in the

opposite order to Drew and Ronnie. I seek to link these differences in views to the teachers' personal beliefs about mathematics in the next sub-section.

Interdependent beliefs about teaching mathematics seen in RAM element 2

I also interpret teachers' views of beliefs about mathematics seen in RAM element 2 as being interdependent. Whilst recognising a dominant belief, Drew suggested Connectionist beliefs linked each of the phases and Jo argued for Transmission. Moreover, Ronnie (Interview 2 [Ronnie], 00:11:31) made a statement I interpret as strongly supporting a Hegelian interpretation when identifying Connectionist and Discovery beliefs do not “happen without the other”. This statement echoes McGowan's (2019, p. 73) view that ‘the isolated subject is dependent on otherness’. More generally, I note Drew and Jo viewed all three categories of beliefs about teaching mathematics as being reflected in the research lesson plan of RAM element 2. My interpretation here is conditional on a qualifier of considering the unit of consideration as the entire original research lesson plan. As shown in sub-section 8.3.2, whilst all three teachers recognised the interdependence, they tended to link a main belief category to a separate phase of the lesson. In general, Transmission beliefs were attributed to the review of previous learning, which led to the Discovery beliefs of the individual problem solving, which led to the Connectionist beliefs of the teacher led discussion. However, taken as a whole lesson, I suggest beliefs were viewed as interdependent.

Once again, the differences in teachers' weightings of interdependence are also of note. I had anticipated RAM element 2 would offer the most potential for consistency in

teacher views since it is a written document. Indeed, as shown in Table 8.3 and Figure 8.2, all three teachers saw the research lesson plan as reflecting an ordered weighting of Connectionist-Discovery-Transmission in terms of beliefs about teaching mathematics. This consistency is further confirmed in the teachers all identifying the drawing together of ideas as contributing to the Connectionist weighting, and the individual problem solving as contributing to the Discovery weighting. However, the three teachers also identified different parts of the research lesson plan as the most important guide for their actions. Drew saw the drawing together of ideas as the most important teacher role, Jo highlighted the importance of reminding students of rules, and Ronnie saw individual problem solving and drawing together as equally important elements of their role. Despite differences between how teachers interpreted the beliefs behind the same teaching activities, at the level of views of beliefs about teaching mathematics there is some consistency. I seek to link these differences in interpretation to a teacher's personal beliefs about teaching mathematics in the next subsection.

Interdependent beliefs about learning mathematics seen in RAM element 3

I also interpret the teachers' views of beliefs about mathematics, as seen in RAM element 3, as being interdependent. Drew, Jo and Ronnie all gave an ordered weighting of Discovery-Connectionist-Transmission and recognised the interdependence of all three beliefs about learning mathematics within their taught research lessons. All three teachers had dominant weightings of Discovery and based this view on seeing the most student learning taking place in the individual problem-solving phase of their research lesson. However, Jo still

recognised the interdependence of the problem-solving phase on the Transmission beliefs seen in the recall starter. Similarly, Drew and Ronnie saw the potential for more Connectionist beliefs in the teacher led discussion was possible based on the strength of the individual problem-solving phase.

I had anticipated RAM element 3 would offer the widest variation in teacher views since I had no direct influence on the live research lessons. However, as shown in Table 8.4 and Figure 8.3, there was considerable consistency between the three teachers' views. Both Drew and Ronnie acknowledged their Connectionist weightings were lower than they should have been after they made in-lesson adjustments to their intentions. Drew suggested their teacher led discussion was curtailed due to time, and Ronnie acknowledged they did not react to student ideas as well as they had hoped. In contrast, Jo saw the planned adjustment of using prompt cards as highly influential to their Connectionist weighting. Hence, although the weightings appear similar, this does not necessarily mean there was a consistent experience of the research lesson. I suggest whilst weightings appear similar, differences remained in the teachers' views of where learning took place. I seek to link these differences in interpretation to a teacher's personal beliefs about learning mathematics in the next sub-section.

Overall comments on views of beliefs about mathematics education seen in the RAM elements

The three RAM elements account for three distinct elements of the RAM programme – a session, a research lesson plan, and a taught research lesson. I have argued all three elements were viewed as revealing interdependence of the three beliefs categories. However, I

also note the professional development session (RAM element 1) led to the least consistency in terms of teachers' views. The research lesson plan (RAM element 2) led to relatively consistent weightings, but with differences in what teachers saw as important. Finally, the teaching of the research lesson (RAM element 3) led to some agreement in weightings, but with different reasons for these weightings. These elements are very different in nature and so some caution should be exercised when interpreting the mean average weightings given in Table 8.5 and Figure 8.4. I see the mean average measure as a helpful proxy for the overall view of the RAM programme as reflecting an interdependent, ordered belief about mathematics education of Connectionist-Discovery-Transmission. In the next sub-section, I move on to suggest the teachers' views of the beliefs about mathematics education seen in the RAM elements are linked to their personal (interdependent) beliefs about mathematics education.

8.4.2 Relative views of the RAM programme compared to personal beliefs

In this sub-section, I make links to Swan's (2006a, 2006b) idea of belief trajectories as discussed in Chapter 2. Swan (2006a, 2006b), and subsequently Calleja (2022), suggested once teachers participate in a Connectionist focused professional development programme, they follow one of three belief trajectories. Teachers may follow a belief trajectory from Transmission to Discovery, from Transmission to Connectionist, or from Discovery to Connectionist (Swan 2006a, Swan 2006b). In effect, Swan (2006a, 2006b) claimed all teachers would eventually follow a trajectory to Connectionist beliefs about mathematics education.

To aid my discussion, I share relative belief weightings in the form of triangular plots from Beliefs [name], to show a comparison between a teacher's view of the beliefs seen in the RAM elements and their personal beliefs. For each teacher, I show four triangular plots, the first compares the mean average beliefs about the three RAM elements with a teacher's beliefs about mathematics education. The next three plots compare a teacher's views of RAM elements 1, 2, and 3 with their beliefs about mathematics, teaching mathematics, and learning mathematics respectively. In addition, as discussed in Chapter 5, I use a black arrow on the axis of a triangular plot to indicate a teacher seeing less of a belief in the RAM element compared to their personal belief. Similarly, I use a green arrow on the axis of a triangular plot to indicate a teacher seeing more of the belief in the RAM element compared to their personal belief.

Drew's views of professional development linked to personal beliefs

According to Swan (2006a, 2006b), Drew would be classified as having a personal belief about mathematics education of Discovery (see Table 6.2). From this perspective, on experiencing a Connectionist dominated professional development programme, Drew could be expected to follow a personal belief trajectory from Discovery to Connectionist. In Figure 8.4, I show the four triangular plots for Drew indicating relative belief weightings by comparing personal beliefs to views of the RAM elements. Looking at the arrows on the axes, a similar pattern can be seen across all four triangular plots with Drew consistently interpreting the RAM elements as showing less Discovery (a black arrow) and more Connectionist (a green arrow) than their personal beliefs. In addition, for RAM elements 3 and

4, Drew had marginal differences in their Transmission weightings. However, I do not interpret these differences as a consistent pattern since the individual Transmission values are relatively low, the differences are relatively low, and the direction of difference is not consistent.

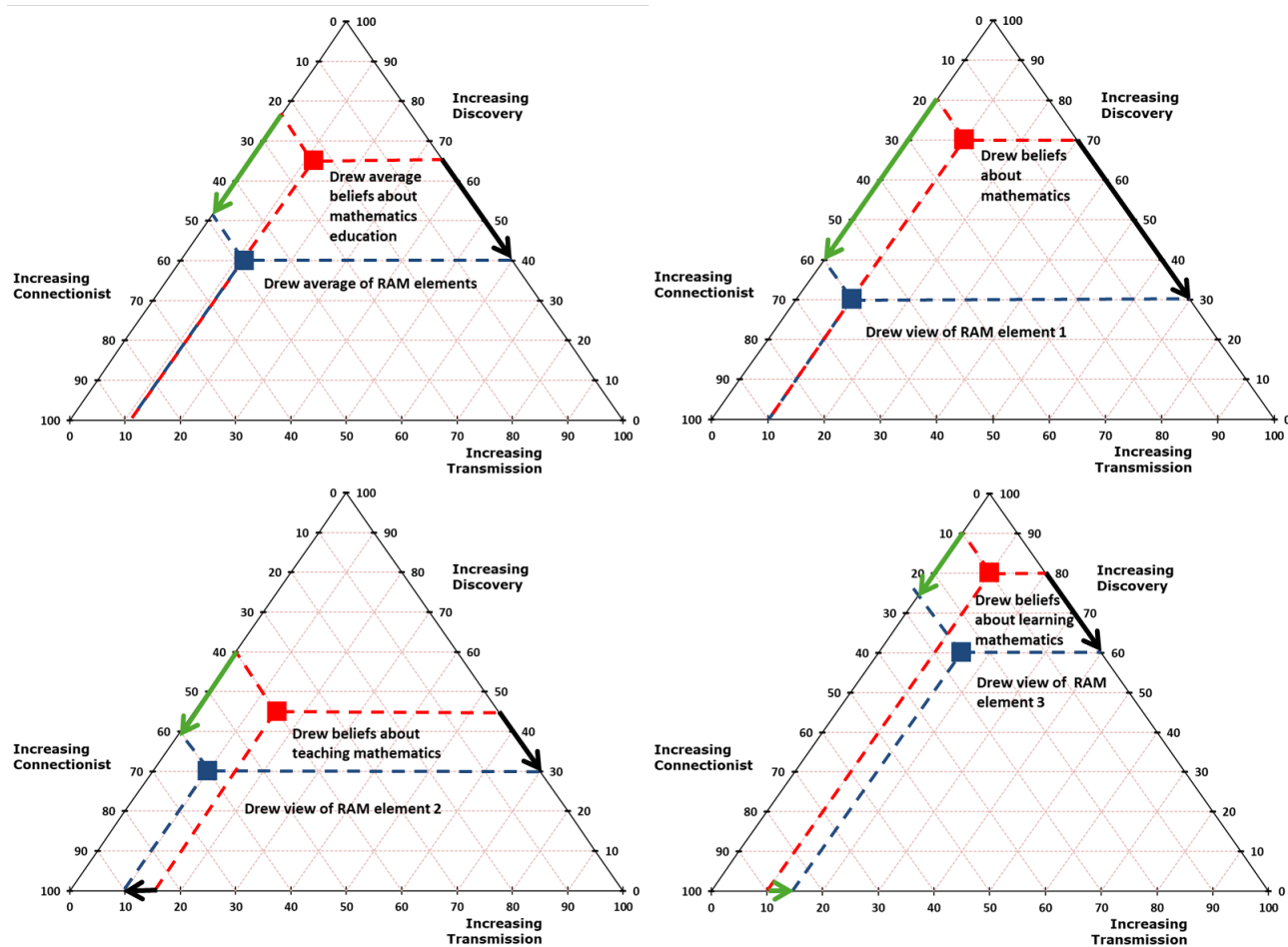


Figure 8.4 Drew's beliefs against their view of the RAM elements

As can be seen in Figure 8.4, Drew viewed RAM elements 1 and 2 as being dominated by Connectionist beliefs (60% for both), but RAM element 3 as being dominated by Discovery beliefs (again at 60%). However, as indicated by the axis arrows, Drew consistently saw more Connectionist beliefs in the RAM elements compared to their personal beliefs across all three RAM elements. Even when RAM element 3 was viewed as dominated by Discovery beliefs, Drew still saw less Discovery (and more Connectionist) relative to their personal beliefs. Hence, I suggest it is not whether an element is consciously seen as reflecting a dominant Connectionist belief that is important, but the unconscious relative relationship between personal belief and the RAM element.

I suggest this interpretation could be seen to support Swan's (2006a, 2006b) understanding of belief trajectories. Swan might have anticipated Drew would move from Discovery beliefs to Connectionist beliefs, with an emphasis on final position. I suggest looking at Drew's relative views provides a possible explanation for the belief trajectory. Since Drew viewed the RAM elements as being more Connectionist than their personal beliefs, then Drew might be on a journey from examining Discovery beliefs to examining Connectionist beliefs.

Jo's views of professional development linked to personal beliefs

According to Swan (2006a, 2006b), Jo would be classified as having a personal belief about mathematics education of Transmission (see Table 6.2). In experiencing a Connectionist dominated professional development programme, Swan might anticipate Jo would be on a belief trajectory of either Transmission to Discovery or Transmission to Connectionist. In

Figure 8.5 I show the four triangular plots for Jo, comparing personal beliefs with views of the beliefs seen in the RAM elements. Jo viewed dominant beliefs of Transmission (45%), Connectionist (60%) and Discovery/Connectionist (45%/45%) for each of RAM elements 1, 2, and 3 respectively. However, it is the consistent pattern seen in the green and black arrows on each triangular plot that I consider to be noteworthy. Jo consistently saw less Transmission (a black arrow) and more Connectionist (a green arrow) in the RAM elements compared to their personal beliefs. In addition, other than for RAM element 2, there is some consistency in pattern with Jo seeing slightly more Discovery beliefs in the RAM elements compared to their personal beliefs. I suggest the change in direction for RAM element 2 is reasonable since, as mentioned in Chapter 7, Jo saw little Transmission beliefs in the RAM research lesson plan.

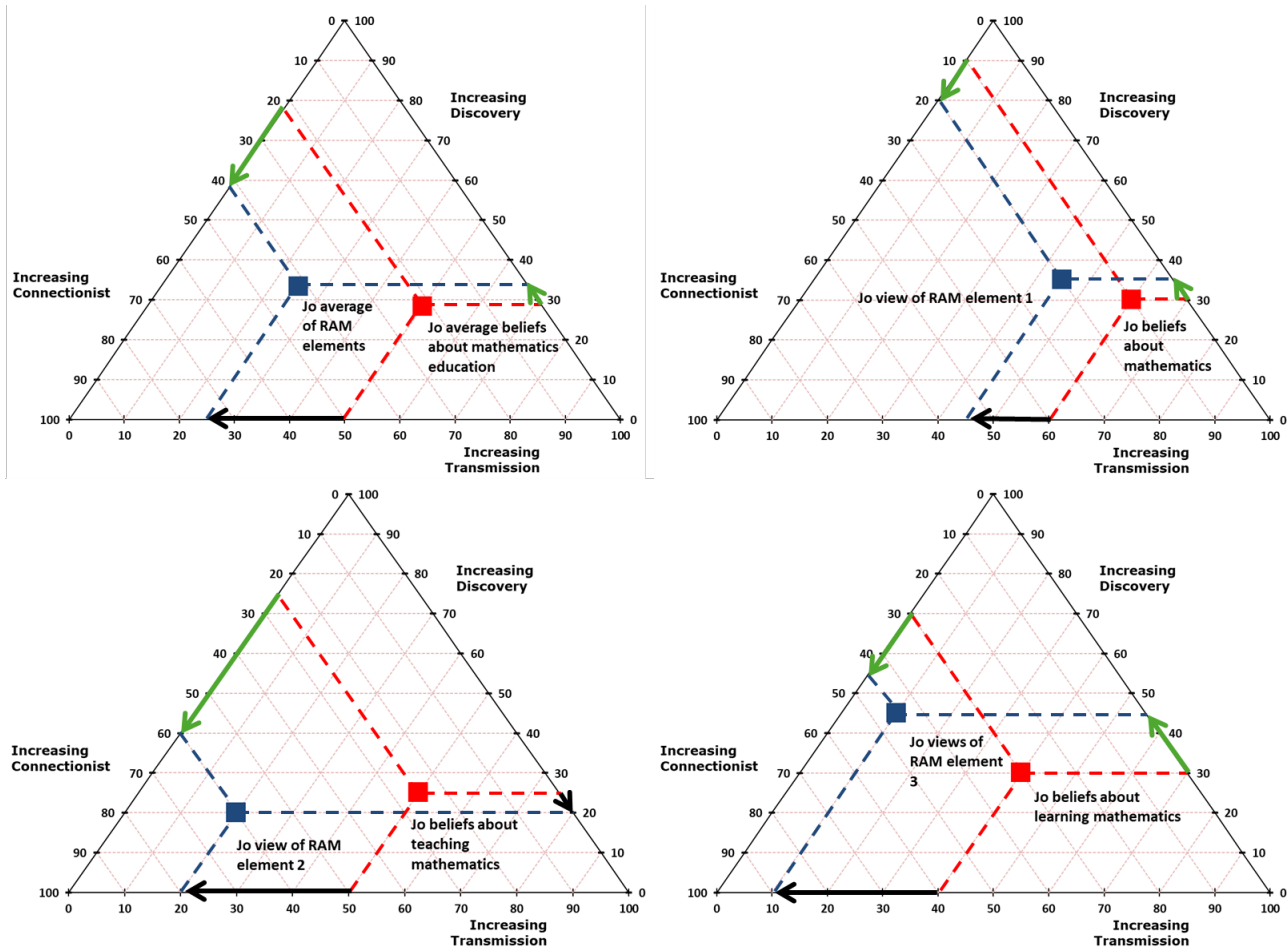


Figure 8.5 Jo's beliefs against their view of the RAM elements

Once again, I interpret this data as showing it is not whether an RAM element is consciously seen as dominated by a Connectionist belief that is important, but the unconscious relative relationship between personal beliefs and the RAM elements. Jo consistently viewed the RAM elements as displaying more Connectionist, more Discovery, and less Transmission beliefs than their personal beliefs. The importance of this relative relationship is made clear in the second plot of Figure 8.5 when considering RAM element 1. Jo consciously saw RAM element 1 as being dominated by Transmission beliefs about mathematics. However, unconsciously, Jo still suggested RAM element 1 had more Connectionist and Discovery (green arrows) and less Transmission (the black arrow) compared to their personal beliefs.

Again, this understanding of relative beliefs could be used to provide a supporting explanation for Swan's (2006a, 2006b) understanding of belief trajectories. If Jo viewed the RAM element as revealing more Connectionist beliefs, then it might be reasonably anticipated Jo would be on a journey from Transmission to Discovery. In addition, the existence of green arrows on the Discovery axes suggest Jo could also be on Swan's (2006a, 2006b) other belief trajectory from Transmission to Discovery.

Ronnie's views of professional development linked to personal beliefs

According to Swan (2006a, 2006b), Ronnie would be classified as having a personal belief about mathematics education of Connectionist (see Table 6.2). If participating in a professional development programme examining Connectionist beliefs, then Swan would not anticipate Ronnie as being on a belief trajectory. Instead, Swan saw participants with Connectionist beliefs remain with Connectionist beliefs after the programme. As per Table

8.5, I note Ronnie slightly varied from the other two teachers in overall seeing the RAM elements as being dominated by Discovery (51%) over Connectionist (47%) beliefs. However, the mean average Discovery weighting was strongly influenced by the high Discovery weighting of 70% in the third RAM element. In Figure 8.6, I show the four relative belief weighting triangular plots for Ronnie and note that Ronnie saw dominant beliefs of Connectionist (65%, 80% and 60%) for each of RAM elements 1, 2, and 3. However, once again I suggest it is important to notice the similar pattern in the relative belief weightings indicated by the arrows on the axes. Ronnie interpreted the RAM elements as showing less Connectionist and Transmission beliefs (black arrows) and more Discovery (green arrow) compared to their personal beliefs.

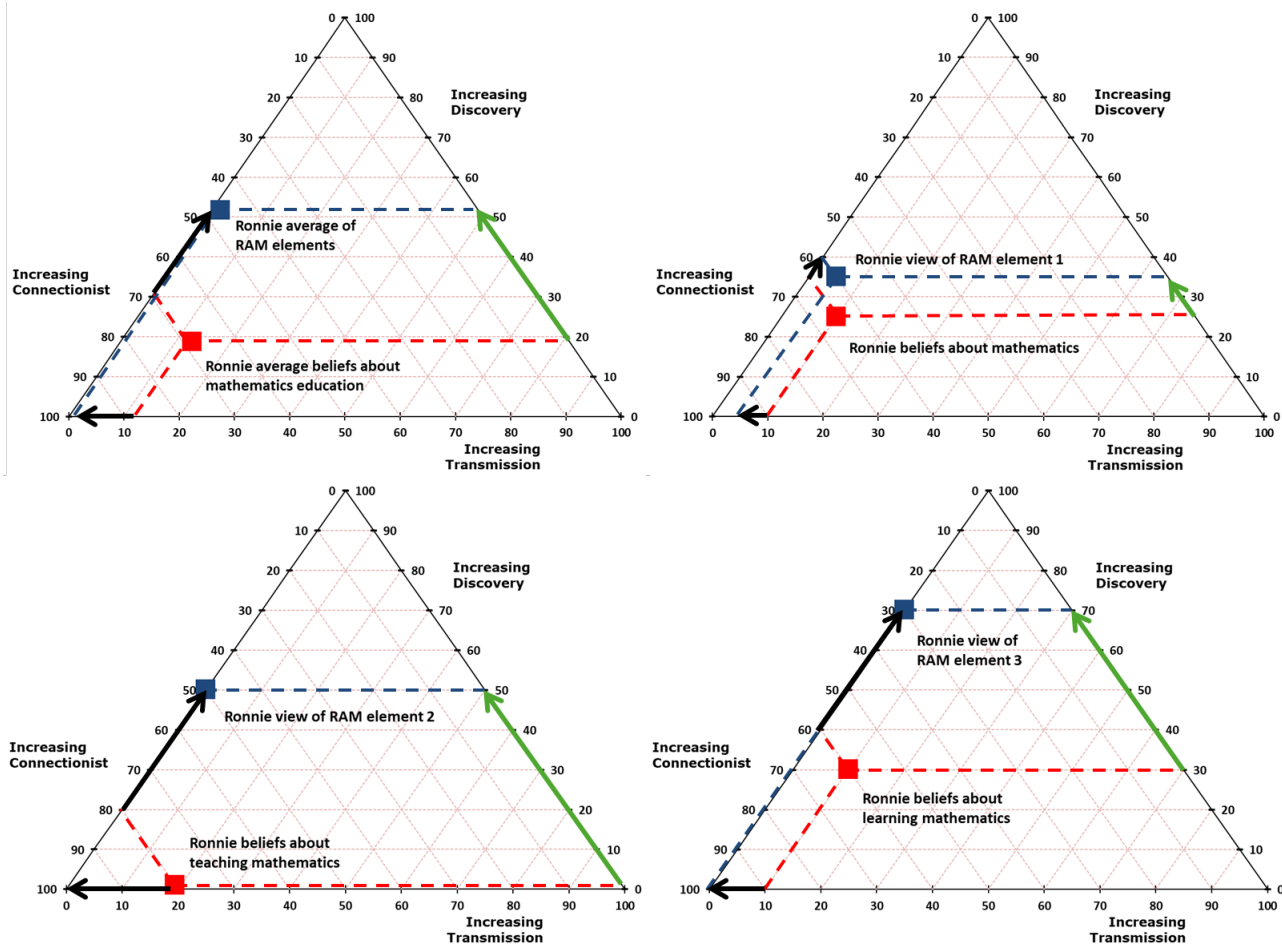


Figure 8.6 Ronnie's beliefs against their view of the RAM elements

Following the same logic applied to Drew and Jo around relative beliefs, I again suggest it is the unconscious relative weightings Ronnie provided that are important, rather than the dominant weightings. Consciously, Ronnie identified the RAM elements as being dominated by Discovery and Connectionist beliefs. However, unconsciously Ronnie saw the RAM elements as displaying more Discovery and less Connectionist beliefs compared to personal beliefs. I argue this data, and the use of relative belief weightings, suggests the possibility of another belief trajectory from Connectionist to Discovery. Rather than reinforcing Connectionist beliefs, Ronnie unconsciously challenges their own Connectionist beliefs by viewing the RAM elements as reflecting more Discovery.

8.5 Implications of this theme

8.5.1 Views of professional development linked to personal beliefs

For all three teachers in this study, I suggest their views of beliefs underpinning the RAM elements are unconsciously shaped by their personal beliefs about mathematics education. For each participant, there appears to be a pattern of seeing less of their dominant belief, and more of a non-dominant belief in the RAM elements compared to their personal beliefs. For example, Drew sees less Discovery, Jo sees less Transmission, and Ronnie sees less Connectionist in the RAM elements compared to their personal beliefs. Similarly, Drew sees more Connectionist, Jo sees more Connectionist (and Discovery), and Ronnie sees more Discovery in the RAM elements compared to their personal beliefs.

Furthermore, I have suggested my interpretation supports, and provides an explanation for, the idea of belief trajectories proposed by Swan (2006a, 2006b). I interpret the belief trajectories as existing because a teacher views the RAM elements as reflecting less of their dominant personal belief and more of a non-dominant belief. This argument means I have identified the potential existence of another belief trajectory from Connectionist to Discovery. The belief trajectory is dependent on the unconscious relative view that the RAM programme has more Discovery beliefs than an individual's personal beliefs. I see this confirmation and extension of belief trajectories as helpful, though additionally suggest a re-interpretation of the meaning of a belief trajectory is necessary according to McGowan's (2019) understanding of contradiction.

8.5.2 Implications for mathematics Dialectic Professional Development (DPD)

My concern with the idea of a belief trajectory is that it places emphasis on moving from belief A to belief B. Drawing on McGowan (2019), I view this perspective as placing emphasis on difference, and so potentially obfuscating the existence of contradiction. Emphasising difference could suggest it is possible to move from one belief about mathematics education to a different noncontradictory belief about mathematics education. According to McGowan (2019, p. 118), the idea of difference would be 'utterly false' since there are no new noncontradictory positions. Instead, I see a belief trajectory as revealing the existence of contradiction in a teacher's dominant belief about mathematics education. I argue a belief trajectory does not indicate the resolution of contradiction, but the journey of reconciliation to contradiction. For example, I interpret Drew's belief trajectory from

Discovery to Connectionist as indicating Drew is in the process of becoming reconciled to contradiction in Discovery beliefs and may move to examine contradiction in Connectionist beliefs next.

This perspective is important for my understanding of mathematics DPD. Firstly, I recognise participants do not view a programme as supporting a single belief about mathematics education, but interdependent beliefs. In addition, I recognise participants have different views of the strength of the interdependent beliefs about mathematics education. These differences are linked to personal beliefs about mathematics education a participant has and may be more important than those a designer wishes to promote. In mathematics DPD, participant should be understood as moving on a journey of being reconciled to the contradictions in their dominant belief.

8.6 Summary of Chapter 8

In this chapter I have provided results and discussion to support my third theme of belief trajectories show journeys not destinations. Developed in response to my research question I firstly presented results around the views of the Drew, Jo, and Ronnie on the beliefs about mathematics education seen in the RAM elements. These results were taken from belief weightings provided at the end of the three main RAM sessions, and from interview comments discussing the weightings. I argue, rather than seeing one belief category the participants recognised the interdependence of all three within the RAM programme. Whilst

there were points of similarity between the teachers, there was variation at the domain level, and variation in the significance of experiences.

Following the recognition of variation between teachers, I presented relative data comparing personal beliefs about mathematics education with views of the RAM programme. In summary, I suggest the participants saw less of their dominant belief about mathematics education and more of a non-dominant belief. By taking this perspective, I then suggested the potential existence of a new belief trajectory from Connectionist to Discovery. However, and more importantly, I also reflected on the importance of a different understanding of a belief trajectory. In line with my contradiction informed perspective, rather than focusing on destination, I suggest a belief trajectory reveals the journey of reconciliation to contradiction.

Chapter 9.

Conclusion

9.1 Synopsis of Chapter 9

In the previous three chapter I presented my analysis of three themes identified in response to my research question. In this final chapter I summarise the themes, make recommendations, and reflect on the contradiction informed perspective that has led to my interpretations. For the Rethinking Approaches in Mathematics (RAM) programme, I suggest the development of opportunities for reflection around moments of variation should be considered in a longitudinal case study. For the more general case of mathematics Dialectic Professional Development (DPD), I commend it as an opportunity to contribute to the formation of teacher beliefs about mathematics education based on an understanding of contradiction. Consequently, I call for education policy makers to distinguish between teacher professional development in the form of student programmes and those in the form of teacher programmes. I suggest this distinction should lead to the allocation of time and money to the development of mathematics teachers through teacher programmes such as those based on mathematics DPD. In doing this an emphasis should be placed on teacher experimentation, beliefs, and reflection grounded in an understanding of contradiction. I suggest this distinction would provide clarity of purpose around the government of the United Kingdom's current tacit support of a mathematics mastery market.

9.2 Recommendations for the RAM programme

I undertook this study based on my desire to gain insight into teachers' beliefs about mathematics education and their actions in a professional development programme. In Chapter 1, I introduced my intention to base this insight on McGowan's (2019) interpretation of Hegelian contradiction. From here I introduced my research question as: *how can mathematics teachers' experiences of a Dialectic Professional Development programme be interpreted through the lens of contradiction?* To facilitate answering this question, I examined conceptions of teacher beliefs in Chapter 2 and introduced a model of beliefs about mathematics education suggested by Swan (2006a, 2006b). In Chapter 3, using a professional development framework (Boylan, Coldwell *et al.* 2018), I discussed Swan's design of a programme and suggested an emerging specification of mathematics DPD characterised by Hegelian contradiction. I exemplified this specification in Chapter 4 in the form of the RAM programme, based on adjusted from of lesson study. In Chapter 5, I introduced the case study methodology I adopted to answer my research question around the RAM programme as experienced by mathematics teachers in a secondary school in the East Midlands of England. I also detailed my approach to data, my original use of triangular plots, and explained the process of Reflexive Thematic Analysis I have adopted. This process led to the analysis of Chapters 6, 7 and 8 where I suggested contradiction can be used to understand the RAM programme through the experiences of three secondary teachers. Therefore, I suggested three themes of beliefs are interdependent, moments of variation are acts of self-sabotage, and belief trajectories show journeys not destinations.

9.2.1 Theme 1: beliefs as interdependent

In my first theme, I suggested contradiction can provide an interpretation of teachers' beliefs about mathematics education in the RAM programme as interdependent. I noted how teacher-provided belief weightings, alongside interview comments, can support the interpretation that teachers have a dominant belief. However, through a lens of contradiction I additionally argued mutual dependence of dominant and non-dominant beliefs can be seen in the same data. I interpreted the data to suggest teachers did not view themselves as having a single dominant belief category, nor as switching between belief categories, but as simultaneously holding interdependent beliefs. Therefore, a set of Transmission, Discovery and Connectionist weightings can all be used in combination to describe a teacher's beliefs about mathematics education. For example, a teacher with dominant Transmission beliefs is understood as also having interdependent Discovery and Connectionist beliefs at the same time. Holding this perspective provides a foundational understanding for the experience of teachers in the RAM programme.

9.2.2 Theme 2: moments of variation are acts of self-sabotage

In my second theme, I suggested contradiction can support an interpretation of some teacher actions as unconscious acts of self-sabotage. I identified these acts as taking place in moments of variation where teachers' unconscious actions conflict with their conscious desires. For example, in RAM session 1, I interpreted Jo speaking to Ronnie during the individual problem-solving phase as the unconscious act of self-sabotage. Similarly, I interpreted Drew reformatting the research lesson into the Task Ladder lesson structure as an

unconscious act of self-sabotage. In a similar fashion, I explained Ronnie's decision not to look at student methods during the live research lesson as an unconscious act of self-sabotage. Therefore, I suggest contradiction in the form of the divided self (McGowan 2019), provides a reasonable interpretation of some teacher actions in the RAM programme.

9.2.3 Theme 3: belief trajectories reveal journeys not destinations

In my third theme I suggested contradiction can be used to understand teachers' belief trajectories as journeys rather than destinations. I firstly identified how participants viewed the RAM programme as simultaneously supporting interdependent beliefs about mathematics education. From this, I suggested the existence of relative relationships between participants' personal beliefs about mathematics education and their views of the beliefs underpinning the RAM programme. I interpreted data as indicating participants saw less of their dominant belief, and more of their non-dominant beliefs, in the RAM programme. For example, Drew viewed the RAM programme as supporting less Discovery and more Connectionist, Jo saw less Transmission and more Connectionist, Ronnie saw less Connectionist and more Discovery. I suggested the first two of these relationships provide a reason Swan saw teachers moving toward Connectionist beliefs in his professional development programme. Since two teachers viewed the RAM programme as reflecting more Connectionist beliefs compared to their personal beliefs, this was their trajectory. However, the third relationship additionally suggests the potential existence of a belief trajectory from Connectionist to Discovery. I claim a relative belief of less Connectionist and more Discovery provides a plausible account for the

existence of this new trajectory. Hence, I suggest belief trajectories reveal participants' views of the RAM programme are strongly influenced by their personal beliefs.

9.2.4 Future development of the RAM programme

Reflecting on the RAM programme, there are some areas I suggest could be further developed. Firstly, as noted in Chapter 2, I am conscious Swan's (2006b) descriptions of beliefs about mathematics incorporated elements of the descriptions of beliefs about teaching mathematics and beliefs about learning mathematics. Returning to descriptions more in line with those suggested by Ernest (1989a), and shown in Table 2.1, may provide a helpful distinction for teachers. This adjustment would provide clarity for teachers when reflecting on their beliefs about mathematics.

Secondly, whilst reflections on beliefs about mathematics education were integrated into the RAM programme, there may be an opportunity for further development of reflection around moments of variation. In this study I suggested McGowan's (2019) theory of the divided self provides a reasonable interpretation of some teacher actions. However, slightly naively, in my initial design of the RAM programme I assumed teachers would simply spot contradiction and realise implications for themselves. The challenge for the RAM programme would be to design clearer opportunities for participants to reflect on their moments of variation, and what these moments may reveal according to an understanding of the divided self. Practically, structuring reflections around the planned adjustments to the research lesson plan, and unplanned adjustments to the research lesson, may be simpler than reflecting on reactions to opportunities for challenge in the programme.

Thirdly, I also suggest there would be benefits of longitudinal research into teachers' experiences of the RAM programme. My study took place over one academic year, with teachers who had already experienced the current system of professional development over many years. I would like to gain further insight into teacher's longer-term experiences and how this relates to the feelings of apathy, confusion, and anger (Woodford, Clapham and Serret 2023) that I suggest teachers may experience in current forms of professional development. The practicalities of studying these long-term effects are not simple, but a case study asking teachers about their views in these areas, in the context of the RAM programme, would be beneficial.

9.3 Recommendation for mathematics DPD

Having applied contradiction to interpret the experiences of participants in the RAM programme, I also consider what insights each of these themes provides for the more general approach of mathematics DPD.

9.3.1 Implications of theme 1 for mathematics DPD

In Chapter 6, I additionally suggested the examples of the three teachers in this study could be used to support the claim that the theoretical belief categories are contradictory. As McGowan (2019, p. 100) argues, 'if we try to keep a concept pure from its negation, we lose the concept itself'. Therefore, I suggest each category of beliefs about mathematics education is 'also what it is not and has its identity in what negates it' (McGowan 2019, p. 85). Hence, I suggest the theoretical categories of Transmission, Discovery, and Connectionist may be

understood as contradictory through their interdependence. This understanding of beliefs about mathematics education has implications for my thinking around mathematics DPD. I see Swan's original use of the model as facilitating the measurement of the development of beliefs. However, my interpretation of the model would facilitate seeing interdependence of beliefs. By providing opportunities for teachers to see interdependence of beliefs, the risks I associate with traditional professional development programmes of teacher apathy, confusion, and anger (Woodford, Clapham and Serret 2023) may be reduced. Hence, if incorporated into mathematics DPD, my interpretation of Swan's belief model could be used to contribute to the formation of teachers' beliefs about mathematics education through an understanding of contradiction.

9.3.2 Implications of theme 2 for mathematics DPD

In Chapter 7, I additionally suggested contradiction can support a general inference on the source of teachers' satisfaction in professional development programmes. Returning to McGowan (2019), I suggested teachers find satisfaction from the experience of professional development, rather than from attaining the object of the programme. In this study, I interpreted teachers as unconsciously deriving satisfaction from the experience of studying Teaching Through Problem-solving (Takahashi 2021), rather than from attaining the approach. Acts of self-sabotage are understood as the unconscious seeking to sustain the conditions for contradiction rather than resolve it. The implication for mathematics DPD of this perspective is to design opportunities for moments of variation for participants to reflect on as part of the programme. In this way teachers' actions, seen in these moments of variation,

become transformed as holding significance. Rather than simply determine their response to mathematics DPD, a participant should be helped to reflect on what that response reveals. Mathematics DPD should not focus on sameness and fidelity, but on facilitating learning through reflection around moments of variation.

9.3.3 Implications of theme 3 for mathematics DPD

In Chapter 8 I suggested contradiction places an emphasis on the journey, rather than the destination, of a mathematics belief trajectory. Swan focused on reporting the destination of the development of Connectionist beliefs using the model he created. As such, development of Connectionist beliefs could be interpreted as the resolution of contradiction. However, in line with McGowan (2019), I argue there are no noncontradictory positions, and I interpret all belief categories as being interdependent. Therefore, I see a relative belief weighting as revealing the contradiction participants are being confronted with and providing an indication of the next belief position they may move to examine. I argue a belief trajectory reveals the journey of reconciliation with contradiction rather than the destination of the resolution of contradiction. The implication for mathematics DPD is to suggest teachers create unconscious mathematics belief trajectories based on their personal beliefs, rather than on any intended beliefs in the programme.

9.3.4 Developing the framework specification for mathematics DPD

In Table 9.1, I summarise my recommendations around mathematics DPD according to the framework suggested by Boylan, Coldwell *et al.* (2018).

Framework element	Summary for mathematics DPD
Philosophical foundations	McGowan's (2019) explanation of Hegelian dialectics with contradiction understood as an ontological feature.
Purpose	The formation of participants' beliefs about mathematics education.
Components	A linear path model from the mathematics DPD programme to teacher experimentation to teacher reflection to the formation of teacher beliefs.
Scope	An adjusted form of a lesson study cycle which include the stages of examination of participants existing beliefs about mathematics education, examination of mathematics, examination of teaching mathematics, teaching the research lesson, and examination of learning mathematics. Throughout the programme, opportunities for participant reflection on: • beliefs about mathematics, • beliefs about teaching mathematics, • beliefs about learning mathematics, • what potential moments of variation may reveal.
Theory of learning	The formation of beliefs takes place through the journey of reconciliation to contradiction, rather than in attaining the examined approach.
Agents of change	Participants understood as agentic, with an acknowledgement of the divided self (2019) in the form of the unconscious and conscious. Potential moments of variation should be included in the design, and opportunities for reflection built around these.

Table 9.1 A summary of recommendations for mathematics DPD

9.4 Further recommendations

Having considered recommendations within the RAM programme, and reflected on implications for mathematics DPD, I now suggest wider recommendations resulting from my study. These recommendations are structured toward policy makers and system leaders, and toward researchers of teacher beliefs.

9.4.1 Recommendations for policy makers and system leaders

My first recommendation to policy makers and system leaders is to acknowledge the distinction between professional development which seeks to change student outcomes, and professional development which seeks to support teacher change. For simplicity, I will refer to the former as a student programme and the latter as a teacher programme. A student programme would be characterised by a sequenced set of lesson resources, would be focused on student outcomes, and may be suitable for evaluation through a randomized controlled trial. In addition, I suggest there are risks associated with a student programme of causing apathy, confusion, and anger in teachers (Woodford, Clapham and Serret 2023). In contrast, a teacher programme would be characterized by experimentation and reflection, with a focus on teacher development. Whilst a student programme is predicated on an ‘emphasis on sameness’ (Strom and Viesca 2021, p. 221), I suggest a teacher programme should be predicated on an emphasis on variation through contradiction. Therefore, I commend mathematics DPD, with its consideration of beliefs about mathematics education and learning from moments of variation, as potentially reducing the risks I associate with student programmes.

I recommend education policy, around teacher professional development, allocates time for mathematics teachers to participate in teacher programmes which contribute to the professionalisation of teaching (Stigler and Hiebert 2009). For example, mathematics DPD, as an adjusted form of lesson study would provide opportunities for the formation of teachers' beliefs, rather than dictating the adoption of a single approach. Indeed, Ofsted (2025), the government agency that inspects schools in England, have proposed distinguishing between schools who use professional development to 'implement' (Ofsted 2025, p. 6), and schools who seek to create a professional learning culture. This language reflects a similar recognition to the distinction I make between student programmes and teacher programmes. Therefore. I argue education policy in England should follow an intentional path of allocating time for mathematics teachers to participate in teacher programmes that seek to create an 'effective professional learning culture, in which staff take responsibility for their professional learning' (Ofsted 2025, p. 6).

One immediate application of this policy in England would be in the arena of the government created mathematics mastery market (Boylan and Adams 2024). I suggest actors in the mathematics mastery market should be distinguished based on whether they offer a student programme or a teacher programme. I suggest actors such as the Ark Mathematics Mastery programme and White Rose Maths provide examples of well-resourced student programmes which may be suitable for evaluation through randomized controlled trials. I therefore suggest a government funded organisation, such as the National Centre for Excellence in the Teaching of Mathematics (NCTEM), has a responsibility to provide teacher

programmes. The nature of these teacher programme would be to support teachers in examining the approaches offered by student programmes, rather than supporting a particular approach. Rather than emphasise the five big ideas of Teaching for Mastery, policy should shift toward supporting teachers to evaluate mastery approaches and understand their responses to these approaches. Mathematics DPD offers one way in which organisations such as the NCETM could examine these different pedagogical approaches to support the formation of beliefs. Most importantly, mathematics DPD would recognise the interdependence of teachers' beliefs, the interdependence of beliefs within the different approaches, and attribute significance to moments of variation.

At the local system level, I have seen an increase in school (and multi academy trust) specified approaches to how teachers should operate in a classroom. This approach is typified by the Task Ladder lesson structure in this study, and I have similarly seen other school leadership teams imposing structures based on behaviour techniques from Teach Like a Champion (Lemov 2021). These lesson structures do not provide the lesson resources of a student programme, but nor do they focus on reflection of a teacher programme. Instead, it appears to me they seek to change teacher beliefs and approaches through what Green (1998) described as indoctrination rather than instruction. As seen in this study, policy recommendations such as the Task Ladder lesson structure can enforce change on teachers. However, teachers should have opportunities to locate themselves, and their response, in relation to any new policy. Therefore, I recommend system leaders return agency and responsibility to mathematics teachers for consideration of the impact of lesson structures on

their pedagogy. Just as the Task Ladder lesson structure in this study revealed the interdependence of beliefs, so too could other classroom approaches be used to support teacher reflection through an understanding of contradiction.

9.4.2 Recommendations for researchers of teacher beliefs

Whilst my study has focused on beliefs about mathematics education, I suggest my work could be extended to teacher beliefs more generally. Traditionally in this area, mutually exclusive statements of belief have acted as statements of predication in assigning attributes to a teacher (Green 1998). Philosophically, Green (1998) argues these statements of predication are synthetic since they are formed through experience rather than logic. This makes possible the view that beliefs can be changed through providing experiences such as those based on the Task Ladder lesson structure, or classroom behaviour management techniques. In contrast, McGowan (2019, p. 223) suggests all statements of predication, including statements of belief should be considered as synthetic based on their ‘involvement with otherness’.

It is this involvement with otherness which provides a different basis for research about teacher beliefs. As already discussed, ascribing only positive statements of beliefs to teachers provides only a partial understanding. Every statement of a teacher’s beliefs can be improved by also identifying ‘the specific way that it fails to be what constitutes it’ (McGowan 2019, p. 27). Therefore, I recommend work on teacher beliefs, from a contradiction informed perspective, should include attempts to identify the negation. In a researcher defined model this might include the use of dominant and non-dominant beliefs. In a teacher defined model, this might include noticing what a teacher claims they are and what

they are not. Teacher beliefs should be viewed through the lens of contradiction as interdependent, as never being stable, and constantly in the process of becoming that which they are not.

9.5 Reflecting on the influence of my philosophical perspective

As part of the process of Reflexive Thematic Analysis, Braun and Clarke (2021b) encourage reflection on the influences that have led to the suggested interpretation. Whilst acknowledging a wide variety of influences, I do not wish to reduce my reflexivity to a self-indulgent ‘feeble confession’ (Parker 2005, p. 28). Instead, I recognise the most important influence on my work has been the adoption of a philosophical perspective aligning to McGowan’s (2019) interpretation of Hegelian contradiction. I review the influence of contradiction on my design of the RAM programme, the data I have prioritised, and the themes I have interpreted. This in turn leads to a brief discussion of the applicability of my work to a different philosophical context.

9.5.1 The influence of my philosophical perspective on my design of professional development

In Chapter 1, I suggested a contradiction informed understanding of teacher professional development would have to look different from current (dualist) approaches. This led to an initial, contradiction informed specification of mathematics DPD in Chapter 3, and an explanation of the RAM programme in Chapter 4. I approached the design of the RAM programme to meet the purpose of the formation of teacher beliefs through an understanding

of contradiction. Furthermore, the design work was influenced by a theory of learning seeking reconciliation to, rather than resolution of, contradiction. Therefore, I designed the RAM programme with the intention of providing an approach to mathematics education for teachers to study rather than suggesting a solution. This subsequently influenced my use of an adjusted form of lesson study, and my use of a Japanese lesson as the central resource for the lesson study cycle.

9.5.2 The influence of my philosophical perspective on data collection

Whilst specifying mathematics DPD in Chapter 3, I also introduced the idea of moments of variation stemming from my reading of McGowan's (2019) work. I suggested teachers should be considered the main agent of change in mathematics DPD, but that their unconscious actions may be interpreted as undermining their stated desires. Therefore, as described in Chapter 4, I designed the RAM programme to include potential moments of variation around which I could collect data. Similarly, my inclusion of Swan's (2006a, 2006b) belief questionnaire in the RAM materials, and an adjusted questionnaire to establish participants views of the RAM elements, were also informed by my contradiction perspective. Consequently, the interview schedules I designed were based on gaining additional insight into the belief weightings, and additional insight into teachers' experiences of the sessions.

9.5.3 The influence of my theoretical perspective on the development of themes

My contradiction informed approach also influenced the process of Reflexive Thematic Analysis which I undertook. In utilising a deductive style, I coded data conscious of

my contradiction informed interpretations of beliefs about mathematics education, moments of variation, and the nature of professional development. Whilst recognising suggestions that Reflexive Thematic Analysis can be driven by a more inductive orientation (Braun and Clarke 2021b), I take the view theory will always ‘be present, leading the researcher’s gaze’ (Malterud 2016, p. 121). I concur with Braun and Clarke (2021b, p. 210), who similarly encourage a researcher to acknowledge ‘the conceptual ideas we (always) come to data and a project with’. Therefore, in employing a deductive approach to Reflexive Thematic Analysis I did not intend to prove a theory, but to provide a ‘theoretical *exploration* of qualitative data’ (Braun and Clarke 2021b, p. 210). I suggest my themes and analysis provide an exploration of understanding teachers’ experiences of the RAM programme according to the theory of contradiction.

9.5.4 Applicability of my interpretations in a different philosophical context

The importance of my philosophical perspective to this study raises a question of the applicability of my interpretation in other philosophical contexts. I firstly acknowledge my preference for challenging the status quo through designing mathematics DPD around Hegelian contradiction. If a fundamental dualist perspective is never critiqued, then the scope for change will always be limited. I have sought to take seriously McGowan’s (2019) claim that contradiction is an ontological feature and offer an interpretation of ‘what others have not yet seen’ (Stake 1995, p. 136). Secondly, I suggest a dialectic approach to learning can operate within a dualist dominated context. McGowan (2019, p. 34) recognises a similar principle operates within the realm of philosophy when stating we must hold to ‘the principle of

noncontradiction in order to show that it ultimately does not hold'. The noncontradictory claims of a dualist approach to professional development should be examined and the contradictions revealed. In terms of mathematics DPD, I do not seek to manufacture false contradictions, but to encourage participants to identify the contradictions within beliefs about mathematics education that already exist. Mathematics DPD is intended to operate in a dualist context and lead to the formation of beliefs through an understanding of contradiction.

9.6 Concluding thoughts

To conclude, I return to some of the wider thoughts around education introduced in Chapter 1 but applied to teacher professional development. Biesta (2020) distinguished between education founded in a paradigm of cultivation and education founded in a paradigm of experience. In line with Biesta (2020), I see attempts to cultivate 'capacities and capabilities' (Biesta 2020, p. 1015) in teachers through new experience as insufficient. Whilst Biesta (2020) identifies this perspective fails to account for locating the self, through a lens of contradiction I additionally suggest it creates unnecessary division. By failing to emphasise contradiction, a cultivation approach to teacher professional development risks creating 'an external enemy to fight and a coherent sense of identity for oneself' (McGowan 2019, p. 13). The creation of opponents allows anger to be directed toward others, rather than examine the contradiction 'stemming from our own failure' (McGowan 2019, p. 12). Therefore, I suggest mathematics DPD is essential because it seeks to return the focus to contradiction through the conception of beliefs about mathematics education as interdependent. In doing this, mathematics DPD seeks to avoid creating enemies, and asks teachers to consider their

experience as ‘one experience of contradiction’ (McGowan 2019, p. 122) rather than as an alternative experience of noncontradictory difference.

In line with Biesta (2020), I therefore suggest the conception of teacher professional development should be supplemented with an approach based in the paradigm of existence. I believe teacher professional development should be ‘concerned with the question of ‘how the ‘I’ exists as ‘I’ (Biesta 2020, p. 1015). Mathematics DPD provides individual teachers with an opportunity ‘not to forget this question’ (Biesta 2020, p. 1020). Within mathematics DPD, participants have opportunities for a personal response to the approach being examined, and opportunities to say no to the approach being examined. The formation of individuals’ beliefs about mathematics education should take place through experimentation and reflection. In addition, through an awareness of contradiction, the opportunity to say no becomes transformed by a consideration of what the act of saying no reveals. As McGowan (2019, p. 43) suggests, ‘what one thinks or claims about oneself falls aside in the face of what one does’. Locating how the I exists as I in teacher professional development is vital, but a consideration of what this ‘I’ reveals about the self may be transformative. In mathematics DPD, reflection on moments of variation has the potential to confront teachers with considering what their reactions and adjustments may reveal about the self.

Conception of teacher professional development matters since it shapes the future of an education system. I believe there is a need to provide ways to support teachers to navigate the education context that feels increasingly dominated by the polarised competing factions of

traditional and progressive. Mathematics DPD, founded in contradiction, offers an approach that seeks more than the development of new capacities and capabilities. By conceiving of beliefs as contradictory the risk of creating opponents rather than focusing on self are mitigated. Furthermore, understanding contradiction in moments of variation can support participants to consider how they relate to the approach being examined. Mathematics DPD does not seek the resolution of systemic contradiction but offers development for teachers through a path to reconciliation with contradiction.

References.

- Aguirre, J., and Speer, N., 1999. Examining the relationship between beliefs and goals in teacher practice. *The Journal of Mathematical Behavior*, 18 (3), 327–356.
- Archibald, M., Ambagtsheer, R., Casey, M. and Lawless, M., 2019. Using zoom video conferencing for qualitative data collection: perceptions and experiences of researchers and participants. *International Journal of Qualitative Methods*, 18, 1–8.
- Aristotle, 2001. *Metaphysics*. Translated by W. Ross. Blacksburg, VA: Virginia Tech.
- Askew, M., Brown, M., Rhodes, V., Johnson, D. and Wiliam, D., 1997. *Effective teachers of numeracy*. London: King's College.
- Asterhan, C., and Lefstein, A., 2024. The search for evidence-based features of effective teacher professional development: a critical analysis of the literature. *Professional Development in Education*, 50 (1), 11–23.
- Baldry, F., Mann, J., Horsman, R., Koiwa, D. and Foster, C., 2023. The use of carefully planned board work to support the productive discussion of multiple student responses in a Japanese problem-solving lesson. *Journal of Mathematics Teacher Education*, 26, 129–153.
- Ball, D., Thames, M. and Phelps, G., 2008. Content knowledge for teaching: what makes it special. *Journal of Teacher Education*, 59 (5), 389–407.
- Beeli-Zimmermann, S., 2019. Extending belief research to adult basic education: an exploration of some adult educators' beliefs about numeracy and its teaching. *Zeitschrift Für Weiterbildungsforschung*, 42 (3), 357–377.
- Berger, R., 2015. Now I see it, now I don't: researcher's position and reflexivity in qualitative research. *Qualitative Research*, 15 (2), 219–234.
- Beswick, K., 2012. Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics*, 79, 127–147.
- Beswick, K., 2011. Knowledge/beliefs and their relationship to emotion. In: *Proceedings of the MAVI-16 conference, 26-29 June 2010*. Tallinn, Estonia: Institute of Mathematics and Natural Sciences, pp. 43–59.

Biesta, G., 2020. Can the prevailing description of educational reality be considered complete? On the Parks-Eichmann paradox, spooky action at a distance and a missing dimension in the theory of education. *Policy Futures in Education*, 18 (8), 1011–1025.

Boaler, J., and Sarah, K., 2017. Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education*, 48 (1), 78–105.

Bourdieu, P., and Ferguson, P., 1999. *The weight of the world: social suffering in contemporary society*. Cambridge: Polity Press.

Boyd, P., and Ash, A., 2018. Mastery mathematics: changing teacher beliefs around in-class grouping and mindset. *Teaching and Teacher Education*, 75, 214–223.

Boylan, M., 2021, Mathematics education in translation. In: G. Ineson, and H. Povey, eds., *Debates in mathematics education*. London: Routledge, 2021, pp. 13–24.

Boylan, M., and Adams, G., 2024. Market mirages and the state's role in professional learning: the case of English mathematics education. *Journal of Education Policy*, 39 (2), 253–275.

Boylan, M., Coldwell, M., Maxwell, B. and Jordan, J., 2018. Rethinking models of professional learning as tools: a conceptual analysis to inform research and practice. *Professional Development in Education*, 44 (1), 120–139.

Boylan, M., and Demack, S., 2018. Innovation, evaluation design and typologies of professional learning. *Educational Research*, 60 (3), 336–356.

Boylan, M., Maxwell, B., Wolstenholme, C., Jay, T. and Demack, S., 2018. The mathematics teacher exchange and 'mastery' in England: the evidence for the efficacy of component practices. *Education Sciences*, 8 (4), 1–31.

Boylan, M., Wolstenholme, C., Maxwell, B., Demack, S., Jay, T., Reaney, S. and Adams, G., 2019. *Longitudinal evaluation of the mathematics teacher exchange: China-England final report*. London: Department for Education.

Braun, V., and Clarke, V., 2021a, Thematic Analysis. In: E. Lyons, and A. Coyle, eds., *Analysing qualitative data in psychology*. London: Sage, 2021a, pp. 128–147.

Braun, V., and Clarke, V., 2021b. *Thematic analysis: a practical guide*. London: Sage.

Brinkmann, S., and Kvale, S., 2015. *Interviews: learning the craft of qualitative research interviewing*. 3rd ed. London: Sage.

British Educational Research Association [BERA], 2024. *Ethical guidelines for educational research*. 5th ed. London: British Educational Research Association.

British Educational Research Association [BERA], 2018. *Ethical guidelines for educational research*. 4th ed. London: British Educational Research Association.

Brown, N., 2022. Scope and continuum of participatory research. *International Journal of Research & Method in Education*, 45 (2), 200–211.

Burnett, C., and Coldwell, M., 2021. Randomised controlled trials and the interventionisation of education. *Oxford Review of Education*, 47 (4), 423–438.

Byrne, D., 2022. A worked example of Braun and Clarke’s approach to reflexive thematic analysis. *Quality & Quantity: International Journal of Methodology*, 56 (3), 1391–1412.

Calleja, J., 2022. Changes in mathematics teachers’ self-reported beliefs and practices over the course of a blended continuing professional development programme. *Mathematics Education Research Journal*, 34, 835–861.

Campbell, S., Greenwood, M., Prior, S., Shearer, T., Walkem, K., Young, S., Bywaters, D. and Walker, K., 2020. Purposive sampling: complex or simple? Research case examples. *Journal of Research in Nursing*, 25 (8), 652–661.

Carrillo, J., 2020, Parallel stories: teachers and researchers searching for mathematics teachers’ specialized knowledge. In: S. Zehetmeier, D. Potari and M. Ribeiro, eds., *Professional development and knowledge of mathematics teachers*. Abingdon: Routledge, 2020, pp. 85–103.

Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P. and Aguilar-González, Á, 2018. The mathematics teacher’s specialised knowledge (MTSK) model. *Research in Mathematics Education*, 20 (3), 236–253.

Chick, H., and Beswick, K., 2018. Teaching teachers to teach Boris: a framework for mathematics teacher educator pedagogical content knowledge. *Journal of Mathematics Teacher Education*, 21 (5), 475–499.

Clapham, A., 2024. Examining Teaching for Mastery as an instance of ‘hyperreal’ cross national policy borrowing. *Oxford Review of Education*, 50 (3), 366–383.

Coe, R., 2021, Inference and interpretation in research. In: R. Coe, ed., *Research methods and methodologies in education*. 4th ed. Washington, MA: Sage, 2021, pp. 43–55.

Cohen, D., 1990. A revolution in one classroom: the case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, 12 (3), 311–329.

Cohen, L., Manion, L. and Morrison, K., 2018. *Research methods in education*. 8th ed. London: Routledge.

Cordingley, P., Higgins, S., Greany, T., Crisp, B., Araviaki, E., Coe, R. and Johns, P., 2020. *Developing Great Leadership of CPDL*. Penrith: Centre for the Use of Research and Evidence in Education.

Cross Francis, D., 2015. Dispelling the notion of inconsistencies in teachers' mathematics beliefs and practices: a 3-year case study. *Journal of Mathematics Teacher Education*, 18 (2), 173–201.

Czerniawski, G., MacPhail, A., Vanassche, E., Ulvik, M., Guberman, A., Oolbekkink-Marchand, H. and Bain, Y., 2021, Researching the professional learning needs of teacher educators: results from international research. In: R. Vanderlinde, K. Smith, J. Murray and M. Lunenberg, eds., *Teacher educators and their professional development*. Abingdon: Routledge, 2021, pp. 28–42.

Darling-Hammond, L., Hyler, M. and Gardner, M., 2017. *Effective teacher professional development*. Palo Alto, CA: Learning Policy Institute.

Deakin, H., and Wakefield, K., 2014. Skype interviewing: reflections of two PhD researchers. *Qualitative Research*, 14 (5), 603–616.

DeBellis, V., and Goldin, G., 1999. Aspects of affect: mathematical intimacy, mathematical integrity. In: *Proceedings of the 23rd conference of the IGPME, 25-30 July 1999*. Haifa, Israel: Israel Institute of Technology, pp. 249–256.

Denzin, N., 2017. *The research act: a theoretical introduction to sociological methods*. Abingdon: Routledge.

Denzin, N., and Lincoln, Y., 2018. *The Sage handbook of qualitative research*. Los Angeles, CA: Sage.

Desimone, L., 2009. Improving impact studies of teachers' professional development: toward better conceptualizations and measures. *Educational Researcher*, 38 (3), 181–199.

Dickinson, P., Eade, F., Gough, S., Hough, S. and Solomon, Y., 2020, Intervening with Realistic Mathematics Education in England and the Cayman Islands - the challenge of clashing educational ideologies. In: M. van den Heuvel-Panhuizen, ed., *International*

reflections on the Netherlands didactics of mathematics : visions on and experiences with Realistic Mathematics Education. Cham, Switzerland: Springer, 2020, pp. 341–366.

Education Endowment Foundation [EEF], 2019. *Maths-for-Life* [online]. Education Endowment Foundation. Available at: https://educationendowmentfoundation.org.uk/projects-and-evaluation/projects/maths-for-life?utm_source=/projects-and-evaluation/projects/maths-for-life&utm_medium=search&utm_campaign=site_search&search_term=Maths-for [Accessed 06 October 2020].

Eichler, A., and Erens, R., 2015. *Domain-specific belief systems of secondary mathematics teachers*. Cham, Switzerland: Springer.

Eichler, A., Erens, R. and Törner, G., 2023. Measuring changes in mathematics teachers' belief systems. *International Journal of Mathematical Education in Science and Technology*, 54 (8), 1490–1508.

Ellis, V., Gatti, L. and Mansell, W., 2024. *The new political economy of teacher education*. Bristol: Bristol University Press.

Engley, R., 2023. The limitation of the bottle episode: Hegel in community. *New Review of Film and Television Studies*, 21 (4), 743–765.

Ernest, P., 2021, The dark side of mathematics. Damaging effects of the overvaluation of mathematics. In: G. Ineson, and H. Povey, eds., *Debates in Mathematics Education*. 2nd ed. Routledge, 2021, pp. 29–42.

Ernest, P., 1989a, The Impact of beliefs on the teaching of mathematics. In: P. Ernest, ed., *Mathematics teaching: the state of the art*. London: Falmer Press, 1989a, pp. 249–254.

Ernest, P., 1989b. The knowledge, beliefs and attitudes of the mathematics teacher: a model. *Journal of Education for Teaching*, 15 (1), 13–33.

European Union, 2016. *Regulation (EU) 2016/679* [online]. EUR-Lex. Available at: <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32016R0679> [Accessed 04 January 2025].

Evans, L., 2014. Leadership for professional development and learning: enhancing our understanding of how teachers develop. *Cambridge Journal of Education*, 44 (2), 179–198.

Fenstermacher, G.D., 1978. A philosophical consideration of recent research on teacher effectiveness. *Review of Research in Education*, 6 (1), 157–185.

- Filges, T., Torgerson, C., Gascoine, L. and Dietrichson, J., 2019. Effectiveness of continuing professional development training of welfare professionals on outcomes for children and young people: a systematic review. *Campbell Systematic Reviews.*, 15 (4), 1–31.
- Finlay, L., 2021. Thematic analysis: the ‘good’, the ‘bad’ and the ‘ugly’. *European Journal for Qualitative Research in Psychotherapy*, 11, 103–116.
- Finlay, L., 2002. “Outing” the researcher: the provenance, process, and practice of reflexivity. *Qualitative Health Research*, 12 (4), 531–545.
- Fletcher-Wood, H., and Zuccollo, J., 2020. *The effects of high-quality professional development on teachers and students: a rapid review and meta-analysis*. London: Education Policy Institute.
- Foster, C., 2023. Problem solving in the mathematics curriculum: from domain-general strategies to domain-specific tactics. *The Curriculum Journal*, 34 (4), 594–612.
- Foster, C., 2022. Implementing confidence assessment in low-stakes, formative mathematics assessments. *International Journal of Science and Mathematics Education*, 20 (7), 1411–1429.
- Foster, C., 2013. Resisting reductionism in mathematics pedagogy. *The Curriculum Journal*, 24 (4), 563–585.
- Francome, T., and Hewitt, D., 2020. “My math lessons are all about learning from your mistakes”: how mixed-attainment mathematics grouping affects the way students experience mathematics. *Educational Review*, 72 (4), 475–494.
- Freeman, M., DeMarrais, K., Preissle, J., Roulston, K. and St. Pierre, E., 2007. Standards of evidence in qualitative research: an incitement to discourse. *Educational Researcher*, 36 (1), 25–32.
- Fujii, T., Matano, H., Amano, H., Ichikawa, S., Ohta, S., Ohtake, K., Ohtani, M., Okada, H., Ginnan, Y., Kudo, M., Kodera, T., Sakaori, F., Sasa, H., Shimizu, Y., Seino, T., Soeda, Y., Takahashi, H. and Takeuchi, A., 2012. *New Mathematics 2*. Translated by A. Takahashi, T. Watanabe and M. Yoshida. Tokyo, Japan: Tokyo Shoseki.
- Furinghetti, F., 1996. A theoretical framework for teachers' conceptions. In: *Proceedings of the MAVI-3 workshop, 23-26 August 1996*. Helsinki, Finland: ERIC, pp. 19–25.
- Furinghetti, F., and Pehkonen, E., 2002, Rethinking characterizations of beliefs. In: *Rethinking characterizations of beliefs. Beliefs: a hidden variable in mathematics education?* Springer, 2002, pp. 39–57.

- Gardner, K., Glassmeyer, D. and Worthy, R., 2019. Impacts of STEM professional development on teachers' knowledge, self-efficacy, and practice. *Frontiers in Education*, 4, 1–10.
- Garner, J.K., and Kaplan, A., 2019. A complex dynamic systems perspective on teacher learning and identity formation: an instrumental case. *Teachers and Teaching*, 25 (1), 7–33.
- Golding, J., 2017. Mathematics teachers' capacity for change. *Oxford Review of Education*, 43 (4), 502–517.
- Gorard, S., See, B. and Siddiqui, N., 2017. *The trials of evidence-based education: the promises, opportunities and problems of trials in education*. Abingdon: Routledge.
- Graham, D.J., and Midgley, N., 2000. Graphical representation of particle shape using triangular diagrams: an Excel spreadsheet method. *Earth Surface Processes and Landforms*, 25 (13), 1473–1477.
- Graven, M., and Heyd-Metzuyanim, E., 2019. Mathematics identity research: the state of the art and future directions. *ZDM Mathematics Education*, 51 (3), 361–377.
- Green, T., 1998. *The activities of teaching*. 2nd ed. New York, NY: Educator's International Press.
- Guba, E., and Lincoln, Y., 1994, Competing paradigms in qualitative research. In: N. Denzin, and Y. Lincoln, eds., *The SAGE handbook of qualitative research*. 2nd ed. London: Sage, 1994, pp. 105–117.
- Guskey, T., 2003. What makes professional development effective? *Phi Delta Kappan*, 84 (10), 748–750.
- Guskey, T., 2002. Professional development and teacher change. *Teachers and Teaching*, 8 (3), 381–391.
- Hamilton, L., Corbett-Whittier, C., Lagrange, M., Birch, J. and Scott, W., 2013. *Using case study in education research*. London: Sage.
- Hammersley, M., 2020, Transcription of speech. In: M. Ward, and S. Delamont, eds., *Handbook of qualitative research in education*. Northampton: Edward Elgar Publishing, 2020, pp. 374–379.
- Hannula, M., Di Martino, P., Pantziara, M., Zhang, Q., Morselli, F., Heyd-Metzuyanim, E., Lutovac, S., Kaasila, R., Middleton, J., Jansen, A. and Goldin, G.A., 2016, Attitudes, beliefs, motivation and identity in mathematics education. In: M. Hannula, P. Di Martino, M.

Pantziara, Q. Zhang, F. Morselli, E. Heyd-Metzuyanim, S. Lutovac, R. Kaasila, J. Middleton, A. Jansen and G. Goldin, eds., *Attitudes, beliefs, motivation and identity in mathematics education: an overview of the field and future directions*. Cham, Switzerland: Springer International Publishing, 2016, pp. 1–35.

Heal, J., 2021. *Maths for life* [online]. Education Endowment Foundation. Available at: https://d2tic4wvo1iusb.cloudfront.net/production/documents/pages/projects/Maths_for_Life_pilot.pdf?v=1705759684 [Accessed 20 January 2023].

Heck, D., Plumley, C., Stylianou, D., Smith, A. and Moffett, G., 2019. Scaling up innovative learning in mathematics: exploring the effect of different professional development approaches on teacher knowledge, beliefs, and instructional practice. *Educational Studies in Mathematics*, 102, 319–342.

Hegel, G., 2018. *The phenomenology of spirit*. Translated by T. Pinkard. Cambridge: Cambridge University Press.

Hegel, G., 2010. *The science of logic*. Translated by G. Di Giovanni. New York, NY: Cambridge University Press.

Hersh, R., 1998, Some proposals for revising the philosophy of mathematics. In: T. Tymoczko, ed., *New directions in the philosophy of mathematics*. Oxford: Princetown University Press, 1998, pp. 9–28.

Hintz, E., and Dean, M., 2020. Best practices for returning research findings to participants: Methodological and ethical considerations for communication researchers. *Communication Methods and Measures*, 14 (1), 38–54.

Hodgen, J., Coe, R., Brown, M. and Küchemann, D., 2014. Improving students' understanding of algebra and multiplicative reasoning: did the ICCAMS intervention work? In: *Proceedings of the 8th British Congress of Mathematics Education, University of Nottingham, 14-17 April 2014*. Nottingham: Taylor & Francis, pp. 167–174.

Hong, J., Cross Francis, D. and Schutz, P., 2024. Reconceptualizing teacher identity development. *Educational Psychologist*, 59 (3), 159–176.

Hordvik, M., MacPhail, A. and Ronglan, L., 2020. Developing a pedagogy of teacher education using self-study: a rhizomatic examination of negotiating learning and practice. *Teaching and Teacher Education*, 88, 1–11.

Inglis, M., 2018. The trials of evidence-based education: the promises, opportunities and problems of trials in education. *Research in Mathematics Education*, 20 (3), 316–321.

- Ingram, N., Hatisaru, V., Grootenboer, P. and Beswick, K., 2020, Researching the affective domain in mathematics education. *In: J. Way, C. Attard, J. Anderson, J. Bobis, H. McMaster and K. Cartwright, eds., Research in Mathematics Education in Australasia 2016–2019*. Singapore, Republic of Singapore: Springer, 2020, pp. 147–175.
- Jones, B., and Ball, S., 2022, Neoliberalism is dead - long live neoliberalism. *In: B. Jones, and S. Ball, eds., Neoliberalism and Education*. Abingdon: Routledge, 2022, pp. 1–16.
- Joubert, M., and Sutherland, R., 2009. *A perspective on the literature: CPD for teachers of mathematics*. Sheffield: National Centre for Excellence in the Teaching of Mathematics.
- Kagan, D., 1992. Implication of research on teacher belief. *Educational Psychologist*, 27 (1), 65–90.
- Kant, I., 2009. *The Critique of Pure Reason*. Translated by J. Meiklejohn. Auckland, New Zealand: The Floating Press.
- Kennedy, A., 2014. Understanding continuing professional development: the need for theory to impact on policy and practice. *Professional Development in Education*, 40 (5), 688–697.
- Kuhs, T., and Ball, D., 1986. *Approaches to teaching mathematics: mapping the domains of knowledge, skills, and dispositions*. East Lansing, MI: National Center for Research on Teacher Education.
- Lemov, D., 2021. *Teach like a champion 3.0: 63 techniques that put students on the path to college*. Hoboken, NJ: Jossey-Bass.
- Lewis, J., and Maisuria, A., 2023. *'Maths to 18' in England*. London: House of Commons Library.
- Lomas, G., Grootenboer, P. and Attard, C., 2012, The affective domain and mathematics education. *In: B. Perry, T. Lowrie, T. Logan, A. MacDonald and J. Greenlees, eds., Research in mathematics education in Australasia 2008-2011*. Rotterdam, The Netherlands: Sense Publishers, 2012, pp. 23–38.
- Lutovac, S., and Kaasila, R., 2018. Future directions in research on mathematics-related teacher identity. *International Journal of Science and Mathematics Education*, 16, 759–776.
- MacCarthy, M., 2021. Using dialectic thematic analysis in dark tourism: combining deductive and inductive reasoning in a modular method. *Journal of Hospitality and Tourism Management*, 48, 468–478.

- MacPhail, A., Ulvik, M., Guberman, A., Czerniawski, G., Oolbekkink-Marchand, H. and Bain, Y., 2019. The professional development of higher education-based teacher educators: needs and realities. *Professional Development in Education*, 45 (5), 848–861.
- Malterud, K., 2016. Theory and interpretation in qualitative studies from general practice: why and how? *Scandinavian Journal of Public Health*, 44 (2), 120–129.
- Martínez-Sierra, G., García-García, J., Valle-Zequeida, M. and Dolores-Flores, C., 2020. High school mathematics teachers' beliefs about assessment in mathematics and the connections to their mathematical beliefs. *International Journal of Science and Mathematics Education*, 18, 485–507.
- Mayo, E., 2000. *The social problems of an industrial civilization*. Abingdon: Routledge.
- McChesney, K., and Aldridge, J., 2021. What gets in the way? A new conceptual model for the trajectory from teacher professional development to impact. *Professional Development in Education*, 47 (5), 834–852.
- McGowan, T., 2019. *Emancipation after Hegel: achieving a contradictory revolution*. New York, NY: Columbia University Press.
- McGowan, T., 2016. *Capitalism and desire: the psychic cost of free markets*. Chichester: Columbia University Press.
- McLeod, D., 1992, Research on affect in mathematics education: a reconceptualization. In: D. Grouws, ed., *Handbook of research on mathematics teaching and learning*. Reston, VA: The National Council of Teachers of Mathematics, 1992, pp. 575–596.
- McMullin, C., 2021. Transcription and qualitative methods: implications for third sector research. *International Journal of Voluntary and Nonprofit Organizations*, 34, 140–153.
- Mears, C., 2021, In depth interviews. In: R. Coe, ed., *Research methods and methodologies in education*. 4th ed. Washington, MA: Sage, 2021, pp. 232–239.
- Mellone, M., Ramploud, A., Di Paola, B. and Martignone, F., 2019. Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. *ZDM Mathematics Education*, 51 (1), 199–212.
- Mero-Jaffe, I., 2011. 'Is that what I said?' Interview transcript approval by participants: an aspect of ethics in qualitative research. *International Journal of Qualitative Methods*, 10 (3), 231–247.

- Mosvold, R., and Fauskanger, J., 2013. Teachers' beliefs about mathematical knowledge for teaching definitions. *International Electronic Journal of Mathematics Education*, 8 (2-3), 43–61.
- Mullis, I., Martin, M., Foy, P., Kelly, D. and Fishbein, B., 2020. *TIMSS 2019 International Results in Mathematics and Science* [online]. Boston College. Available at: <https://timss2019.org/reports/achievement/#math-4> [Accessed 01 June 2021].
- National Center for Education Statistics, 1999. *Japan mathematics lessons* [online]. TIMSSVIDEO. Available at: <http://www.timssvideo.com/japan-mathematics-lessons> [Accessed 13 November 2020].
- National Centre for Excellence in the Teaching of Mathematics [NCETM], 2024. *The five big ideas – coherence* [online]. National Centre for Excellence in the Teaching of Mathematics. Available at: <https://www.ncetm.org.uk/features/the-five-big-ideas-coherence/> [Accessed 28 January 2025].
- National Centre for Excellence in the Teaching of Mathematics [NCETM], 2023. *Five big ideas in teaching for mastery* [online]. National Centre for Excellence in the Teaching of Mathematics. Available at: <https://www.ncetm.org.uk/teaching-for-mastery/mastery-explained/five-big-ideas-in-teaching-for-mastery/> [Accessed 20 January 2024].
- Nguyen, D., and Tran, D., 2023. High school mathematics teachers' changes in beliefs and knowledge during lesson study. *Journal of Mathematics Teacher Education*, 26 (6), 809–834.
- Noyes, A., Brignall, C., Jacques, L., Powell, J. and Adkins, M., 2023. *The mathematics pipeline in England: patterns, interventions and excellence*. Nottingham: University of Nottingham.
- O'Brien, J., and Jones, K., 2014. Professional learning or professional development? Or continuing professional learning and development? Changing terminology, policy and practice. *Professional Development in Education*, 40 (5), 683–687.
- Obilor, E., 2023. Convenience and purposive sampling techniques: are they the same. *International Journal of Innovative Social and Science Education Research*, 11 (1), 1–7.
- Ofsted, 2025. *School inspection toolkit: draft for consultation*. London: UK government.
- Op't Eynde, P., de Corte, E. and Verschaffel, L., 2002, Framing students' mathematics-related beliefs. A quest for conceptual clarity and a comprehensive categorization. In: G.C. Leder, E. Pehkonen and G. Törner, eds., *Beliefs: a hidden variable in mathematics education?* New York, NY: Springer, 2002, pp. 13–38.

- Opfer, D., and Pedder, D., 2011. Conceptualizing teacher professional learning. *Review of Educational Research*, 81 (3), 376–407.
- Pajares, M., 1992. Teachers' beliefs and educational research: cleaning up a messy construct. *Review of Educational Research*, 62 (3), 307–332.
- Pampaka, M., Williams, J., Quinn, J., Harris, D., Swanson, D., Wo, L., Kenna, A. and Hutcheson, G., 2021. *Increasing competence and confidence in algebra and multiplicative structures (ICCAMS): evaluation report*. London: Education Endowment Foundation.
- Parker, I., 2005. *Qualitative psychology: introducing radical research*. Maidenhead: Open University Press.
- Pearson, M., Albon, S. and Hubball, H., 2015. Case study methodology: flexibility, rigour, and ethical considerations for the scholarship of teaching and learning. *Canadian Journal for the Scholarship of Teaching and Learning*, 6 (3), 1–8.
- Pouwels, J., and Biesta, G., 2017. With Socrates on your heels and Descartes in your hand: on the notion of conflict in John Dewey's democracy and education. *Education Sciences*, 7 (1), 7.
- Rolka, K., and Roesken-Winter, B., 2015, Networking theories to understand beliefs and their crucial role in mathematics education. In: B. Pepin, and B. Roesken-Winter, eds., *From beliefs to dynamic affect systems in mathematics education: exploring a mosaic of relationships and interactions*. Cham, Switzerland: Springer, 2015, pp. 73–93.
- Rubel, L., and Stachelek, A., 2018. Tools for rethinking classroom participation in secondary mathematics. *Mathematics Teacher Educator*, 6 (2), 8–25.
- Sancar, R., Atal, D. and Deryakulu, D., 2021. A new framework for teachers' professional development. *Teaching and Teacher Education*, 101, 1–12.
- Schleicher, A., 2019. *PISA 2018: insights and interpretations*. Paris, France: OECD Publishing.
- Schleicher, A., 2018. *World class how to build a 21st-century school system: how to build a 21st-century school system*. Paris, France: OECD Publishing.
- Schoenfeld, A., 2020. Reframing teacher knowledge: a research and development agenda. *ZDM Mathematics Education*, 52 (2), 359–376.
- Schoenfeld, A., 2019, The what and the why of modeling. In: S. Chamberlin, and B. Sriraman, eds., *Affect in Mathematical Modeling*. Cham, Switzerland: Springer, 2019, pp. 89–97.

- Schoenfeld, A., 2015, How we think: a theory of human decision-making, with a focus on teaching. In: S. Cho, ed., *The proceedings of the 12th international congress on mathematical education*. Cham, Switzerland: Springer, 2015, pp. 229–246.
- Schoenfeld, A., 2010. *How we think: a theory of goal-oriented decision making and its educational applications*. Abingdon: Routledge.
- Schoenfeld, A.H., 2004. The math wars. *Educational Policy*, 18 (1), 253–286.
- Schuck, S., and Grootenboer, P., 2000. Affective issues in mathematics education. *Review of Mathematics Education in Australasia*, 2003, 53–74.
- Seidman, I., 2019. *Interviewing as qualitative research: a guide for researchers in education and the social sciences*. 4th ed. New York, NY: Teachers College Press.
- Seleznyov, S., 2019. Lesson study beyond Japan: evaluating impact. *International Journal for Lesson and Learning Studies*, 8 (1), 2–18.
- Seleznyov, S., Goei, S. and Ehren, M., 2024. International policy borrowing and the case of Japanese Lesson Study: culture and its impact on implementation and adaptation. *Professional Development in Education*, 50 (1), 59–73.
- Shah, M., and Campbell, C., 2023. A state-of-the-art review of Canadian literature on teacher professional learning and leadership (2010-2020). *International Journal of Leadership in Education*, , 1–20.
- Shayer, M., and Adhami, M., 2007. The impact of a thinking skills approach (CAME) on students' mathematical ability. *Educational Studies in Mathematics*, 64, 265–291.
- Shimizu, Y., and Kang, H., 2022. Discussing students' thinking and perspectives for improving teaching: an analysis of teachers' reflection in post-lesson discussions in lesson study cycles. *ZDM*, 54 (2), 419–431.
- Shulman, L.S., 1986. Those who understand: knowledge growth in teaching. *Educational Researcher*, 15 (2), 4–14.
- Simons, H., 1996. The paradox of case study. *Cambridge Journal of Education*, 26 (2), 225–240.
- Sims, S., Anders, J., Inglis, M. and Lortie-Forgues, H., 2023. Quantifying “promising trials bias” in randomized controlled trials in education. *Journal of Research on Educational Effectiveness*, 16 (4), 663–680.

Sims, S., Fletcher-Wood, H., O'Mara-Eves, A., Cottingham, S., Stansfield, C., Van Herwegen, J. and Anders, J., 2021. *What are the characteristics of teacher professional development that increase pupil achievement? A systematic review and meta-analysis*. London: Education Endowment Foundation.

Sims, S., and Fletcher-Wood, H., 2021. Identifying the characteristics of effective teacher professional development: a critical review. *School Effectiveness and School Improvement*, 32 (1), 47–63.

Singer, P., 2001. *Hegel: A very short introduction*. New York, NY: Oxford University Press.

Skott, J., 2022. Conceptualizing individual-context relationships in teaching: Developments in research on teachers' knowledge, beliefs and identity. In: *Twelfth Congress of the European Society for Research in Mathematics Education, 2 - 6 February 2022*. Bozen-Bolzano, Italy: .

Skott, J., 2015, The promises, problems, and prospects of teacher related belief research. In: H. Fives, and M.G. Gill, eds., *International handbook on teacher's beliefs*. New York, NY: Routledge, 2015, pp. 13–30.

Stake, R., 2010. *Qualitative research studying how things work*. New York, NY: Guilford Press.

Stake, R., 1995. *The art of case study research*. London: Sage.

Stigler, J., and Hiebert, J., 2016. Lesson study, improvement, and the importing of cultural routines. *ZDM Mathematics Education*, 48 (4), 581–587.

Stigler, J., and Hiebert, J., 2009. *The teaching gap: best ideas from the world's teachers for improving education in the classroom*. New York, NY: Simon and Schuster.

Stojanov, K., 2018. *Education, self-consciousness and social action: bildung as a neo-Hegelian concept*. Abingdon: Routledge.

Strom, K., Martin, A. and Villegas, A., 2018. Clinging to the edge of chaos: the emergence of practice in the first year of teaching. *Teachers College Record*, 120 (7), 1–32.

Strom, K., and Viesca, K., 2021. Towards a complex framework of teacher learning-practice. *Professional Development in Education*, 47 (2-3), 209–224.

Swain, J., and Swan, M., 2007. *Maths4Life*. London: National Research and Development Centre for Adult Literacy and Numeracy.

Swan, M., 2014, Improving the alignment between values, principles and classroom realities. *In: Y. Li, and G. Lappan, eds., Mathematics Curriculum in School Education*. Dordrecht, Netherlands: Springer, 2014, pp. 621–636.

Swan, M., 2011, Designing tasks that challenge values, beliefs and practices: a model for the professional development of practicing teachers. *In: O. Zaslavsky, and P. Sullivan, eds., Constructing knowledge for teaching secondary mathematics*. London: Springer, 2011, pp. 57–71.

Swan, M., 2007. The impact of task-based professional development on teachers' practices and beliefs: a design research study. *Journal of Mathematics Teacher Education*, 10, 217–237.

Swan, M., 2006a. *Collaborative learning in mathematics: a challenge to our beliefs and practices*. London: National Research and Development Centre for Adult Literacy and Numeracy.

Swan, M., 2006b. Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, 75 (1), 58–70.

Swan, M., Pead, D., Doorman, M. and Mooldijk, A., 2013. Designing and using professional development resources for inquiry-based learning. *ZDM Mathematics Education*, 45, 945–957.

Swan, M., and Swain, J., 2010a. The impact of a professional development programme on the practices and beliefs of numeracy teachers. *Journal of further and Higher Education*, 34 (2), 165–177.

Swan, M., and Swain, J., 2010b. The impact of a professional development programme on the practices and beliefs of numeracy teachers. *Journal of further and Higher Education*, 34 (2), 165–177.

Takahashi, A., 2021. *Teaching mathematics through problem-solving: a pedagogical approach from Japan*. Abingdon: Routledge.

Takahashi, A., and McDougal, T., 2016. Collaborative lesson research: maximizing the impact of lesson study. *ZDM Mathematics Education*, 48 (4), 513–526.

Terry, G., and Hayfield, N., 2020, Reflexive thematic analysis. *In: M. Ward, and S. Delamont, eds., Handbook of qualitative research in education*. Cheltenham: Edward Elgar Publishing Limited, 2020, pp. 430–441.

Thompson, A., 1992, Teachers' beliefs and conceptions: a synthesis of the research. *In: D. Grouws, ed., Handbook of research on mathematics teaching and learning: a project of the*

National Council of Teachers of Mathematics. Charlotte, NC: MacMillan Publishing, 1992, pp. 127–146.

Thompson, A., 1984. The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15 (2), 127–146.

Tight, M., 2019. *Documentary research in the social sciences*. London: Sage.

Treacy, M., and Leavy, A., 2023. Student voice and its role in creating cognitive dissonance: the neglected narrative in teacher professional development. *Professional Development in Education*, 49 (3), 458–477.

Trivedi, S., 2022. *Meeting the challenge of providing high quality continuing professional development for teachers*. Leicester: CFE Research.

Van den Brande, J., and Zuccollo, J., 2021. *The cost of high-quality professional development for teachers in England*. London: Educational Policy Institute.

Vesga-Bravo, G., Angel-Cuervo, Z. and Chacón-Guerrero, G., 2022. Beliefs about mathematics, its teaching, and learning: contrast between pre-service and in-service teachers. *International Journal of Science and Mathematics Education*, 20 (4), 769–791.

Wake, G., 2018, A case study of theory-informed task design: what might we, as designers, learn? In: L. Rodríguez-Muñiz, L. Muñiz-Rodríguez, Á Aguilar-González, P. Alonso, F. García and A. Bruno, eds., *Investigación en educación matemática XXII*. Gijón, Spain: Universidad de Oviedo, 2018, pp. 94–109.

Wake, G., Adkins, M., Dalby, D., Hall, J., Joubert, M., Lee, G. and Noyes, A., 2023. *Centres for excellence in maths teaching for mastery randomised controlled trial*. Nottingham: University of Nottingham.

Wake, G., 2024, Issues in adopting and adapting lesson study. In: A. Takahashi, and G. Wake, eds., *The Mathematics Practitioner's Guidebook for Collaborative Lesson Research*. Abingdon: Routledge, 2024, pp. 12–18.

Wake, G., 2022. Designing lesson study for individual and collective learning: networking theoretical perspectives. *International Journal for Lesson and Learning Studies*, 12 (1), 7–20.

Walker, M., Worth, J. and Van den Brande, J., 2019. *Teacher Workload Survey 2019*. London: Government Social Research.

Wenger, E., 1998. *Communities of practice: learning, meaning, and identity*. Cambridge: Cambridge University Press.

Williams, J., and Ryan, J., 2020. On the compatibility of dialogism and dialectics: the case of mathematics education and professional development. *Mind, Culture, and Activity*, 27 (1), 70–85.

Williams, J., and Ryan, J., 2014, Bakhtinian dialogue and Hegelian dialectic in mathematical and professional education. In: C. Nicol, S. Oesterle, P. Liljedahl and D. Allan, eds., *Proceedings of the Joint Meeting of PME 38*. Vancouver, BC: ERIC, 2014, pp. 377–384.

Willis, R., Lynch, D., Peddell, L., Yeigh, T., Woolcott, G., Bui, V., Boyd, W., Ellis, D., Markopoulos, C. and James, S., 2023. Development of a teacher of mathematics identity (ToMI) scale. *Mathematics Education Research Journal*, 35 (1), 107–132.

Woodford, M., 2024, What we learned by researching lesson study. In: A. Takahashi, and G. Wake, eds., *The mathematics practitioner's guidebook for collaborative lesson research*. Abingdon: Routledge, 2024, pp. 196–203.

Woodford, M., Clapham, A. and Serret, N., 2023. 'Pedagogic oppression': the need for emancipation through contradiction in teacher professional development. *Professional Development in Education*, 50 (6), 1148–1161.

Woodford, M., and Wake, G., 2023. Designing lessons for dialogic learning. *Mathematics Teaching*, 285, 12–17.

Wright, P., 2012. The math wars: tensions in the development of school mathematics curricula. *For the Learning of Mathematics*, 32 (2), 7–13.

Wright, P., Fejzo, A. and Carvalho, T., 2022. Progressive pedagogies made visible: implications for equitable mathematics teaching. *The Curriculum Journal*, 33 (1), 25–41.

Xie, S., and Cai, J., 2021. Teachers' beliefs about mathematics, learning, teaching, students, and teachers: perspectives from Chinese high school in-service mathematics teachers. *International Journal of Science and Mathematics Education*, 19, 747–769.

Yin, R.K., 2018. *Case study research and applications: design and methods*. Los Angeles, CA: Sage.

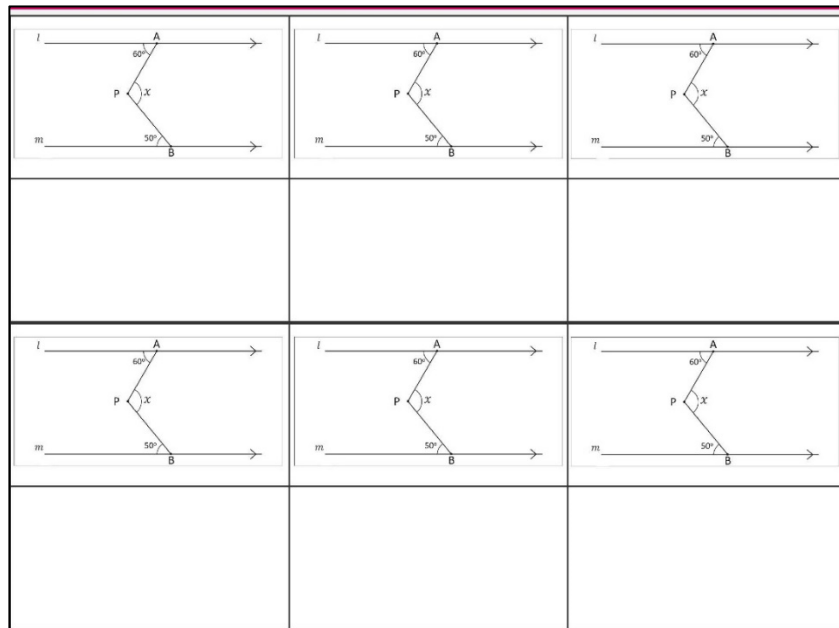
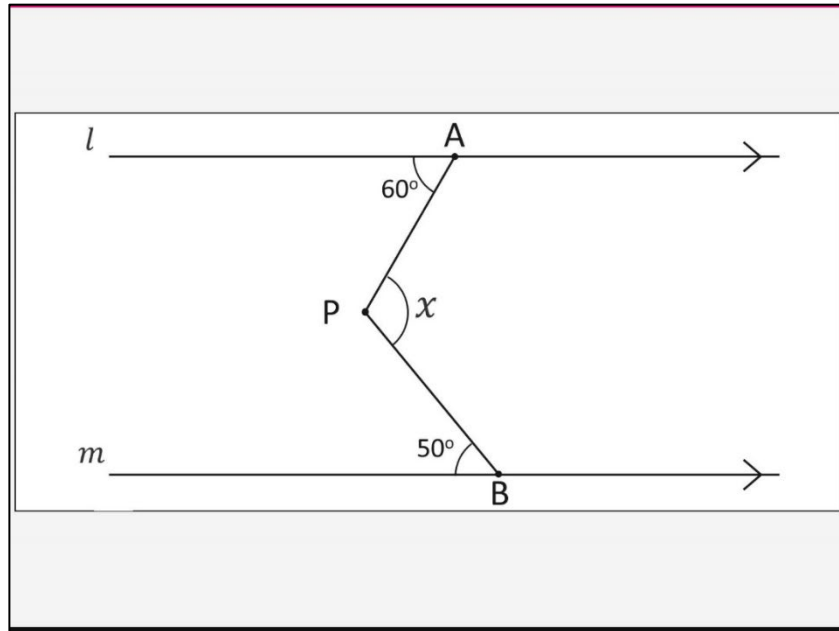
Žižek, S., 2023. *Too late to awaken: what lies ahead when there is no future?* London: Allen Lane.

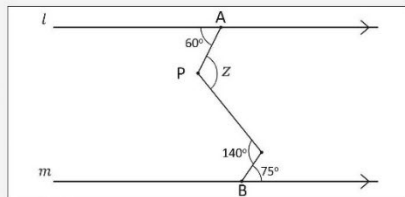
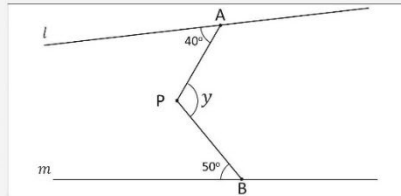
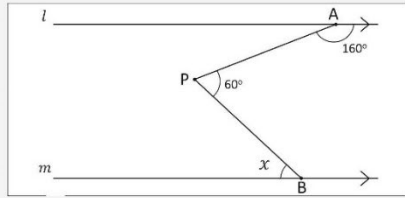
Žižek, S., 2012. *Less than nothing: Hegel and the shadow of dialectical materialism*. London: Verso.

Appendices.

- Appendix 1 - RAM lesson slides
- Appendix 2 - RAM introductory slides
- Appendix 3 - RAM session 1 slides
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- Appendix 5 - RAM session 3 slides
- Appendix 6 - Participant resources for the RAM introductory sessions
- Appendix 7 - Participant resources for RAM session 1
- Appendix 8 - Participant resources for RAM session 2
- Appendix 9 - Participant resources for RAM session 3
- Appendix 10 - Example of completed resources from RAM introductory session
- Appendix 11 - Example of completed resources from RAM session 1
- Appendix 12 - Example of completed resources from RAM session 2
- Appendix 13 - Example of completed resources from RAM session 3
- Appendix 14 - The project information sheet and consent form
- Appendix 15 - A copy of Belief [Drew]
- Appendix 16 - Interview guides
- Appendix 17 - Example transcript excerpt from interview 1
- Appendix 18 - Daa imported into NVIVO 14
- Appendix 19 - Initial comments on data
- Appendix 20 - Generated codes

Appendix 1 – RAM lesson slides





Appendix 2 – RAM introductory session slides

Rethinking approaches in Mathematics

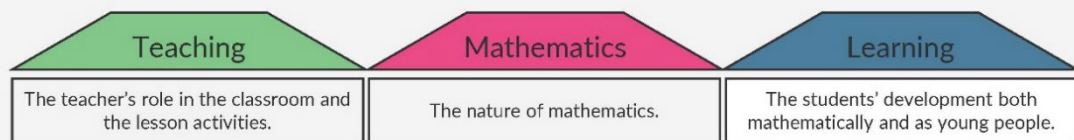
matt.woodford@ntu.ac.uk

Purpose of RaM

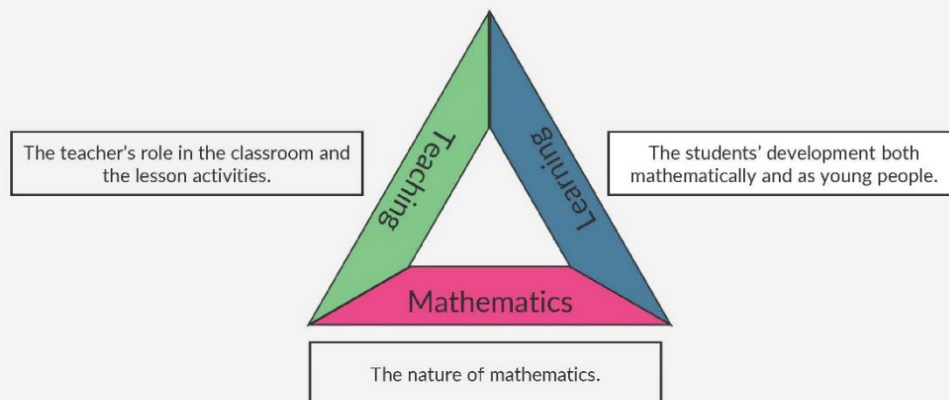
1. To encourage the consideration of our beliefs, and therefore approach, toward mathematics teaching.
2. It is **not** to try and force you to teach in one particular style!

Rethinking approaches in Mathematics

Beliefs toward:



3 sessions and a research lesson



Developing a research theme

What?	Respect • for others • for self • self discipline
--------------	--

What?	Responsibility • for actions • for learning
--------------	---

What?	Resilience • confidence • courage • self belief
--------------	--

How?	Fluency • develop conceptual understanding • recall and apply knowledge
-------------	---

How?	Reasoning • follow a line of enquiry • conjecture relationships and generalisations • develop an argument, justification or proof
-------------	--

How?	Problem solving • break down problems • persevere in seeking solutions
-------------	--

Current beliefs

Activity 3-1: Beliefs about Mathematics

Why? I am following belief statements according to your view of the nature of mathematics.

Statement	Percentage
A ... a given body of knowledge and standard set of procedures. It is not of value or interest to students.	%
B ... a subject which is taught by the teacher. It is not of value or interest to students.	%
C ... of the accumulated knowledge of the world. It is not of value or interest to students.	%
	100%

Give each statement a weighting in this table to reflect how strongly you agree with it by making the three scores sum to 100%.

For example, if you allocated weightings of 25% and 75% to two of the statements then the first statement must be allocated a weighting of 0%.

Activity 3-2: Beliefs about Learning

Why? I am following belief statements according to your view of the nature of learning.

Statement	Percentage
A ... an individual activity based on practice, repetition, and memorisation of knowledge.	%
B ... an individual activity based on practical application and reflection.	%
C ... a personal activity in which students are challenged and motivated to understand through discussion.	%
	100%

Give each statement a weighting in this table to reflect how strongly you agree with it by making the three scores sum to 100%.

For example, if you allocated weightings of 25% and 75% to two of the statements then the first statement must be allocated a weighting of 0%.

Activity 3-3: Beliefs about Teaching

Why? I am following belief statements according to your view of the nature of teaching.

Statement	Percentage
A ... teaching is a transmission of knowledge from teacher to student. The teacher is the source of knowledge and the student is the recipient.	%
B ... teaching is a process in which the teacher provides a structured environment for students to learn. The teacher is the source of knowledge and the student is the recipient.	%
C ... teaching is a process in which the teacher provides a structured environment for students to learn. The teacher is the source of knowledge and the student is the recipient.	%
	100%

Give each statement a weighting in this table to reflect how strongly you agree with it by making the three scores sum to 100%.

For example, if you allocated weightings of 25% and 75% to two of the statements then the first statement must be allocated a weighting of 0%.

Beliefs about mathematics education

Activity 4-1: Beliefs about mathematics education
Calculate your mean average weights for the three domains A, B and C.

Belief	Percentage
A	%
B	%
C	%
	100%

Plot the position of your beliefs about mathematics education on the axes provided below.

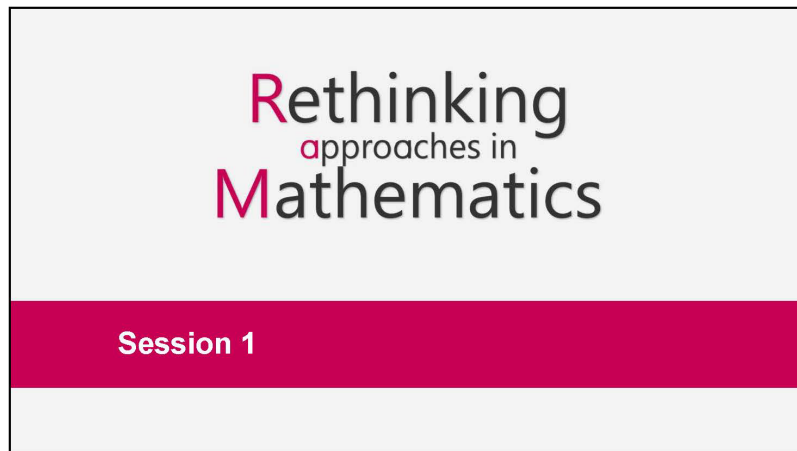
4/11

In addition

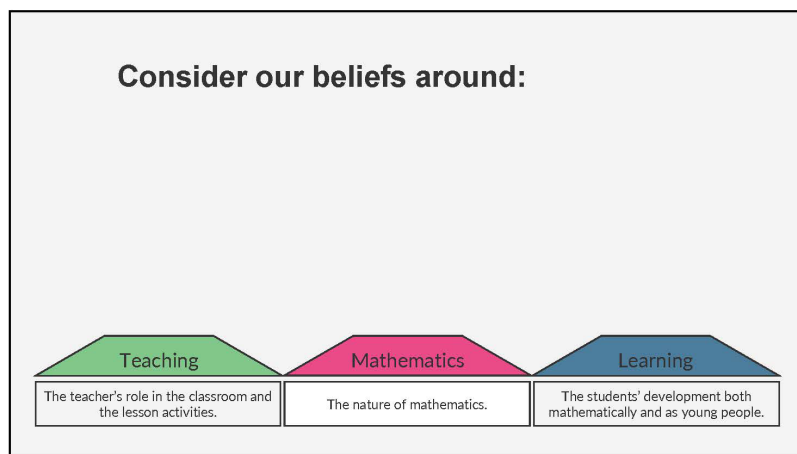
3 (or more) people who would be happy to volunteer to offer deeper views. An online discussion with me after each session at a time that is convenient to you.

- Views around teaching.
- Thoughts on the PD session and materials.

Appendix 3 – RAM session 1 slides



1



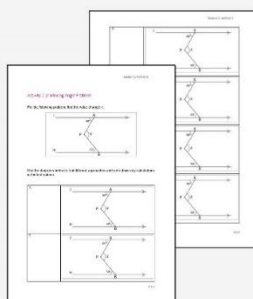
2

1

1-2: Missing Angle Problem

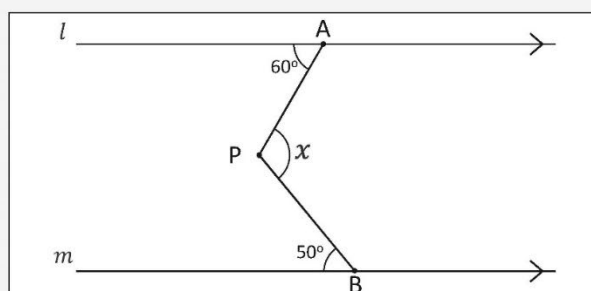
Try to find as many **different** methods as you can, writing each one in a separate box.

If your method doesn't result in a solution please leave your workings out and simply move on to another box.



3

You have 8 minutes...



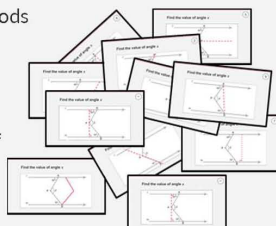
4

2

In groups of 3 or 4

Identify and **explain** any methods that you found and **make sense** of any methods that you didn't find.

Classify the methods into groups of similar strategies.



5

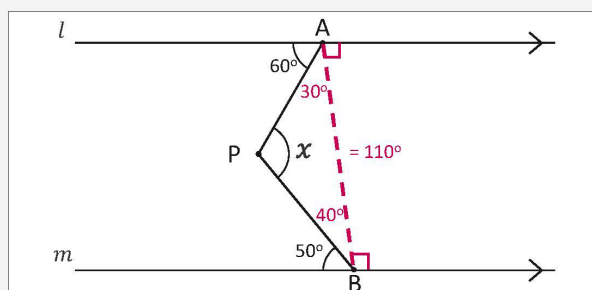
Find the value of angle x . 	Find the value of angle x . 	Find the value of angle x . 	Find the value of angle x .
Find the value of angle x . 	Find the value of angle x . 	Find the value of angle x . 	Find the value of angle x .
Find the value of angle x . 	Find the value of angle x . 	Find the value of angle x . 	Find the value of angle x .

6

3

Find the value of angle x

10



7

Method 10

$$x + a + b = 180^\circ \text{ (angles of a triangle sum to } 180^\circ\text{)}$$

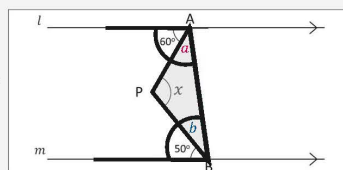
$$(60^\circ + a) + (50^\circ + b) = 180^\circ \text{ (co-interior angles sum to } 180^\circ\text{)}$$

$$\therefore a + b = 70^\circ$$

Combining the above,

$$x + 70^\circ = 180^\circ$$

$$\therefore x = 110^\circ$$



8

4

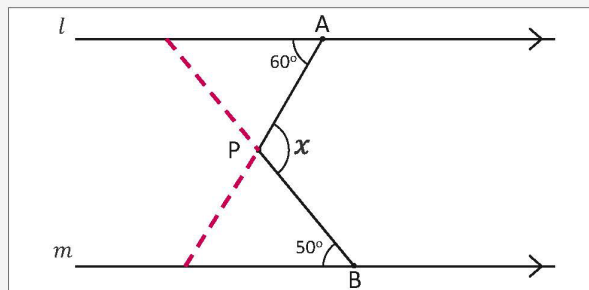
Find the value of angle x 1	Find the value of angle x 2	Find the value of angle x 3	Find the value of angle x 4
Find the value of angle x 5	Find the value of angle x 6	Find the value of angle x 7	Find the value of angle x 8
Find the value of angle x 9	Find the value of angle x 10	Find the value of angle x 11	Find the value of angle x 12

9

Add a parallel line Find the value of angle x 1	Extend a line Find the value of angle x 2	Add a perpendicular line Find the value of angle x 3	Add a perpendicular line Find the value of angle x 4
Find the value of angle x 5	Find the value of angle x 6	Find the value of angle x 7	Find the value of angle x 8
Find the value of angle x 9	Connect two points Find the value of angle x 10	Find the value of angle x 11	Find the value of angle x 12

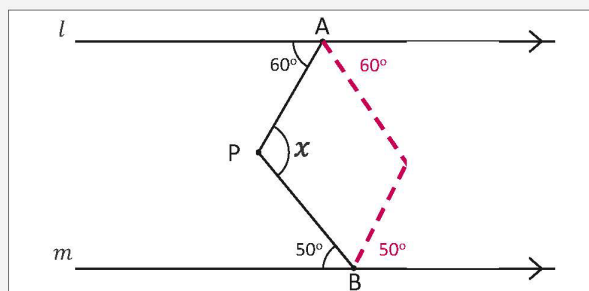
10

What would you bring out for this method?



11

What would you bring out for this method?



12

6

1-3: Session Reflection

Individually, please complete the session reflection sheet.

Activity 1-3: Session Reflection

Thinking about all of the work around the missing angle problem, what are the most important ideas that you have learned today? Please write them in the table below.

Question	Mathematical Idea	Knowledge
A	A relationship of angles that we need to be confident in.	
B	A strategy to help us find the missing angle.	
C	Any other ideas that we have learned today.	

Now have the opportunity to write the ideas that you have learned today in a paragraph. You have 10 minutes to write your reflection.

.....

13

1-4: Lesson Plan

Create the outline of a lesson based around the original missing angle problem, including any 'next questions'.

Lesson Plan

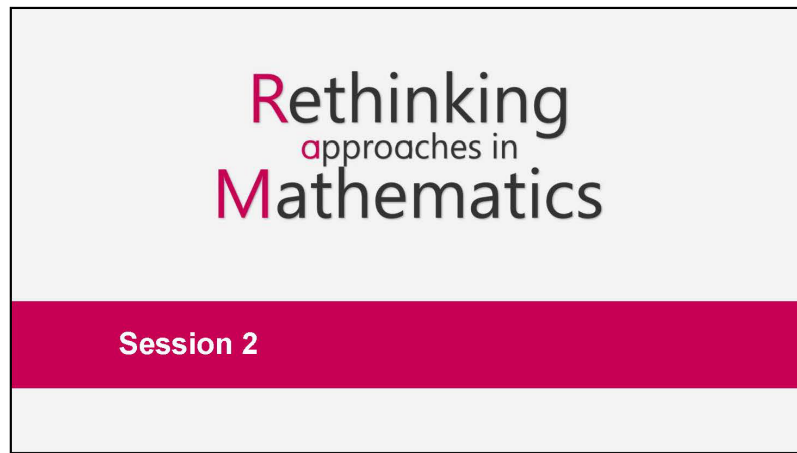
Activity 1-4: Lesson Plan

Write the outline of a lesson based around the original missing angle problem, including any 'next questions'.

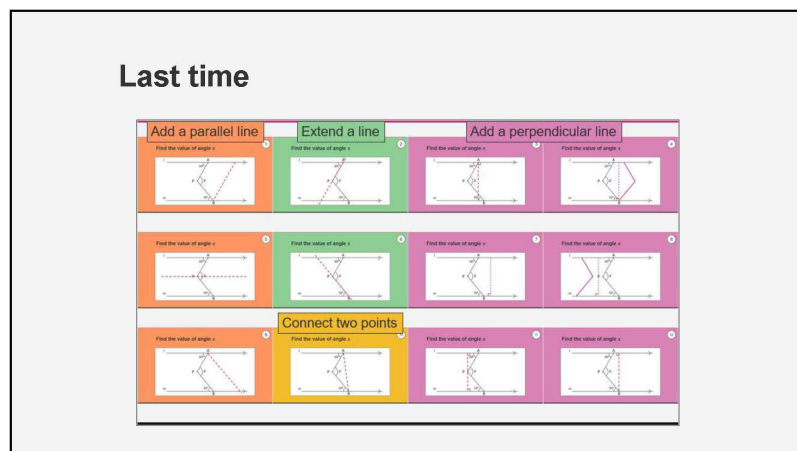
.....

14

Appendix 4 – RAM session 2 slides



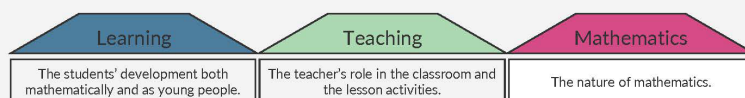
1



2

1

Consider our orientations around:



3

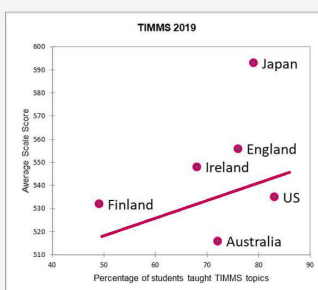
Orientations around the nature of teaching

Orientation	Teaching is ...
A	... structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to grasp what is taught.
B	... assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.
C	... a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.

4

2

Mathematics in Japan



Selected data for Grade 8 from: <https://timss2019.org/reports/achievement/#math-4>

5

Teaching Through Problem Solving

A lesson for the missing angle problem has been designed based on a style of “**Teaching Through Problem-solving**” commonly seen in Japan.

1. Review previous learning
2. Present the problem
3. Individual problem solving
4. Whole class discussion of solution methods
5. Highlight the major points
6. Extend

6

Phase	Approx. duration (mins)	Key actions
1. Review previous learning	5	
2. Present the problem	5	
3. Individual problem solving	10	
4. Whole class discussion of solution methods	15	
5. Highlight the major points	10	
6. Extend	15	

7

		<i>Extend a line</i>
$\angle BCP = 60^\circ$ $\angle BPC = 70^\circ$ $\angle X = 180^\circ - 70^\circ = 110^\circ$ (alternate to 60° at A) (angles of a triangle sum to 180°) (supplementary angles sum to 180°)	$\angle ADP = 50^\circ$ $\angle APD = 70^\circ$ $\angle Z = 180^\circ - 70^\circ = 110^\circ$ (alternate to 50° at B) (angles of a triangle sum to 180°) (supplementary angles sum to 180°)	
		<i>Add a perpendicular line</i>
$\angle X_1 = 60^\circ$ (alternate to $\angle A$) $\angle X_2 = 50^\circ$ (alternate to $\angle B$) $\angle X = 50^\circ + 60^\circ = 110^\circ$ <i>Add parallel line</i>	$\angle PAE = 120^\circ$ (supplementary to 60°) $\angle PBE = 40^\circ$ (complementary to 50°) $\angle X = 360^\circ - 120^\circ - 40^\circ - 90^\circ = 110^\circ$ (angles of a quadrilateral sum to 360°)	

8

Important points about the lesson

1. Try to teach this lesson **as closely as you can** to the lesson plan. This will allow us to discuss the learning in the lesson from a position of experience in the next RaM session.
2. But if you want to make adjustments that's okay – please explain them on the lesson plan.

9

2-2: Missing Angle Lesson Plan

In groups of 3 (or 4) discuss each phase of the lesson plan.

Please respond to the pink questions by writing on the sheet.

A group discussion, but an individual decision.

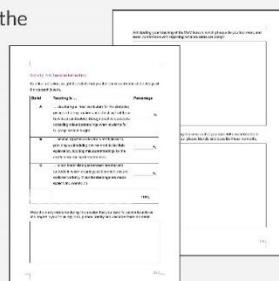
Activity 2-2: Missing Angle Lesson Plan		
Lesson Objective: To enable students to develop problem solving strategies involving angle facts.		
Phase 1: Review previous learning (5 minutes). To remind students of previous learning that may be useful in this lesson.		
Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher facilitates a whole class recall activity. "Can you tell me some angle related facts that we know?"	How will the teacher set a whole class recall activity?	What students are expected to put down to the whole class suggestions or to write about ideas?
Write the important suggestions on the board as students suggest ideas.		What are the important angle facts that you expect students to suggest?

10

5

2-3: Session Reflection

Individually, please complete the session reflection sheet.



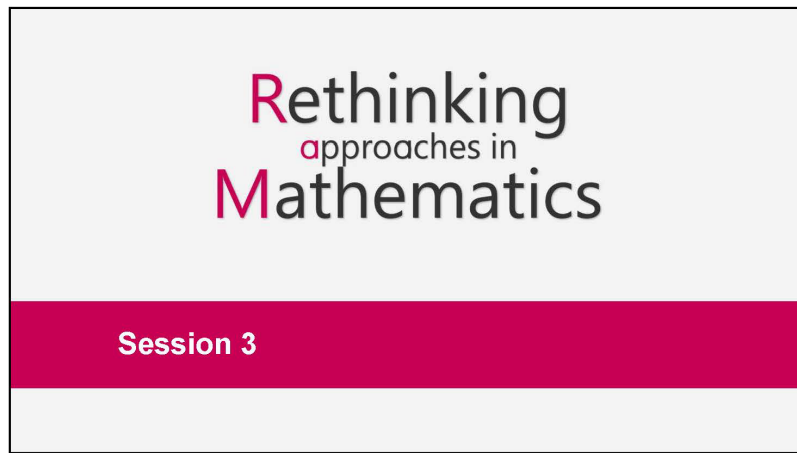
11

Follow-Up

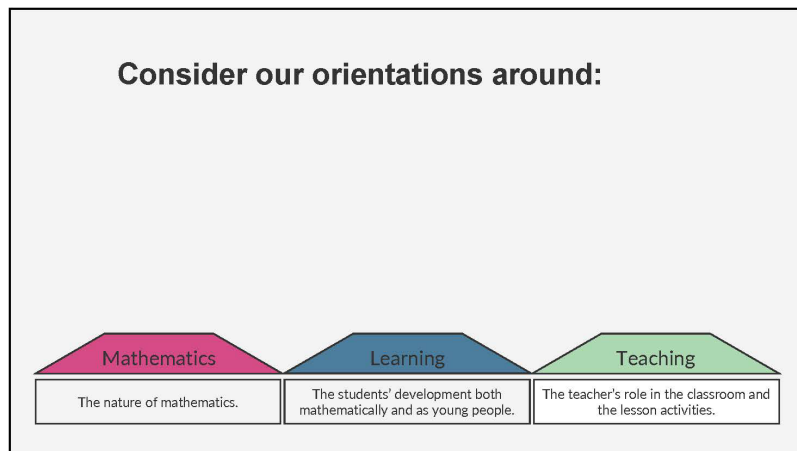
Teach the missing angle lesson before the next RaM session.

12

Appendix 5 – RAM session 3 slides



1



2

1

Beliefs around the nature of learning

Belief	Learning is ...
A	... an individual activity based on watching, listening and imitating until fluency is attained.
B	... an individual activity based on practical exploration and reflection.
C	... an interpersonal activity in which students are challenged and arrive at understanding through discussion.

3

3-2: Reflecting on the research lesson

Activity 3-2: Reflecting on the research lesson

After the lesson, the teacher will:

1. Reflect on the lesson and discuss it with the research team.

2. Reflect on the lesson and discuss it with the research team.

3. Reflect on the lesson and discuss it with the research team.

4. Reflect on the lesson and discuss it with the research team.

5. Reflect on the lesson and discuss it with the research team.

6. Reflect on the lesson and discuss it with the research team.

7. Reflect on the lesson and discuss it with the research team.

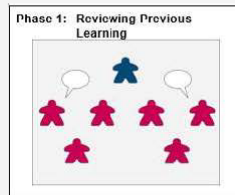
8. Reflect on the lesson and discuss it with the research team.

9. Reflect on the lesson and discuss it with the research team.

10. Reflect on the lesson and discuss it with the research team.

4

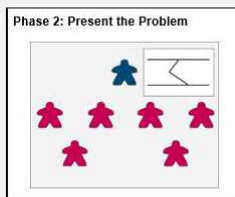
Phase 1: Review



How did you manage the whole class recall activity or an alternative? What learning took place?

5

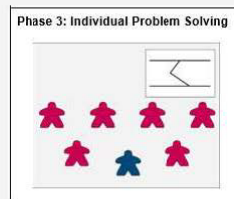
Phase 2: Present Problem



Did you draw a careful copy of the problem and ask students to draw it or an alternative? What learning took place?

6

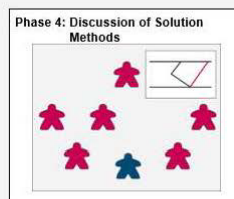
Phase 3: Individual Work



Did you get students to work individually in silence for 10 minutes whilst you noted solutions or an alternative? What learning took place?

7

Phase 4: Discussion

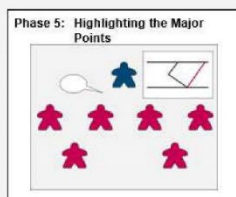


Did you base only introduce methods that students found or follow an alternative? What learning took place?

8

4

Phase 5: Drawing together



Did you ask students to write notes or follow an alternative?
What learning took place?

9

3-3: Session Reflection

Individually, please complete the session reflection sheet.

10

Appendix 6 – Participant resources for the RAM introductory session

Activity 1-1: Beliefs about Mathematics

Weight the following belief statements according to your view of the nature of mathematics.

Belief	Mathematics is ...	Percentage
A	 a given body of knowledge and standard set of procedures. A set of universal truths and rules which need to be conveyed to students.	----- %
B	... a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	----- %
C	... an interconnected body of ideas which the teacher and the student create together through discussion	----- %
		100%

Give each belief statement in the table a weighting to reflect how strongly you agree with it by making **the three scores sum to 100%**.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

1:1:1

Activity 2-1: Beliefs about Teaching

Weight the following belief statements according to your view of the nature of teaching.

Belief	Teaching is ...	Percentage
A	... structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.	----- %
B	... assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	----- %
C	... a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	----- %
		100%

Give each orientation statement in the table a weighting to reflect how strongly you agree with it by making **the three scores sum to 100%**.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

3:1:1

Activity 3-1: Beliefs about Learning

Weight the following belief statements according to your view of the nature of learning.

Belief	Learning is ...	Percentage
A	... an individual activity based on watching, listening, and imitating until fluency is attained.	----- %
B	... an individual activity based on practical exploration and reflection.	----- %
C	... an interpersonal activity in which students are challenged and arrive at understanding through discussion.	----- %
		100%

Give each orientation statement in the table a weighting to reflect how strongly you agree with it by making **the three scores sum to 100%**.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

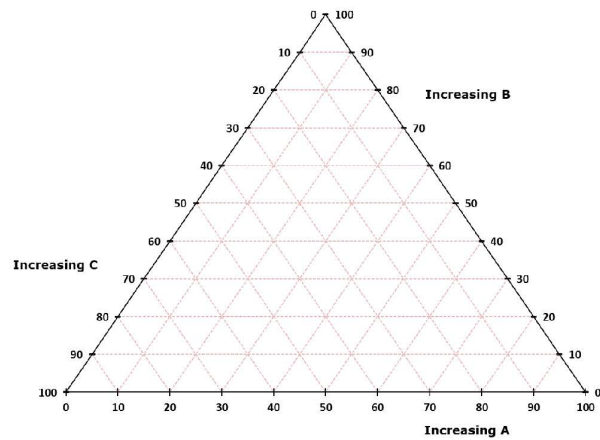
3:1:1

Activity 4-1: Beliefs about mathematics education

Calculate your mean average weightings across the three domains A, B and C.

Belief	Percentage
A	_____ %
B	_____ %
C	_____ %
	100%

Plot the position of your beliefs about mathematics education on the axes provided below.

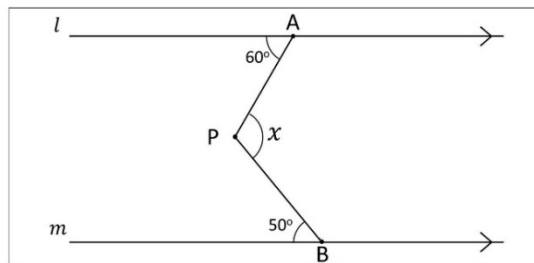


4:1:1

Appendix 7 - Participant resources for RAM session 1

Activity 1-2: Missing Angle Problem

For the following problem, find the value of angle x .



Use the diagrams below to trial different approaches and write down any calculations in the first column.

1.	
2.	

1:2:1

3.	
4.	
5.	
6.	

1:2:2

Activity 1-3: Session Reflection

As a final reflection, weight the beliefs that you think have underpinned the design of the session.

Belief	Mathematics is ...	Percentage
A	 a given body of knowledge and standard set of procedures. A set of universal truths and rules which need to be conveyed to students.	----- %
B	... a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	----- %
C	... an interconnected body of ideas which the teacher and the student create together through discussion	----- %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

Appendix 8 - Participant resources for RAM session 2

Session 2: Activity 2	
Activity 2-2: Missing Angle Lesson Plan	
Lesson Objective:	To enable students to develop problem solving strategies involving angle facts.
Phase 1: Review previous learning (5 minutes). To remind students of previous learning that may be useful in this lesson.	
Teacher Actions	Decisions around student actions
During this phase the teacher facilitates a whole class recall activity. “Can you tell me some angle related facts that we know?”	Will students be expected to just listen to the whole class suggestions or to write down ideas?
Write the important suggestions on the board as students suggest ideas.	What are the important angle facts that you expect students to suggest?

2.2.1

Phase 2: Presenting the Problem (5 minutes). To ensure that all students understand the problem.

Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher leads the class in modelling how to make an accurate and careful copy of the problem in their books. “Please carefully copy this diagram into your books as I draw it.” Each of these points helps ensure students understand the problem: • Draw and label the parallel lines l and m. • Mark point P equidistant between l and m. • Mark point A on line l and point B on line m. (Not quite vertically opposite each other). • Connect P with A, then P with B. • Mark $\angle A = 60^\circ$, $\angle B = 50^\circ$.	<i>Will you ask students to make an accurate copy of the diagram in their book?</i>	<i>What do you want the page to look like in the student's books?</i>

Phase 3: Individual Problem Solving (10 minutes). To give students time to individually engage with the problem and allow the teacher to plan phase 4.

Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase students work individually, whilst the teacher notes which methods could be shared. "You have 10 minutes to try and find angle x in as many ways as you can on your own. If you solve it in one way, please try to find an alternative method and <u>show your reasoning</u> on the worksheet." Hand out a copy of Student Worksheet to each student. As students work the teacher should walk around the classroom and carry out two tasks: - Mark on the Method Recording Sheet the names of students that employ the various methods. - Start to plan which student methods you would like shared during phase 4.	<i>Will you ask students to work individually for 10 minutes?</i> <i>Will you stress the importance of students writing their reasoning on the sheet?</i>	<i>What correct methods do you anticipate from students? What incorrect methods do you anticipate from students?</i>

2:2:3

Phase 4: Discussion of Solution Methods (15 minutes). To ensure all students understand the reasoning behind different methods.

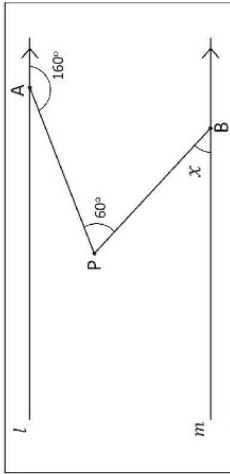
Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher leads the whole class in reviewing the methods whilst students listen to each other and copy the methods on to their Student Worksheet. Note that during this phase you should work with the ideas that students have and not introduce alternative methods that you know. Using the information gathered on the Method Recording Sheet select a student to come to the board and add <u>auxiliary lines and reasoning</u> on to the session presentation slide. Invite 4 to 6 students to the board so that a range of methods are shared (within as many of the method groups as possible). Ask questions of the students to gain clarity <u>but avoid</u> commenting on whether the answer is correct.	<i>Will you only introduce methods that students have found?</i> <i>What do you anticipate your board to look like? (Please write on the second side of the Example of final board sheet).</i>	An example of how the board <i>may</i> look at the end can be seen on the Example of final board sheet. How your board actually looks will depend on your class and who you select. <i>Will you ask the student to write their reasoning on the board?</i>

Phase 6: Highlighting the Major Points (10 minutes): To create a deeper understanding of the chosen methods.

Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher orchestrates a whole class discussion to gain deeper insight into the methods whilst students make notes on the important points. There are three important features to draw out: - The addition of auxiliary lines to diagrams can help. “What do all of our methods have in common?” - The identification of groups of methods. “Can anyone group any of our methods, for example, what do methods {x} and {y} have in common?” - The recognition of angle facts that could be used by the addition of each auxiliary line. “Which angle facts did this allow us to use?” - Identification of any additional auxiliary lines that may not have been seen so far. “Could there be any other auxiliary lines that we could add?”	<i>Will you ask students to write notes?</i>	<i>What do you want the page to look like in the students' books?</i>

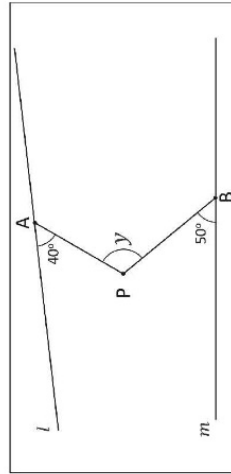
2:2:5

Phase 6: Extending (15 minutes). To give students an opportunity for creativity and to deepen their confidence.

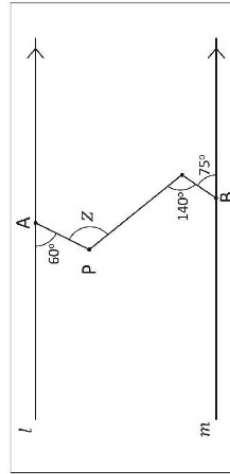
Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher asks students to adjust the problem, find their own solution, then switch problems with a partner to solve each others question, then compare strategies. Three potential ideas are provided to illustrate what could be done and the teacher should suggest them to the class if necessary. a. Change which angles are given. 	<i>Will you prepare additional questions for the class or rely on their creativity?</i> <i>Which methods are best for each of the three suggested problems?</i>	<i>Will you ask students to work out their own solution first before switching with a partner?</i>

2:2:6

b. "Stop the lines being parallel."



c. "Increase the number of turns."



d.

Activity 2-4: Session Reflection

As a final reflection, weight the beliefs that you think have underpinned the design of the research lesson.

Belief	Teaching is ...	Percentage
A	... structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.	----- %
B	... assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	----- %
C	... a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	----- %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

2:3:1

Appendix 9 - Participant resources for RAM session 3

Activity 3-2: Reviewing the research lesson

Phase 1: Review previous learning

How did you manage the whole class recall activity or an alternative? What learning took place?

Phase 2: Present Problem

Did you draw a careful copy of the problem and ask students to draw it or an alternative? What learning took place?

Phase 3: Individual Work

Did you get students to work individually in silence for 10 minutes whilst you noted solutions or an alternative? What learning took place?

Phase 4: Discussion

Did you base only introduce methods that students found or follow an alternative? What learning took place?

Phase 5: Drawing together

Did you ask students to write notes or follow an alternative? What learning took place?

3:2:1

Activity 3-4: Session Reflection

As a final reflection, weight the beliefs that you think were apparent in the learning that you saw take place in the research lesson.

Belief	Learning is ...	Percentage
A	... an individual activity based on watching, listening, and imitating until fluency is attained.	----- %
B	... an individual activity based on practical exploration and reflection.	----- %
C	... an interpersonal activity in which students are challenged and arrive at understanding through discussion.	----- %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

3:4:1

Appendix 10 – Example of completed resources from RAM introductory session

Activity 1-1: Beliefs about Mathematics

Weight the following belief statements according to your view of the nature of mathematics.

Belief	Mathematics is ...	Percentage
A	 a given body of knowledge and standard set of procedures. A set of universal truths and rules which need to be conveyed to students.	10 %
B	... a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	20 %
C	... an interconnected body of ideas which the teacher and the student create together through discussion	70 %
		100%

Give each belief statement in the table a weighting to reflect how strongly you agree with it by making the three scores sum to 100%.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

Activity 2-1: Beliefs about Teaching

Weight the following belief statements according to your view of the nature of teaching.

Belief	Teaching is ...	Percentage
A	... structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.	15 %
B	... assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	40 %
C	... a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	45 %
		100%

Give each orientation statement in the table a weighting to reflect how strongly you agree with it by making **the three scores sum to 100%**.

For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

3:1:1

Activity 3-1: Beliefs about Learning

Weight the following belief statements according to your view of the nature of learning.

Belief	Learning is ...	Percentage
A	 an individual activity based on watching, listening, and imitating until fluency is attained.	10 %
B	... an individual activity based on practical exploration and reflection.	10 %
C	... an interpersonal activity in which students are challenged and arrive at understanding through discussion.	80 %
		100%

Give each orientation statement in the table a weighting to reflect how strongly you agree with it by making **the three scores sum to 100%**.

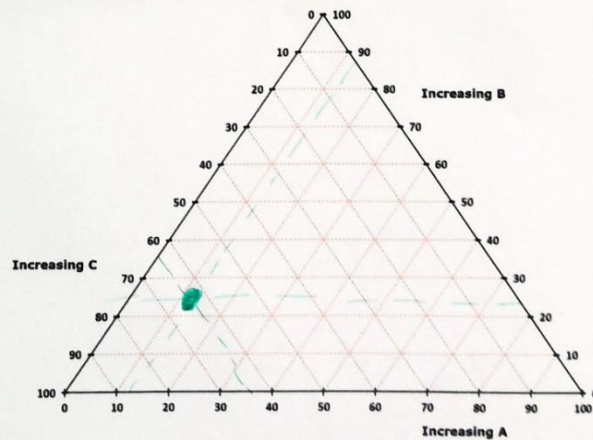
For example, if you allocated weightings of 30% and 10% to two of the statements then the final statement must be allocated a weighting of 60%.

Activity 4-1: Beliefs about mathematics education

Calculate your mean average weightings across the three domains A, B and C.

Belief	Percentage
A	11.7 %
B	23.3 %
C	65.0 %
100%	

Plot the position of your beliefs about mathematics education on the axes provided below.



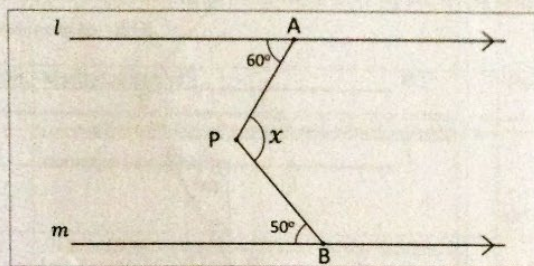
4:1:1

Appendix 11 – Example of completed resources from RAM session 1

Session 1: Activity 2

Activity 1-2: Missing Angle Problem

For the following problem, find the value of angle x .

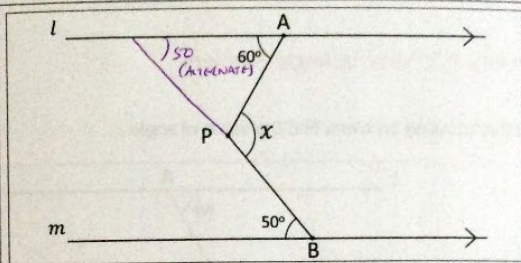


Use the diagrams below to trial different approaches and write down any calculations in the first column.

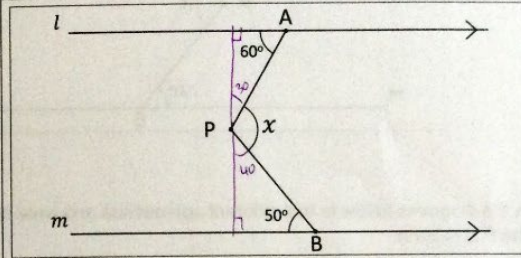
1. $x = 60 + 50 = 110^\circ$	
2. EXT. ANGLE = SUM OF TWO OPPOSING INT. ANGLES $\therefore x = 50 + 60$ $= 110^\circ$	

1:2:1

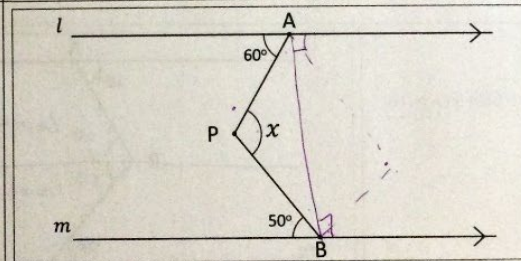
3.
See previous
 $\therefore x = 60 + 50 =$
 110°



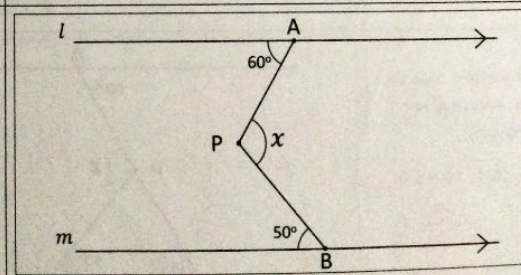
4.
 $x = 180 - 70$
 $=$ 110°



5.



6.



Activity 1-3: Session Reflection

As a final reflection, weight the beliefs that you think have underpinned the design of the session.

Belief	Mathematics is ...	Percentage
A	 a given body of knowledge and standard set of procedures. A set of universal truths and rules which need to be conveyed to students.	10 %
B	... a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	60 %
C	... an interconnected body of ideas which the teacher and the student create together through discussion	30 %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

I was worried about how I see
maths education is wildly different
to other peoples - am I wrong?

Appendix 12 – Example of completed resources from RAM session 2

Session 2: Activity 2

Activity 2-2: Missing Angle Lesson Plan

Lesson Objective:		To enable students to develop problem solving strategies involving angle facts.	
Phase 1: Review previous learning (5 minutes). To remind students of previous learning that may be useful in this lesson.			
Teacher Actions	Decisions around teacher actions	Decisions around student actions	
During this phase the teacher facilitates a whole class recall activity. "Can you tell me some angle related facts that we know?"	Will you do this phase as a whole class recall activity? Polite! pick up -Diagrams key	Will students be expected to just listen to the whole class suggestions or to write down ideas? Keep on board Notes encouraged	
Write the important suggestions on the board as students suggest ideas.		What are the important angle facts that you expect students to suggest? $\neq$ Δ $\square$	

2:2:1

Phase 2: Presenting the Problem (5 minutes). To ensure that all students understand the problem.

Session 2: Activity 2

Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher leads the class in modelling how to make an accurate and careful copy of the problem in their books. "Please carefully copy this diagram into your books as I draw it." Each of these points helps ensure students understand the problem: • Draw and label the parallel lines l and m. • Mark point P equidistant between l and m. • Mark point A on line l and point B on line m. (Not quite vertically opposite each other). • Connect P with A, then P with B. • Mark $\angle A = 60^\circ$, $\angle B = 50^\circ$.	Will you ask students to make an accurate copy of the diagram in their book? Draw out as explaining but give them copies	What do you want the page to look like in the student's books? Rough notes

2.2.2

Phase 3: Individual Problem Solving (10 minutes). To give students time to individually engage with the problem and allow the teacher to plan phase 4.

Session 2: Activity 2

Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase students work individually, whilst the teacher notes which methods could be shared. "You have 10 minutes to try and find angle x in as many ways as you can on your own. If you solve it in one way, please try to find an alternative method and <u>show your reasoning on the worksheet</u>." Hand out a copy of Student Worksheet to each student. As students work the teacher should walk around the classroom and carry out two tasks: - Mark on the Method Recording Sheet the names of students that employ the various methods. - Start to plan which student methods you would like shared during phase 4.	Will you ask students to work individually for 10 minutes? Yes but give out prompts cards. Explain their use. Will you stress the importance of students writing their reasoning on the sheet? Yes (despite not myself)	What correct methods do you anticipate from students? What incorrect methods do you anticipate from students? Get them thinking but not jells

Phase 4: Discussion of Solution Methods (15 minutes). To ensure all students understand the reasoning behind different methods.

Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher leads the whole class in reviewing the methods whilst students listen to each other and copy the methods on to their Student Worksheet. Note that during this phase you should work with the ideas that students have and not introduce alternative methods that you know. Using the information gathered on the Method Recording Sheet select a student to come to the board and add <u>auxiliary lines and reasoning</u> on to the session presentation slide. Invite 4 to 6 students to the board so that a range of methods are shared (within as many of the method groups as possible). Ask questions of the students to gain clarity <u>but avoid</u> commenting on whether the answer is correct.	Will you only introduce methods that students have found? No though use of prompt cards have student's talk through What do you anticipate your board to look like? (Please write on the second side of the Example of final board sheet).	An example of how the board may look at the end can be seen on the Example of final board sheet. How your board actually looks will depend on your class and who you select. Will you ask the student to write their reasoning on the board? No - I'd ask them to talk me through (not a lot of students comfortable)

Phase 5: Highlighting the Major Points (10 minutes): To create a deeper understanding of the chosen methods.

Teacher Actions	Decisions around teacher actions	Decisions around student actions
During this phase the teacher orchestrates a whole class discussion to gain deeper insight into the methods whilst students make notes on the important points. There are three important features to draw out: - The addition of auxiliary lines to diagrams can help. "What do all of our methods have in common?" - The identification of groups of methods. "Can anyone group any of our methods, for example, what do methods {x} and {y} have in common?" - The recognition of angle facts that could be used by the addition of each auxiliary line. "Which angle facts did this allow us to use?" - Identification of any additional auxiliary lines that may not have been seen so far. "Could there be any other auxiliary lines that we could add?"	Will you ask students to write notes? No - Pens down listening No keep steps or notes to copy just ideas	What do you want the page to look like in the students' books?

Activity 2-4: Session Reflection

As a final reflection, weight the beliefs that you think have underpinned the design of the research lesson.

Belief	Teaching is ...	Percentage
A	... structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.	10 %
B	... assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	60 %
C	... a non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	30 %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

2:3:1

Appendix 13 – Example of completed resources from RAM session 3

Activity 3-2: Reviewing the research lesson

Phase 1: Review previous learning

How did you manage the whole class recall activity or an alternative? What learning took place?

Give students a recall activity of 4 questions.

Phase 2: Present Problem

Did you draw a careful copy of the problem and ask students to draw it or an alternative? What learning took place?

I did. I feel like they focused on that rather than listening.

Phase 3: Individual Work

Did you get students to work individually in silence for 10 minutes whilst you noted solutions or an alternative? What learning took place?

Yes - I handed out cards (prompts) after 5. There were no corrects in the first 5. Students learnt how scary the 1st step can be.

Phase 4: Discussion

Did you base only introduce methods that students found or follow an alternative? What learning took place?

Post panned - Did groupwork 1st. Then worked through on board. Kids are petrified of getting things wrong.

Phase 5: Drawing together

Did you ask students to write notes or follow an alternative? What learning took place?

Attempted

3:2:1

Activity 3-4: Session Reflection

As a final reflection, weight the beliefs that you think were apparent in the learning that you saw take place in the research lesson.

Belief	Learning is ...	Percentage
A	... an individual activity based on watching, listening, and imitating until fluency is attained.	15 %
B	... an individual activity based on practical exploration and reflection.	25 %
C	... an interpersonal activity in which students are challenged and arrive at understanding through discussion.	60 %
		100%

Were there any moments during this session that you have felt uncomfortable or challenged in your thinking? If so, please identify and describe these moments.

3:4:1

Appendix 14 – The project information sheet and consent form

Nottingham Trent University
50 Shakespeare Street
Nottingham
NG1 4FQ



Project Information Sheet

[Orientations and Dissonance: Exploring Teachers' Experiences of the Rethinking Approaches in Mathematics Intervention](#)

Thank you for your consideration of participating in this research study.

The purpose of the Project Information Sheet is to provide you with information about the research, explain what your participation will involve and inform you of how the collected data will be handled.

I would be grateful if you would take time to read the following information carefully. Please do not hesitate to contact me with any questions you may have.

[What is the research about?](#)

Research suggests that Mathematics teachers' decisions in the classroom are influenced by their resources, goals and orientations. However, professional development materials tend to focus only on resources and goals. This research focuses on how orientations (shorthand for teachers' dispositions, beliefs, values, tastes and preferences) underpin and affect all other decisions.

The one-year Rethinking Approaches to Mathematics (RAM) intervention has been designed to encourage teachers to consider what they believe about how Mathematics should be taught. Materials have been developed based on resources taken from Japan – one of the highest performing countries in the 2018 PISA Mathematics rankings.

The RAM intervention consists of four stages:

1. Completion of a whole department PD session exploring Mathematical knowledge and approaches in geometry.
2. Completion of a whole department PD session exploring lesson design.
3. Teaching a lesson.
4. Completion of a whole department PD session reflecting on the taught lesson and learning over the year.

It is intended that this study will help to define an alternative approach to teacher professional development for the benefit of individual teachers, local institutions and national organisations. This approach would place the consideration of orientations at the centre of development and encourage teachers to determine direction.

What will I be asked to do if I agree to take part?

Your participation is entirely voluntary. If you do decide to take part, you will be given this information sheet to keep, and be asked to sign a consent form.

You will be asked to participate in the RAM intervention as part of normal departmental development time.

In addition, you will be asked to have three of your Mathematics lessons recorded. The first and third lesson will be based on your resources, the second will be delivered using the RAM intervention materials. You will be asked to take part in a brief interview after each of the delivered lessons. There is no time limit to the interview and it may be as long or short as you wish. If you change your mind about participating in the study you should contact up to 10 weeks after the interview date to withdraw. If you withdraw after this point anonymised data may be retained as part of the study as it will have already formed part of my analysis.

What are the benefits of taking part?

Thank you for your participation in the RAM intervention and associated research. The intervention is designed to deepen teachers' pedagogy and will ultimately support the development of young people in their understanding and attainment in Mathematics.

How will my confidentiality be protected?

With your consent I will record the interview to allow me to accurately reflect what is said. The recording will be transcribed (written out) and anonymised. This will be done by changing your name and disguising any details of my interview which may reveal the name of your institution, your identity or the identity of people we speak about.

Data will be stored on NTU secure servers under a pseudonym (false name) that I will give to you. During the project, data will only be accessible to myself and my supervisory team via our NTU login details using password protected computers. However, only I will be authorised to access personal data, such as the recordings and contact details.

What will happen to the data I provide?

This study will adhere to the General Data Protection Regulations and Data Protection Act with Nottingham Trent University acting as the data controller. Therefore, personal information for academic research is collected on the basis of public interest. This means that if you agree to take part in this study, I will only use your data in the ways needed to conduct the research study.

Data will be analysed to provide information for the overall findings and conclusions of the research. Findings will be reported in my thesis and potentially in other academic forums such as academic journals. Direct quotations from the interviews or copies of the completed resources may be used, but

not in a way that would identify you. Resultant publications will be openly accessible through the Nottingham Trent University Institutional Repository, IRep.

Who should I contact with questions or problems?

If you have any questions about the study or require more information in order to help you decide if you would like to take part in this research, then please contact me using the following details:

Matt Woodford
Nottingham Institute of Education
Ada Byron King
College Drive
Clifton
Nottingham
NG11 8NS
0115 848 6329
matt.woodford@ntu.ac.uk

Any complaint or concern about any aspect of the way you have been dealt with during the course of the study will be addressed; please contact me in the first instance, or my Supervisors:

Dr Andrew Clapham andrew.clapham@ntu.ac.uk

Dr Natasha Serret natasha.serret@ntu.ac.uk

If you wish to raise a complaint on how I have handled your personal data, you can contact the NTU Data Protection Officer who will investigate the matter using the email address dpo@ntu.ac.uk

Who has reviewed the project?

This study has been reviewed and received a favourable ethical opinion from the School of Social Sciences Research Ethics Committee. They are an independent group of people whose concern is to protect your interests.

Many thanks for your consideration and potential participation in this project.

Matt Woodford

Consent Form

Statement	Please tick to indicate agreement
I confirm that I have read and understood the Participant Information Sheet.	
I have been given the opportunity to consider the information provided, ask questions and have had these questions answered to my satisfaction.	
I understand that my participation is voluntary and that I can ask to withdraw without giving a reason at any point.	
I consent to my participation in professional development activities being recorded.	
I consent to digital copies of any resources produced as part of the RAM intervention PD sessions being taken and that they will be anonymised.	
I consent to my interview being recorded and understand that the audio file will be retained at Nottingham Trent University for a period of 10 years after the project start.	
I consent to my lessons being recorded and understand that the video file will be destroyed once the data has been transcribed.	
I consent to digital copies of any resources produced as part of lessons being taken and that they will be anonymised.	
I understand that my data will be processed in accordance with Data Protection Law as explained in the Participant Information Sheet.	
I understand that in any report on the results of this research my identity will remain anonymous.	
I understand that anonymised transcript and data analysis files will be publicly available for future reuse from Zenodo and that it will not be possible to identify either myself or my school.	
I agree to take part in this study.	

Signature of participant: _____ Date: _____

I believe the participant is giving informed consent to participate in this study

Signature of researcher: _____ Date: _____

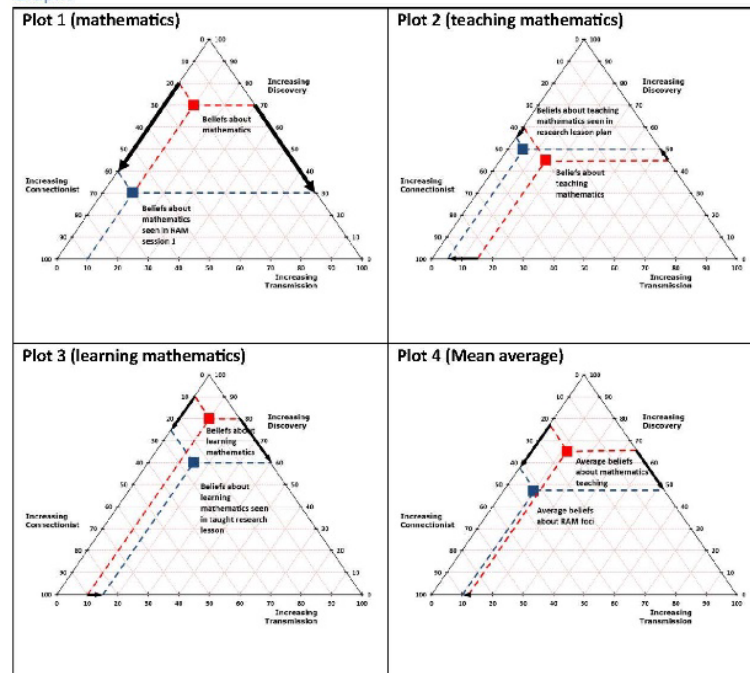
Appendix 15 – A copy of Beliefs [Drew]

Beliefs [Drew]

Tabulated weightings

Activity document	Domain	Transmission weighting	Discovery weighting	Connectionist weighting
1-1	Mathematics	10	70	20
2-1	Teaching mathematics	15	45	40
3-1	Learning mathematics	10	80	10
	Mean average of personal beliefs	12	65	23
1-3	Mathematics (of RAM session 1)	10	30	60
2-4	Teaching mathematics (of the RAM research lesson plan)	5	50	45
3-4	Learning mathematics (in the taught RAM research lesson)	15	60	25
	Mean average of RAM foci	10	47	43

Graphs



Observations

Looking at plot 4:

- Drew's dominant personal belief is Discovery (with a weighting of 65%).
- Drew has an ordered personal belief cluster about mathematics teaching of Discovery, Connectionist, Transmission.
- Drew has a dominant mean average interpretation of the RAM foci of Discovery.
- Drew has an ordered mean average interpretation of the cluster of beliefs about mathematics teaching shown in the RAM foci of Discovery, Connectionist, Transmission.
- There is an overall pattern of very similar levels of Transmission, more Connectionist, and less Discovery between personal beliefs and average beliefs seen in RAM foci.

Looking at plots 1, 2 and 3:

- Each plot follows the same general pattern shown in plot 4 with very similar levels of Transmission, more Connectionist, and less Discovery between personal beliefs and beliefs seen in RAM foci.
- The biggest difference is seen in plot 1 (beliefs about mathematics).
- The smallest difference is seen in plot 3 (beliefs about learning mathematics).
- Plots 1 and 3 have the red dot above the blue dot. However, plot 2 (beliefs about teaching mathematics) saw much lower levels of Discovery, and more Transmission in personal beliefs compared to the other domains.

Appendix 16 – Interview guides

Interview 0

Introduce myself, explain the purpose of the RAM programme, and my reasons for wanting to complete these interviews.

1. Can you tell me a bit about your background?
 - How long have you been teaching?
 - Can you tell me about any previous jobs or education?
 - How would you describe yourself as a teacher?
2. Can you tell me about what you hope gain from participating in the RAM programme?
 - Any good PD experiences?
 - Any bad PD experiences?
 - What is the purpose of PD?

Interview 1

Thank teacher for their involvement in the session.

1. Can you explain the weightings you gave for your beliefs about mathematics from the introductory session?
 - Read out weightings.
 - Do you see any tension in holding these beliefs?
2. Can you tell me your thoughts on the session?
 - Tell me about the individual problem solving.
 - Tell me about the group activity where you looked at the solutions.
 - Tell me about the whole group activity,
3. Can you explain the weightings you gave to the beliefs about mathematics you felt were seen in the session?

Interview 2

Thank teacher for their involvement in the session.

1. Can you explain the weightings you gave for your beliefs about teaching mathematics from the introductory session?
 - Read out weightings.
 - Do you see any tension in holding these beliefs?
2. What did you think of the Teaching Through Problem Solving style of lesson we looked at?
3. Tell me about your group discussion and what you thought about the research lesson plan.
 - Which parts of the lesson plan did you commit to?
 - Which parts did you change?
4. Could you explain the weightings you gave to beliefs about teaching mathematics you felt the research lesson plan was based on?

Interview 3

Thank teacher for their involvement in the session.

1. Can you explain the weightings you gave for your beliefs about learning mathematics from the introductory session?
 - Read out weightings.
 - Do you see any tension in holding these beliefs?
2. Can you tell me a little about the research lesson that you taught?
 - Which year group and set did you try it with?
 - Were there any moments where you changed what you had planned?
 - What did you learn from teaching the lesson?
 - Is there anything you would do different if you taught it again?
3. Could you explain the weightings you gave to beliefs about learning mathematics you felt you saw in the research lesson you taught?

Appendix 17 – Example transcript excerpt from interview 1

00:04:42

Drew

I don't know. I don't know. I don't see enough of that. You're right. It does seem a lot more process driven, but they probably get better results.

00:04:50

Drew

But depends what you define as a result of course.

00:04:53

Matt

Right, so, so, so you were massively strong on that second one?

How do you feel about A and C? So A was about, kind of, solutions told to the students by the teacher.

00:05:05

Drew

Laws and procedures.

00:05:10

Matt

C was about, sort of, connecting through discussion, enabled by the teacher. Which are a little bit contradictory to B.

00:05:20

Drew

I don't think A and C are contradictory to B at all.

Because ultimately the qualification is not in discovering mathematics, the qualification is in applying mathematics, really, really. And there is a difference there. The the two, you know you can talk about problem solving in GCSE's and six markers all you like, but really they're just interleaved problems. Really if we're, right, so that's not what the qualification is. So yeah, there's going to be, there's going to be a thing, but I feel like a complete outlier. I think most people would go with 80% on A.

00:06:09

Drew

OK, OK, here's a question for you. 20 years ago, do you think A would have been the most popular?

00:06:16

Matt

No.

00:06:19

Matt

Appendix 18 – Data imported into NVIVO 14

Files

Name	Codes	References
Beliefs [Jo]	6	14
Beliefs [Red]	5	13
Interview 0 [Jo]	5	12
Interview 0 [Red]	5	16
Beliefs [Drew]	5	11
Interview 0 [Drew]	4	23
Lesson plan [Drew], Page 4	2	2
Lesson plan [Drew], Page 1	1	1
Lesson plan [Drew], Page 2	1	1
Lesson plan [Drew], Page 3	1	1
Lesson plan [Drew], Page 5	1	1
Lesson Plan [Jo], Page 1	1	1
Lesson Plan [Jo], Page 3	1	1
Lesson Plan [Jo], Page 4	1	2
Lesson Plan [Jo], Page 5	1	1
Lesson Plan [Red], Page 1	1	1
Lesson Plan [Red], Page 2	1	1
Lesson Reflection [Drew], Page 1	1	3
Lesson Reflection [Drew], Page 3	1	1
Lesson Reflection [Jo], Page 1	1	1
Lesson Reflection [Jo], Page 3	1	1
Lesson Reflection [Red], Page 1	1	2
Problem [Drew], Page 1	0	0
Problem [Drew], Page 2	0	0
Problem [Drew], Page 3	0	0
Problem [Jo], Page 1	0	0
Problem [Jo], Page 2	0	0
Problem [Jo], Page 3	0	0
Problem [Red], Page 1	0	0
Problem [Red], Page 2	0	0
Lesson plan [Drew], Page 6	0	0
Lesson Plan [Red], Page 3	0	0

Beliefs [Drew]

Tabulated weightings

Activity document	Domain	Transmission weighting	Discovery weighting	Connectionist weighting
1-1	Mathematics	10	70	20
2-1	Teaching mathematics	15	45	40
3-1	Learning mathematics	10	80	10
	Mean average of personal beliefs	12	65	23
1-3	Mathematics (of RAM session 1)	10	30	60
2-4	Teaching mathematics (of the RAM research lesson plan)	10	30	60
3-4	Learning mathematics (in the taught RAM research lesson)	15	60	25
	Mean average of RAM foci	12	40	48

Graphs

Plot 1 (mathematics)

Plot 2 (teaching mathematics)

Files

Name	Codes	References
Lesson Plan [Jo], Page 6	0	0
Lesson Plan [Red], Page 3	0	0
Lesson Plan [Jo], Page 4	0	0
Lesson Plan [Red], Page 5	0	0
Lesson Plan [Red], Page 6	0	0
Lesson Reflection [Drew], Page 2	0	0
Lesson Reflection [Jo], Page 2	0	0
Lesson Reflection [Red], Page 2	0	0
Lesson Reflection [Red], Page 3	0	0

Beliefs [Drew]

Tabulated weightings

Activity document	Domain	Transmission weighting	Discovery weighting	Connectionist weighting
1-1	Mathematics	10	70	20
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	Mean average of personal beliefs	12	65	23
1-3	Mathematics (of RAM session 1)	10	30	60
2-4	Teaching mathematics (of the RAM research lesson plan)	10	30	60
3-4	Learning mathematics (in the taught RAM research lesson)	15	60	25
	Mean average of RAM foci	12	40	48

Graphs

Plot 1 (mathematics)

Plot 2 (teaching mathematics)

Note: When first analysing data, I named one of the teachers as Red, however in my final write-up this teacher was renamed to Ronnie.

Appendix 19 – Initial comments on data

<u>DREW</u>	<u>Belgio</u> Drew to DCT Saw RAM as CDT Similar patterns at home lead. → RAM is more C less D. 	<u>Belgio</u> To in TDC Saw RAM as CDT Similar patterns at home lead. → RAM is less T more C more D. 	<u>Belgio</u> Rel is CDT Saw RAM as DCT Similar patterns at home lead. → RAM is less T less C more D. 	<u>RED</u>
<u>Lesson Plan</u> - Adapt starter. - Draw diagram. - Individual problem solving. - Draw together, using answers first. - Intention to read rather than plan the book. - Uncover on notes. - Open to extension.	<u>Interview 0</u> - Background. - Goal PD informed classroom practice. - Goal PD: intention giving. - Task ladder and knowledge retrieval. - Concerns around quality. - Lots of conflicting feelings. - Expresses how ladder but not convinced by it. - Teaching more amongst responsibilities.	<u>Interview 0</u> - Background. - Teacher "very structured". - Task ladder reflects own approach. - Goal PD experience during lesson feedback. - Goal PD also saw a calm art teacher. - PD is about being the best version of self. - Uses a number of guides from previous sessions.	<u>Interview 0</u> - Background (connection). - Literacy making connections. - (and Arthur Strong). - Goal PD: looking for the one thing that will make a difference. - Task ladder (pupil experience, not convinced by it). - A job that applied for (a negative experience).	<u>Lesson Plan</u> - Intention to recap facts. - Intention to ask students to sketch diagram. - Intention to individual problem solve. - Intention to highlight named materials in bracketed phase.
<u>Lesson Reflection</u> - Diagram open correctly. - Individual problem solving led to resilience and creativity. - Spent too long during solutions. - Brief bit of reflection.	<u>Interview 1</u> - Experience of RAM 1 (problem solving). - Comments on how members of best get on. - Belgio: weightings. - "Belgio's are a bit of a load". - Classification of methods. - Belgio about RAM 1. - Individual problem solving.	<u>Interview 1</u> - Session 1 and differentiation with individual problem solving. - Still unclear about the task. - Belgio about notes. - "Can't be rules and algorithms". - Expressed working with others, shared more than just rules. - Expects the session.	<u>Interview 1</u> - Expects during the problem. - A good pattern, but "notes" for the classroom. - Importance of experience. - Belgio about notes. - "Longway (connected?)". - Lack of self-confidence. - Belgio about RAM 1. - Expects tomorrow.	<u>Lesson Reflection</u> - Did not get pupils to copy the diagram. - Did not make notes whilst students problem solved.
<u>Interview 2</u> - Belgio about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Interview 2</u> - Belgio about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Interview 2</u> - Belgio about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Interview 2</u> - Belgio about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Lesson Reflection</u> - Did not get pupils to copy the diagram. - Did not make notes whilst students problem solved.
<u>Interview 3</u> - Belgio weighting about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Interview 3</u> - Belgio weighting about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Interview 3</u> - Belgio weighting about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Interview 3</u> - Belgio weighting about learning notes. - "Structure, lesson". - "Structure, lesson". - Link TIPS. - "Seems hard to get into it, but then some slight change (clarifier)". - Belgio about RAM 2. - "Facilitation for the day".	<u>Lesson Reflection</u> - Did not get pupils to copy the diagram. - Did not make notes whilst students problem solved.
<u>Lesson Plan</u> - Intention for starter questions. - Teacher draw diagram, students get sheets. - Intention to use prompt cards to support individual work. - Prompt cards used to bring lesson together.	<u>Lesson Reflection</u> - Draw diagram, but students didn't really listen. - Prompt cards used, noticed how students found first step difficult. - Used cards to discuss solutions. - Attempted to bring together.	<u>Lesson Reflection</u> - Draw diagram, but students didn't really listen. - Prompt cards used, noticed how students found first step difficult. - Used cards to discuss solutions. - Attempted to bring together.	<u>Lesson Reflection</u> - Draw diagram, but students didn't really listen. - Prompt cards used, noticed how students found first step difficult. - Used cards to discuss solutions. - Attempted to bring together.	<u>Lesson Reflection</u> - Draw diagram, but students didn't really listen. - Prompt cards used, noticed how students found first step difficult. - Used cards to discuss solutions. - Attempted to bring together.

1. Task Ladder appears across many documents. Teachers recognise the effect it has had on their teaching. Drew caught between disliking it and needing to ensure it is followed in the maths department. Jo sees it as fitting their teaching. Red very angry about it, but now seems to accept it as helpful.
2. Good PD needs to affect practice and bad PD is enforced with little explanation. All have experience of being told, instead they want to think and apply. (They all show signs of considering how things could be improved anyway).
3. All teachers see more of the second belief in RAM, and less of their primary belief. So you interpret PD as the thing you are not?
4. Dominant personal beliefs get confirmed, but I also think secondary (and tertiary) beliefs are confirmed in interviews. I don't think the teachers see them as contradictory, and happy to hold all the beliefs. But a dominant belief is clear.
5. Some variation of beliefs at domain level. Perhaps the task ladder has had some influence on beliefs about teaching mathematics.
6. Teachers open to trying the new lesson. I'm a little surprised that they were so happy to make adjustments to the plan. I think I thought they would try to stick as closely as possible. Linked to this, when I was in the session, I didn't think they committed to thinking through the reasons for the different practice. Instead, they preferred to say what they wanted.
7. Generally seemed to want, and did, put in starters to remind students of previous work.
8. All attempted the individual work, Drew made notes, Jo turned it into a prompt card activity, Red, did not make notes.
9. All attempted some form of bringing it together.
10. Little focus on grouping strategies. I think this was more of a time issue – certainly for Drew and Red.

Note: When first analysing data, I named one of the teachers as Red, however in my final write-up this teacher was renamed to Ronnie.

Appendix 20 – Generated codes

Early set of codes

Name	Files	References	Created on	Created by	Modified on	Modified by
○ Dominant Discovery	3	9	21/08/2024 15:47	MW	22/08/2024 09:19	MW
○ Dominant Transmission	1	1	21/08/2024 15:48	MW	22/08/2024 08:48	MW
○ Dominant Connectionist	3	3	21/08/2024 15:48	MW	22/08/2024 08:57	MW
○ More Transmission	1	1	21/08/2024 15:49	MW	22/08/2024 08:58	MW
○ More Connectionist	2	8	21/08/2024 15:49	MW	22/08/2024 08:58	MW
○ More Discovery	2	7	21/08/2024 15:50	MW	22/08/2024 08:45	MW
○ Less Discovery	2	5	21/08/2024 15:50	MW	22/08/2024 08:59	MW
○ Less Connectionist	1	4	21/08/2024 15:50	MW	21/08/2024 15:57	MW
○ Less Transmission	2	7	21/08/2024 15:50	MW	22/08/2024 08:56	MW
○ Drew Transmission	2	8	22/08/2024 09:09	MW	22/08/2024 13:57	MW
○ Drew T (LM)	1	3	22/08/2024 10:06	MW	22/08/2024 14:31	MW
○ Drew T (M)	0	0	22/08/2024 10:06	MW	22/08/2024 10:06	MW
○ Drew T (TM)	1	3	22/08/2024 10:06	MW	22/08/2024 14:20	MW
○ Drew Connectionist	2	5	22/08/2024 09:09	MW	22/08/2024 13:55	MW
○ Drew C (LM)	1	1	22/08/2024 10:06	MW	22/08/2024 14:31	MW
○ Drew C (M)	0	0	22/08/2024 10:05	MW	22/08/2024 10:05	MW
○ Drew C (TM)	1	3	22/08/2024 10:05	MW	22/08/2024 14:20	MW
○ Drew Discovery	2	11	22/08/2024 09:09	MW	22/08/2024 13:53	MW
○ Drew D (LM)	1	1	22/08/2024 10:05	MW	22/08/2024 14:30	MW
○ Drew D (M)	1	1	22/08/2024 10:02	MW	22/08/2024 10:10	MW
○ Drew D (TM)	1	5	22/08/2024 10:05	MW	22/08/2024 14:20	MW
○ Jo Transmission	1	3	22/08/2024 09:23	MW	22/08/2024 09:39	MW
○ Jo Transmission (LM)	1	1	22/08/2024 09:58	MW	22/08/2024 10:41	MW
○ Jo Transmission (TM)	1	2	22/08/2024 09:58	MW	22/08/2024 14:23	MW
○ Jo Transmission (M)	1	2	22/08/2024 09:57	MW	24/08/2024 14:54	MW
○ Jo Discovery	0	0	22/08/2024 09:23	MW	22/08/2024 09:23	MW
○ Jo Discovery (LM)	1	1	22/08/2024 09:57	MW	22/08/2024 10:42	MW
○ Jo Discovery (M)	1	1	22/08/2024 09:57	MW	22/08/2024 14:14	MW
○ Jo Discovery (TM)	0	0	22/08/2024 09:57	MW	22/08/2024 09:57	MW
○ Jo Connectionist	1	2	22/08/2024 09:23	MW	22/08/2024 09:24	MW
○ Jo Connectionist (LM)	1	2	22/08/2024 09:56	MW	22/08/2024 10:42	MW
○ Jo Connectionist (M)	1	2	22/08/2024 09:55	MW	24/08/2024 14:54	MW

Name	Files	References	Created on	Created by	Modified on	Modified by
○ Teaching Ladder	6	16	22/08/2024 09:27	MW	22/08/2024 14:34	MW
○ Good PD	3	5	22/08/2024 09:28	MW	22/08/2024 13:59	MW
○ Bad PD	3	6	22/08/2024 09:28	MW	22/08/2024 13:58	MW
○ Red Transmission	1	1	22/08/2024 09:41	MW	22/08/2024 09:46	MW
○ Red T (LM)	1	1	22/08/2024 10:00	MW	22/08/2024 10:51	MW
○ Red T (M)	0	0	22/08/2024 09:59	MW	22/08/2024 09:59	MW
○ Red T (TM)	2	6	22/08/2024 10:00	MW	22/08/2024 10:51	MW
○ Red Connectionist	2	3	22/08/2024 09:41	MW	22/08/2024 14:07	MW
○ Red C (LM)	1	5	22/08/2024 10:01	MW	22/08/2024 14:38	MW
○ Red C (M)	1	2	22/08/2024 10:01	MW	22/08/2024 11:27	MW
○ Red C (TM)	1	5	22/08/2024 10:01	MW	22/08/2024 10:33	MW
○ Red Discovery	0	0	22/08/2024 09:41	MW	22/08/2024 09:41	MW
○ Red D (LM)	1	1	22/08/2024 10:00	MW	22/08/2024 10:48	MW
○ Red D (M)	0	0	22/08/2024 10:00	MW	22/08/2024 10:00	MW
○ Red D (TM)	0	0	22/08/2024 10:00	MW	22/08/2024 10:00	MW
○ Influence of exams	2	3	22/08/2024 10:14	MW	24/08/2024 14:54	MW
○ Contradiction	3	3	22/08/2024 10:22	MW	22/08/2024 14:31	MW
○ Classroom wisdom	3	7	22/08/2024 10:43	MW	22/08/2024 14:02	MW
○ Negative experience	2	6	22/08/2024 11:17	MW	24/08/2024 14:54	MW
○ Noticed	3	5	22/08/2024 11:18	MW	24/08/2024 14:54	MW
○ Positive experience	2	8	22/08/2024 11:18	MW	22/08/2024 14:06	MW
○ TTPS	3	5	22/08/2024 11:39	MW	22/08/2024 14:20	MW
○ Change to lesson plan	15	34	22/08/2024 11:43	MW	22/08/2024 14:39	MW
○ Commitment to lesson plan	2	2	22/08/2024 11:43	MW	22/08/2024 14:32	MW
○ Student attitudes	2	2	22/08/2024 11:52	MW	22/08/2024 14:24	MW
○ Lesson feature	3	13	22/08/2024 11:54	MW	22/08/2024 14:35	MW
○ What the lesson has caused thought about	8	21	22/08/2024 11:54	MW	22/08/2024 14:35	MW
○ RAM as Discovery	7	7	22/08/2024 12:20	MW	22/08/2024 14:35	MW
○ RAM as Connectionist	8	13	22/08/2024 12:20	MW	22/08/2024 14:35	MW
○ RAM as Transmission	7	8	22/08/2024 12:21	MW	24/08/2024 14:54	MW
○ Maths as a language	1	3	22/08/2024 14:03	MW	22/08/2024 14:03	MW
○ Doubts over the lesson	3	5	22/08/2024 14:08	MW	22/08/2024 14:27	MW

Final set of codes downloaded from NVivo

Name	Files	References	Modified by		
Beliefs about learning mathematics seen in RAM element 3	8	20	MW		
Beliefs about mathematics education seen in RAM elements	3	25	MW		
Beliefs about mathematics seen in RAM element 1	10	25	MW		
Beliefs about teaching mathematics seen in RAM element 2	9	19	MW		
Simultaneous beliefs	8	20	MW		
External pressure on beliefs	9	38	MW		
In line with own research lesson plan	8	22	MW		
I also believe this	9	34	MW		
Less of my dominant belief	3	22	MW		
More of my non-dominant beliefs	3	22	MW		
My development as a teacher	3	8	MW		
My main belief about learning mathematics	3	17	MW		
My main belief about mathematics	8	26	MW		
My main belief about mathematics education	3	19	MW		
My main belief about teaching mathematics	4	23	MW		
Negative features of PD	3	16	MW		
Negative reaction to an element of the PD	3	18	MW		
Planned changes to the research lesson plan	17	37	MW		
Positive features of PD	3	8	MW		
Unplanned changes to the live research lesson	8	34	MW		